Beam-Beam Effects and Luminosity Optimization for e+e- Colliders at High Energies

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IHEP-BINP CEPC Accelerator Collaboration Workshop
Beijing, 12 January 2016

Outline

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- ❑ Beamstrahlung lifetime
- ❑ Optimization of β_y^*
- ❑ Bunch lengthening and impact of hour-glass
- ❑ Crab Waist collision scheme
- ❑ Flip-flop at high and low energies, optimization of β_x^*
- ❑ Summary

Luminosity

For flat beams (both head-on and crossing angle collision):

$$L = \frac{\gamma}{2er_e} \cdot \frac{I_{tot}\xi_y}{\beta_y^*} \cdot R_H \quad \propto \quad \frac{\xi_y}{\beta_y^*}$$

I_{tot} – total beam current (defined by SR power, e.g. 50 MW)

ξ_y – vertical betatron tune shift, its limit depends on the collision scheme

R_H – hour-glass factor: $R_H \approx$ [0.86, 0.71, 0.60] for L_i / β_y^* = [1, 2, 3]

L_i – length of the interaction area:

$$L_i = \frac{\sigma_z}{\sqrt{1+\phi^2}} \qquad\qquad \phi = \frac{\sigma_z}{\sigma_x} tg\left(\frac{\theta}{2}\right) \quad \text{– Piwinski angle}$$

β_y^* should be minimized, but there are restrictions:

- Beta-function at the final quads raises as $1/\beta_y^*$, that affects dynamic aperture and creates problems with chromaticity corrections.
- L_i should be squeezed to $L_i \sim \beta_y^*$. On the other hand, too short L_i may cause problems with beamstrahlung lifetime.

When performing optimizations, we do not care about the bunch population N_p and the number of bunches N_b. Namely, N_p is adjusted according to beam-beam limit, and it defines N_b (since the total beam current I_{tot} is fixed).

Beamstrahlung

At very high energies, the luminosity is limited by the beamstrahlung lifetime:

$$\tau_{bs} \propto \exp\left(\frac{2\alpha\eta\rho}{3r_e\gamma^2}\right)\cdot\frac{\rho\sqrt{\eta\rho}}{L_i\gamma^2}$$

α – fine structure constant
η – energy acceptance
ρ – average bending radius of particle's trajectory at the IP

Obviously, the major tool for reducing the negative effect of beamstrahlung is making ρ larger. For flat beams, ρ is inversely proportional to the surface charge density:

$$\frac{1}{\rho} \propto \frac{N_p}{\gamma\sigma_x\sigma_z} \propto \frac{\xi_y}{L_i}\sqrt{\frac{\varepsilon_y}{\beta_y^*}} \propto L\cdot\sqrt{\frac{\varepsilon_y}{\beta_y^*}}$$

The last transformation is based on assumption that $L_i \sim \beta_y^*$. We want to increase the luminosity L while keeping the lifetime (and therefore ρ) large enough. It follows that:

- The vertical emittance (i.e. both the betatron coupling and the horizontal emittance) should be minimized.

- β_y^* (together with L_i) should be *maximized*! What does it mean? Increase of β_y^* by a factor *k* may result in luminosity gain by $k^{1/2}$ (with ρ unchanged), but ξ_y will grow by $k^{3/2}$. *We can do this until ξ_y remains below the beam-beam limit.*

What is the optimum β_y?

The general rule for optimization: ***if there are multiple limiting factors, optimum performance happens when all limits are reached simultaneously*.**

In our case it means that β_y^* (together with L_i) should be adjusted in such a way that both τ_{bs} and ξ_y achieve their limits.

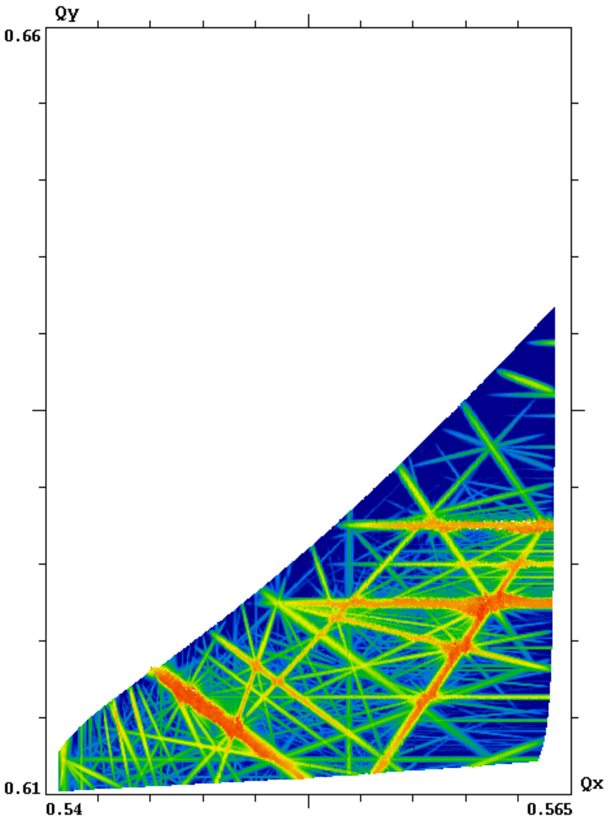
It follows that shifting the balance towards "limit by the beamstrahlung lifetime" (e.g. decrease of η, increase of ε_y and γ) will require increase of L_i (together with β_y^*), and vise versa.

Example: E = 175 GeV, η = 0.02, ε_y = 2.6 pm => $\beta_y^* \approx$ 2 mm

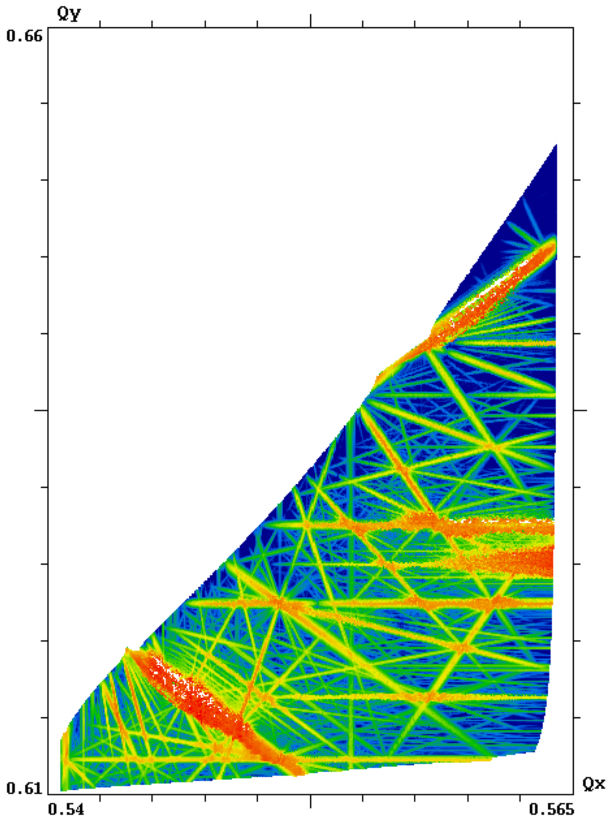
At "low" energies (80, 45.5 GeV) τ_{bs} allows to squeeze β_y^* below 1 mm; then we reach another limitation: lattice of IR, chromaticity correction and DA require $\beta_y^* \geq$ 1 mm.

On the other hand, bunch lengthening due to beamstrahlung becomes significant **at low energies**. **We need crossing angle**, if we want to keep β_y^* small (about 1 mm).

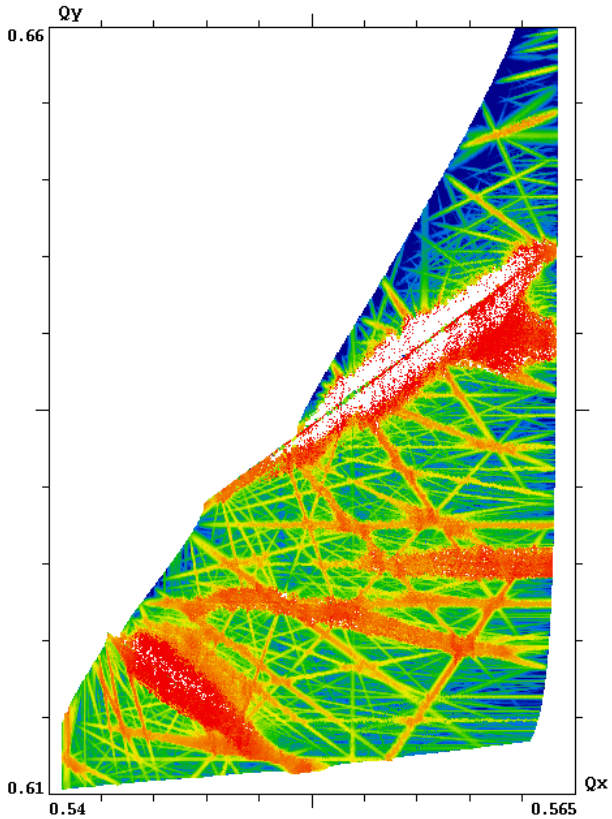
Impact of Hour-glass: Tune shift



$\sigma_z = \beta_y$



$\sigma_z = 2\beta_y$

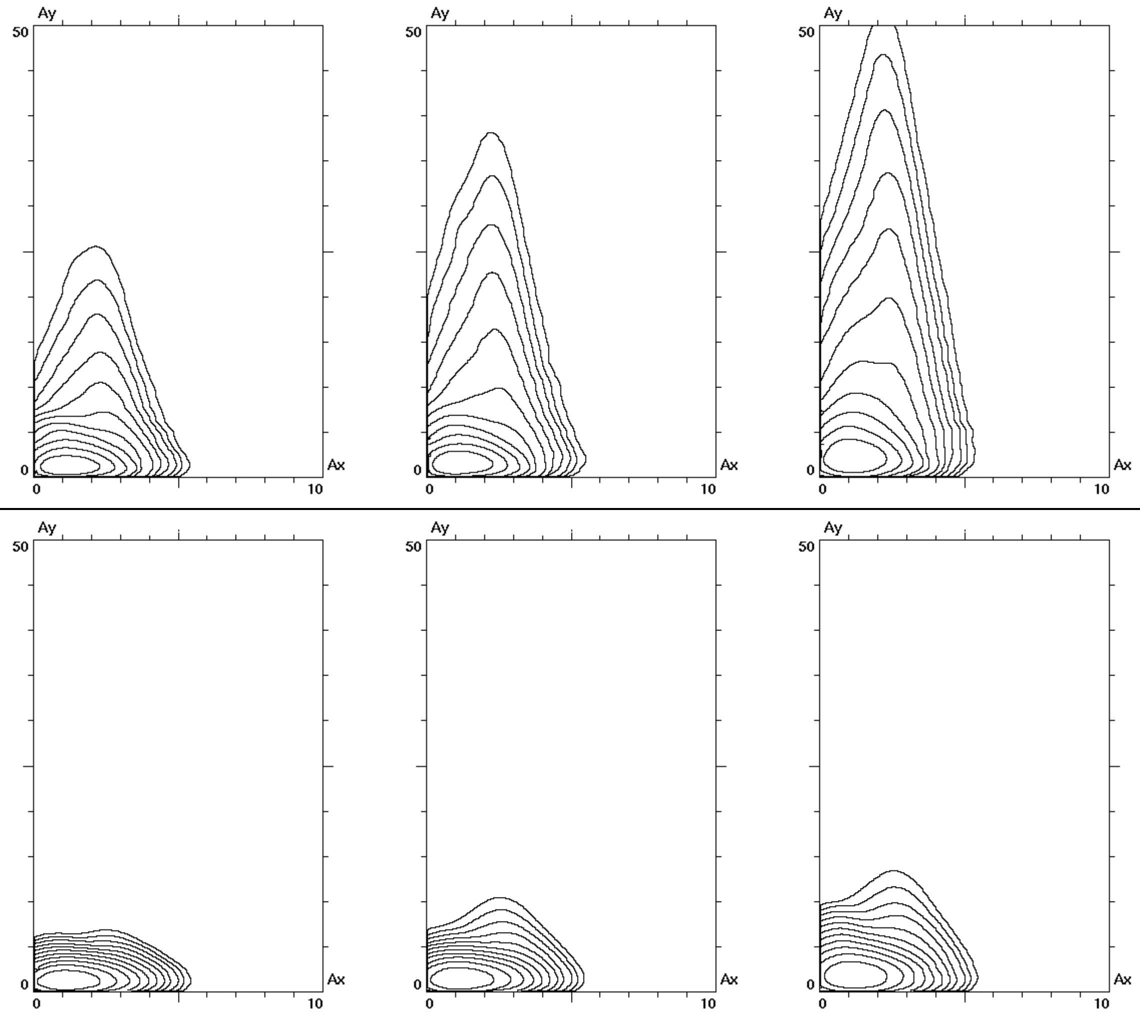


$\sigma_z = 3\beta_y$

FMA footprints in the plane of betatron tunes, synchrotron amplitude: $A_s = 1$ sigma.

Parameters as for TLEP Z from FCC-ACC-SPS-0004, $\xi_x \approx \xi_y \approx 0.03$ (nominal).

Impact of Hour-glass vs. Damping



Damping as for 45.5 GeV

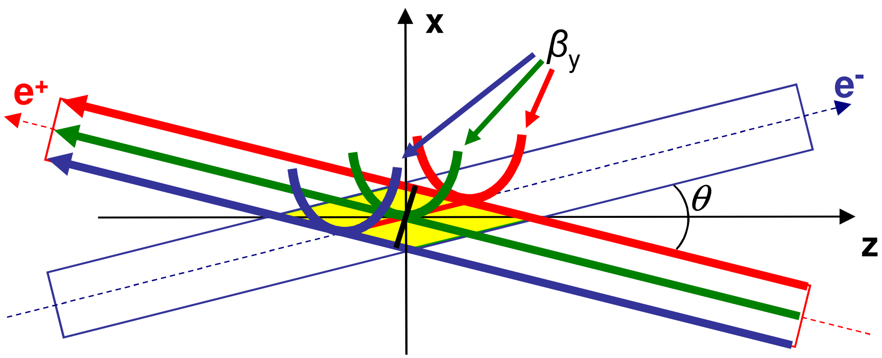
Damping as for 120 GeV
(20 times stronger)

$\sigma_z = \beta_y$, $R_H \approx 0.86$ $\quad$ $\sigma_z = 2\beta_y$, $R_H \approx 0.71$ $\quad$ $\sigma_z = 3\beta_y$, $R_H \approx 0.60$

Contour plots of equilibrium distribution in the space of normalized betatron amplitudes.
Density between successive contour lines decreases by a factor of *e*.

Crab Waist Scheme

P. Raimondi, 2006



$$\phi = \frac{\sigma_z}{\sigma_x} tg\left(\frac{\theta}{2}\right)$$ – Piwinski angle

1) Large Piwinski angle: ϕ >> 1

2) β_y approx. equals to overlapping area: $\beta_y \approx \sigma_z / \phi$

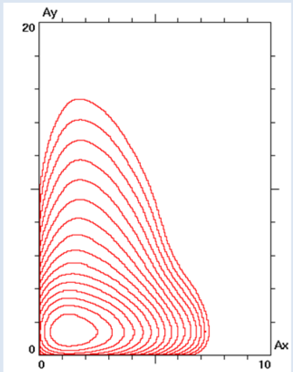
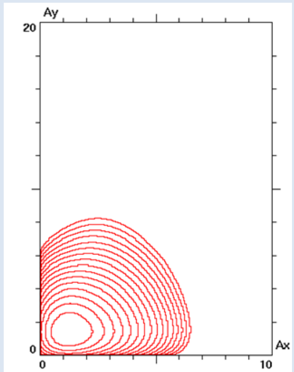
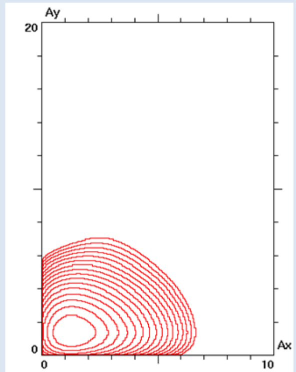
3) Crab Waist: minimum of β_y along the axis of the opposite beam

Advantages:

- ✓ Impact of hour-glass is small and does not depend on bunch lengthening
- ✓ Suppression of betatron coupling resonances allows to achieve ξ_y ~ 0.2

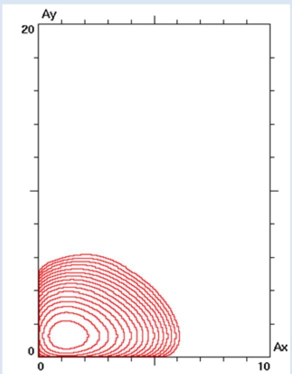
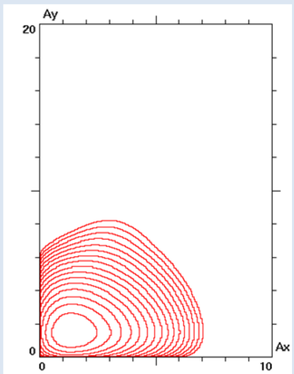
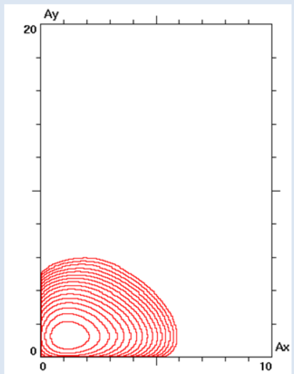
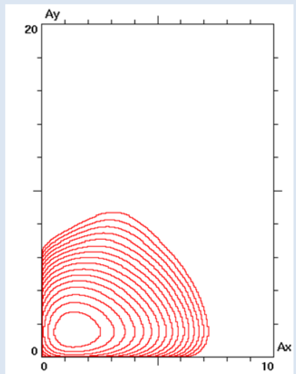
As a result, luminosity can be significantly increased!

FCC-ee @175 GeV, Different Collision Schemes

Collision scheme	**Head-on**	**30 mrad**	**Crab Waist**
N_p	$1.8{\cdot}10^{11}$	$2.1{\cdot}10^{11}$	$2.2{\cdot}10^{11}$
N_b	66	57	54
σ_z / σ_{zbs} [mm]	2.41 / 2.80	2.41 / 2.87	2.41 / 2.89
ν_x / ν_y	0.56 / 0.61	0.54 / 0.57	0.54 / 0.57
$\Delta\nu_x$ / $\Delta\nu_y$	0.126 / 0.141	0.056 / 0.084	0.057 / 0.092
L [$cm^{-2}s^{-1}$]	$1.35{\cdot}10^{34}$	$1.06{\cdot}10^{34}$	$1.23{\cdot}10^{34}$
τ_{bs} [min]	30	30	30
Density contour plots $10\sigma_x \times 20\sigma_y$			

Flip-Flop @ 175 GeV (30 mrad, crab waist)

triggered by asymmetry in bunch currents and beamstrahlung bunch lengthening

Asymmetry	10 %		15 %	
Bunch	"strong"	"weak"	"strong"	"weak"
σ_{zbs} [mm]	2.68	3.06	2.62	3.11
L [$cm^{-2}s^{-1}$]	$1.09 \cdot 10^{34}$		$1.02 \cdot 10^{34}$	
τ_{bs} [min]	~900	5	> 3000	3
Density contour plots $10\sigma_x \times 20\sigma_y$				

To work at the maximum luminosity, the bunch currents asymmetry must be < 10%.

Flip-Flop @ Low Energies

Asymmetry in the bunch currents leads to asymmetry in the bunch lengths (due to beamstrahlung).

Asymmetry in the bunch lengths leads to the "weak" ε_x growth (depends on crossing angle and ξ_x).

Due to betatron coupling, vertical emittance of the "weak" bunch also increases.

The vertical emittance blowup enhances beamstrahlung for the "weak" bunch, and its lengthening.

To suppress flip-flop, we need to reduce the horizontal emittance growth. This can be achieved by decrease of β_x (and ξ_x decreases in the same proportion).

In simulations, this effect depends on the model: how ε_y is affected by ε_x.

Summary

- Crab Waist collision scheme provides higher luminosity than head-on, especially at "low" energies (80, 45.5 GeV) .
- Beamstrahlung is one of the most important factors that affect the collider performance. At "high" energies (175, 120 GeV) this is manifested mainly in limiting the lifetime; at "low" energies (80, 45.5 GeV) – in the bunch lengthening.
- Flip-flop instability, which is enhanced by the bunch lengthening due to beamstrahlung, also may limit the luminosity.
- The general recipe to reduce beamstrahlung: decrease of N_p, therefore increase of N_b and decrease of emittances.
- At "high" energies (175, 120 GeV) β_y^* (together with L_i) should be optimized in order to reach both beam-beam and beamstrahlung limits simultaneously. At "low" energies (80, 45.5 GeV) β_y^* is limited by the lattice of IR and DA.
- To avoid flip-flop instability, β_x^* (which is proportional to ξ_x when ϕ >> 1) should be reduced.
- The whole optimization process is rather complicated and should be performed with the help of beam-beam simulations in strong-strong (or quasi-strong-strong) model.