

*Towards precision theoretical description on high  
energy phenomenology for the CEPC-SppC  
–Progress of FDC project*

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## Introduction

- The propose of the CEPC-SppC is to find new physics or/and do precise measurement on Higgs properties
- It is well known that precision theoretical description on high energy phenomenology for CEPC-SppC must be achieved.
- Therefore, higher-order perturbative calculations in QFT for SM are required for signal and background.
- FDC project is aimed at automatic calculation on these calculation and already can do next-leading-order(NLO) calculation automatically.
- Based on FDC, there are already many hard works been achieved in last 8 years.
- Recent progress for FDC project will be introduced in this talk.

## Brief Introduction to FDC package

Feynman Diagram Calculation(FDC).

This first version of FDC was presented at AIHENP93 workshop,1993.

FDC Homepage::

[http://www.ihep.ac.cn/lunwen/wjx/public\\_html/index.html](http://www.ihep.ac.cn/lunwen/wjx/public_html/index.html)

***FDC-LOOP***

FDC-PWA

FDC-EMT

FDC-SM-and-Many-Extensions

FDC-NRQCD

FDC-MSSM

**Written in REDUCE,  
RLISP,C++.  
To generate Fortran**

**Event Generator**

## The calculations by using FDC-loop in last 8 years

- Our work concentrate on QCD correction to heavy quarkonium production and polarization in B-factory, z boson decay,  $\Upsilon$  decay, HERA, Tevatron, LHC.
- It is found that that QCD corrections to these processes are very important.

## Recent Progress in FDC project

- Event Generator available at LO (BASES and PYTHIA)
- automatic counter terms generation, automatic calculation of renormalization constant (scheme dependent). . . .
- Geometric method for sector decomposition in multi-loop calculation

# Muti-loop Calculation in QFT

In last ten years, there are very active developments in methods for higher-order perturbative calculation in QFT.

- Unitary Cut, On shell Helicity Amplitude, IBP, Symbol, Sector Decomposition, Mellin-Barnes(MB), Differential Equation(DE)
- Key step: to calculate loop integral (master integral, in more than one-loop case, master integral basis have not yet config out and therefore, sclar integral or even tensor integral)
- Sector Decomposition is a better way to separate all UV and IR divergences in loop integral in analytic form and passes the finite part to numerical calculation.

## Geometric method for sector decomposition

- Sector Decomposition is an old method and has been very actively developed. The most recently development is the Geometric method by T. Kaneko and T. Ueda on 2010, which beautifully and effectively translate the problem into a Geometric problem:  
"to construct and triangulate convex polyhedral cone"
- They (T. Kaneko and T. Ueda) construct a computer program for their method by utilizing an algorithm from mathematician and they can finish the triple box(3-loop) by 53 hours CPU time.
- We develop a new algorithm and construct a computer program. We can finish the same work within 3-minutes CPU time on the same CPU.

# Geometric method for sector decomposition

Sector Decomposition is a method used to separate divergences in loop integral.  
With  $\alpha$  presentation of a propagator

$$\frac{1}{D_i^{a_i}} = -i \int_0^\infty d\alpha_i \exp(iD_i \alpha_i a_i),$$

An  $h$ -loop integral with  $N$  propagators can be expressed as

$$G = \int \frac{d^d k_1 d^d k_2 \cdots d^d k_h}{D_1^{a_1} D_2^{a_2} \cdots D_N^{a_N}} = \int d^d k \int d^N \alpha \exp\left(i \sum_{l=1}^N D_l \alpha_l a_l\right)$$

After integration on loop momenta, it becomes

$$G = C \int_0^\infty d^N \alpha \prod_l \alpha_l^{a_l - 1} U^{-d/2} e^{-iF/U} \quad (1)$$

where  $U$  and  $F$  are homogeneous polynomials of  $\alpha_i$  with the homogeneity degrees  $h$  and  $h + 1$ , and  $C$  is a constant.



Let  $\eta = \sum \alpha_l$ , insert  $\delta(\eta - \sum \alpha_l)$  into the integral, and make the transformation  $\alpha_l = \eta \alpha'_l$ . After the integration over  $\eta$ , the integral becomes

$$G = C' \int_0^1 d^N \alpha \delta\left(1 - \sum \alpha_l\right) \prod_l \alpha_l^{a_l-1} \frac{U^{a-(h+1)d/2}}{F^{a-hd/2}} \quad (2)$$

with  $a = \sum a_l$ .

One can always reach Eq.(2) with usual loop integral techniques. And this is where sector decomposition starts. In this integral, only the integration over  $\alpha_i$  is remained, and the interval is now limited to  $[0, 1]$  due to the delta function. And this is how sector decomposition works on it:

- separate the integration domain into  $N$  sectors  $\Delta_{k,k=1,2,\dots,N}$ , where  $\Delta_k$  is defined by  $\alpha_i \leq \alpha_k, i \neq k$ .
- do the transformation  $\alpha'_i = \alpha_i/\alpha_k, i \neq k$  in  $\Delta_k$ , and integrate over  $\alpha_k$  with the delta function
- now, the integral in the integration domain  $\Delta_k$  (labelled with  $G_k$ ) becomes

$$G_k = C' \int_0^1 d^{N-1} \alpha \prod_l \alpha_l^{a_l-1} \frac{U_k^{a-(h+1)d/2}}{F_k^{a-hd/2}} \quad (3)$$

where  $U_k$  and  $F_k$  are obtained by setting  $\alpha_k$  to 1 in  $U$  and  $F$ .

- Usually these  $\Delta_k$  are called primary sectors. But they are not sufficient since the divergences are still hidden inside.
- Further decomposition is needed.
- Here we introduce the geometric method [[?]], which can separate the divergence after one more decomposition (free from infinite recursion).
- For convenience, we rewrite  $G_k$  into

$$G_k = C' \int_0^1 d^{N-1} \alpha \alpha^{\nu} U_k^{\beta} F_k^{\gamma} \quad (4)$$

with  $\nu = \{a_1 - 1, a_2 - 1, \dots, a_{N-1} - 1\}$ ,  $\alpha^{\nu} = \prod_l \alpha_l^{a_l - 1}$ ,  
 $\beta = a - (h + 1)d/2$  and  $\gamma = -(a - hd/2)$

This is how the geometric method works:

- A monomial in  $U_k(F_k)$  has the form  $c_b \alpha^b = c_b \alpha_1^{b_1} \alpha_2^{b_2} \cdots \alpha_{N-1}^{b_{N-1}}$ , and can be characterized by a vector  $b = \{b_1, b_2, \dots, b_{N-1}\}$ , thus we have two vector sets  $Z^U$  and  $Z^F$
- do transformation  $\alpha_i = e^{-y_i}$ , the Jacobian is  $e^{-\sum_i y_i}$ , the integral becomes

$$G_k = C' \int_0^\infty d^{N-1} y e^{-v \cdot y} U_k^\beta F_k^\gamma \quad (5)$$

- now  $U_k = \sum_{b \in Z^U} c_b e^{-b \cdot y}$ ,  $F_k = \sum_{b' \in Z^F} c_{b'} e^{-b' \cdot y}$
- suppose  $e^{-b(b') \cdot y}$  is maximal in  $U_k(F_k)$ , label this domain as  $\Delta_{bb'}$ .
- extract these two term from  $U_k$  and  $F_k$ , and the integral can be further decomposed into

$$G_k = \sum_{b \in Z^U} \sum_{b' \in Z^F} \int_{\Delta_{bb'}} d^{N-1} y e^{-(v + b\beta + b'\gamma) \cdot y} \times \left[ c_b + \sum_{d \in Z^U / \{b\}} c_d e^{-(d-b) \cdot y} \right]^\beta \times \left[ c_{b'} + \sum_{d' \in Z^F / \{b'\}} c_{d'} e^{-(d'-b') \cdot y} \right]^\gamma \quad (6)$$

where  $Z^U / \{b\}$  denotes subset of  $Z^U$  obtained by removing  $b$  from  $Z^U$

- We have supposed that in the domain  $\Delta_{bb'}$ ,  $e^{-b(b') \cdot y}$  is maximal in  $U_k(F_K)$  for certain  $b(b')$ .
- $e^{-(d-b) \cdot y} \leq 1$  and  $e^{-(d'-b') \cdot y} \leq 1$  for all  $d(d')$
- $(d-b) \cdot y \geq 0$ ,  $(d'-b') \cdot y \geq 0 \rightarrow$  restrictions on  $\Delta_{bb'}$
- Last two terms in Eq.(6) is always finite when  $\alpha_i = 0$
- All physical poles (if exist) have been factorized into first term
- Now we can expand last two term with respect to  $\epsilon = (D-4)/2$  to required order and obtained a Laurent series of the whole integral in terms of  $\epsilon$ , whose coefficients are expressed by finite integrals
- The question is how to obtain the domain  $\Delta_{bb'}$

- Define  $Z_b^{U(F)} \equiv \{v - b | v \in Z^{U(F)} / \{b\}\}$ ,  $S \equiv Z_b^U \cup Z_{b'}^F \cup R^{N-1}$ , where  $R^{N-1}$  is the standard basis of  $(N - 1)$ -dimensional Euclidean space.
- Let  $C(S)$  be the vector space generated by  $S$ ,  $C(S) \equiv \{\sum_i r_i v_i | r_i \geq 0, v_i \in S\}$ , then  $C(S)$  is just a convex polyhedral cone in  $(N - 1)$ -dimensional Euclidean space.
- The dual cone of  $C(S)$  is defined by  $C(S)^V \equiv \{y | v \cdot y \geq 0, \forall v \in C(S)\}$ ,  $C(S)^V$  is also a convex polyhedral cone in  $(N - 1)$ -dimensional Euclidean space.
- $\Delta_{bb'} = \{y | v \cdot y \geq 0, \forall v \in S\} = C(S)^V$ .  $R^{N-1}$  is also in  $S$  because of the fact  $y_i \geq 0$ .
- Let  $V$  be the set of all edges of  $C(S)^V$ , then  $C(S)^V$  can also be written as  $\{\sum_i u_i v_i | u_i \geq 0, v_i \in V\}$
- If  $C(S)^V$  is simplicial (has only  $N - 1$  edges), change the integration variables from  $y_i$  into  $u_i$ ,  $\Delta_{bb'}$  is obtained.
- If not, triangulate  $C(S)^V$  (split it into several simplicial sub-cones), and change integration variables in each sub-cones (more sub-sectors).
- This is why this method is called 'geometric'

		Double box		Triple box	
Package	Strategy	No. of Sectors	Time(s)	No. of Sectors	Time(s)
FIESTA4	S	362	1.4	23783	831
	B	586	0.5	121195	127
	X	282	0.3	10259	44
	KU	266	13.7	6822	7472
	KU0	326	4.5	10556	6487
	KU2	266	61.9	-	-
SecDec3.0	X	320	1.4	11384	700
	G1	270	4.2	7871	574
T. Kaneko and T. Ueda		266	29.2	6568	123280
FDC-SD (Our result)		266	0.6	6568	116

Comparison with FIESTA4 [[?]], SecDec3.0 [[?]] and method proposers [[?]].  
 Strategies KU, KU0, KU2, G1 and G2 are all based on the geometric method,  
 while G2 is different in the strategy of primary sectors.

decomposition only, without the integration of finite coefficients

# Summary



Thanks for your attention!