## Importance of a Model Independent Measurement of $BR(H \rightarrow BSM)$

Tim Barklow (SLAC) Sep 03, 2016 CEPC-SppC Study Group Meeting Systematic Error on  $\sigma(ZH)_{hadronic}$  Measurement from Model Dependence of Analysis

$$\left(\frac{\Delta\sigma(ZH)_{hadronic}}{\sigma(ZH)_{hadronic}}\right)^{2} = \frac{S+B}{S^{2}} \left\{ 1 + \frac{(L\sigma_{ZH})^{2}}{S+B} \sum_{i} BR_{i}^{2} \left[ \left(\frac{\Delta\sigma \cdot BR_{i}}{\sigma \cdot BR_{i}}\right)^{2} (\xi_{i} - \langle \xi \rangle)^{2} + \Delta\xi_{i}^{2} \right] \right\}$$

Penalty for non-uniform efficiency

MC calculation

- $\xi_i = efficiency$  for events from H decay i to pass  $\sigma(ZH)_{hadronic}$  analysis
- S = Number of signal events in  $\sigma(ZH)_{hadronic}$  analysis
- B = Number of background events in  $\sigma(ZH)_{hadronic}$  analysis

For the sys error from unknown BSM decays we must assume  $BR_{BSM} \ge \Delta BR_{BSM}$ . From the range of efficiencies in Mark Thomson's ILC analysis at  $\sqrt{s}$ =350 GeV we get  $\Delta \xi_{BSM} = .035$ .

 $\Delta BR_{BSM} = 0.04$  gives a sys error of 11% of the stat error for the  $\sqrt{s}$ =350 GeV  $\sigma(ZH)_{hadronic}$  measurement.

### Systematic Error on $\sigma \cdot BR_i$ from $\Delta BR_{BSM}$

Neglecting non-Higgs background, the number of events  $N_i$  passing Higgs decay channel *i* selection criteria is

$$N_i = \sum_j \boldsymbol{\sigma} \boldsymbol{\cdot} \boldsymbol{B} \boldsymbol{R}_j \boldsymbol{\varepsilon}_{ij} \boldsymbol{L}$$

 $\varepsilon_{ij}$  = efficiency for Higgs decay mode j to pass Higgs decay channel i selection For SM decays the efficiencies  $\varepsilon_{ij}$  can be calculated with MC. But what if decay mode j is a BSM decay? To account for this possibility a conservative systematic error can be assigned assuming  $\varepsilon_{ij} = 1$ . This leads to a systematic error of  $\Delta N_i = L\sigma \Delta BR_{BSM}$ 

Improvement in Higgs Coupling Errors if  $\Delta BR_{BSM}$  is small.

Further improvement in the Higgs coupling measurements can be obtained using the constraint  $\sum_{i} BR_{i} = 1$  as first noted by Michael Peskin. This constraint is model independent so long as the error in  $BR(H \rightarrow BSM)$  is included in the fit. What error in  $BR(H \rightarrow BSM)$  is required to produce an improvement in Higgs coupling measurements ?

In the following the ILC H-20 scenario is a 20 year run plan with operation at 250+350+500 GeV with 2000+200+4000 fb<sup>-1</sup>

Perform coupling fit with  $\sum_{i} BR_{i} = 1$  including  $\Delta BR(H \rightarrow BSM)$ (the constraint  $\sum_{i} BR_{i} = 1$  is model independent if  $\Delta BR(H \rightarrow BSM)$  is included in the fit)

$\Delta BR(H \to BSM)$	~	8	4	2	1	0.1
$\Delta BR(H \rightarrow Invis)_0$	$\sim$					
ZZ	0.31%	0.29%	0.26%	0.22%	0.20%	0.19%
WW	0.38%	0.36%	0.31%	0.25%	0.21%	0.19%
bb	0.60%	0.57%	0.52%	0.46%	0.42%	0.40%
$oldsymbol{ au}^+oldsymbol{ au}^-$	0.88%	0.86%	0.83%	0.79%	0.77%	0.76%
<i>gg</i>	0.92%	0.91%	0.88%	0.86%	0.85%	0.84%
сс	1.1%	1.1%	1.1%	1.1%	1.1%	1.0%
YY	3.1%	3.1%	3.1%	3.1%	3.1%	3.1%
$\Gamma_{tot}$	1.7%	1.6%	1.3%	1.0%	0.84%	0.74%

ILC Higgs Coupling Precision assuming 20 year H20 scenario

 $\Delta BR(H \rightarrow Invis)_0$  corresponds to the H-20 ILC 95% C.L. limit of 0.34%

Perform coupling fit with  $\sum_{i} BR_{i} = 1$  including  $\Delta BR(H \rightarrow BSM)$ (the constraint  $\sum_{i} BR_{i} = 1$  is model independent if  $\Delta BR(H \rightarrow BSM)$  is included in the fit)

$\Delta BR(H \to BSM)$	~	8	4	2	1	0.1
$\Delta BR(H \rightarrow Invis)_0$					I	0.1
ZZ	0.26%	0.24%	0.22%	0.19%	0.18%	0.17%
WW	1.2%	1.2%	1.2%	1.2%	1.2%	1.2%
bb	1.3%	1.3%	1.2%	1.2%	1.2%	1.2%
$oldsymbol{ au}^+oldsymbol{ au}^-$	1.4%	1.4%	1.4%	1.4%	1.3%	1.3%
gg	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
сс	1.7%	1.7%	1.7%	1.6%	1.6%	1.6%
γγ	4.7%	4.7%	4.7%	4.7%	4.7%	4.7%
$\Gamma_{tot}$	2.8%	2.7%	2.5%	2.4%	2.3%	2.3%

CEPC Higgs Coupling Precision assuming 5 ab<sup>-1</sup>

 $\Delta BR(H \rightarrow Invis)_0$  corresponds to the 5 ab<sup>-1</sup> CEPC 95% CL limit of 0.28% 40% improvement in  $\Delta g_z$  if  $\Delta BR(H \rightarrow BSM) \approx \Delta BR(H \rightarrow Invis)$ 

Perform coupling fit with  $\sum_{i} BR_{i} = 1$  including  $\Delta BR(H \rightarrow BSM)$  for (the constraint  $\sum_{i} BR_{i} = 1$  is model independent if  $\Delta BR(H \rightarrow BSM)$  is included in the fit)

$\frac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0}$	$\infty$	8	4	2	1	0.1
ZZ	0.20%	0.19%	0.17%	0.14%	0.13%	0.12%
WW	0.26%	0.25%	0.22%	0.19%	0.17%	0.17%
bb	0.47%	0.%	0.41%	0.37%	0.34%	0.33%
$oldsymbol{ au}^+oldsymbol{ au}^-$	0.65%	0.63%	0.61%	0.58%	0.57%	0.56%
<i>gg</i>	0.70%	0.69%	0.68%	0.66%	0.66%	0.65%
cc	0.86%	0.85%	0.83%	0.82%	0.81%	0.80%
YY	2.6%	2.6%	2.6%	2.6%	2.6%	2.6%
$\Gamma_{tot}$	1.2%	1.1%	0.96%	0.76%	0.64%	0.58%

### Higgs Coupling Precision 5 $ab^{-1}CEPC + H-20$ ILC

 $\Delta BR(H \rightarrow Invis)_0$  corresponds to either the H-20 ILC 95% C.L. limit of 0.34% or the 5 ab<sup>-1</sup> CEPC 95% CL limit of 0.28%



WW

ILC 250+350+500 GeV with 2000+200+4000 fb<sup>-1</sup> (H-20 scenario full run  $\Rightarrow$  20.2 yrs) CEPC 250 GeV with 5000  $fb^{-1}$ 

mumu

invis

GamTot



99

cc

bb



tautau

ZZ

gamgam

 $\frac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0} = 1$ 

## How Do You Measure $\sigma \cdot BR_{BSM}$ ?

• Use  $BR_{BSM} = 1 - \sum_{SM \text{ decays } i} BR_i$ 

This can be used for estimating the systemtatic errors for  $\sigma$ (ZH) and the SM  $\sigma$ -BR's. It can't of course be used to improve Higgs couplings through the constraint  $\sum BR_i = 1$ .

This approach assumes negligible contimination of SM  $\sigma$ -BR analyses by BSM events, and so is model dependent.

The error in this case is

$$(\Delta BR_{BSM})^{2} = \sum_{\text{SM decays i}} \left[ \left( \frac{\Delta \sigma \cdot BR_{i}}{\sigma \cdot BR_{i}} \right)^{2} + \left( \frac{\Delta \sigma_{ZH}}{\sigma_{ZH}} \right)^{2} \right] (BR_{i})^{2}$$

which can be pretty good given that  $\sigma \cdot BR_i$  is measured well for decay channels with large BR's. This technique was used to obtain  $\Delta BR_{BSM} = 0.04$  in the discussion of the  $\sqrt{s} = 350 \text{ GeV } \sigma(\text{ZH})_{hadronic}$  systematic error.

## How Do You Measure $\sigma \cdot BR_{BSM}$ ?

• Include a BSM contribution in each of the SM  $\sigma \cdot BR_i$  analyses when doing the correlated global fit of all SM  $\sigma \cdot BR_i$ 

The problem here is the unknown efficiency for BSM decays to pass the selection for each decay channel. One might also gain additional information by performing the ZH leptonic and hadronic recoil analysis on an event sample from which all events passing SM  $\sigma \cdot BR_i$  analyses have been removed. Work is ongoing to develop this kind of procedure.

• Go through a long list of BSM decay topologies, perform a dedicated search for each, and then convince yourself that all possible BSM decay topologies have been covered.

Sort of a brute force approach, but it might be the only way. Tricky part is proving that all topologies have been accounted for.

#### 215 page "Exotic Decays of the 125 GeV Higgs Boson" arXiv:1312.4992

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#### Summary

- BSM decays of the Higgs are of course important in their own right. However, even if no evidence for BSM decays is found, the model independent limit that can be placed on BSM decays affects the SM coupling measurements. It is important in evaluating the systematic errors for σ(ZH) and the σ•BR's, and strong limits on BSM decays are needed to squeeze the last bit of Higgs coupling precision out of the data (the CEPC Δg<sub>z</sub>=0.26% improves to 0.18% if ΔBR(H → BSM) ≈ ΔBR(H → Invis)).
- Work is ongoing to estimate BR(H → BSM) at CEPC using several techniques.

# **Backup Slides**

#### Higgs Physics Systematic Errors

#### Model Independence of ZH Recoil Measurements

In order to use the hadronic ZH recoil measurement in our Higgs analyses we have to quantify the penalty associated with the fact that  $\sigma(ZH \rightarrow q \,\overline{q} + X)$  is "almost model independent". By how much must we blow up  $\Delta\sigma(ZH \rightarrow q \,\overline{q} + X)$  to account for the fact that the efficiencies differ by as much as 7%?



#### ★ Combining visible + invisible analysis: wanted M.I.

i.e. efficiency independent of Higgs decay mode

Decay mode	$arepsilon_{\mathscr{L}>0.65}^{\mathrm{vis}}$	$arepsilon_{\mathscr{L}>0.60}^{\mathrm{vis}}$	$\varepsilon^{\mathrm{vis}} + \varepsilon^{\mathrm{invis}}$	_	
$\begin{array}{l} H \rightarrow \text{invis.} \\ H \rightarrow q\overline{q}/gg \\ H \rightarrow WW^* \\ H \rightarrow ZZ^* \\ H \rightarrow \tau^+\tau^- \\ H \rightarrow \gamma\gamma \end{array}$	<0.1 % 22.2 % 21.6 % 20.2 % 24.7 % 25.8 %	22.0% <0.1% 0.1% 1.0% 0.3% <0.1%	22.0 % 22.2 % 21.7 % 21.2 % 24.9 % 25.8 %		Very similar efficiencies
$H \rightarrow Z\gamma$	18.5%	0.3 %	18.8 %		
$\begin{array}{c} H \rightarrow WW^* \rightarrow q\overline{q}q\overline{q} \\ H \rightarrow WW^* \rightarrow q\overline{q}l\nu \\ H \rightarrow WW^* \rightarrow q\overline{q}\tau\nu \\ H \rightarrow WW^* \rightarrow l\nu l\nu \\ H \rightarrow WW^* \rightarrow l\nu \tau\nu \\ H \rightarrow WW^* \rightarrow \nu \tau\nu \tau\nu \end{array}$	21.3 % 21.9 % 22.1 % 24.8 % 20.5 % 16.4 %	<0.1 % <0.1 % <0.1 % 0.1 % 0.8 % 2.5 %	21.3 % 21.9 % 22.1 % 25.0 % 22.1 % 18.9 %	}	Look at wide range of WW topologies

It is not sufficient to vary the SM Higgs branching ratios to estimate this systematic error. The problem is the BSM decays; they cannot be accounted for in this way.

To handle the BSM decays we have used an approach where we use all of our  $\sigma \cdot BR$  measurements for SM Higgs decays to obtain an estimate of the average signal efficiency for  $\sigma(ZH \rightarrow q \bar{q} + X)$ . It is then straightforward to propagate the  $\sigma \cdot BR_i$  errors, as well as the systematic errors on the individual decay mode efficiencies for the  $\sigma(ZH \rightarrow q \bar{q} + X)$  selection, to the error on  $\sigma(ZH \rightarrow q \bar{q} + X)$ .

Let

- $\Psi \equiv \sigma(ZH \to q\,\overline{q} + X)$
- $\Omega$  = Number of signal + background events in  $\sigma(ZH \rightarrow q \,\overline{q} + X)$  analysis
- B = Predicted number of background events in  $\sigma(ZH \rightarrow q \,\overline{q} + X)$  analysis
- $\Xi$  = Average efficiency for signal events to pass  $\sigma(ZH \rightarrow q \,\overline{q} + X)$  analysis L = luminosity

$$\Psi = \frac{\Omega - B}{L\Xi} = \frac{1}{\Xi} \sum_{i} \psi_i \xi_i = \sum_{i} \psi_i \quad \text{where}$$

 $\psi_i = \sigma(ZH) \cdot BR_i$ 

 $\xi_i = e$ fficiency for events from Higgs decay i to pass  $\sigma(ZH \rightarrow q \,\overline{q} + X)$  analysis

$$\Xi = \frac{\sum_{i} \psi_i \xi_i}{\sum_{i} \psi_i}$$

$$\psi_i = \frac{\omega_i - \beta_i}{L\eta_i}$$

 $\omega_i$  = Number of signal + background events in  $\sigma(ZH) \cdot BR_i$  analysis  $\beta_i$  = Predicted number of background events in  $\sigma(ZH) \cdot BR_i$  analysis  $\eta_i$  = efficiency for Higgs decay i to pass  $\sigma \cdot BR_i$  analysis

- $K_i$  = number of signal + background events common to had Z recoil and  $\sigma \cdot BR_i$  analyses
- E = number of signal + background events unique to had Z recoil analysis  $\varepsilon_i$  = number of signal + background events events unique to  $\sigma \cdot BR_i$  analysis

$$\Omega = E + \sum_{i} K_{i} \qquad S \equiv \Omega - B \qquad T \equiv \frac{\sqrt{S + B}}{S}$$

$$\omega_{i} = K_{i} + \varepsilon_{i} \qquad S_{i} \equiv \omega_{i} - \beta_{i} \qquad \tau_{i} \equiv \frac{\sqrt{S_{i} + \beta_{i}}}{S_{i}}$$

$$\lambda_{i} \equiv \frac{K_{i}}{\omega_{i}} \qquad N \equiv L \sigma_{ZH} \qquad r_{i} \equiv BR_{i} \qquad \delta_{i} \equiv \xi_{i} - \Xi$$

$$\left(\frac{\Delta \Psi}{\Psi}\right)^{2} = T^{2} \left\{ 1 + \frac{N^{2}}{\Omega} \sum_{i} r_{i}^{2} \left[ \tau_{i}^{2} \left( \delta_{i}^{2} - 2\lambda_{i}\eta_{i}\delta_{i} \right) + \Delta\xi_{i}^{2} \right] \right\} \qquad \text{This is our result for the error on}$$

$$\sigma(ZH \to q \overline{q} + X)$$

$$\left(\frac{\Delta\sigma(ZH\to q\,\overline{q}+X)}{\sigma(ZH\to q\,\overline{q}+X)}\right)^2 = \mathrm{T}^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right] \right\} \quad \text{i.e. sys err} = \frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right]$$

Assume  $\sqrt{s} = 350$  GeV and L=500 fb<sup>-1</sup>

$$\mathsf{N} = L\,\sigma_{ZH} = 45383 \quad r_i = BR_i = (1 - BR_{BSM})BR_i(SM) \quad \tau_i(SM) = \frac{\Delta\sigma \bullet \mathsf{BR}_i(SM)}{\sigma \bullet \mathsf{BR}_i(SM)} = \frac{\sqrt{s_i + \beta_i}}{s_i}$$

Assume 
$$T = \frac{\sqrt{S+B}}{S} = 0.014$$
  $\Omega = S+B = 17738$  and  $\xi_i(SM)$  given in the table four pages back.

We assume that the vis+invis efficiency values in the table four pages back cover all possible BSM decays since they cover all SM decays from completely invisible to fully hadronic multi-jet decays. Assuming no knowledge of the properties of the BSM decays we can then set

$$\xi_{BSM} = 0.5 * [\xi_{vis+invis}(max) + \xi_{vis+invis}(min)] = 0.5 * [0.258 + 0.188] = 0.22$$
  
$$\Delta \xi_{BSM} = 0.5 * [\xi_{vis+invis}(max) - \xi_{vis+invis}(min)] = .035$$

$$\left(\frac{\Delta\sigma(ZH\to q\,\overline{q}+X)}{\sigma(ZH\to q\,\overline{q}+X)}\right)^2 = \mathrm{T}^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right] \right\} \quad \text{i.e. sys err} = \frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right]$$

We next obtain the error  $\frac{\Delta \sigma \bullet BR_{BSM}}{\sigma \bullet BR_{RSM}}$  from Michael Peskin's Higgs coupling fit program. We do not use the  $\sum_{i} BR_{i} = 1$  constraint, and to begin with we only use the leptonic recoil  $\sigma_{ZH}$  measurement. This provides a model independent measurement of  $g_{BSM}$ . For  $\sqrt{s} = 350$  GeV, L=500 fb<sup>-1</sup> Michael's program gives  $\frac{\Delta g_{BSM}}{g_{BSM}} = 0.032$  which we multiply by two to get  $\frac{\Delta \sigma \bullet BR_{BSM}}{\sigma \bullet BR_{BSM}} = 0.064$ . We take this error to mean that  $0 < BR(H \rightarrow BSM) < 2 \times 0.064$ , and set the measured  $BR(H \rightarrow BSM) = 0.064$ . This gives a model independent hadronic recoil cross section error of  $\frac{\Delta\sigma(ZH \to q\,\overline{q} + X)}{\sigma(ZH \to q\,\overline{q} + X)} = 0.014 * 1.27 = 0.018.$ 

We then add this new model indepdendent hadronic recoil  $\sigma_{\rm ZH}$  measurement as input to Michael's program to obtain a new error  $\frac{\Delta \sigma \bullet BR_{BSM}}{\sigma \bullet BR_{RSM}} = 0.041$ . Setting  $BR(H \to BSM) = 0.041$  we then obtain a new model independent hadronic recoil  $\sigma_{ZH}$  error of  $\frac{\Delta\sigma(ZH \rightarrow q\,q + X)}{\sigma(ZH \rightarrow q\,\overline{q} + X)} = 0.014 * 1.12 = 0.016$ .

Iterating again we arrive at  $BR(H \rightarrow BSM) = 0.039$  and  $\frac{\Delta\sigma(ZH \rightarrow q\,q+X)}{\sigma(ZH \rightarrow q\,\overline{q}+X)} = 0.014 \times 1.11 = 0.016$ . Further interations don't give any improvement.

$$\left(\frac{\Delta\sigma(ZH\to q\,\overline{q}+X)}{\sigma(ZH\to q\,\overline{q}+X)}\right)^2 = \mathrm{T}^2 \left\{ 1 + \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right] \right\} \quad \text{i.e. sys err} = \frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right]$$

We have shown that  $\frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[ \tau_i^2 \delta_i^2 + \Delta \xi_i^2 \right] = 0.11$  for  $\sqrt{s} = 350$  GeV, L=500 fb<sup>-1</sup>.

How does this scale with luminosity?

$$\frac{N^2}{\Omega} \propto L \quad \tau_i^2 \propto L^{-1} \quad r_i^2 \text{ is independent of lumi except } r_{BSM}^2 = \tau_{BSM}^2 \propto L^{-1} \text{ .}$$
  
If we assume  $\Delta \xi_i = 0$  except  $\Delta \xi_{BSM} = 0.035$  then  
$$\frac{1}{2} \frac{N^2}{\Omega} \sum_i r_i^2 \left[ \tau_i^2 \delta_i^2 + \Delta \xi_i^2 \right] = 0.11 \text{ independent of the luminosity at } \sqrt{s} = 350 \text{ GeV.}$$

Caveats for hadronic recoil systematic error calculation:

(1) This systematic error analysis was only done at  $\sqrt{s} = 350$  GeV; it has not yet been done for  $\sqrt{s} = 250$  & 500 GeV

(2) These results assume that the true  $r_{BSM} = BR(H \rightarrow BSM)$  is small. As the true  $r_{BSM}$  grows we need to keep the product  $r_{BSM}\Delta\xi_{BSM}$  constant to maintain the same systematic error, where  $\xi_{BSM}$  is the efficiency for BSM Higgs decays to pass the hadronic recoil analysis. For example

true  $r_{BSM}$  required  $\Delta \xi_{BSM}$ 

.05	0.027
.10	0.014
.15	0.0091
.20	0.0068

These  $\Delta \xi_{BSM}$  requirements may seem stringent for the larger values of true  $r_{BSM}$ . However as  $r_{BSM}$  grows we will have more *BSM* decays to analyze and the required improvement in Monte Carlo modelling of the *BSM* decays should follow.  $\Psi \equiv \sigma(ZH) \cdot BR(visible)$ 

- $\Omega$  = Number of signal + background events in  $\sigma$ (*ZH*)•*BR*(*visible*) analysis
- B = Predicted number of background events in  $\sigma(ZH)$ •BR(visible) analysis
- $\Xi$  = Average efficiency for signal events to pass  $\sigma(ZH)$ •BR(visible) analysis L = luminosity

$$\Psi = \frac{\Omega - B}{L\Xi} = \frac{1}{\Xi} \sum_{i} \psi_i \xi_i = \sum_{i} \psi_i \quad \text{where}$$

 $\psi_i = \sigma(ZH) \cdot BR_i$ 

 $\xi_i = e$ fficiency for events from Higgs decay i to pass  $\sigma(ZH) \cdot BR(visible)$  analysis

$$\Xi = \frac{\sum_{i} \psi_i \xi_i}{\sum_{i} \psi_i}$$

$$\psi_i = \frac{\omega_i - \beta_i}{L\eta_i}$$

 $\omega_i$  = Number of signal + background events in  $\sigma(ZH) \cdot BR_i$  analysis  $\beta_i$  = Predicted number of background events in  $\sigma(ZH) \cdot BR_i$  analysis  $\eta_i$  = efficiency for Higgs decay i to pass  $\sigma \cdot BR_i$  analysis

## $K_i$ = number of signal + background events common to had Z recoil and $\sigma \cdot BR_i$ analyses

E = number of signal + background events unique to had Z recoil analysis  $\varepsilon_i$  = number of signal + background events events unique to  $\sigma \cdot BR_i$  analysis

$$\Omega = E + \sum_{i} K_{i} \qquad S \equiv \Omega - B \qquad T \equiv \frac{\sqrt{S + B}}{S}$$
$$\omega_{i} = K_{i} + \varepsilon_{i} \qquad S_{i} \equiv \omega_{i} - \beta_{i} \qquad \tau_{i} \equiv \frac{\sqrt{S_{i} + \beta_{i}}}{S_{i}}$$

 $\lambda_{i} \equiv \frac{K_{i}}{\omega_{i}} \qquad \qquad N \equiv L \sigma_{ZH} \qquad r_{i} \equiv BR_{i} \qquad \delta_{i} \equiv \xi_{i} - \Xi$ 

$$(\Delta \Psi)^{2} = \left(\frac{\partial \Psi}{\partial \Omega}\right)^{2} V_{\Omega\Omega} + \left(\frac{\partial \Psi}{\partial \Xi}\right)^{2} V_{\Xi} + 2\frac{\partial \Psi}{\partial \Omega}\frac{\partial \Psi}{\partial \Xi} V_{\Omega\Xi}$$
$$\frac{\partial \Psi}{\partial \Omega} = \frac{1}{L\Xi} = \frac{\Psi}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-1} \qquad \qquad \frac{\partial \Psi}{\partial \Xi} = -\frac{\Omega - B}{L\Xi^{2}} = -\frac{\Psi}{\Xi}$$

$$V_{\Omega\Omega} = E + \sum_{i} K_{i} = \Omega$$
$$V_{\Xi\Xi} = \frac{1}{L^{2} \Psi^{2}} \sum_{i} \frac{(\xi_{i} - \Xi)^{2}}{(\eta_{i})^{2}} (\varepsilon_{i} + K_{i})$$
$$V_{\Omega\Xi} = \frac{1}{L \Psi} \sum_{i} \frac{\xi_{i} - \Xi}{\eta_{i}} K_{i}$$

$$\begin{split} \left(\frac{\Delta\Psi}{\Psi}\right)^2 &= \frac{1}{\Omega^2} \left(1 - \frac{B}{\Omega}\right)^{-2} V_{\alpha\alpha} + \frac{1}{\Xi^2} V_{\Xi\Xi} - \frac{2}{\Omega\Xi} \left(1 - \frac{B}{\Omega}\right)^{-1} V_{\alpha\Xi} \\ &= \frac{1}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-2} + \frac{1}{L^2 \Xi^2 \Psi^2} \sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2} (\varepsilon_i + K_i) - \frac{2}{L\Omega\Xi\Psi} \left(1 - \frac{B}{\Omega}\right)^{-1} \sum_i \frac{\xi_i - \Xi}{\eta_i} K_i \\ &= \frac{1}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-2} + \frac{1}{L^2 \Xi^2 \Psi^2} \sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2} (L\eta_i \psi_i + \beta_i) - \frac{2}{L\Omega\Xi\Psi} \left(1 - \frac{B}{\Omega}\right)^{-1} \sum_i \frac{\xi_i - \Xi}{\eta_i} \lambda_i (L\eta_i \psi_i + \beta_i) \\ &= \frac{1}{\Omega} \left(1 - \frac{B}{\Omega}\right)^{-2} \left[1 + \frac{L}{\Omega} \sum_i \frac{(\xi_i - \Xi)^2}{\eta_i} \psi_i \left(1 + \frac{\beta_i}{s_i}\right) - \frac{2L}{\Omega} \sum_i (\xi_i - \Xi) \psi_i \lambda_i \left(1 + \frac{\beta_i}{s_i}\right)\right] \\ &= \frac{S + B}{S^2} \left\{1 + \frac{L}{\Omega} \sum_i (\xi_i - \Xi) \psi_i \left(\frac{s_i + \beta_i}{s_i^2}\right) \left[(\xi_i - \Xi) L\psi_i - 2\lambda_i s_i\right]\right\} \end{split}$$

$$= \mathrm{T}^{2} \left\{ 1 + \frac{N^{2}}{\Omega} \sum_{i} r_{i}^{2} \tau_{i}^{2} \left[ \delta_{i}^{2} - 2\lambda_{i} \eta_{i} \delta_{i} \right] \right\}$$

What if we don't do a hadronic Z recoil measurement and instead only use  $\sigma(ZH) \cdot BR_i$  to calculate  $\sigma(ZH) \cdot BR(visible) = \sum_i \sigma(ZH) \cdot BR_i$ ?

$$\Psi' = \sum_{i} \psi_{i} \qquad \qquad \psi_{i} = \frac{\omega_{i} - \beta_{i}}{L \xi_{i}}$$
$$(\Delta \Psi')^{2} = \sum_{i} \left(\frac{\partial \Psi'}{\partial \omega_{i}}\right)^{2} \omega_{i} , \qquad \frac{\partial \Psi'}{\partial \omega_{i}} = \frac{1}{L \eta'_{i}}$$
$$(\Delta \Psi')^{2} = \frac{1}{L^{2}} \sum_{i} = \frac{1}{L^{2}} \sum_{i} \frac{s_{i} + \beta_{i}}{\xi_{i}^{2}}$$

$$\left(\frac{\Delta \Psi'}{\Psi'}\right)^2 = \left(\sum_i \frac{\omega_i - \beta_i}{L\xi_i}\right)^{-2} \frac{1}{L^2} \sum_i \frac{s_i + \beta_i}{\xi_i^2}$$
$$= \frac{S + B}{S^2} \frac{L}{\Omega} \Xi^2 \sum_i \frac{\psi_i}{\xi_i} \left(1 + \frac{\beta_i}{s_i}\right)$$

Compare this now with our formula for  $\left(\frac{\Delta\Psi}{\Psi}\right)^2$  for  $\lambda_i = 1$ :

$$\left(\frac{\Delta\Psi}{\Psi}\right)^{2} = \frac{S+B}{S^{2}} \left\{ 1 + \frac{1}{\Omega} \sum_{i} \omega_{i} \left[ \left(1 - \frac{\Xi}{\xi_{i}}\right)^{2} - 2\left(1 - \frac{\Xi}{\xi_{i}}\right) \right] \right\}$$
$$= \frac{S+B}{S^{2}} \left\{ 1 + \frac{1}{\Omega} \sum_{i} \omega_{i} \left[ 1 - \frac{2\Xi}{\xi_{i}} + \left(\frac{\Xi}{\xi_{i}}\right)^{2} - 2 + 2\frac{\Xi}{\xi_{i}} \right] \right\} = \left(\frac{\Delta\Psi'}{\Psi'}\right)^{2}$$