$D^0-\overline{D^0}$ mixing and other charm decays at Belle*

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Abstract The $D^0-\overline{D}^0$ mixing in different decays with corresponding method is measured in this paper, there is a clear evidence for non-zero y D^0 mixing parameter, and measurement of D^0 mixing parameter x is still a challenge. CP violation in the decays is not observed. Branching fractions of other charm decays are presented.

Key words $D^0-\overline{D}^0$ mixing, CP violation, charm decays, CP eigenstates

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1 Introduction

The B factory KEKB is located at the High Energy Accelerator Research Organization (KEK) in Japan, it is an energy-asymmetric e^+e^- collider running at the $\Upsilon(4S)$ resonance. The data is accumulated by the Belle detector, it is a large-solid-angle magnetic spectrometer that consists of subdetectors with better performance: a silicon vertex detector(SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and a electromagnetic calorimeter (ECL) comprised of CSI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. It's the B factory with the largest accumulated luminosity in the world. For the reaction $\Upsilon(4S)\to b\overline{b}$, its cross section is 1.05 nb. At the same energy the $e^+e^-\to q\overline{q}$ (q=u, d, s, or c) continuum process has a cross section of 3.7 nb, containing $c\overline{c}\sim 1.30$ nb. So, it also provides a largest sample of charm hadrons beside B meson.

Mixing phenomena, the oscillation of a neutral meson into its corresponding anti-meson as a function of time, has been observed in the K^0 and B^0 systems. It can be described by the difference in mass (ΔM) and lifetime ($\Delta\Gamma$) of the two charge-parity (CP) eigenstates in the neutral meson system. The mixing in the D^0 system with Standard Model (SM) prediction is substantially smaller than that for the K^0 and B^0. It is consequence that $D^0-\overline{D^0}$ mixing is dominated by light quark intermediate states, a feature unique to the D^0 system among the neutral mesons. Moreover, the small Cabibbo factor $\tan^2\theta_C$ and the Glashow-Illiopolous-Maiani(GIM) cancellatin will further suppress the D^0 mixing rate.

The time evolution of the D^0-$\overline{D}^0$ system is described by the Schrödinger equation

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t)\end{pmatrix} = \left(\boldsymbol{M}-\frac{\mathrm{i}}{2}\boldsymbol{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t)\end{pmatrix},$$

where the $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ matrices are Hermitian, and CPT invariance requires $M_{11}=M_{22}\equiv M$ and $\Gamma_{11}=\Gamma_{22}\equiv\Gamma$. The off-diagonal elements of these matrices describe the dispersive (or long distance) and absorptive (or short-distance) contributions to D^0-$\overline{D}^0$ mixing.

The two eigenstates D_1 and D_2 of the effective Hamiltonian matrix $\left(\boldsymbol{M}-\frac{\mathrm{i}}{2}\boldsymbol{\Gamma}\right)$ are given by

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle, \quad p^2+q^2=1.$$

the corresponding eigenvalues are

$$\lambda_{1,2}\equiv m_{1,2}-\frac{\mathrm{i}}{2}\Gamma_{1,2}=\left(M-\frac{\mathrm{i}}{2}\Gamma\right)\pm\frac{q}{p}\left(M_{12}-\frac{\mathrm{i}}{2}\Gamma_{12}\right),$$

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where $m_{1,2}$, $\Gamma_{1,2}$ are the masses and decay widths and

$$\frac{q}{p}=\sqrt{\frac{M_{12}^*-\frac{\mathrm{i}}{2}\Gamma_{12}^*}{M_{12}-\frac{\mathrm{i}}{2}\Gamma_{12}}}.$$

the proper time evolution of the eigenstates is

$$|D_{1,2}(t)\rangle=e_{1,2}(t)|D_{1,2}\rangle,\quad e_{1,2}(t)=\mathrm{e}^{-\mathrm{i}(m_{1,2}-(\mathrm{i}\Gamma_{1,2}/2))t}.$$

A state that is prepared as a flavor eigenstate $|\mathrm{D}^0\rangle$ or $|\overline{\mathrm{D}}^0\rangle$ at $t=0$ will evolve according to

$$\begin{aligned}|\mathrm{D}^0(t)\rangle &= \frac{1}{2p}[p(e_1(t)+e_2(t))|\mathrm{D}^0\rangle+\\ &\quad q(e_1(t)-e_2(t))|\overline{\mathrm{D}}^0\rangle],\\ |\overline{\mathrm{D}}^0(t)\rangle &= \frac{1}{2q}[p(e_1(t)-e_2(t))|\mathrm{D}^0\rangle+\\ &\quad q(e_1(t)+e_2(t))|\overline{\mathrm{D}}^0\rangle].\end{aligned}\tag{1}$$

The time-dependent terms are given explicitly by

$$\begin{aligned}|e_{1,2}(t)|^2 &= \exp(2\Im(\lambda_{1,2})t)=\exp(-\Gamma_{1,2}t)\\ &= \exp(-\overline{\Gamma}(1\pm y)t),\\ e_1(t)e_2^*(t) &= \exp(-\mathrm{i}\lambda_1 t)\exp(+\mathrm{i}\lambda_2 t)\\ &= \exp(-\overline{\Gamma}(1+\mathrm{i}x)t).\end{aligned}\tag{2}$$

where

$$\overline{\Gamma}=\frac{\Gamma_1+\Gamma_2}{2},\quad x=\frac{m_2-m_1}{\overline{\Gamma}}=\frac{\Delta\mathrm{M}}{\overline{\Gamma}},$$
$$y=\frac{\Gamma_2-\Gamma_1}{2\overline{\Gamma}}=\frac{\Delta\Gamma}{2\overline{\Gamma}}.$$

Therefore, D^0 mixing is characterized by the two dimensionless mixing parameters x and y. Calculations within the SM typically estimates $|x|,|y|\lesssim 10^{-3}$ [1]. $SU(3)$ flavor-symmetry breaking and long-distance effects in SM may raise both parameters to $\sim 10^{-2}$ [2], however, it is difficult to calculate it. New physics effects may enhance x, but are not expected to y, just like top quark enhances the rate of B^0 mixing. Consequently, a discovery of $|x|\gg|y|$ implies New Physics.

CPV of charm decays in the SM is strongly suppressed by CKM(Cabiboo-Kobayashi-Maskawa), negligible. SM predictions for direct CPV in SCS decays are at most of the order of 10^{-3}. So, observation of large $\mathrm{O}(10^{-2})$ CPV in charm-decays would be a sign of new physics, similar as other FCNC (Flavor Changing Neutral Current) processes. The classification of CP-violating effects is the following:

$$A_{\mathrm{CP}}=\frac{\Gamma(\mathrm{D}\to\mathrm{f})-\Gamma(\overline{\mathrm{D}}\to\overline{\mathrm{f}})}{\Gamma(\mathrm{D}\to\mathrm{f})+\Gamma(\overline{\mathrm{D}}\to\overline{\mathrm{f}})}=a_{\mathrm{f}}^d+a_{\mathrm{f}}^m+a_{\mathrm{f}}^i,\tag{3}$$

where

a_{f}^d is CP violation in decay:

$$\left|\frac{A_{\mathrm{f}}}{\overline{A}_{\overline{\mathrm{f}}}}\right|\equiv 1+\frac{A_{\mathrm{D}}}{2}(A_{\mathrm{D}}\neq 0),$$

a_{f}^m is CP violation in mixing:

$$\left|\frac{q}{p}\right|\equiv 1+\frac{A_{\mathrm{M}}}{2}(A_{\mathrm{M}}\neq 0),$$

a_{f}^i is CP violation in interference ($f=f$):

$$\phi\equiv arg\left(\frac{q}{p}\frac{\overline{A}_{\mathrm{f}}}{A_{\mathrm{f}}}\right)(\phi\neq 0).$$

2 D^0 mixing measurement

2.1 D^0 WS hadronic decay $\mathrm{D}^0\to\mathrm{K}^+\pi^-$

The “wrong-sign”(WS) process, $\mathrm{D}^0\to\mathrm{K}^+\pi^-$, can proceed either through direct doubly-Cabibbo-suppressed (DCS) decay or through mixing followed by the “right-sign” (RS) Cabibbo-favored(CF) decay $\mathrm{D}^0\to\overline{\mathrm{D}}^0\to\mathrm{K}^+\pi^-$. The two decays can be distinguish by the decay-time distribution. The differential WS rate relative to RS process is the following:

$$\begin{aligned}r_{\mathrm{WS}}(t) &= \frac{\Gamma(\mathrm{D}^0(\mathrm{t})\to\mathrm{K}^+\pi^-)}{\Gamma(\overline{\mathrm{D}^0}\to\mathrm{K}^+\pi^-)}\\ &= [R_{\mathrm{D}}+\sqrt{R_{\mathrm{D}}}y'\overline{\Gamma}t+\frac{1}{4}(x'^2+y'^2)\overline{\Gamma}^2t^2]\mathrm{e}^{-\overline{\Gamma}t},\end{aligned}\tag{4}$$

where R_{D} is DCS/CF rate, δ is a strong phase between DCS and CF,

$$\begin{aligned}x' &= x\cos\delta+y\sin\delta,\\ y' &= y\cos\delta-x\sin\delta.\end{aligned}\tag{5}$$

The first term in brackets is due to the DCS(CF) amplitude, the middle term is due to interference between the two processes, the last term is due to mixing. The relative mixing and DCS decay are

$$R_{\mathrm{M}}\equiv\frac{1}{2}(x^2+y^2)\equiv\frac{1}{2}(x'^2+y'^2)$$

and R_{D}.

The D^0 must be from D^*, $\mathrm{D}^{*+}\to\mathrm{D}^0\pi_{\mathrm{s}}^+$, the flavor tagging is used with slow π's charge. The backgrounds are suppressed with $Q=M_{\mathrm{D}^*}-M_{\mathrm{D}^0}-M_\pi$. The D^0 lifetime is measured using the 3-dimensional distance between its decay and production vertex. The decay vertex of D^0 is obtained by vertex fit to the two daughters K, π, and the D^0 production vertex is got by extrapolating the D^0 trajectory back to the IP profile.

If $(R_D^+,\ x'^{+2},\ y'^{+})$ is for D^0 and $(R_D^-,\ x'^{-2},\ y'^{-})$ for $\overline{D}^0$, then, CPV in decay is

$$A_D = \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-},$$

CPV in mixing is

$$A_M = \frac{R_M^+ - R_M^-}{R_M^+ + R_M^-}.$$

The analysis results [3] are shown in the Table 1 and 2. R_D without D^0 mixing is $3.77\pm0.08(\text{stat.})\pm0.05(\text{syst.})$. The results show that CPV is not observed and the no-mixing point $x'^2 = y' = 0$ corresponds to a 1-C.L of 3.9%.

Table 1. D^0–$\overline{D}^0$ mixing measurement with WS $D^0 \to K^+\pi^-$, No CPV.

Parameter	Fit result	95% C.L interval
$R_D(\times10^{-3})$	3.64 ± 0.17	(3.3, 4.0)
$x'^2(\times10^{-3})$	$0.18^{+0.21}_{-0.23}$	<0.72
$y'(\times10^{-2})$	$0.06^{+0.40}_{-0.39}$	(-0.99, 0.68)
$R_M(\times10^{-3})$	—	$(0.63\times10^{-5}, 0.40)$

Table 2. D^0–$\overline{D}^0$ mixing measurement with WS $D^0 \to K^+\pi^-$, CPV is allowed.

Parameter	Fit result	95% C.L interval
$x'^2(\times10^{-3})$	—	<0.72
$y'(\times10^{-2})$	—	$(-2.8, 2.1)$
$R_M(\times10^{-3})$	—	<0.40
A_D	0.023 ± 0.047	$(-0.076, 0.107)$
A_M	0.67 ± 1.20	$(-0.995, 1.0)$
$\lvert\phi\rvert(^\circ)$	$9.4(84.5)\pm25.3$	No limits

2.2 CP eigenstates $D^0 \to K^+K^-,\ \pi^+\pi^-$

The decay rates to CP eigenstates $D^0 \to K^+K^-$, $\pi^+\pi^-$ are sensitive to y, the measurement of lifetime difference between $D^0 \to K^-\pi^+$ (CP-mixed) and $D^0 \to K^+K^-, \pi^+\pi^-$ (CP-even) decay is used to obtain the mixing parameter y.

$$\Gamma(D^0,\ \overline{D}^0 \to K^{-,+}\pi^{+,-}) \propto e^{-t/\tau_{D^0}},$$

$$\Gamma(D^0,\ \overline{D}^0 \to K^{-,+}\pi^{+,-}) \propto e^{-(1+y_{CP})t/\tau_{D^0}}, \quad (6)$$

then,

$$y_{CP} \equiv \frac{\tau_{K^\mp,\pi^\pm}}{\tau_{K^+K^-,\ \pi^+\pi^-}} - 1 =$$

$$\frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) x\sin\phi. \quad (7)$$

In limit of no CPV, $y_{CP} = y$. moreover, the decays have the high signal purity ($>90\%$). The CP violation in this decay is the following:

$$A_\Gamma = \frac{\tau(\overline{D}^0 \to f_{CP}) - \tau(D^0 \to f_{CP})}{\tau(\overline{D}^0 \to f_{CP}) + \tau(D^0 \to f_{CP})} =$$

$$\frac{1}{2}\left(|\frac{q}{p}| - |\frac{p}{q}|\right) y\cos\phi - \frac{1}{2}\left(|\frac{q}{p}| + |\frac{p}{q}|\right) x\sin\phi. \quad (8)$$

The proper decay time distribution is fitted using the following formula:

$$\frac{dN}{dt} \propto \int e^{-t'/\tau} \cdot R(t-t')dt' + B(t), \quad (9)$$

where

$$R(t-t') = \sum_i^N f_i \sum_{k=1}^3 w_k G(t-t', \sigma_{ik}, t_0).$$

The results [4] are the following:

$$A_\Gamma = (0.01\pm0.30\pm0.15)\%,$$
$$y_{CP} = (1.31\pm0.32\pm0.25)\%,$$

therefore, CPV in the decays is not observed, the significance of $D^0-\overline{D}^0$ mixing is 3.2σ, it is the first evidence of D^0 mixing.

2.3 D^0 mixing in the decay $D^0 \to K_S^0\phi$

The measurement of lifetime difference between CP-even and CP-odd eigenstates is used to obtain the mixing parameter y_{CP}. For this decay $D^0 \to K_S^0\phi$, $\phi \to K^+K^-$, the $\sqrt{s_0} = m_{K^+K^-}$ is dependent CP mixture, the ON region is mainly CP-odd ($\phi(1020)$) and the OFF region mainly CP-even($a_0(980)^0$).

$$\frac{d^2N(s_0,t)}{ds_0dt} \propto a_1(s_0)e^{-(1+y_{CP})t/\tau_{D^0}}(\text{CP}-\text{even}) +$$

$$a_2(s_0)e^{-(1-y_{CP})t/\tau_{D^0}}(\text{CP}-\text{odd}), \quad (10)$$

then, effective lifetimes in ON and OFF regions are the following:

$$\tau_{\rm ON} = [1+(1-2f_{\rm ON})y_{\rm CP}]\tau_{\rm D^0},$$

$$\tau_{\rm OFF} = [1+(1-2f_{\rm OFF})y_{\rm CP}]\tau_{\rm D^0},$$

$$y_{\rm CP} = \frac{1}{f_{\rm ON}-f_{\rm OFF}}\left(\frac{\tau_{\rm OFF}-\tau_{\rm ON}}{\tau_{\rm OFF}+\tau_{\rm ON}}\right), \qquad (11)$$

where, $f_{\rm ON}$, $f_{\rm OFF}$ are CP-even fractions in ON and OFF regions. In order to reduce effects of resolution function, there are topologically equal events in ON and OFF regions, they all have high purity (Table 3).

Table 3. Untagged sample used to increase the statistics.

Region	ON	OFF
signal ($\times 10^3$)	72	62
Purity	97%	91%

The background is estimated from sidebands in ($m_{\rm K^0_S K^+K^-}$, $m_{\rm K^0_S}$) plane, $f_{\rm ON}$ and $f_{\rm OFF}$ are from fit to $m_{\rm K^+K^-}$ using 8-resonance Dalitz model, $\tau_{\rm ON}$ and $\tau_{\rm OFF}$ determined from mean proper decay times of all events and background events using the following equation:

$$\tau_{\rm ON,OFF}+t_0 = \frac{\langle t\rangle_{\rm ON,OFF}-(1-p_{\rm ON,OFF})\langle t\rangle_{\rm b}^{\rm ON,OFF}}{p_{\rm ON,OFF}}. \qquad (12)$$

Therefore, we get [5]:

$$y_{\rm CP} = +(0.11\pm 0.61({\rm stat.})\pm 0.52({\rm syst.}))\%,$$

it is consistent with above result (section 2.2).

2.4 $\mathrm{D^0}$ mixing in the decay $\mathrm{D^0\to K^0_S\pi^+\pi-}$

By measuring the time evolution of the Dalitz plot one can measure, or constrain the values of the standard mixing parameters x and y. This method enables the measurement of x and y separately. $\mathrm{D^0}$ decays to final states:

$$\langle s|H|{\rm D^0}(t)\rangle = e_1(t)A_1+e_2(t)A_2 = M,$$

$$\langle \overline{s}|H|\overline{{\rm D^0}(t)}\rangle = e_1(t)\overline{A_1}+e_2(t)\overline{A_2} = \overline{M}, \qquad (13)$$

where

$$\mathrm{D^0}: A(m_-^2,m_+^2) = \sum_r a_r {\rm e}^{{\rm i}\phi_r} A_r(m_-^2,m_+^2)+a_{nr}{\rm e}^{{\rm i}\phi_{nr}}$$

$$\overline{\mathrm{D}}^0: \overline{A}(m_-^2,m_+^2) = \sum_r \overline{a}_r {\rm e}^{{\rm i}\phi_r} \overline{A}_r(m_-^2,m_+^2)+a_{nr}{\rm e}^{{\rm i}\phi_{nr}},$$

therefore, the decay rate of $\mathrm{D^0}$ is a function of time, including mixing parameters x and y, where t is in unit of $\mathrm{D^0}$ lifetime.

$$|\mathcal{M}|^2 = |e_1(t)|^2|A_1|^2+|e_2(t)|^2|A_2|^2+ 2\mathcal{R}[e_1(t)e_2^*(t)A_1A_2^*],$$

$$|\mathcal{M}|^2 = |A_1|^2{\rm e}^{-yt}+|A_2|^2{\rm e}^{yt}+2\mathcal{R}[A_1A_2^*]\cos(xt)+ 2\mathcal{I}[A_1A_2^*]\sin(xt){\rm e}^{-t},$$

$$|\overline{\mathcal{M}}|^2 = |\overline{A}_1|^2{\rm e}^{-yt}+|\overline{A}_2|^2{\rm e}^{yt}+2\mathcal{R}[\overline{A}_1\overline{A}_2^*]\cos(xt)+ 2\mathcal{I}[\overline{A}_1\overline{A}_2^*]\sin(xt){\rm e}^{-t}. \qquad (14)$$

So, we can get the $\mathrm{D^0-\overline{D}^0}$ mixing parameter x and y. As CP symmetry is conserved ($|q/p|=1$ and $\phi=0$), the results [6] are the following:

$$x = (0.80\pm 0.29^{+0.13}_{-0.16})\%,$$

$$y = (0.33\pm 0.24^{+0.10}_{-0.14})\%.$$

When CPV is allowed, we get:

$$|q/p| = 0.86\pm 0.30\pm 0.09,$$

$$\phi = -0.24\pm 0.30\pm 0.09.$$

Therefore, the significance of $\mathrm{D^0}$ mixing is 2.2 σ for x, the current best measurement of x, CPV is not observed.

3 Other charm decay

3.1 search for $D^0\to l^+l^-$

FCNC does not appear in SM on tree level (higher order below allowed), different models give their predictions (Table 4), Certain new physics scenarios allows this process: new particle replacing W boson.

Table 4. Different decay model prediction.

Model	Branch fraction($\mathrm{D^0\to\mu^+\mu^-}$)
Experiment (CDF preliminary)	$\leq 4.3\times10^{-7}$
Standard model (SD)	$\sim 10^{-18}$
Standard model (LD)	$\sim$ several$\times10^{-13}$
$Q=+2/3$ Vector-like Singlet	4.3×10^{-11}
$Q=-1/3$ Vector-like Singlet	$1\times10^{-11}(m_{\rm S}/500\ {\rm GeV})^2$
$Q=-1/3$ Fourth Family	$1\times10^{-11}(m_{\rm S}/500\ {\rm GeV})^2$
Z' Standard model (LD)	$2.3\times10^{-12}/(M_{Z'}\,({\rm TeV}))^2$
Family Symmetry	0.7×10^{-18}
RPV-SUSY	$4.8\times10^{-9}(300\ {\rm GeV}/m_{\rm d})^2$

Except Family symmetry, all NP exceed the SM prediction. Belle has the largest data, most sensitive to RPV-SUSY scenario.

There are two kinds of background(Fig. 1, the combinatorial background is described using 2D estimation with $a(1-bm)/\sqrt{q}$. The ratio of combinatorial background in the signal to the number in the sideband is the following (Table 5):

Table 5. Combinatorial background.

Channel	p(%)
(a) $\mu^+\mu^-$	1.08
(b) e^+e^-	1.49
(c) $e^\pm\mu^\mp$	1.43

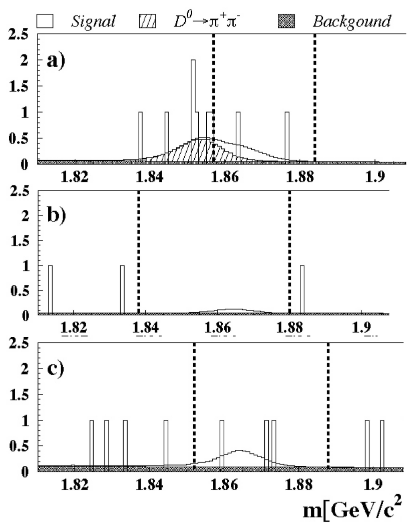

Fig. 1. Estimation of background.

The reflection background from $D^0\to\pi^+\pi^-$, the number of reflection in the signal window is in Table 6.

Table 6. Number of reflection in signal window.

Channel	$N_{\rm refl}^{\rm Data}$
(a) $\mu^+\mu^-$	1.81 ± 0.002
(b) e^+e^-	0.0372 ± 0.0002
(c) $e^\pm\mu^\mp$	0.1935 ± 0.0006

Table 7. Events in signal window.

Channel	events	background
(a) $\mu^+\mu^-$	2	3.1 ± 0.1
(b) e^+e^-	0	1.7 ± 0.2
(c) $e^\pm\mu^\mp$	3	2.6 ± 0.2

Therefore, the result is obtained using event counting at the signal window(Table 7). The new best upper limits(95% C.L, preliminary) for leptonic decay of D^0 are obtained:

$$B(D^0\to\mu^+\mu^-)<1.4\times10^{-7},$$

$$B(D^0\to e^+e^-)<7.9\times10^{-8},$$

$$B(D^0\to e^\pm\mu^\mp)<2.6\times10^{-7}.$$

3.2 Study of $D^+_{(s)}\to K_sh^+$

In order to look for ratios of CS to CF in $D^+_{(s)}$ decays, the yields are measured, shown in the Table 8.

Table 8. Events of different mode in the decay $D^+_{(s)}\to K_sh^+$.

Modes	Yield
$D^+\to K_SK^+$	100855 ± 561
$D_s^+\to K_SK^+$	204093 ± 768
$D^+\to K_S\pi^+$	566105 ± 1159
$D_s^+\to K_S\pi^+$	16817 ± 448

Therefore, we get the new best measurements, shown in the Table 9.

Table 9. Ratios of CS to CF in $D^+_{(s)}\to K_sh^+$ decays.

Mode	PDG2008
$B(D^+\to K_SK^+)/B(D^+\to K_S\pi^+)$	$0.189\pm0.016\pm0.007$
$B(D_s^+\to K_S\pi^+)/B(D_s^+\to K_SK^+)$	$0.082\pm0.009\pm0.002$
Mode	**Belle 2009**
$B(D^+\to K_SK^+)/B(D^+\to K_S\pi^+)$	$0.190\pm0.001\pm0.002$
$B(D_s^+\to K_S\pi^+)/B(D_s^+\to K_SK^+)$	$0.077\pm0.002\pm0.002$

3.3 Observation of $D_s^+\to K^+K^+\pi^-$

The decay $D_s^+\to K^+K^+\pi^-$ has not observed yet, moreover, one can look at the double ratio to test $SU(3)$ flavor symmetry.

$$\frac{B(D_s^+\to K^+K^+\pi^-)}{B(D_s^+\to K^+K^-\pi^+)}\frac{B(D^+\to K^+\pi^+\pi^-)}{B(D^+\to K^-\pi^+\pi^-)}=\tan^8\theta_c, \quad (15)$$

here, differences in the phase space cancel in the ratios and $SU(3)$ breaking effects due to resonant intermediate states in the 3-body will violate the equation above.

Finally, we get the double ratios:

$$\frac{B(D_s^+\to K^+K^+\pi^-)}{B(D_s^+\to K^+K^-\pi^+)}\frac{B(D^+\to K^+\pi^+\pi^-)}{B(D^+\to K^-\pi^+\pi^+)}=$$
$$\tan^8\theta_c\cdot(1.57\pm0.21), \quad (16)$$

it is OK, the $SU(3)$ breaking effect is not observed. The branching fraction is shown in the Table 10. It is the first observation of this decay with 9.1σ [7].

Table 10. Branching fraction.

Branching fraction($\times 10^{-4}$)	Belle	World average
$B(D^+ \to K^+\pi^+\pi^-)$	$5.2\pm0.2\pm0.1$	6.2 ± 0.7
$B(D_s^+ \to K^+K^+\pi^-)$	$1.3\pm0.2\pm0.1$	2.9 ± 1.1

4 Conclusions

From the above study with Belle's data, there are some results as the following:

- It seems that there is a clear evidence for non-zero y (D^0 mixing parameter);
- The measurement of D^0 mixing parameter x is still a challenge, it needs more studies;
- No evidence of CP violation in D^0 decays is observed;
- The best limits are achieved for leptonic decays of D^0 (preliminary);
- The most precise branch ratios of $D^+_{(s)} \to K_s h^+$ are obtained (preliminary);
- It is the first observation of DCSD $D_s^+ \to K^+K^+\pi^-$

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