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Hadronic light-by-light scattering contribution to the muon $g-2$*

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Abstract We review recent developments concerning the hadronic light-by-light scattering contribution to the anomalous magnetic moment of the muon. We first discuss why fully off-shell hadronic form factors should be used for the evaluation of this contribution to the $g-2$. We then reevaluate the numerically dominant pion-exchange contribution in the framework of large-N_C QCD, using an off-shell pion-photon-photon form factor which fulfills all QCD short-distance constraints, in particular, a new short-distance constraint on the off-shell form factor at the external vertex in $g-2$, which relates the form factor to the quark condensate magnetic susceptibility in QCD. Combined with available evaluations of the other contributions to hadronic light-by-light scattering this leads to the new result $a_\mu^{\mathrm{LbyL;had}} = (116 \pm 40) \times 10^{-11}$, with a conservative error estimate in view of the many still unsolved problems. Some potential ways for further improvements are briefly discussed as well. For the electron we obtain the new estimate $a_e^{\mathrm{LbyL;had}} = (3.9 \pm 1.3) \times 10^{-14}$.

Key words muon, anomalous magnetic moment, hadronic contributions, effective field theories, large-N_C

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1 Introduction

The muon g–2 has served over many decades as an important test of the Standard Model (SM). It is also sensitive to contributions from New Physics slightly above the electroweak scale. For several years now a discrepancy of about three standard deviations has existed between the SM prediction and the experimental value, see the recent reviews Refs. [1–4] on the muon $g-2$. The main error in the theoretical SM prediction comes from hadronic contributions, i.e. hadronic vacuum polarization and hadronic light-by-light (had. LbyL) scattering. Whereas the hadronic vacuum polarization contribution can be related to the cross section $e^+e^- \to$ hadrons, no direct experimental information is available for had. LbyL scattering. One therefore has to rely on hadronic models to describe the strongly interacting, nonperturbative dynamics at the relevant scales from the muon mass up to about 2 GeV. This leads to large uncertainties, see Refs. [3, 5, 6] for recent reviews on had. LbyL scattering.

The still valid picture of had. LbyL scattering as proposed some time back in Ref. [7] is shown in Fig. 1.

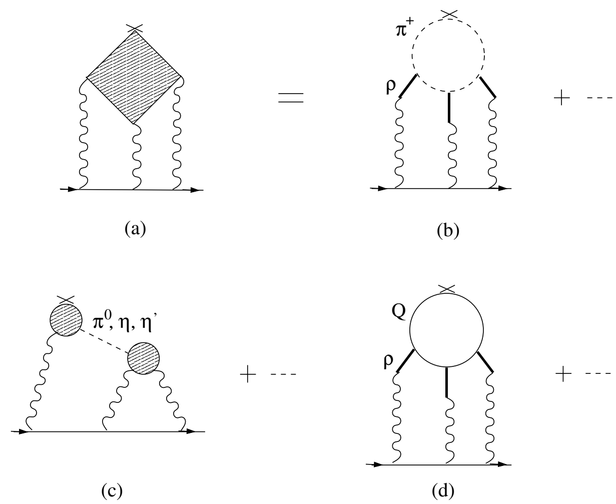

Fig. 1. The hadronic light-by-light scattering contribution to the muon $g-2$.

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There are three classes of contributions to the relevant hadronic four-point function $\langle VVVV\rangle$ [Fig. 1(a)] which can also be understood within an effective field theory approach to had. LbyL scattering: (1) a charged pion and Kaon loop [Fig. 1(b)], where the coupling to photons is dressed by some form factor (ρ-meson exchange, e.g. via vector meson dominance (VMD)), (2) pseudoscalar exchange diagrams [Fig. 1(c)] together with the exchanges of heavier resonances ($\mathrm{f}_0, \mathrm{a}_1, \ldots$) and, finally, (3) the irreducible part of the four-point function which was modeled in Ref. [7] and later works [8, 9] by a constituent quark loop dressed with VMD-type form factors [Fig. 1(d)]. The latter contribution can also be viewed as a short-distance complement of the employed low-energy hadronic models. According to quark-hadron duality, the (constituent) quark loop also models the contribution from the exchanges and loops of heavier resonances, like $\pi', \mathrm{a}_0', \mathrm{f}_0', \mathrm{p}, \mathrm{n}, \ldots$, if they are not explicitly included in the other terms.

One can try to reduce the model dependence and the corresponding uncertainties by relating the hadronic form factors at low energies to results from chiral perturbation theory and at high energies (short distances) to the operator product expansion. In this way, one connects the form factors to the underlying theory of QCD. This has been done in Refs. [8–13] for the numerically dominant contribution from the exchange of light pseudoscalars π^0, η and η'. In Ref. [11] also important short-distance constraints on the axial-vector pole contribution have been imposed.

2 On-shell versus off-shell form factors

It was pointed out recently in Ref. [2], that one should use fully off-shell form factors for the evaluation of the LbyL scattering contribution. This seems to have been overlooked in the recent literature, in particular, in Refs. [5, 6, 10–12]. The on-shell form factors as used in Refs. [10, 12] actually violate four-momentum conservation at the external vertex, as observed already in Ref. [11].

For illustration, we consider the contribution of the lightest intermediate state, the neutral pion. The key object which enters the diagram in Fig. 1(c) is the off-shell form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ which can be defined via the QCD Green's function $\langle \mathrm{VVP}\rangle$ [8, 9, 13]

$$\begin{aligned}&\int \mathrm{d}^4x\, \mathrm{d}^4y\, \mathrm{e}^{\mathrm{i}(q_1\cdot x + q_2\cdot y)}\, \langle 0|T\{j_\mu(x) j_\nu(y) P^3(0)\}|0\rangle = \\ &\left[\varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta\, \frac{\mathrm{i}\langle\overline{\psi}\psi\rangle}{F_\pi}\, \frac{\mathrm{i}}{(q_1+q_2)^2 - m_\pi^2} \times \right. \\ &\left. \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)\right] + \ldots, \end{aligned} \tag{1}$$

up to small mixing effects with the states η and η' and neglecting exchanges of heavier states like $\pi^{0\prime}, \pi^{0\prime\prime}, \ldots$. Here $j_\mu(x)$ is the light quark part of the electromagnetic current and $P^3(x) = \left(\overline{\psi}\mathrm{i}\gamma_5 \frac{\lambda^3}{2}\psi\right)(x)$. Note that for off-shell pions, instead of $P^3(x)$, we could use any other suitable interpolating field, like $\partial^\mu A_\mu^3(x)$ or even a fundamental pion field $\pi^3(x)$.

The corresponding contribution to the muon $g-2$ may be worked out with the result [10]

$$\begin{aligned} a_\mu^{\mathrm{LbyL};\pi^0} = & -e^6 \int \frac{\mathrm{d}^4q_1}{(2\pi)^4} \frac{\mathrm{d}^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m_\mu^2][(p-q_2)^2 - m_\mu^2]} \times \\ & \left[\frac{\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2)\ \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2, q_2^2, 0)}{q_2^2 - m_\pi^2}\ T_1(q_1, q_2; p) + \right. \\ & \left. \frac{\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)\ \mathcal{F}_{\pi^{0*}\gamma^*\gamma}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - m_\pi^2}\ T_2(q_1, q_2; p)\right], \end{aligned} \tag{2}$$

where the external photon has now zero four-momentum. See Ref. [10] for the expressions for T_i. Note that for general form factors a compact three-dimensional integral representation for $a_\mu^{\mathrm{LbyL};\pi^0}$ has been derived in Ref. [3].

Instead of the representation in Eq. (2), Refs. [10, 12] considered on-shell form factors which would yield the so called pion-pole contribution, e.g. for the term involving T_2, one would write

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2)\ \times\ \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, (q_1+q_2)^2, 0). \tag{3}$$

Although pole dominance might be expected to give a reasonable approximation, it is not correct as it was used in those references, as stressed in Refs. [2, 11]. The point is that the form factor sitting at the external photon vertex in the pole approximation

$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,(q_1+q_2)^2,0)$ for $(q_1+q_2)^2 \neq m_\pi^2$ violates four-momentum conservation, since the momentum of the external (soft) photon vanishes. The latter requires $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}((q_1+q_2)^2,(q_1+q_2)^2,0)$. Ref. [11] then proposed to use instead

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2,q_1^2,q_2^2) \times \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,m_\pi^2,0). \qquad (4)$$

Note that putting the pion on-shell at the external vertex automatically leads to a constant form factor, given by the Wess-Zumino-Witten (WZW) term [14]. However, this prescription does not yield the pion-exchange contribution with off-shell form factors, which we calculate with Eq. (2).

Strictly speaking, the identification of the pion-exchange contribution is only possible, if the pion is on-shell. If one is off the mass shell of the exchanged particle, it is not possible to separate different contributions to the $g-2$, unless one uses some particular model where elementary pions can propagate. In this sense, only the pion-pole contribution with on-shell form factors can be defined, at least in principle, in a model-independent way. On the other hand, the pion-pole contribution is only a part of the full result, since in general, e.g. using some resonance Lagrangian, the form factors will enter the calculation with off-shell momenta.

3 Pseudoscalar exchange contribution

After the observation in Ref. [2] that off-shell form factors should be used, the numerically dominant pion-exchange contribution was reanalyzed in detail in our paper [13]. First we derived a new QCD short-distance constraint on the off-shell pion-photon-photon form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}$ from Eq. (1) at the external vertex in had. LbyL scattering. It arises in the limit when the space-time argument of one of the vector currents in $\langle \mathrm{VVP}\rangle$ approaches the argument of the pseudoscalar density [15].

In the chiral limit, assuming octet symmetry and up to corrections of order α_s, one then obtains the relation [13]

$$\lim_{\lambda\to\infty} \mathcal{F}_{\pi^{0*}\gamma^*\gamma}((\lambda q_1)^2,(\lambda q_1)^2,0) = \frac{F_0}{3}\,\chi + \mathcal{O}\left(\frac{1}{\lambda}\right), \quad (5)$$

where F_0 is the pion-decay constant in the chiral limit and χ is the quark condensate magnetic susceptibility in QCD in the presence of a constant external electromagnetic field, introduced in Ref. [16]: $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_{\mathrm{F}} = e\,e_{\mathrm{q}}\,\chi\,\langle\overline{\psi}\psi\rangle_0\,F_{\mu\nu}$, with $e_{\mathrm{u}} = 2/3$ and $e_{\mathrm{d}} = -1/3$. Note that there is no falloff in Eq. (5) in this limit, unless χ vanishes.

Unfortunately there is no agreement in the literature what the actual value of χ should be. Note that χ actually depends on the renormalization scale μ. Most recent estimates yield values $\chi(\mu = 1~\mathrm{GeV}) \approx -3~\mathrm{GeV}^{-2}$ [17], although other approaches give a much larger absolute value of $\chi(\mu = 0.5~\mathrm{GeV}) \approx -9~\mathrm{GeV}^{-2}$ [16, 18]. While the running with μ can explain part of the difference, it seems likely that the different models used are not fully compatible.

In Ref. [13] we then reevaluated the pion-exchange contribution using an off-shell form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2,q_1^2,q_2^2)$ in the framework of large-N_{C} QCD. In the spirit of the minimal hadronic Ansatz [19] for Green's functions in large-N_{C} QCD, such a form factor had already been constructed in Ref. [15]. It generalizes the usual VMD form factor and contains the two lightest multiplets of vector resonances, the ρ and the ρ' (lowest meson dominance (LMD) +V). In contrast to the VMD ansatz, the LMD+V form factor fulfills all the relevant short-distance constraints derived in Refs. [11, 13, 15], including the new one from Eq. (5). In Ref. [13] we assumed that the LMD/LMD+V framework is self-consistent, therefore the estimate $\chi^{\mathrm{LMD}} = -2/M_{\mathrm{V}}^2 = -3.3~\mathrm{GeV}^{-2}$ was used (with a typical large-N_{C} uncertainty of 30%), which is compatible with other estimates [17].

Other model parameters are fixed by normalizing the form factor to the pion decay amplitude $\pi^0 \to \gamma\gamma$ and by reproducing experimental data [20] for the on-shell form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$, see Ref. [13] for all the details. Recently, BABAR [21] has published new data for this form factor which does not show the characteristic falloff for large Euclidean momentum, $\lim_{Q^2\to\infty} \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \sim 1/Q^2$ [22]. Some implications of this new experimental result have been been discussed in Ref. [23]. As shown in Ref. [24], using the BABAR data to fit some of the LMD+V model parameters, does, however, not change the final result given below.

Varying all the LMD+V model parameters in reasonable ranges and adding all uncertainties linearly to cover the full range of values obtained with the scan of parameters, we get the new estimate [13]

$$a_\mu^{\mathrm{LbyL};\pi^0} = (72 \pm 12) \times 10^{-11}. \qquad (6)$$

As far as the contribution to a_μ from the exchanges of the other light pseudoscalars η and η' is concerned, a simplified approach was adopted in Ref. [13], as was done earlier in other works [8–11]. We took a simple VMD form factor, normalized to the experimental decay width $\Gamma(\mathrm{PS} \to \gamma\gamma)$. In this way one obtains the results $a_\mu^{\mathrm{LbyL};\eta} = 14.5\times10^{-11}$ and $a_\mu^{\mathrm{LbyL};\eta'} = 12.5\times10^{-11}$. Adding up the contributions from all the light pseudoscalar exchanges, we obtain the new estimate [13]

$$a_\mu^{\mathrm{LbyL;PS}} = (99\pm16)\times10^{-11}, \quad (7)$$

where we have assumed a 16% error, as inferred above for the pion-exchange contribution.

For comparison, we have listed in Table 1 some evaluations of the pion- and pseudoscalar-exchange contribution to had. LbyL scattering by various groups. The model used by each group has also been indicated in the first column of the table, see the corresponding references for all the details (we used the abbreviations: ENJL = Extended Nambu-Jona-Lasinio model; HLS = Hidden Local Symmetry model; χQM = chiral quark model; FF = form factor; h_2 is one of the LMD+V model parameters).

Table 1. Results for the π^0,η and η' exchange contributions obtained by various groups.

model for $\mathcal{F}_{\mathrm{P}^{(*)}\gamma^*\gamma^*}$	$a_\mu^{\mathrm{LbyL};\pi^0}\times10^{11}$	$a_\mu^{\mathrm{LbyL;PS}}\times10^{11}$
point coupling	$+\infty$	$+\infty$
modified ENJL (off-shell) [BPP] [8]	59(9)	85(13)
VMD/HLS (off-shell) [HKS, HK] [9]	57(4)	83(6)
nonlocal χQM (off-shell) [DB] [25]	65(2)	–
AdS/QCD (off-shell ?) [HoK] [26]	69	107
LMD+V (on-shell, $h_2=0$) [KN] [10]	58(10)	83(12)
LMD+V (on-shell, $h_2=-10$ GeV^2) [KN] [10]	63(10)	88(12)
LMD+V (on-shell, constant FF at external vertex) [MV] [11]	77(7)	114(10)
LMD+V (off-shell) [N] [13]	72(12)	99(16)

Our results for the pion and the sum of all pseudoscalar exchanges are about 20% larger than the values in Refs. [8, 9] which used other hadronic models that presumably do not obey the new short-distance constraint from Eq. (5) and thus have a stronger damping at large momentum. Within the non-local χQM used in Ref. [25] there is a strong, exponential suppression for large pion virtualities. According to Ref. [26], the estimate with the AdS/QCD model has an error of at most 30%. On the other hand, our result is smaller than the pion- and pseudoscalar-pole contribution calculated in Ref. [11]. Since only the *pion-pole* contribution is considered in Ref. [11], their short-distance constraint cannot be directly applied to our approach. However, our ansatz for the pion-exchange contribution agrees qualitatively with the short-distance behavior of the quark-loop derived in Ref. [11], see the discussion in Refs. [3, 13]. Note, however, that the numerical value for the pion-pole contribution listed as [MV] in Table 1 should rather be 80×10^{-11}, see Refs. [5, 13, 25].

4 Summary of other contributions

In Table 2 we have collected the results for all the contributions to had. LbyL scattering according to Fig. 1 obtained by various groups in recent times, including some "guesstimates" for the total value. In the following, we highlight the main features of the numbers given and point out some critical issues regarding each contribution. A more detailed discussion can be found in Ref. [3].

Table 2. Summary of the most recent results for the various contributions to $a_\mu^{\mathrm{LbyL;had}}\times10^{11}$. The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0,η,η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	–	22±5	–	15±10	22±5
scalars	−6.8±2.0	–	–	–	–	−7±7	−7±2
π,K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π,K loops +subl. N_C	–	–	–	0±10	–	–	–
quark loops	21±3	9.7±11.1	–	–	–	2.3	21±3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

As one can see from Table 2, the different models used by various groups lead to slightly different results for the individual contributions. The final result is dominated by the pseudoscalar exchange contribution, which is leading in large-$N_{\rm C}$, but subleading in the chiral counting. The other contributions are smaller, but not negligible. Furthermore, they cancel out to some extent, in particular the dressed pion and Kaon loops and the dressed quark loops.

In Ref. [11], new QCD short-distance constraints were derived for the axial-vector pole contribution with on-shell form factors $\mathcal{F}_{{\rm A}\gamma^*\gamma^*}$ at both vertices. A huge enhancement of a factor of ten was observed compared to the earlier estimates in Refs. [8, 9] which assumed nonet symmetry for the states $\rm a_1, f_1$ and $\rm f_1'$. It was shown that the result is very sensitive to the mass of the exchanged axial-vector resonance. Since the form factors include light vector mesons like the ρ, this leads to a smaller effective mass of the exchanged resonance, compared to $M_{\rm A} \sim 1300$ MeV. The result is also sensitive to the mixing of the states f_1 and $\rm f_1'$. The result given in Table 2 corresponds to ideal mixing. If $\rm f_1$ is a pure octet state and f_1' a pure singlet, the final result goes down to $a_\mu^{\rm LbyL;a_1,f_1,f_1'} = 17 \times 10^{-11}$. The procedure adopted in Ref. [11] is an important improvement over Refs. [8, 9] and we have therefore taken the result for the axial-vectors from that reference for our final estimate for the full had. LbyL scattering contribution. This despite the fact that only on-shell form factors have been used in Ref. [11]. As we argued above, we think that one should use consistently off-shell form factors at the internal and the external vertex.

Within the ENJL model used in Ref. [8], the scalar exchange contribution is related via Ward identities to the constituent quark loop. In fact, Ref. [9] argued that the effect of the exchange of scalar resonances below several hundred MeV might already be included in the sum of the (dressed) quark loops and the (dressed) pion and Kaon loops. Such a potential double-counting is definitely an issue for the broad sigma meson $\rm f_0(600)$. It is also not clear which scalar resonances are described by the ENJL model used in Ref. [8]. The parameters were determined from a fit to various low-energy observables and resonance parameters, among them a scalar multiplet with mass $M_{\rm S} = 983$ MeV. However, with those fitted parameters, the ENJL model actually predicts a rather low mass of $M_{\rm S}^{\rm ENJL} = 620$ MeV.

The (dressed) charged pion- and Kaon-loops from Fig. 1(b) yield the leading contribution in the chiral counting, but are subleading in $N_{\rm C}$. We note that the result without dressing (scalar QED) is actually finite: $a_\mu^{{\rm LbyL};\pi^\pm} = -46 \times 10^{-11}$. The dressing with form factors then leads to a rather huge and very model dependent suppression (compare the results for Refs. [8] and [9] in Table 2), so that the final result is much smaller than the one obtained for the pseudoscalars. This effect was studied in Ref. [11] for the HLS model used in Ref. [9], in an expansion in $(m_\pi/M_\rho)^2$. Ref. [11] observed a large cancellation between the first few terms in the series and the expansion converges only very slowly. The main reason is that typical momenta in the loop integral are of order $\mu = 4m_\pi \approx 550$ MeV and the effective expansion parameter is μ/M_ρ. The authors of Ref. [11] took this as an indication that the final result is very likely suppressed, but also very model dependent and that the chiral expansion looses its predictive power. The pion and Kaon loops contribution is then only one among many potential contributions of $\mathcal{O}(1)$ in $N_{\rm C}$ and they lump all of these into the guesstimate $a_\mu^{{\rm LbyL};N_{\rm C}^0} = (0 \pm 10) \times 10^{-11}$. However, since this estimate does not even cover the results for the pion and Kaon loops given in Refs. [8, 9], we think this procedure is not very appropriate.

The (dressed) constituent quark loops from Fig. 1(d) are also leading in large-$N_{\rm C}$. The result with point-like couplings is finite: $a_\mu^{\rm LbyL;quarks} = 62 \times 10^{-11}$. The dressing with form factors then leads again to a large and very model dependent suppression of the final result, compare Refs. [8] and [9] in Table 2.

In the recent review [6] the central values of some of the individual contributions to had. LbyL scattering were adjusted and some errors were enlarged to cover the results obtained by various groups which used different models, see Table 2. Finally, the errors were added in quadrature. Maybe the resulting small error masks some of the uncertainties we still face in had. LbyL scattering. Note that the dressed light quark loops are not included as a separate contribution in Ref. [6] (only the contribution from a bare c-quark is included in Table 2). The light quark loops are assumed to be already covered by using the short-distance constraint from Ref. [11] on the pseudoscalar-pole contribution. Although numerically the final estimate from Ref. [11] is very close to our result given in Table 2, in view of the interpretation given for this term in the Introduction, we do not see any reason, why the contribution from the dressed quark loops should be discarded completely. At least in large-$N_{\rm C}$ QCD, only the sum of all resonance exchanges should be dual to the quark loops.

5 Conclusions

Combining our result for the pseudoscalars with the evaluation of the axial-vector contribution in Ref. [11] and the results from Ref. [8] for the other contributions, we obtain the new estimate [3, 13]

$$a_{\mu}^{\mathrm{LbyL;had}} = (116 \pm 40) \times 10^{-11} \tag{8}$$

for the total had. LbyL scattering contribution to the anomalous magnetic moment of the muon1). The variation of the results for the individual contributions listed in Table 2 reflects our inherent ignorance of strong interaction physics in had. LbyL scattering. One can take the differences between those values as an indication of the model uncertainty and, to be conservative, all the errors have been added linearly, as was done earlier in Refs. [1, 5, 8, 10].

Certainly, more work on the had. LbyL scattering contribution is needed to fully control all the uncertainties, in particular, if we want to fully profit from a potential future $g-2$ experiment with an expected error of about 15×10^{-11} [27]. Maybe at some point we will get an estimate from lattice QCD [28], although the relevant QCD Green's function $\langle \mathrm{VVVV} \rangle$, to be integrated over the phase space of three off-shell photons, is a very complicated object.

In the meantime we suggest the following way forward [3]. It is very important to have a unified framework (hadronic model) which deals with all the contributions to had. LbyL scattering. A purely phenomenological approach would be to use some resonance Lagrangian where all couplings are fixed from experiment. Since such Lagrangians are in general non-renormalizable it is, however, not clear how to achieve a proper matching with QCD at short distances. Such a matching can be achieved within the large-N_{C} framework, however, the corresponding resonance Lagrangians in general contain many unknown coefficients and it will be difficult to fix all of them theoretically or experimentally. In any case, in both of these approaches any additional experimental information on various hadronic form factors would be very useful to constrain the theoretical models. In this respect, $\mathrm{e}^+\mathrm{e}^-$ colliders running at energies around 0.50–2 GeV could help to measure some of the form factors relevant for had. LbyL scattering [29].

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1) Applying the same procedure to the electron, we get $a_{\mathrm{e}}^{\mathrm{LbyL};\pi^0} = (2.98\pm0.34)\times10^{-14}$ [13]. Note that the naive rescaling $a_{\mathrm{e}}^{\mathrm{LbyL};\pi^0}(\text{rescaled}) = (m_{\mathrm{e}}/m_{\mu})^2\ a_{\mu}^{\mathrm{LbyL};\pi^0} = 1.7\times10^{-14}$ yields a value which is almost a factor of 2 too small. Our estimates for the other pseudoscalars contributions are $a_{\mathrm{e}}^{\mathrm{LbyL};\eta} = 0.49\times10^{-14}$ and $a_{\mathrm{e}}^{\mathrm{LbyL};\eta'} = 0.39\times10^{-14}$. Therefore we get $a_{\mathrm{e}}^{\mathrm{LbyL;PS}} = (3.9\pm0.5)\times10^{-14}$. Assuming that the pseudoscalar contribution yields the bulk of the result of the total had. LbyL scattering correction, we obtain $a_{\mathrm{e}}^{\mathrm{LbyL;had}} = (3.9\pm1.3)\times10^{-14}$, with a conservative error of about 30%, see Ref. [3]. This value was later confirmed in the published version of Ref. [6] where a leading logs estimate yielded $a_{\mathrm{e}}^{\mathrm{LbyL;had}} = (3.5\pm1.0)\times10^{-14}$

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