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Two-photon-exchange contribution to proton form factors in both space-like and time-like regions*

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Abstract The two two-photon exchange corrections to the unpolarized cross section and polarized observable $P_{\rm T}$, $P_{\rm L}$ in elastic ep scattering are discussed in a simple hadronic model. Comparing with previous results, the Δ contribution are re-analysed. And the similar corrections in $e^+e^- \to p\overline{p}$ are also discussed.

Key words two-photon exchange, form factor, proton

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1 Introduction

Since 2003, the Two-Photon Exchange (TPE) correction has attracted many interest due to its success in the explanation of discrepant measurements of $R=G_{\rm E}/G_{\rm M}$ by Rosenbluth method and polarized method [1–5]. The Rosenbluth method extracts the electromagnetic form factors of proton $G_{\rm E}, G_{\rm M}$ from the un-polarized elastic ep scattering while the polarized method extracts the ratio R from the polarized observables $P_{\rm T}, P_{\rm L}$ in the polarized ep scattering. The very different results for R by the two methods are surprised at the first sight and then partly explained by considering the TPE effects in the unpolarized ep scattering. Up to now, many model dependent methods [2–5] have been developed to estimate the TPE contribution and also many model-independent [6–8] analysis are proposed to extract the TPE effects directly. These estimates give similar results for the corrections to unpolarized cross sections while give different results for the corrections to polarized observables. This situation calls for the effort to understand the TPE effects more accurately. In this letter, we give a re-analysis of TPE effects based on a simple hadronic model [2]. The TPE effects to the polarized observables from the Δ intermediate are estimated and the TPE effects to un-polarized cross section when including Δ is improved comparing with the previous results [9]. Similarly, the two-photon annihilation contributions in the $e^+e^- \to p\overline{p}$ are reviewed [10].

2 Two-photon exchange contribution for ep → ep

In the simple hadronic model, we take the hadron as the elemental freedom and use the momentum dependent effective vertexes to describe the structure of hadron. For simplicity, the well-defined on-shell form factors of hadron are taken as the effective vertexes. In such model, the Feynamn diagram for elastic ep scattering with one-photon exchange(OPE) is showed

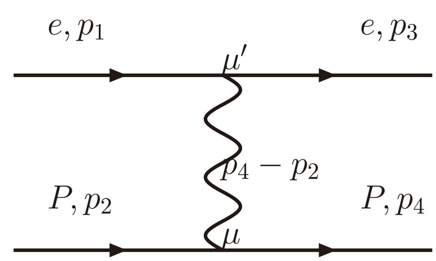

Fig. 1. One-photon exchange diagram.

as Fig. 1 and the unpolarized ep scattering cross section at tree level can be expressed as

$$d\sigma_0 = A[\tau G_{\rm M}^2(Q^2)+\epsilon G_{\rm E}^2(Q^2)], \quad (1)$$

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where

$$\tau = \frac{Q^2}{4M^2},\ Q^2 = -q^2 = -(p_4-p_2)^2,$$
$$\epsilon = [1+2(1+\tau)\tan^2(\theta/2)]^{-1},$$

θ is the scattering angle of the electron in the laboratory frame, A only depends on kinematic variables, and

$$G_{\rm E} = F_1 - \tau F_2,$$
$$G_{\rm M} = F_1 + F_2, \qquad (2)$$

with F_1, F_2 defined by the electromagnetic current matrix element

$$\langle p'|J^{\gamma}_{\mu}|p\rangle = \overline{u}(p')\left[F_1\gamma_\mu + F_2\frac{{\rm i}\sigma_{\mu\nu}}{2M}q^\nu\right]u(p). \qquad (3)$$

From Eq. (1), it is easy to see the form factors $G_{\rm E}$, $G_{\rm M}$ at fixed Q^2 can be extracted by the measurement of the unpolarized cross section at different ϵ, and the ratio R can be extracted from the slope of the ${\rm d}\sigma_0 - \epsilon$ curve.

Similarly, under the OPE approximation, two of the polarized observables in the polarized ep scattering can be expressed as

$$I_0 P_{\rm T} = -2\sqrt{\tau(1+\tau)}G_{\rm E}G_{\rm M}\tan\frac{\theta}{2},$$
$$I_0 P_{\rm L} = \frac{1}{M}(E_{\rm e}+E_{\rm e'})\sqrt{\tau(1+\tau)}\ G^2_{\rm M}\tan^2\frac{\theta}{2}, \qquad (4)$$

where

$$I_0 = G^2_{\rm E} + \frac{\tau}{\epsilon}G^2_{\rm M},$$

$E_{\rm e}, E_{\rm e'}$ are the energy of initial and final electron, respectively. Combination of Eqs. (4) gives:

$$\frac{G_{\rm E}}{G_{\rm M}} = -\frac{P_{\rm T}}{P_{\rm L}}\frac{(E_{\rm e}+E_{\rm e'})}{2M}\tan\left(\frac{\theta}{2}\right). \qquad (5)$$

Eq. (5) shows the ratio R can be extracted by the measurement of $P_{\rm T}$, $P_{\rm L}$ at fixed Q^2 and ϵ.

About 2000, the results by the polarized method were got and show very different behavior of R comparing with the results by Rosenbluth method. Such discrepancy was explained by TPE effect firstly in a simple hadronic model [2]. Later, the TPE effects were studied by GPDs method [3], dispersive relation method [4] and pQCD methods [5].

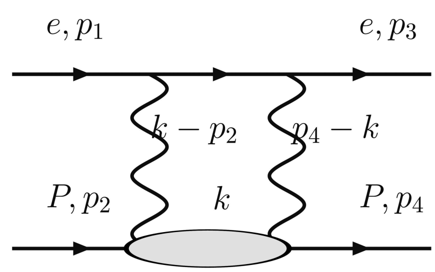

Fig. 2. Two-photon exchange diagram, the cross diagram is implied.

In the hadronic model, the TPE diagrams are showed as Fig. 2, where the intermediate states can be N, Δ. The TPE correction to unpolarized cross section from the Δ contribution has been discussed in [9] and the following relation is used

$$\Gamma^{\gamma}_{\Delta\to{\rm N}}(p,q) = \gamma_0[\Gamma^{\gamma}_{{\rm N}\to\Delta}(p,q)]^\dagger\gamma_0, \qquad (6)$$

where Γ's are the effective vertexes and defined by the following matrix elements

$$\overline{u}(p+q)\Gamma^{\mu\alpha,\gamma}_{\Delta\to{\rm N}}(p,q)u^{\Delta}_{\alpha}(p) = -{\rm i}e\langle N(p+q)|J^{\mu}_{\rm em}|\Delta(p)\rangle,$$
$$\overline{u}^{\Delta}_{\beta}(p)\Gamma^{\beta\nu,\gamma}_{{\rm N}\to\Delta}(p,q)u(p-q) = -{\rm i}e\langle\Delta(p)|J^{\nu}_{\rm em}|N(p-q)\rangle, \qquad (7)$$

with the matrix elements expressed as

$$\langle N(p')|J^{\rm em}_{\mu}|\Delta(p)\rangle = \frac{1}{M^2_{\rm N}}\overline{u}(p')[g_1F^{(1)}_{\Delta}(q^2)(g^{\alpha}_{\mu}\not p\not q - p_\mu\gamma^\alpha\not q - \gamma_\mu\gamma^\alpha p\cdot q + \gamma_\mu\not p q^\alpha) + g_2F^{(2)}_{\Delta}(q^2)(p_\mu q^\alpha - p\cdot q g^{\alpha}_{\mu}) +$$
$$g_3F^{(3)}_{\Delta}(q^2)/M_{\rm N}(q^2(p_\mu\gamma^\alpha - g^{\alpha}_{\mu}\not p) + q_\mu(q^\alpha\not p - \gamma^\alpha p\cdot q))]\gamma_5 T^{\dagger}_3 u_\alpha(p), \qquad (8)$$

In our opinion, such relation should be modified as [11]

$$\Gamma^{\gamma}_{\Delta\to{\rm N}}(p,q) = -\gamma_0[\Gamma^{\gamma}_{{\rm N}\to\Delta}(p,-q)]^\dagger\gamma_0. \qquad (9)$$

Also we take the recent results for the coupling and form factor which express as

$$g_1 = 1.91,\ g_2 = 2.63,\ g_3 = 1.59$$

and

$$G_{\rm M}/\mu_{\rm p} = \left(\frac{\Lambda^2_1}{Q^2+\Lambda^2_1}\right)^2,$$
$$G_{\rm E} = \left(\frac{\Lambda^2_1}{Q^2+\Lambda^2_1}\right)^2\frac{\Lambda^2_3}{Q^2+\Lambda^2_3},$$
$$F^{(i)}_{\Delta} = \left(\frac{\Lambda^2_1}{Q^2+\Lambda^2_1}\right)^2\frac{\Lambda^2_4}{Q^2+\Lambda^2_4}. \qquad (10)$$

with $\Lambda_1 = 0.84$, $\Lambda_2 = 2$, $\Lambda = \sqrt{2}$ GeV.

Using these as input, we have the corrections to the unpolarized cross section and polarized observables showed as Figs. 3, 4, 5.

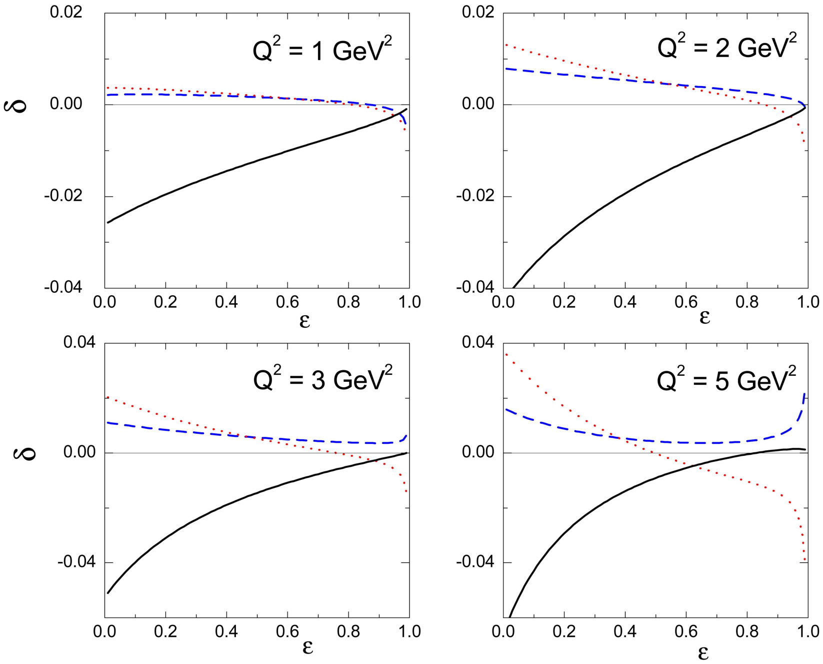

Fig. 3. TPE corrections to the unpolarized cross section where $\delta = \sigma^{1\gamma\otimes 2\gamma}/\sigma^{1\gamma\otimes 1\gamma}$. The solid line denotes to the TPE correction from N, the dot line denotes to the TPE correction from Δ with $g_3 = 1.59$ and the dash line denotes to the TPE correction from Δ with $g_3 = 0$.

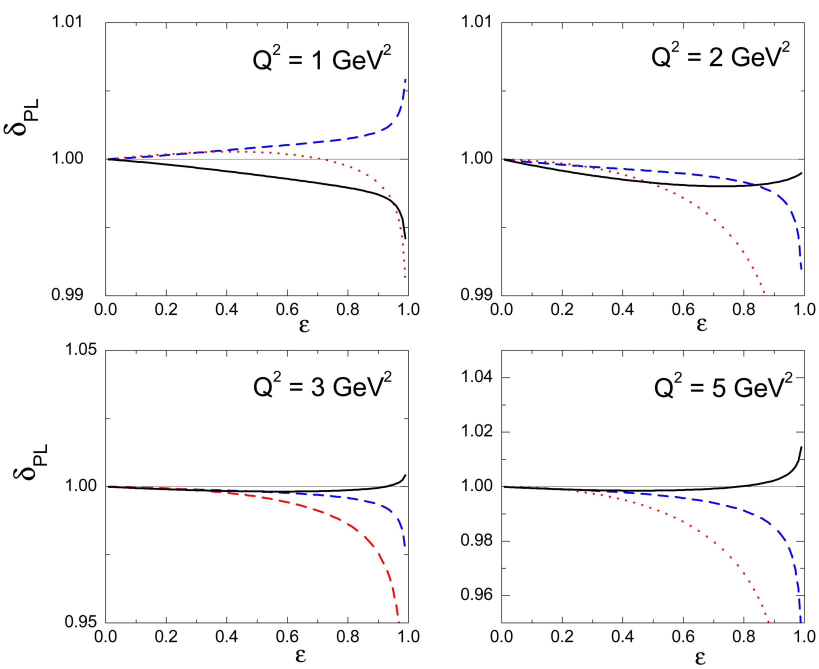

Fig. 4. TPE corrections to the polarized observable $P_{\rm L}$ where $\delta_{PL} = P_{\rm L}^{1\gamma+2\gamma}/P_{\rm L}^{1\gamma}$. The notations are same with with Fig. 3.

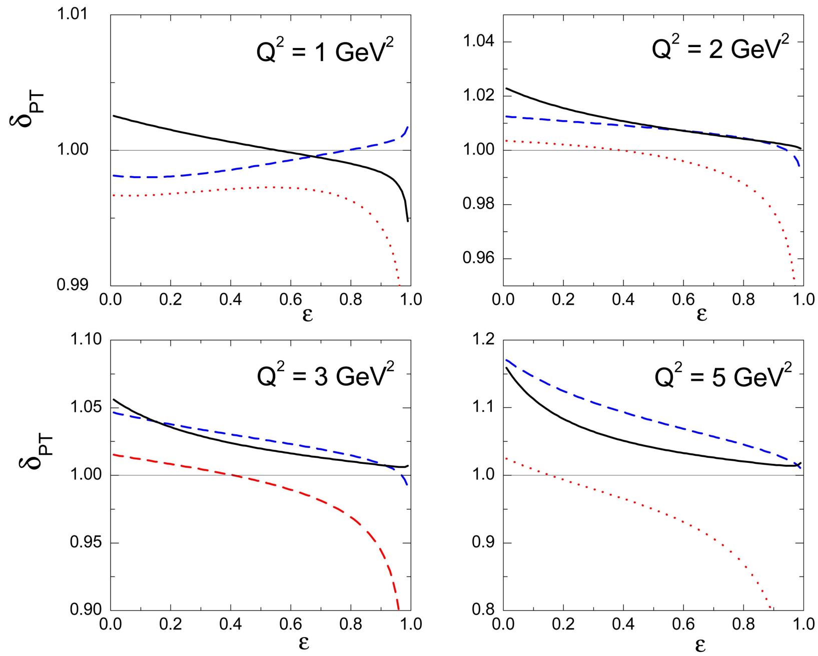

Fig. 5. TPE corrections to the polarized observable $P_{\rm T}$ where $\delta_{PT} = P_{\rm T}^{1\gamma+2\gamma}/P_{\rm T}^{1\gamma}$. The notations are same with with Fig. 3.

From Fig. 3, we see the TPE corrections to unpolarized cross section from Δ are always opposite to N's. And when we modified the coupling g_3 form 0 to 1.59, the contributions from Δ become larger at small ϵ which means its contribution will cancel the contribution from N more. For the polarized case, we see the TPE corrections to $P_{\rm L}$ from Δ are small while the TPE corrections to $P_{\rm T}$ from Δ are relatively larger and even show un-physical behavior at large Q^2 and ϵ. In our practical calculation, we also add an additional form factor to regular such divergence and found such un-physical behavior can be regularized, while the behavior at the region with $\epsilon \leq 0.6$ does not change. The combined corrections result in the positive correction the ration R for the polarized method at small ϵ. This is still opposite with the results by the GDPs. Such results calls for further understanding of the TPE corrections by different methods, and the coming experimental data proposed by [12] may help us to distinguish different methods.

3 Two-photon annihilation contribution for $e^+e^- \to p\overline{p}$

Since the important pole played by the TPE effects in the ep scattering, it is nature to ask how about the two photon annihilation effects in the $e^+e^- \to p\overline{p}$ which provides the measurement of proton's form factors in the time-like region. Such effects have been estimated in the simple hadronic model [10] and in this letter we just give a short review. For simplicity, we only used the monopole form for the form factors to give the estimate and the results are showed as Figs. 6, 7.

Fig. 6 shows the correction to unpolarized cross section from Δ is always opposite to the N's and the total result is weakly dependent on the angle at small s. Fig. 7, 8 show the correction to P_x is mainly

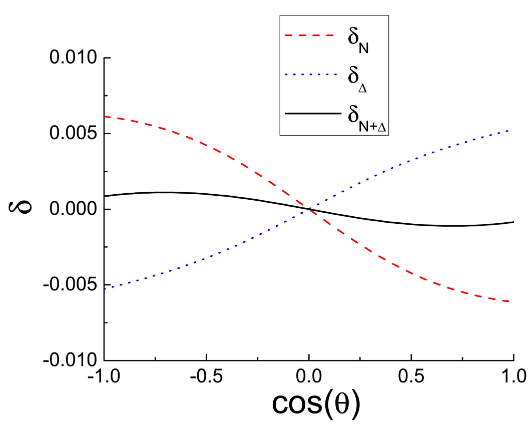

Fig. 6. Two-photon annihilation correction to the unpolarized cross section of $e^+e^- \to p\overline{p}$ at $s = 4$ GeV^2. The dashed and dotted lines denote to the corrections from N and Δ, respectively, and their sum is given by the solid lines. θ is the angle between the momentum of finial antiproton and initial electron in the center of mass frame.

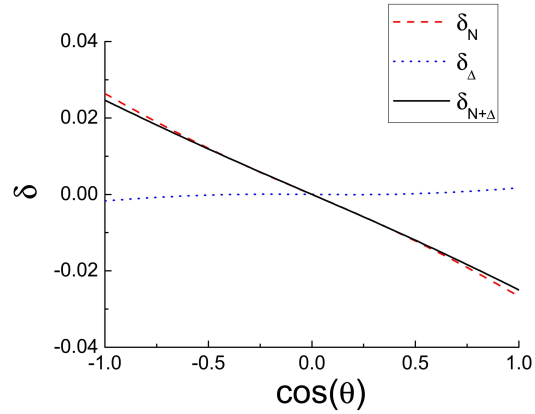

Fig. 7. Two-photon annihilation correction to P_x at $s=4$ GeV^2. The notations are same with with Fig. 6.

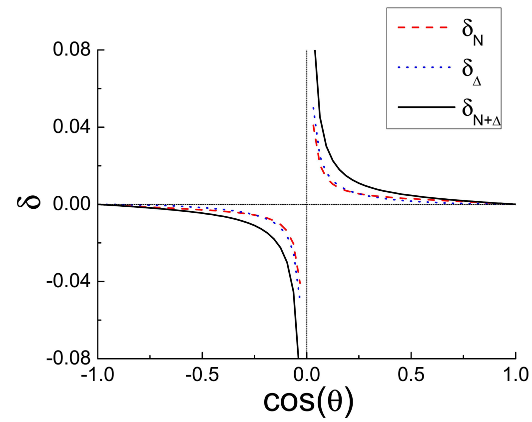

Fig. 8. Two-photon annihilation correction to P_z at $s=4$ GeV^2. The notations are same with with Fig. 6.

dominated by the N intermediate while the correction to P_z is enhanced by the Δ contribution. Such property suggests the nonzero P_z at $\theta=\pi/2$ may be a good place to measure the two-photon exchange like effects directly. From another hand, like the ep scattering case, other methods are needed to estimate the correction more accu.

4 Conclusion

The TPE effects in the hadronic model are reviewed and some modifications are improved comparing with the old results. And the corrections to the polarized observables from Δ intermediate state are estimated. The new results still show some different results with GDPs method for the polarized observable. It is still an open question to estimate the TPE effects more precisely.

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