

# Rare Kaon Decays from NA48/2

Cristina Morales

on behalf of the **NA48/2** Collaboration:

**Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz,  
Northwestern, Perugia, Pisa, Saclay,  
Siegen, Torino, Vienna**



PHIPSI'09



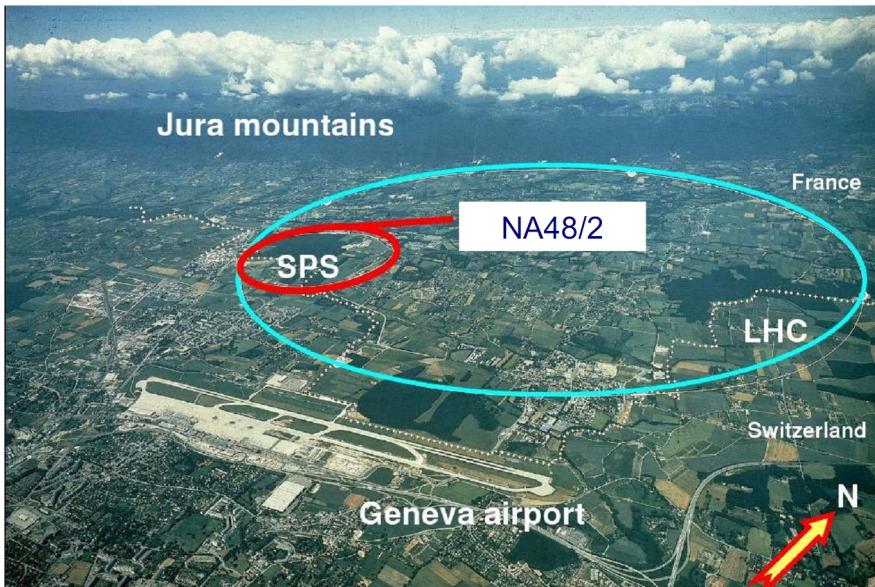
# Outline

## New Measurements of Radiative Kaon Decays

- $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ :
  - First Measurement of IB-DE Interference
  - Search for Direct CP Violation
- $K^\pm \rightarrow \pi^\pm \gamma\gamma$ :
  - Precise Measurement of the Decay Rate
- $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ :
  - First Observation and Measurements of BR and Decay Distribution

# The NA48/2 Experiment

- Fixed target experiment in North Area of SPS



- Devoted to charge asymmetry studies in  $K^\pm \rightarrow 3\pi$
- Simultaneous  $K^\pm$  beams with  $p_K = 60 \pm 3 \text{ GeV}/c$

NA48	1997	$\varepsilon'/\varepsilon$ run	$K_L + K_S$
	1998	$\varepsilon'/\varepsilon$ run	$K_L + K_S$
	1999	$\varepsilon'/\varepsilon$ run	$K_S$ Hi. Int.
	2000	$K_L$ only	$K_S$ High Intensity NO Spectrometer
NA48/1	2001	$\varepsilon'/\varepsilon$ run	$K_S$ High Int.
	2002	$K_S$ High Intensity	
NA48/2	2003	$K^\pm$ High Intensity	
	2004	$K^\pm$ High Intensity	

# The NA48/2 Detector

## Main detector components:

- Magnet spectrometer (4 DCHs):  
1% resolution for  $p=20$  GeV/c
- Liquid Krypton EM calorimeter  
1.4% energy resolution for  $E_\gamma = 2$  GeV/c

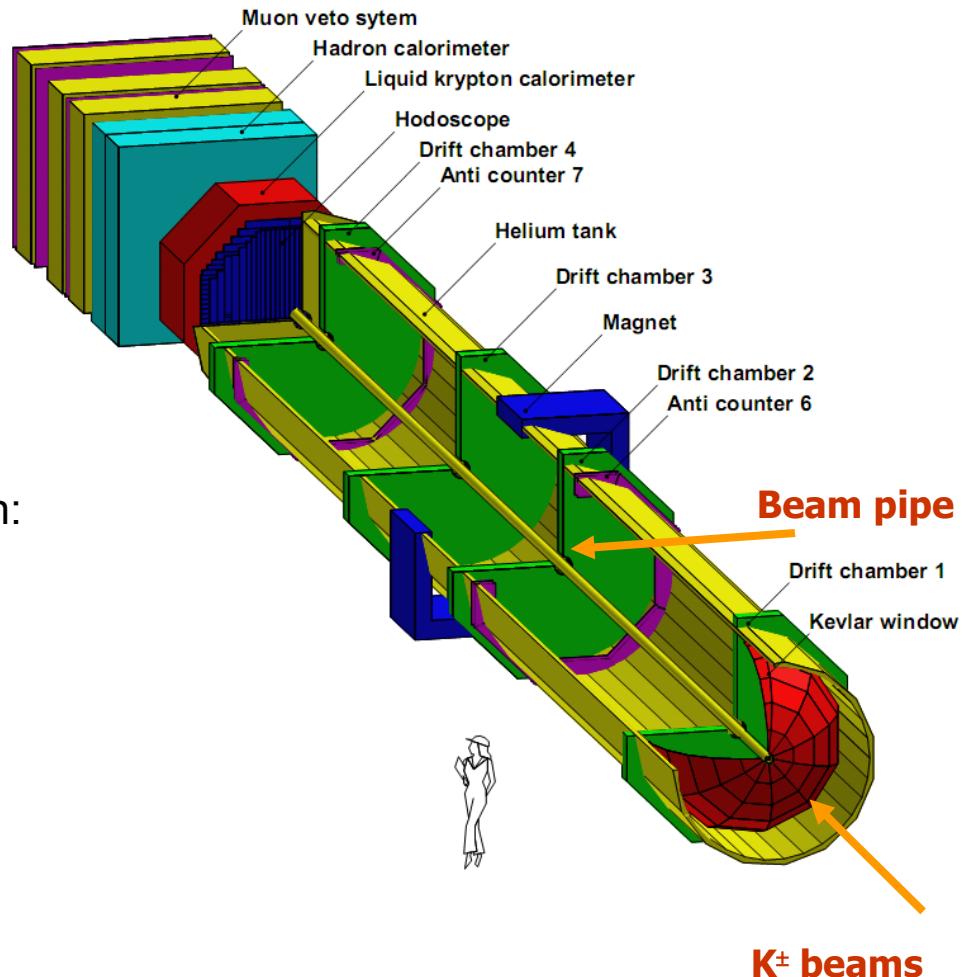
## Two main trigger modes:

- Charged. Devoted to  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  selection:  
3 charged tracks
- Neutral. Devoted to  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  selection:  
> 2 em clusters in LKr x or y projection

## Total statistics:

$$K^\pm \rightarrow \pi^- \pi^+ \pi^\pm: \sim 4 \cdot 10^9$$

$$K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm: \sim 1 \cdot 10^8$$

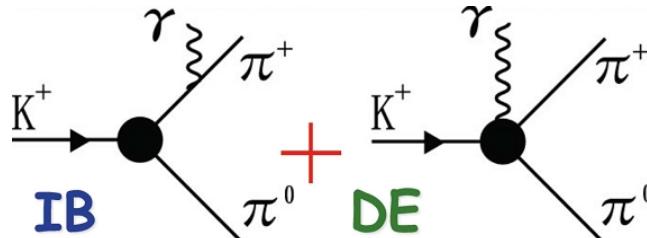


$$K^\pm \rightarrow \pi^\pm \pi^0 \gamma$$

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Theory

Two sources of  $\gamma$  radiation:

**Inner Bremsstrahlung (IB)** and **Direct Emission (DE)**



Kinematic variable:

$$W^2 = \frac{(p_\pi \cdot p_\gamma)(p_K \cdot p_\gamma)}{m_K^2 m_\pi^2}$$

$$\frac{\partial \Gamma^\pm}{\partial W} = \underbrace{\frac{\partial \Gamma_{IB}^\pm}{\partial W}}_{\text{Inner Bremsstrahlung (IB)}} \left[ 1 + \underbrace{2 \cos(\pm\phi + \delta_1^1 - \delta_0^2) |X_E|}_{\text{Interference (INT)}} W^2 + \underbrace{m_\pi^4 m_K^4 (|X_E|^2 + |X_M|^2)}_{\text{Direct Emission (DE)}} W^4 \right]$$

known from  $K^\pm \rightarrow \pi^\pm \pi^0$   
and QED

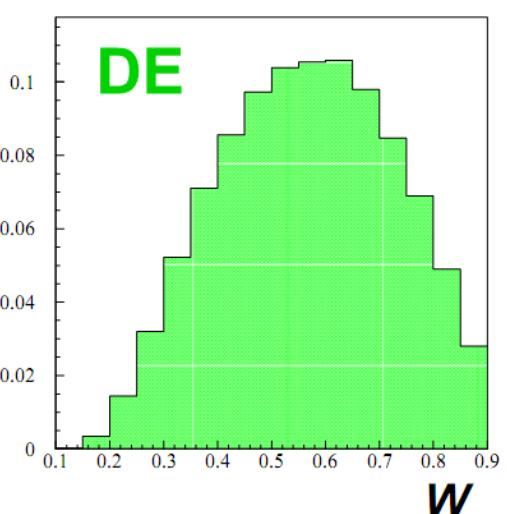
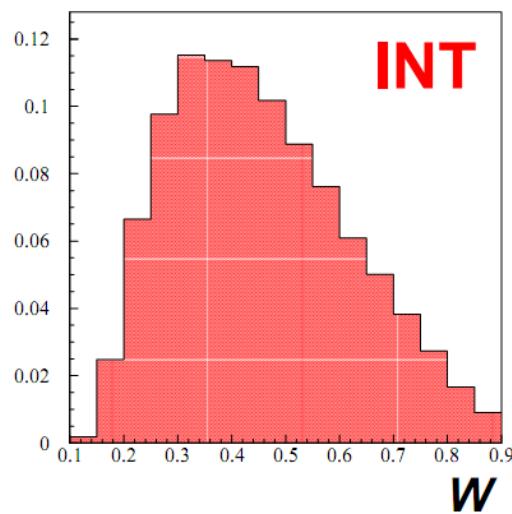
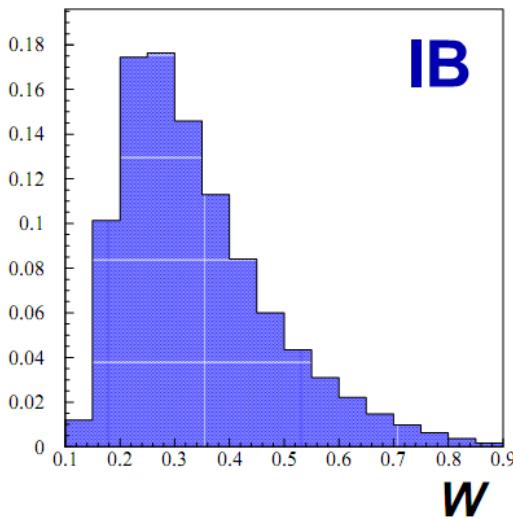
Interference of IB  
and  
electric DE.  
No prediction.

- two terms ( $\mathcal{O}(p^4)$  ChPT):
  - $X_M$ : magnetic part has two contributions:  
reducible WZW functional ( $\sim 260 \text{ GeV}^{-4}$ )  
+ direct (not known)
  - $X_E$ : no prediction

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Theory

$$\frac{\partial \Gamma^\pm}{\partial W} = \underbrace{\frac{\partial \Gamma_{IB}^\pm}{\partial W}}_{\text{Inner Bremsstrahlung (IB)}} + \underbrace{2 \cos(\pm\phi + \delta_1^1 - \delta_0^2) |X_E| W^2}_{\text{Interference (INT)}} + \underbrace{m_\pi^4 m_K^4 (|X_E|^2 + |X_M|^2) W^4}_{\text{Direct Emission (DE)}}$$

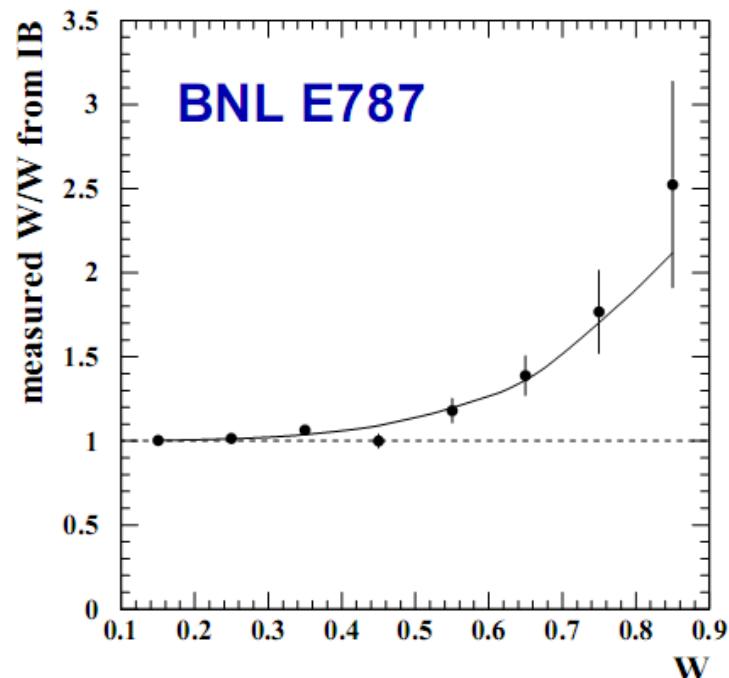
$$W^2 = \frac{(p_\pi \cdot p_\gamma)(p_K \cdot p_\gamma)}{m_K^2 m_\pi^2}$$



# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Experimental Status

## Previous Measurements:

	$\text{Br(DE)} \times 10^6$	Stat.
<b>E787</b>	$4.7 \pm 0.9$	20 k
<b>E470</b>	$3.8 \pm 1.1$	10 k
<b>ISTRAP+</b>	$3.7 \pm 4.0$	930

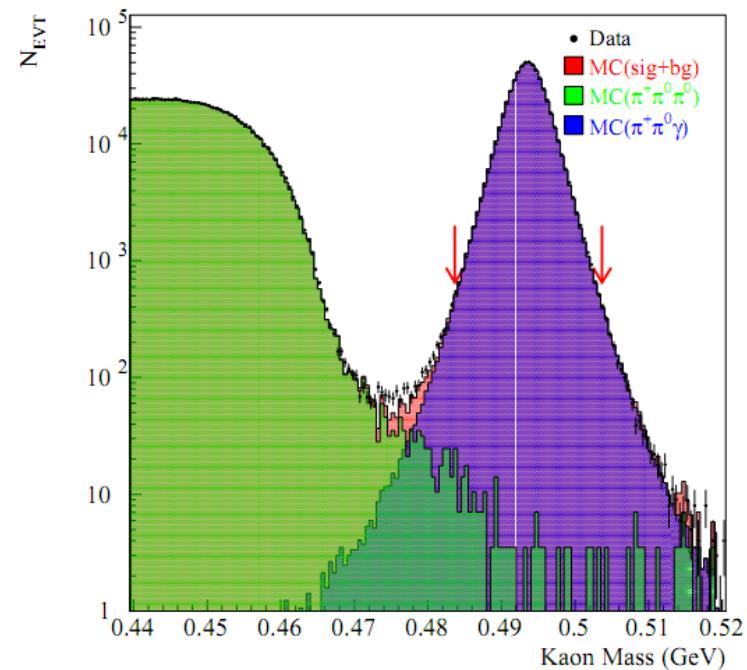


- All previous DE measurements:
  - Kinematic range  $55 < T_\pi^* < 90$  MeV
  - Assumption  $\text{INT} = 0$
- So far **neither INT nor CP violation** observed
  - E787.  $\text{INT} = (-0.4 \pm 1.6)\%$

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Data Sample

## New NA48/2 Measurement:

- Simultaneous  $K^+ / K^-$  beams  
     $\Rightarrow$  CPV check possible
- Larger  $T_\pi^*$  region available  
 $0 < T_\pi^* < 80$  MeV
- Background negligible:  
 $< 1\% \times DE$  (mainly  $\pi^\pm \pi^0 \pi^0$ )
- $O(10^{-3})$  mistagging probability for odd  $\gamma$



## Total $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ data sample:

- More than 1 million events
- For the fit: restrict to  $0.2 < W < 0.9$  and  $E_\gamma > 5$  GeV  
     $\Rightarrow$  Still 600k  $\pi^\pm \pi^0 \gamma$  candidates

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Fit

## ■ Extended Maximum Likelihood Fit

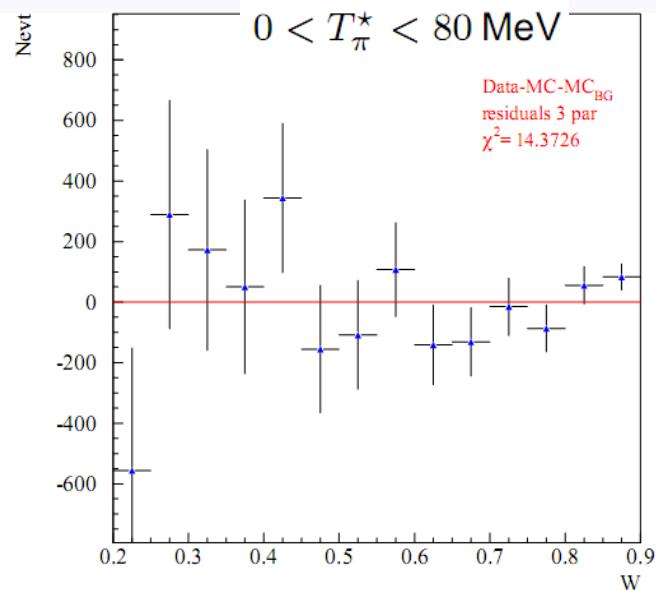
Correct for acceptances with MC

$$\text{Data}(i) = N_0[(1 - \alpha - \beta) \cdot \mathbf{IB}_{\text{MC}}(i) + \alpha \cdot \mathbf{INT}_{\text{MC}}(i) + \beta \cdot \mathbf{DE}_{\text{MC}}(i)]$$

$$\text{Frac(DE)} = (3.32 \pm 0.15) \times 10^{-2}$$

$$\text{Frac(INT)} = (-2.35 \pm 0.35) \times 10^{-2}$$

$$\text{Frac(DE)} = \frac{\text{Br(DE)}}{\text{Br(IB)}} \quad \text{Frac(INT)} = \frac{\text{Br(INT)}}{\text{Br(IB)}}$$

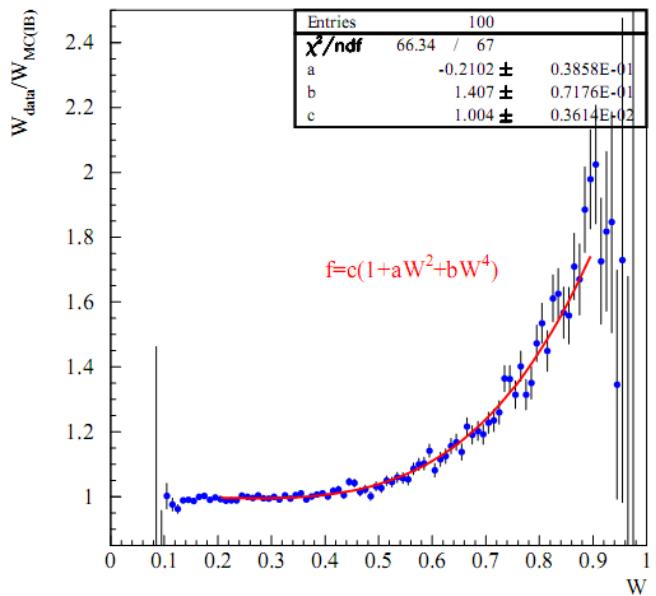


## ■ Polynomial Fit

Fit ratio W (Data)/ W (IBMC) with:

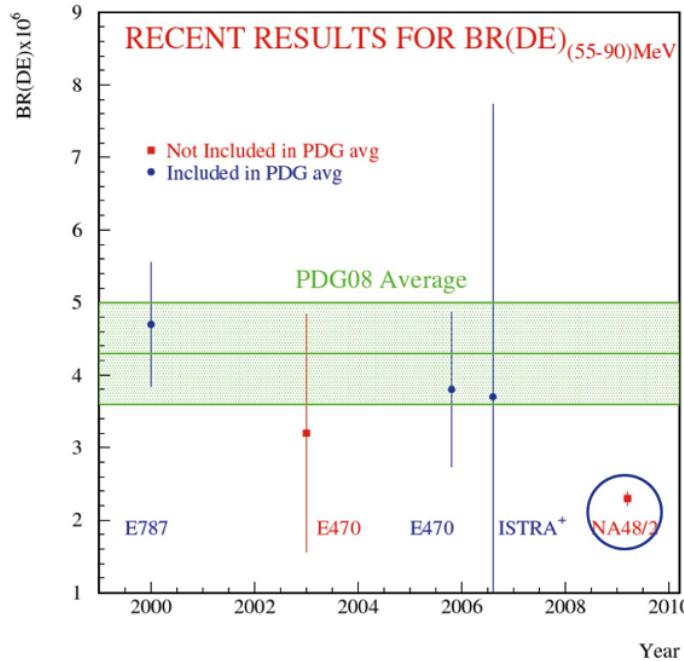
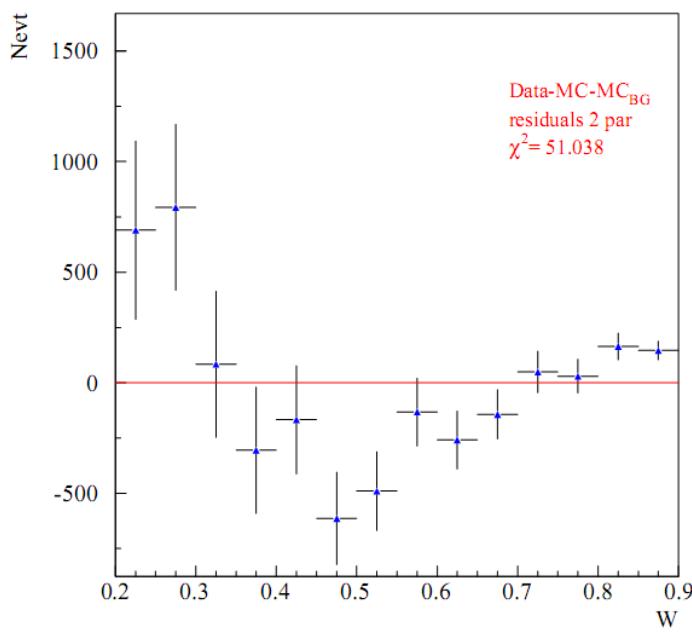
$$F = c \cdot (1 + aW^2 + bW^4) \implies \text{Frac(DE)}, \text{Frac(INT)}$$

$$\begin{aligned} \text{Frac(DE)} &= (3.19 \pm 0.16) \times 10^{-2} \\ \text{Frac(INT)} &= (-2.21 \pm 0.41) \times 10^{-2} \end{aligned}$$



# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Exp. Comparison

Fit with **INT = 0** and **extrapolation to  $55 < T_\pi^* < 90$  MeV:**



$$\text{Br(DE)}_{55 < T_\pi^* < 90 \text{ MeV}}^{\text{INT}=0} = (2.32 \pm 0.05_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-6}$$

⇒ **Clear disagreement with INT = 0 hypothesis!**  
**Need to fit with non-vanishing interference term!**

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Final Results

Final NA48/2 results on  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  fractions:

$$\text{Frac(DE)}_{0 < T_\pi^* < 80 \text{ MeV}} = (3.32 \pm 0.15_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-2}$$

$$\text{Frac(INT)}_{0 < T_\pi^* < 80 \text{ MeV}} = (-2.35 \pm 0.35_{\text{stat}} \pm 0.39_{\text{syst}}) \times 10^{-2}$$

Approximations for extracting  $X_E$  and  $X_M$ :

$$\rho = -0.93$$

■  $\phi = 0$

■  $\cos(\delta_1^1 - \delta_0^2) = \cos 6.5^\circ \approx 1$

$$X_E = \frac{\text{Frac(INT)}}{2 \cdot 0.105 \cdot m_K^2 m_\pi^2}, \quad X_M = \sqrt{\frac{\text{Frac(DE)} - m_K^4 m_\pi^4 |X_E|^2 \cdot 0.0227}{0.0227 \cdot m_K^4 m_\pi^4}}$$

Magnetic and electric components:

$$X_E = (-24 \pm 4_{\text{stat}} \pm 4_{\text{syst}}) \text{ GeV}^{-4}$$

$$X_M = (254 \pm 11_{\text{stat}} \pm 11_{\text{syst}}) \text{ GeV}^{-4}$$

WZW reducible anomaly prediction:  $X_M \approx 270 \text{ GeV}^{-4}$

⇒ NA48/2 measurement points to reducible anomaly only

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : CPV Studies

## ■ Asymmetry in the total rate

$$A_N = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} = \frac{N_{\pi^+\pi^0\gamma} - R \cdot N_{\pi^-\pi^0\gamma}}{N_{\pi^+\pi^0\gamma} + R \cdot N_{\pi^-\pi^0\gamma}}$$

with  $R = N_{K^+}/N_{K^-} = 1.7998(4)$  from  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

$$\begin{aligned} A_N &= (0.0 \pm 1.0_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-3} \\ |A_N| &< 1.5 \times 10^{-3} \quad (90\% \text{ CL}) \end{aligned}$$

⇒ First limit on  $\sin \phi$ :  
 $\sin \phi = -0.01 \pm 0.43$   
 $|\sin \phi| < 0.56 \quad (90\% \text{ CL})$

## ■ Asymmetry in the Dalitz plot

$$\frac{d\Gamma^\pm}{dW} = \frac{d\Gamma_{\text{IB}}^\pm}{dW} (1 + (a \pm e)W^2 + bW^4)$$

$$A_W = e \int \frac{\text{INT}}{\text{IB}} = (-0.6 \pm 1.0) \times 10^{-3}$$

⇒ No CP asymmetry observed in  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ !

$$K^\pm \rightarrow \pi^\pm \gamma\gamma$$

$$K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$$

# $K^\pm \rightarrow \pi^\pm \gamma\gamma$ : Theory

Differential  $K^\pm \rightarrow \pi^\pm \gamma\gamma$  decay rate

$$\frac{\partial^2 \Gamma}{\partial y \partial z} = \frac{m_K}{2^9 \pi^3} \left[ z^2 (|A + B|^2 + |C|^2) + \left( y^2 - \frac{1}{4} \lambda(1, r_\pi^2, z) \right)^2 (|B|^2 + |D|^2) \right]$$

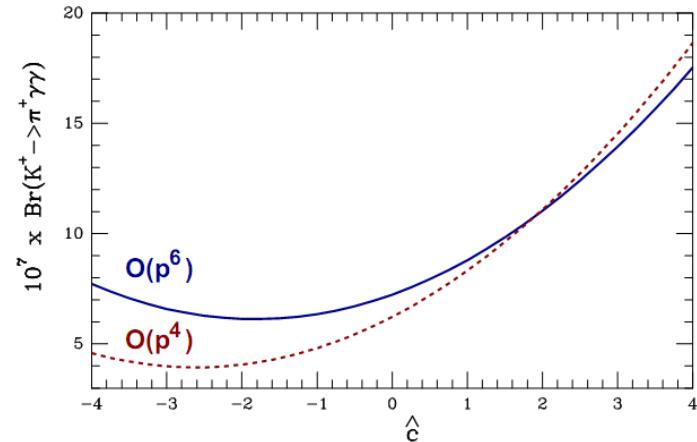
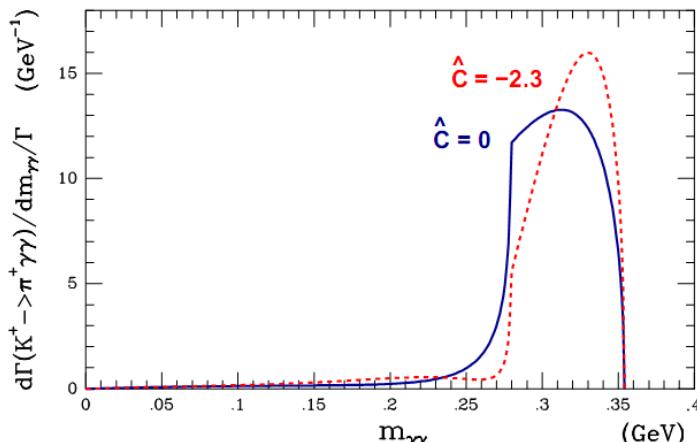
At  $\mathcal{O}(p^4)$ : (Ecker, Pich, de Rafael, Nucl. Phys. B 303 (1988) 665)

- $A(z, \hat{c})$  contains loops and  $\hat{c}$  of  $\mathcal{O}(1)$ .
- $C(z)$  contains poles and tadpoles.

(Gerard, Smith, Trine, Nucl. Phys. B 730 (2005) 1)

At  $\mathcal{O}(p^6)$ : Unitarity corrections, could increase Br by 30 – 40%.

(D'Ambrosio, Portolés, Nucl. Phys. B 386 (1996) 403)



# $K^\pm \rightarrow \pi^\pm \gamma\gamma$ : Trigger

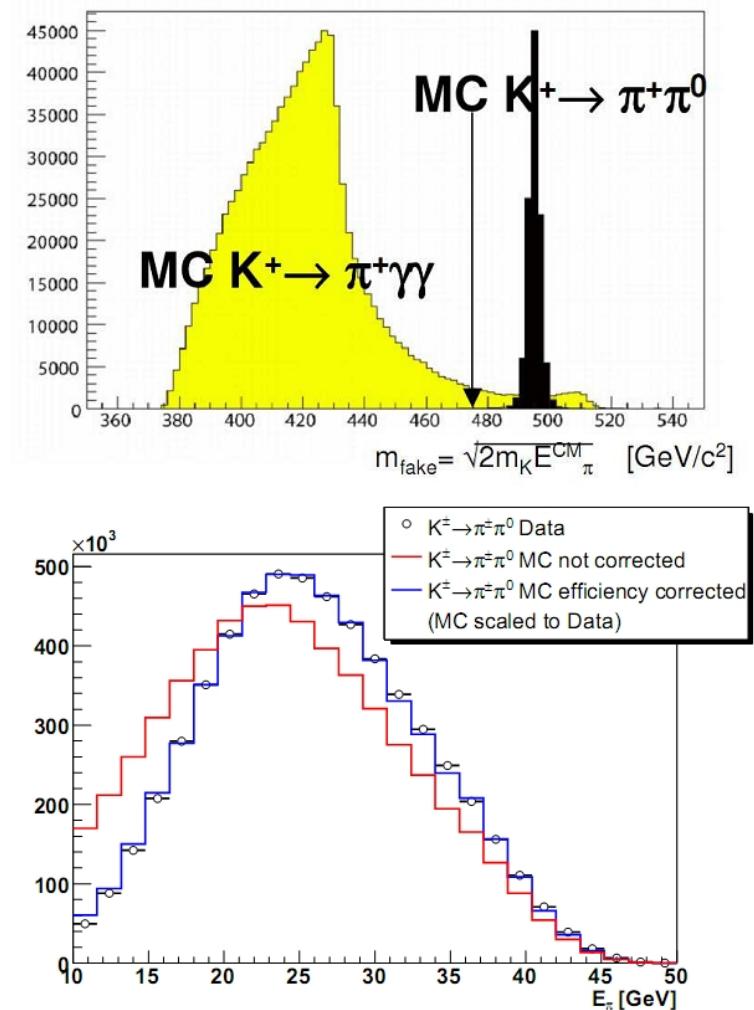
## Trigger Efficiency

- $K^\pm \rightarrow \pi^\pm \gamma\gamma$  selected through neutral trigger.
- **L1:** More than 2 e.m. clusters required.  
 $\Rightarrow \approx 50\% \text{ efficiency}$
- **L2:** Rejection of  $K^\pm \rightarrow \pi^\pm \pi^0$  by cutting on  $E_\pi^*$ .  
 $\Rightarrow \approx 80\% \text{ efficiency}$

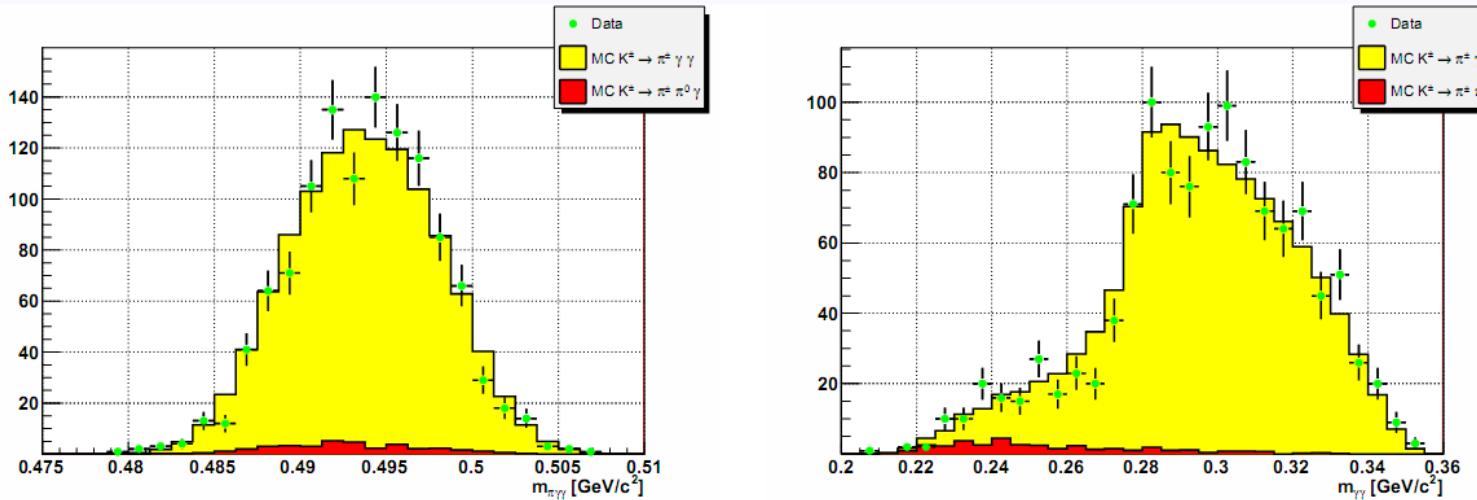
Statistics too low to measure trigger efficiencies from  $K^\pm \rightarrow \pi^\pm \gamma\gamma$ .



**Use background events and correct for different kinematics.**



# $K^\pm \rightarrow \pi^\pm \gamma\gamma$ : Branching Fraction



- **1164  $K^\pm \rightarrow \pi^\pm \gamma\gamma$  candidates** in 40% of NA48/2 data.  
(About 40 times more than previous world sample!)
- **Background:** **3.3%**, mainly from  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ .
- **Systematics:** Mainly from trigger efficiency determination.

Assume ChPT  $\mathcal{O}(p^6)$  and  $\hat{c} = 2$ : *(preliminary)*

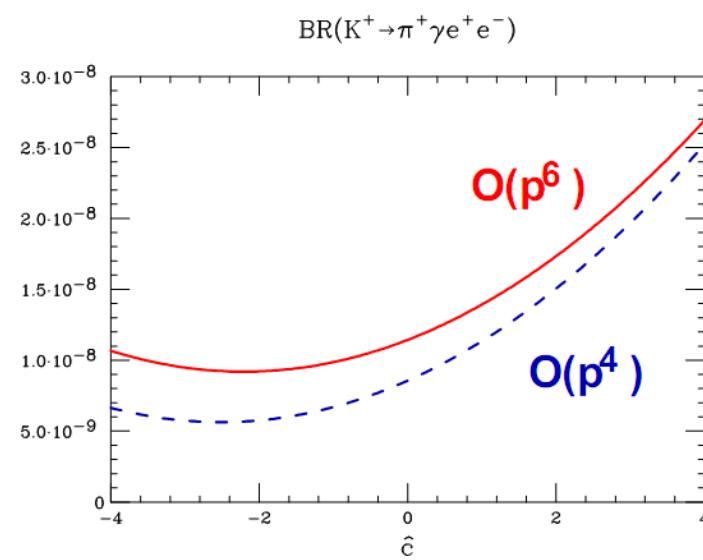
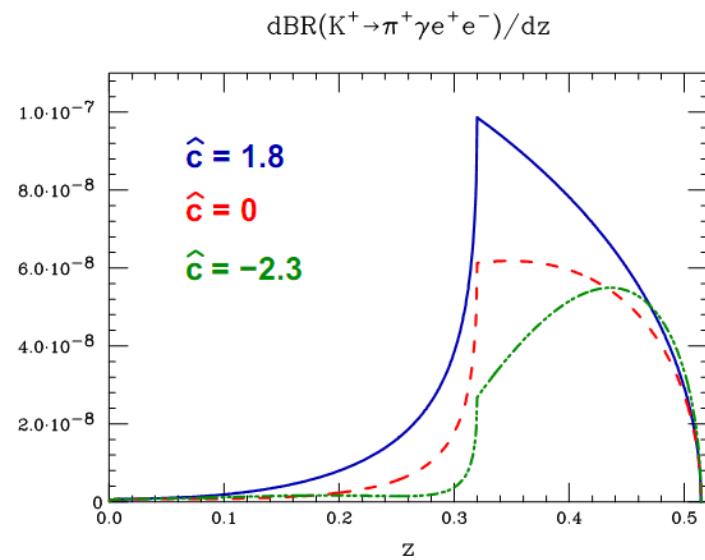
$$\text{Br}(K^\pm \rightarrow \pi^\pm \gamma\gamma)_{\hat{c}=2, \mathcal{O}(p^6)} = (1.07 \pm 0.04_{\text{stat}} \pm 0.08_{\text{syst}}) \cdot 10^{-6}$$

Model independent measurement and  $\hat{c}$  extraction in preparation.

# $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ : Branching Fraction

Same as  $K^\pm \rightarrow \pi^\pm \gamma\gamma$  with an internal  $\gamma$  conversion.

- $\mathcal{O}(p^4)$ : BR and  $m_{ee\gamma}$  determined by  $\hat{c}$
- $\mathcal{O}(p^6)$ : Unitarity corrections  $\Rightarrow$  change in BR by 30 – 40%.  
(Gabbiani, Phys. Rev. Lett. D 59 (1999) 094022)

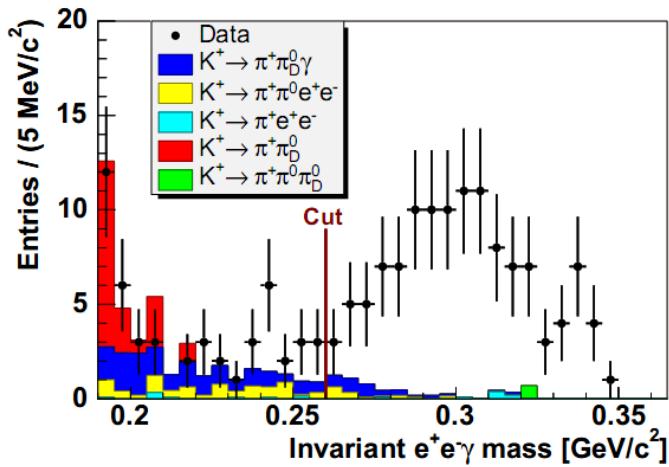
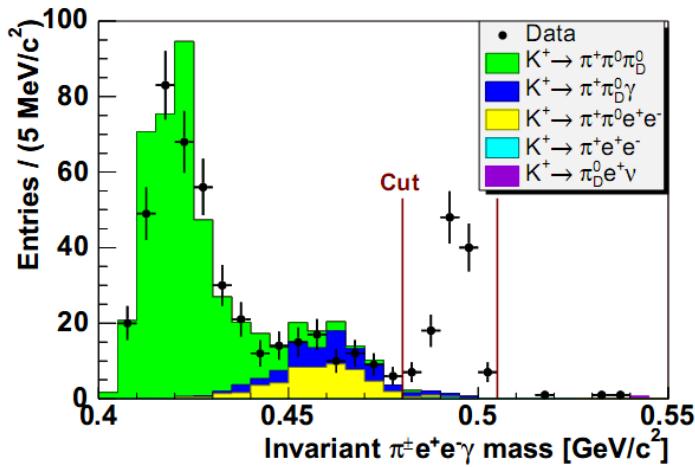


# $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ : Fit of $m_{ee\gamma}$

## Model Independent Measurement:

- **120  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$  candidates** (selection through 3-track-trigger).
  - Normalization to  $K^\pm \rightarrow \pi^\pm \pi_D^0 \rightarrow \pi^\pm e^+ e^- \gamma$ .
  - Computing BR in bins of  $m_{ee\gamma}$ .
- ⇒ **No assumption on  $m_{ee\gamma}$  distribution used!**

$$\text{Br}(K^\pm \rightarrow \pi^\pm e^+ e^- \gamma)_{m_{ee\gamma} > 260 \text{ MeV}} = (1.19 \pm 0.12_{\text{stat}} \pm 0.04_{\text{syst}}) \cdot 10^{-8}$$



# Conclusions

## ■ $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ :

- More than 1 million reconstructed events with tiny background.
- First observation and measurement of interference between IB and DE amplitudes.
- Limits of  $\mathcal{O}(10^{-3})$  on direct CP violation in  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ .

## ■ $K^\pm \rightarrow \pi^\pm \gamma\gamma$ :

- More than  $40\times$  the statistics of previous experiments.
- Preliminary measurement of the branching fraction.

## ■ $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ :

- First observation of the decay with 120 events.
- Measurements of the branching fraction and the  $ee\gamma$  decay distribution.

Thank You !

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Fit

## ■ Extended Maximum Likelihood Fit

Correct for acceptances with MC

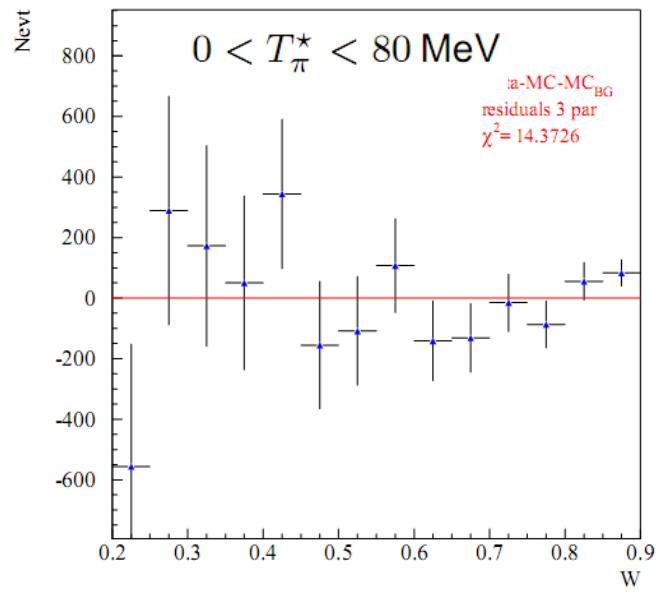
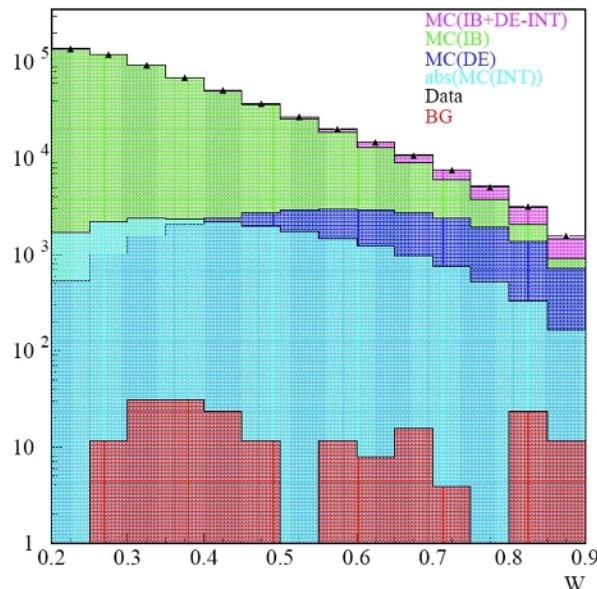
$$\text{Data}(i) = N_0[(1 - \alpha - \beta) \cdot \mathbf{IB}_{\text{MC}}(i) + \alpha \cdot \mathbf{INT}_{\text{MC}}(i) + \beta \cdot \mathbf{DE}_{\text{MC}}(i)]$$

$$\text{Frac(DE)} = (3.32 \pm 0.15) \times 10^{-2}$$

$$\text{Frac(INT)} = (-2.35 \pm 0.35) \times 10^{-2}$$

$$\text{Frac(DE)} = \frac{\text{Br(DE)}}{\text{Br(IB)}}$$

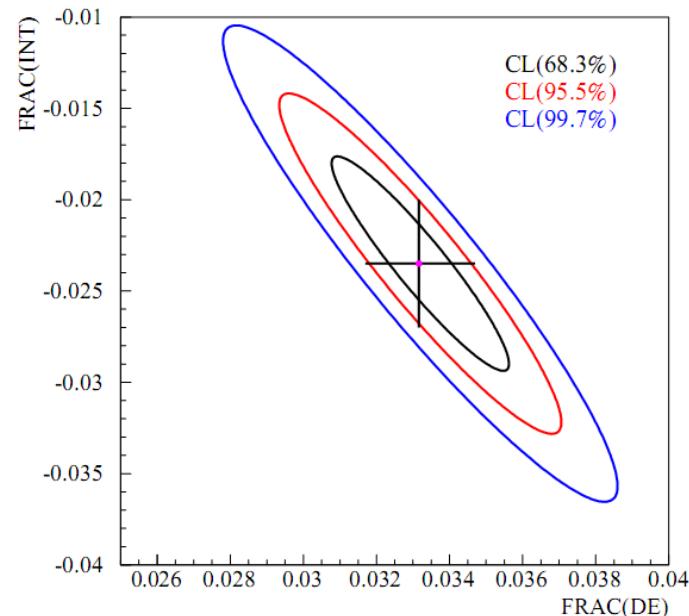
$$\text{Frac(INT)} = \frac{\text{Br(INT)}}{\text{Br(IB)}}$$



# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : Final Results

## Systematics:

Source	DE $\times 10^2$	INT $\times 10^2$
Acceptance	0.10	0.15
L1 Trigger	0.01	0.03
L2 Trigger	—	0.30
Energy Scale	0.09	0.21
<b>Total</b>	<b>0.14</b>	<b>0.39</b>



Final NA48/2 results on  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  fractions:

$$\text{Frac(DE)}_{0 < T_\pi^* < 80 \text{ MeV}} = (3.32 \pm 0.15_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-2}$$

$$\text{Frac(INT)}_{0 < T_\pi^* < 80 \text{ MeV}} = (-2.35 \pm 0.35_{\text{stat}} \pm 0.39_{\text{syst}}) \times 10^{-2}$$

Correlation:  $\rho = -0.93$

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : CPV Studies

Decay rate may depend on kaon charge:

$$\frac{\partial \Gamma^\pm}{\partial W} = \frac{\partial \Gamma_{IB}^\pm}{\partial W} \left[ 1 + \underbrace{2 \cos(\pm\phi + \delta_1^1 - \delta_0^2)}_{INT} |X_E| W^2 + m_\pi^4 m_K^4 (|X_E|^2 + |X_M|^2) W^4 \right]$$

- If  $\phi \neq 0$ :  $\Gamma(K^+ \rightarrow \pi^+ \pi^0 \gamma) \neq \Gamma(K^- \rightarrow \pi^- \pi^0 \gamma)$ !  
     $\Rightarrow$  CP violation!
- SM prediction on asymmetry:  $2 \cdot 10^{-6} - 10^{-5}$  for  $50 < E_\gamma^* < 170$  MeV.
- Possible SUSY contributions can push the asymmetry up to  $10^{-4}$  in some  $W$  regions.
- Two possible measurements:
  - Asymmetry in the total rate  $\Rightarrow$  need normalization ( $K_{3\pi}$ )
  - Asymmetry in the Dalitz plot

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : CPV Studies

## ■ Asymmetry in the total rate

For CP asymmetry analysis: Remove cuts on  $W$  range and  $E_\gamma^{\min}$

⇒ 1.08 million events for CPV analysis.

Measurement of rate asymmetry:

$$A_N = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} = \frac{N_{\pi^+\pi^0\gamma} - R \cdot N_{\pi^-\pi^0\gamma}}{N_{\pi^+\pi^0\gamma} + R \cdot N_{\pi^-\pi^0\gamma}}$$

with  $R = N_{K^+}/N_{K^-} = 1.7998(4)$  from  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ .



$$A_N = (0.0 \pm 1.0_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-3}$$

$$|A_N| < 1.5 \times 10^{-3} \quad (90\% \text{ CL})$$

⇒ First limit on  $\sin \phi$ :

$$\sin \phi = -0.01 \pm 0.43, \quad |\sin \phi| < 0.56 \quad (90\% \text{ CL})$$

# $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : CPV Studies

## ■ Asymmetry in the Dalitz plot

Fit of asymmetry in  $W$  spectrum:

$$\frac{d\Gamma^\pm}{dW} = \frac{d\Gamma_{IB}^\pm}{dW} (1 + (a \pm e)W^2 + bW^4)$$



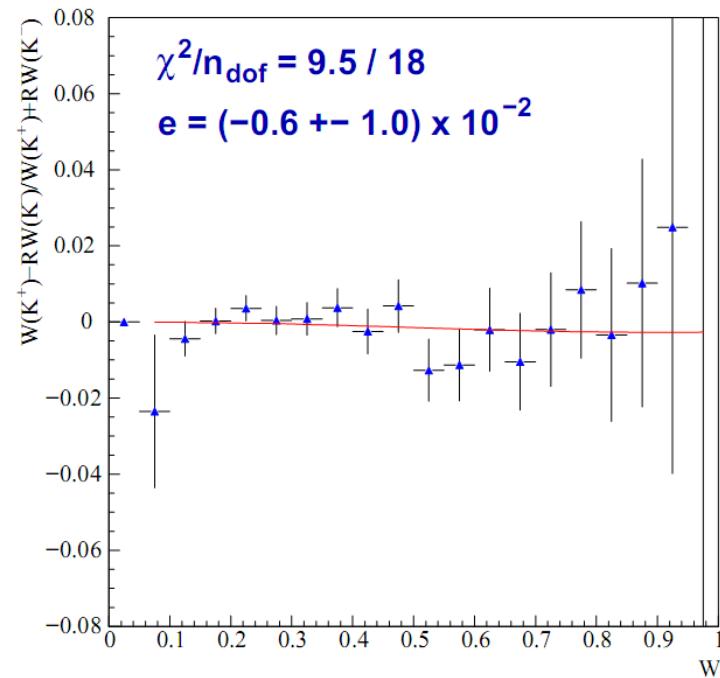
Single parameter fit to:

$$\frac{dA_W}{dW} = \frac{e \cdot W^2}{1 - 0.247 W^2 + 1.463 W^4}$$



$$A_W = e \int \frac{\text{INT}}{\text{IB}} = (-0.6 \pm 1.0) \times 10^{-3}$$

compatible with  $A_N$ .



⇒ No CP asymmetry observed in  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ !

# $K^\pm \rightarrow \pi^\pm \gamma\gamma$ Decays

$$\frac{\partial^2 \Gamma}{\partial y \partial z} = \frac{m_K}{2^9 \pi^3} \left[ z^2 (|A+B|^2 + |C|^2) + \left( y^2 - \frac{1}{4} \lambda(1, r_\pi^2, z) \right)^2 \cdot (|B|^2 + |D|^2) \right]$$

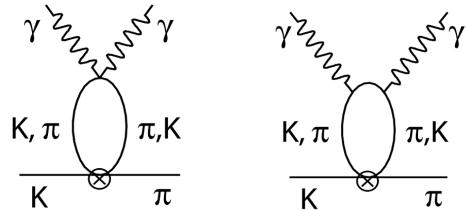
$$y = \frac{p_K (q_1 - q_2)}{m_K^2}, \quad z = \frac{(q_1 + q_2)^2}{m_K^2}.$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca), \quad r_\pi = \frac{m_\pi}{m_K}$$

**O( $p^4$ )**

[Nucl.Phys.B303(1988) 665; hep-ph/0508189]

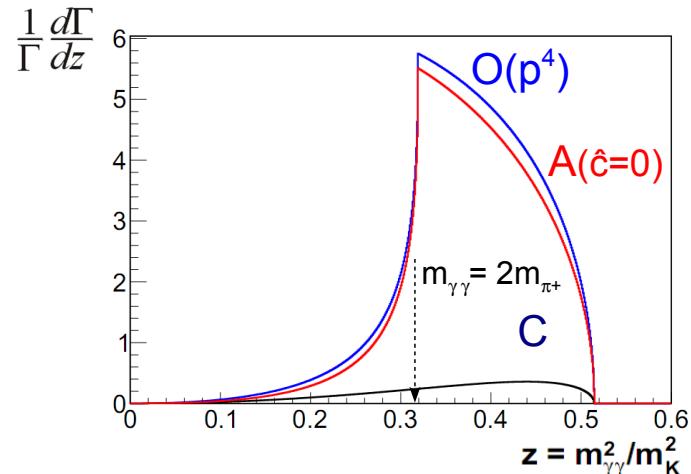
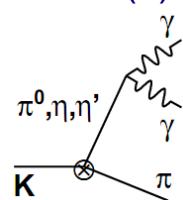
- A: • Loop diagrams  $\Rightarrow$  **cusp** at  $\pi^+ \pi^-$  threshold:  $m_{\gamma\gamma} = 2m_{\pi^+}$



- Tree level **counterterms**  $\Rightarrow \hat{c}$  parameter

Model dependent. Predicted of O(1)

- C: • Poles and tadpoles

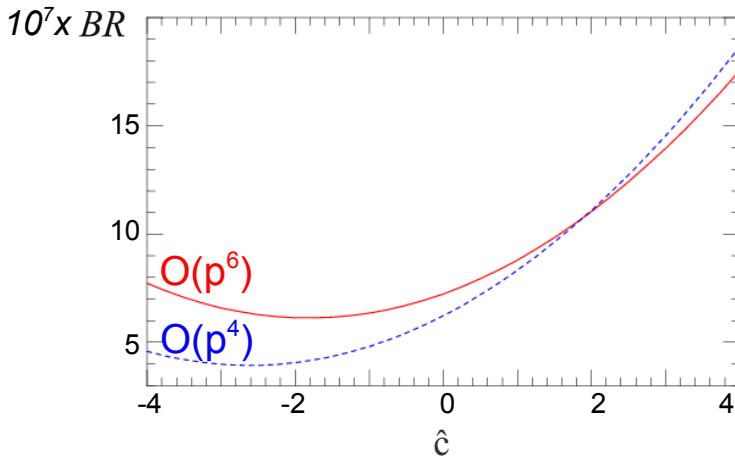
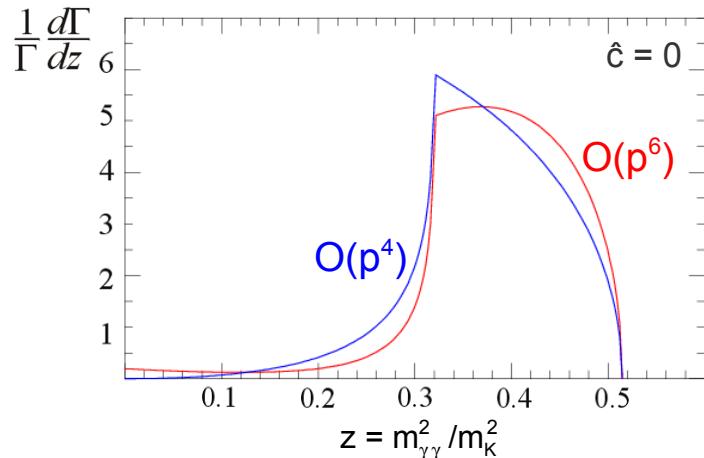


**O( $p^6$ )**

[Nucl.Phys. B386 (1996), 403]

- **Unitarity corrections** by  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  at  $O(p^4)$  can increase BR by 30- 40% and modify spectrum

# $K^\pm \rightarrow \pi^\pm \gamma\gamma$ Decays



Measurement of branching ratio and spectrum determine model dependent  $\hat{c}$  and whether  $O(p^6)$  corrections explain observed rate and shape

**E787 (Brookhaven)** (1997)  $K^\pm \rightarrow \pi^\pm \gamma\gamma$   
selected events: 31 ( $0.16 < z < 0.39$ )

$\hat{c} = 1.8 \pm 0.6$   
 $BR(K^\pm \rightarrow \pi^\pm \gamma\gamma) = (1.10 \pm 0.32) \times 10^{-6}$

**E949 (Brookhaven)** (2005)  $K^\pm \rightarrow \pi^\pm \gamma\gamma$   
No event observed ( $z < 0.04$ )

$BR(K^\pm \rightarrow \pi^\pm \gamma\gamma, z < 0.04) < 8.3 \times 10^{-9}$

**NA48/2 (CERN)** (2008)  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$   
selected events: 120. Full spectrum

$\hat{c} = 0.90 \pm 0.45$