

Can the estimate of $g-2$ be improved?

A HLS based approach

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LPNHE Paris 6/7

OUTLINE

- Model & Method
- Isospin Breaking : **dynamical (ρ, ω, ϕ) mixing**
- Absolute scale of spectra
- Processes to fit
- Global fit
- $\pi\pi$ contribution to $g-2$
- Conclusions and outlook

HLS : A VMD Model

- **The Hidden Local Symmetry model is :**

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

- A unified **VMD** framework including

$e^+e^- \rightarrow \pi\pi / K\bar{K} / \pi\gamma / \eta\gamma / \pi\pi\pi$ & $\tau \rightarrow \pi\pi \nu_\tau$
& **PV** γ couplings & $\eta/\eta' \rightarrow \gamma\pi\pi/\gamma\gamma$ &

- Few parameters : $e, f_\pi, V_{ud}, V_{us}, a (\approx 2), g, \dots$

Still not an operating model : **needs breaking**

- **breaking** $SU(3)/U(3)$

M.Bando *et al.* Phys. Rep. 164 (1988) 217

A. Bramon *et al.* PL B 345 (1995) 263

- **breaking** Isospin Symmetry

M. Benayoun *et al.* EPJ C 55 (2008) 139

HLS : A VMD Model

- **The Hidden Local Symmetry model is :**
 - A unified **VMD** framework including
 $e^+e^- \rightarrow \pi\pi / K\bar{k} / \pi\gamma / \eta\gamma / \pi\pi\pi$ & $\tau \rightarrow \pi\pi \nu_\tau$
& **PV** γ couplings & $\eta/\eta' \rightarrow \gamma\pi\pi/\gamma\gamma$ &
 - Few parameters : $e, f_\pi, V_{ud}, V_{us}, a (\approx 2), g, \dots$
Still not an operating model : **needs breaking**
 - **breaking** $SU(3)/U(3)$: $z_A, z_V, z_T, (x/\theta)$
 - **breaking** Isospin Symmetry : **s-dependent** (8 par.)

The Largest Possible Data Set

- **Overconstrained parametrization provided by:**
 - $e^+e^- \rightarrow \pi\pi$ & 12 $PV\gamma$ decays & $\eta/\eta' \rightarrow \gamma\gamma + \dots$
 - $\pi^0 V\gamma$ & $\eta V\gamma$ couplings $\longrightarrow e^+e^- \rightarrow (\pi^0/\eta)\gamma$ [$/\pi\pi\pi$]
 - ++ $\tau \rightarrow \pi\pi \nu_\tau \rightarrow$ only a constraint \rightarrow should improve param. accuracy & solve the e^+e^-/τ puzzle
 - **Present Limits :** M. Benayoun *et al.* EPJ C 55 (2008) 139
 - ✓ the $\approx \phi$ mass region (1.05 GeV)
 - ✓ No scalars mesons, no ρ' , no ρ''
 - **Analysis Method : Global (over)constrained fit**

Analysis Method

- **A Global Fit to the largest possible data set**
- **Why ?**
- **Check VMD constraints well accepted by DATA?**
(correct lineshapes & yields with Proba. OK)
- **IF YES ↔ theoretical correlations fulfilled by DATA**
→→ improved parameters
- **form factors & fit parameters values & errors & covariance matrix → contributions to g-2**
- **Procedure fully blind to g-2**

Isospin Breaking : Vector Field Mixing

- tree level:: **ideal vector fields \equiv mass *eigenstates***
- At one loop, the HLS Lagrangian piece

$$\left(\rho_I + \omega_I - \sqrt{2} z_V \varphi_I\right) K^{-\vec{\partial}} K^+ + \left(\rho_I - \omega_I + \sqrt{2} z_V \varphi_I\right) K^{0\vec{\partial}} \bar{K}^0$$

induces **transitions among ideal fields**

ideal fields $\not\equiv$ mass *eigenstates*

isospin symmetry breaking :

$$m_{K^\pm} \neq m_{K^0}$$

Transitions at one loop

- Define the loops :

$$\text{Red Oval} = K^+ K^- , \quad \text{Blue Oval} = K^0 \bar{K}^0$$

VVP Lagrangian \Rightarrow K^*K loops // Yang-Mills \Rightarrow $K^*\bar{K}^*$ loops

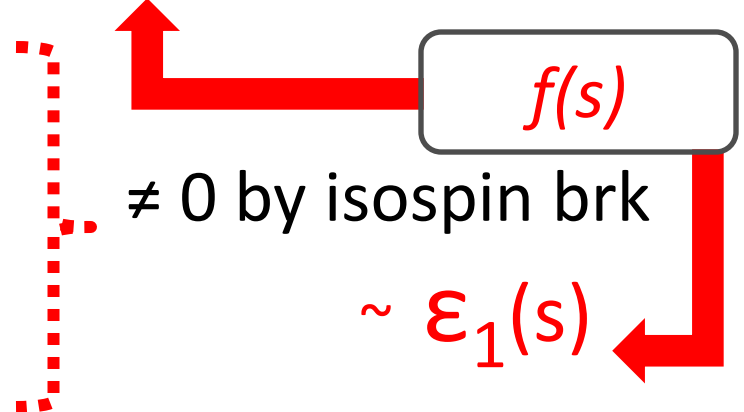
Then, **beside self-masses** :

$$\Pi_{\omega\phi}(s) \rightarrow \text{Red Oval} + \text{Blue Oval} \sim \epsilon_2(s) \neq 0 \text{ always}$$

$$\Pi_{\rho\omega}(s) \rightarrow \text{Red Oval} - \text{Blue Oval}$$

$\neq 0$ by isospin brk

$$\Pi_{\rho\rho}(s) \rightarrow \text{Red Oval} - \text{Blue Oval} \sim \epsilon_1(s)$$



The Mass Matrix Eigen System

$$M^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & \varepsilon_1(s) & -\mu\varepsilon_1(s) \\ \varepsilon_1(s) & m^2 + \varepsilon_2(s) & -\mu\varepsilon_2(s) \\ -\mu\varepsilon_1(s) & -\mu\varepsilon_2(s) & z_V m^2 + \mu^2 \varepsilon_2(s) \end{pmatrix}$$

As :

$$(m^2, \Pi_{\pi\pi}(s)) \gg \varepsilon_2(s) \gg \varepsilon_1(s)$$

Solve perturbatively

$$M^2(s) = M_0^2(s) + \delta M^2(s)$$

From Ideal To Physical Fields

$$\begin{pmatrix} \rho_I^0 \\ \omega_I \\ \varphi_I \end{pmatrix} = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & \gamma \\ -\beta & -\gamma & 1 \end{bmatrix} \begin{pmatrix} \rho^0 \\ \omega \\ \varphi \end{pmatrix}$$

$$R(s+i\varepsilon)\tilde{R}(s+i\varepsilon) = 1$$

Ideal
Fields

$$\begin{bmatrix} \alpha(s) \\ \beta(s) \\ \gamma(s) \end{bmatrix} =$$

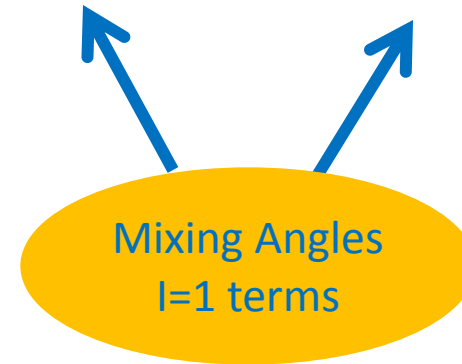
$$\begin{bmatrix} \frac{\varepsilon_1(s)}{\Pi_{\pi\pi}^\rho(s) - \varepsilon_2(s)} \\ \frac{\sqrt{2} z_V \varepsilon_1(s)}{(1 - z_V) m^2 + \Pi_{\pi\pi}^\rho(s) - 2 z_V^2 \varepsilon_2(s)} \\ \frac{\sqrt{2} z_V \varepsilon_2(s)}{(1 - z_V) m^2 + (1 - 2 z_V^2) \varepsilon_2(s)} \end{bmatrix}$$

Physical
Fields

$$+ O(\varepsilon_i^2)$$

V $\pi\pi$ Couplings: $I=1$ part of ω and ϕ

$$\frac{ia g}{2} \rho_I \cdot \pi^- \vec{\partial} \pi^+ \Rightarrow \frac{ia g}{2} [\rho^0 - \alpha(s) \omega + \beta(s) \phi] \cdot \pi^- \vec{\partial} \pi^+$$



*At leading order : **ρ term unchanged** ($I=1$ part)

*small **s -dependent ω and ϕ** ($I=1$ part)

$\gamma - V$ Couplings: $I=0$ part of ρ^0

- In terms of Ideal Fields

$$-e agf_{\pi}^2 \left[\rho_I^0 + \frac{1}{3} \omega_I - \frac{\sqrt{2}}{3} z_V \phi_I \right]_{\mu} \bullet A^{\mu} \Rightarrow$$

- Becomes $\Rightarrow -e \left[f_{\rho}^{\gamma} \rho^0 + f_{\omega}^{\gamma} \frac{1}{3} \omega - f_{\phi}^{\gamma} \frac{\sqrt{2}}{3} z_V \phi \right]_{\mu} \bullet A^{\mu}$

With

$$agf_{\pi}^2 \Rightarrow f_V^{\gamma} \equiv f_V^{\gamma}(s) = agf_{\pi}^2 [1 + O(\epsilon_i(s))] !!!$$

ρ couplings to γ/W

- (γ/W) V transitions :

$$\text{wavy line} \rightarrow \text{black circle} \rightarrow \text{solid line} + \text{wavy line} = \text{wavy line} \rightarrow \text{black line} \rightarrow \text{black circle} + \text{wavy line} \rightarrow \text{white circle} \rightarrow \text{black line}$$

- tree : $l=0$ terms in ρ , $l=1$ terms in ω and ϕ

$$\frac{f_{\rho}^{\gamma}}{f_{\rho}^W} = \left[1 + \frac{\alpha(s)}{3} + \frac{\sqrt{2} z_V}{3} \beta(s) \right] \leftarrow (f_{\rho}^W = a g f_{\pi}^2)$$

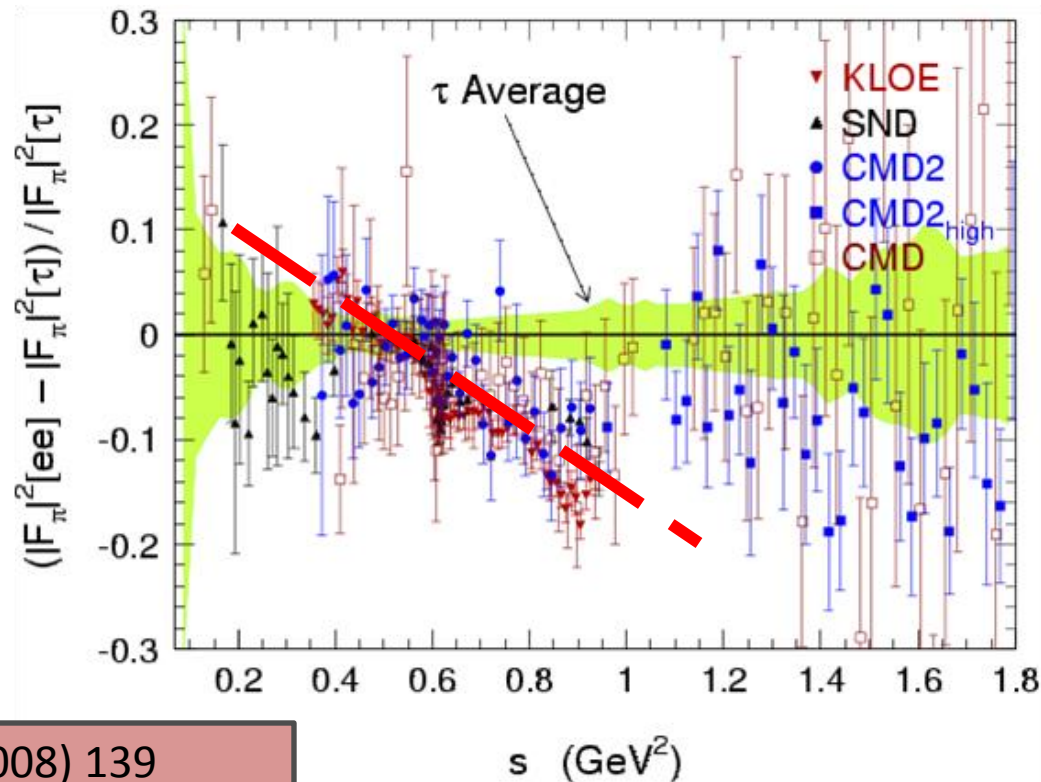
Dipion Mass Spectrum : The Latest Account (2007)

M. Davier NP Proc. Supp. 169 (2007) 288

- **Inv. Mass dependent
(missing) effect !**

Is it isospin breaking?

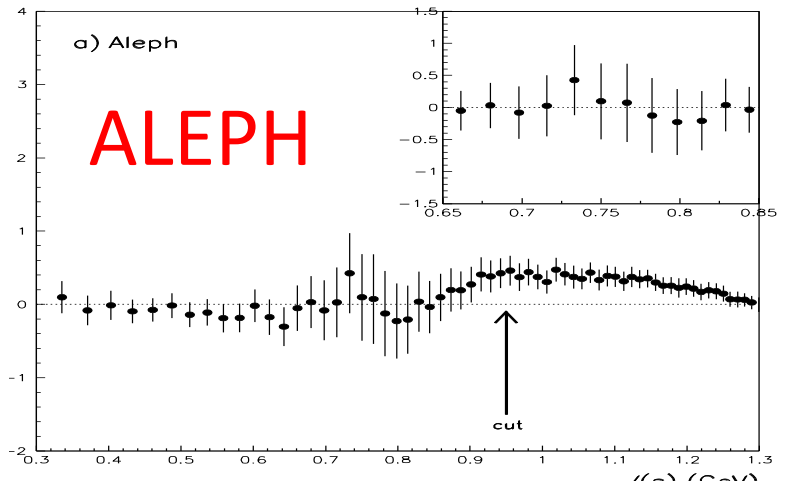
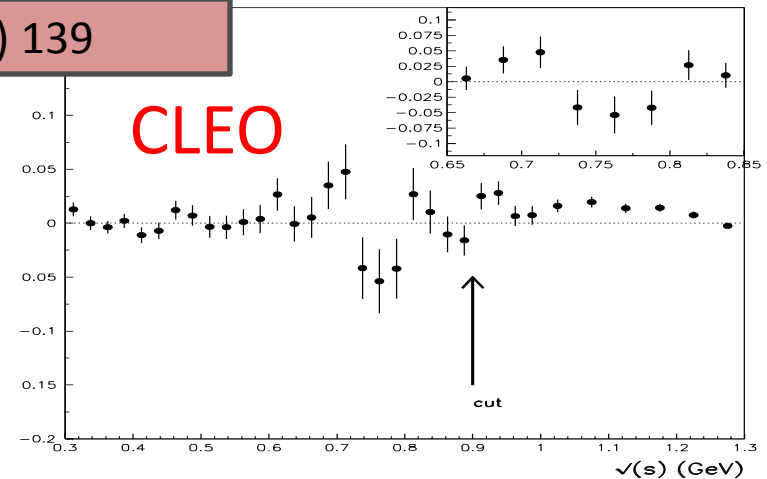
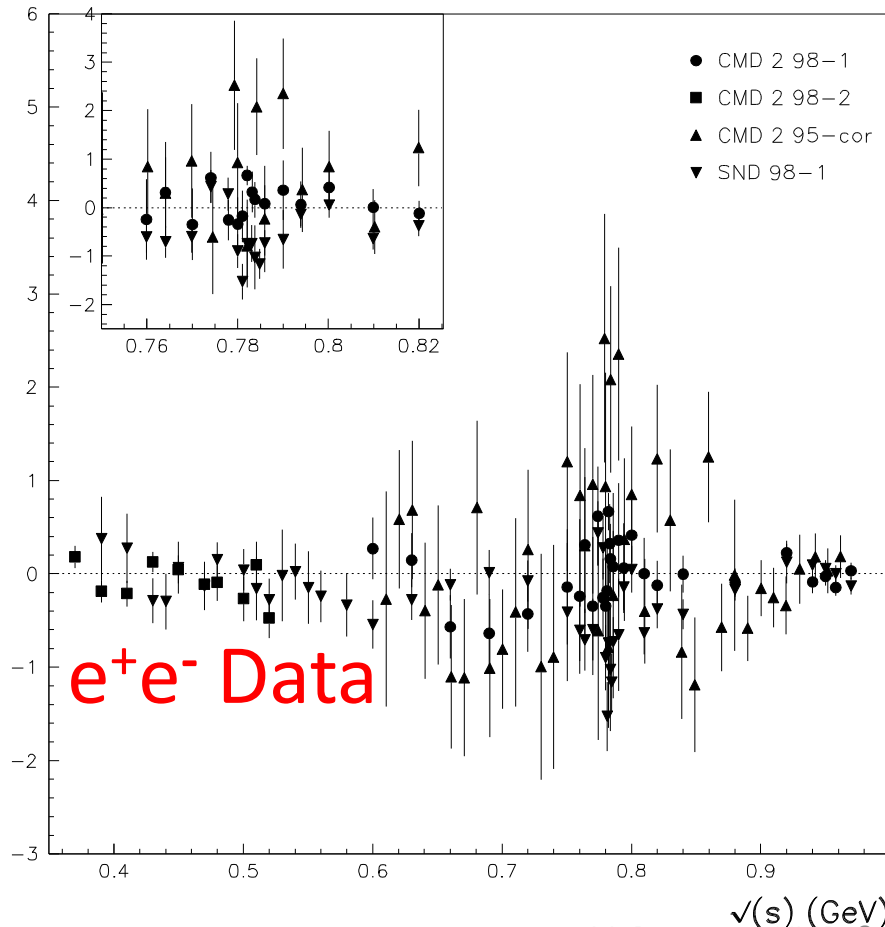
M. Benayoun et al . EPJ C 55 (2008) 139



Result : Structureless Fit Residuals

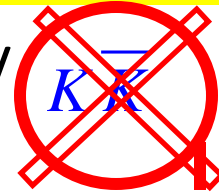
M. Benayoun et al . EPJ C 55 (2008) 139

Residuals



Annihilations/Decays Considered

- Non-Anomalous annihilations : $e^+e^- \rightarrow \pi\pi /$
or decays : $\tau \rightarrow \pi\pi \nu_\tau$



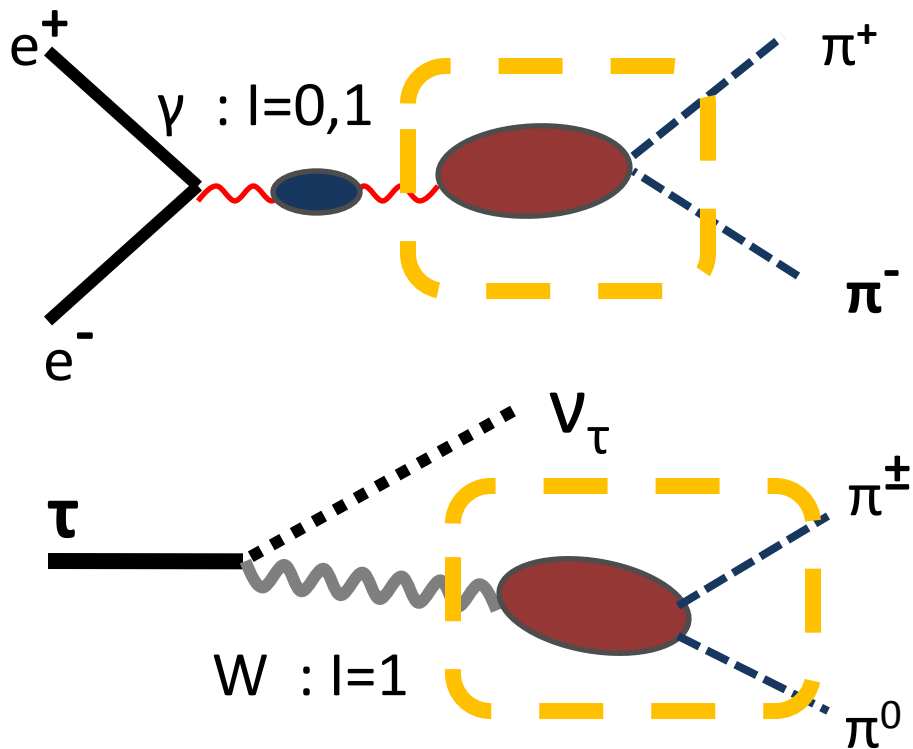
- Anomalous Processes : $e^+e^- \rightarrow \pi\gamma / \eta\gamma / \pi\pi\pi$
(Anomalous Effective Lagrangian pieces)

- Radiative decays widths ($VP\gamma, \eta/\eta' \rightarrow \pi\pi\gamma$)

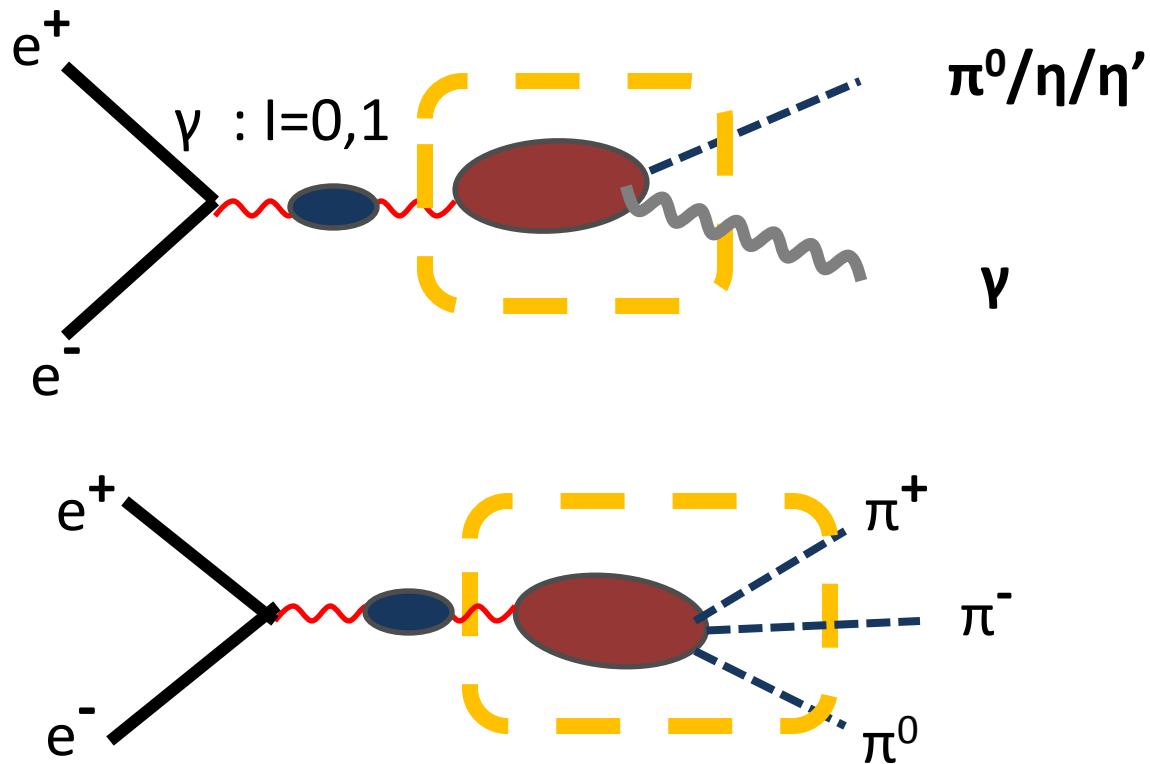
A. Bramon *et al.* Phys. Lett. B486 (406) 2000

OVERCONSTRAINED MODEL & Breaking Scheme

Non-Anomalous Processes



Anomalous Processes



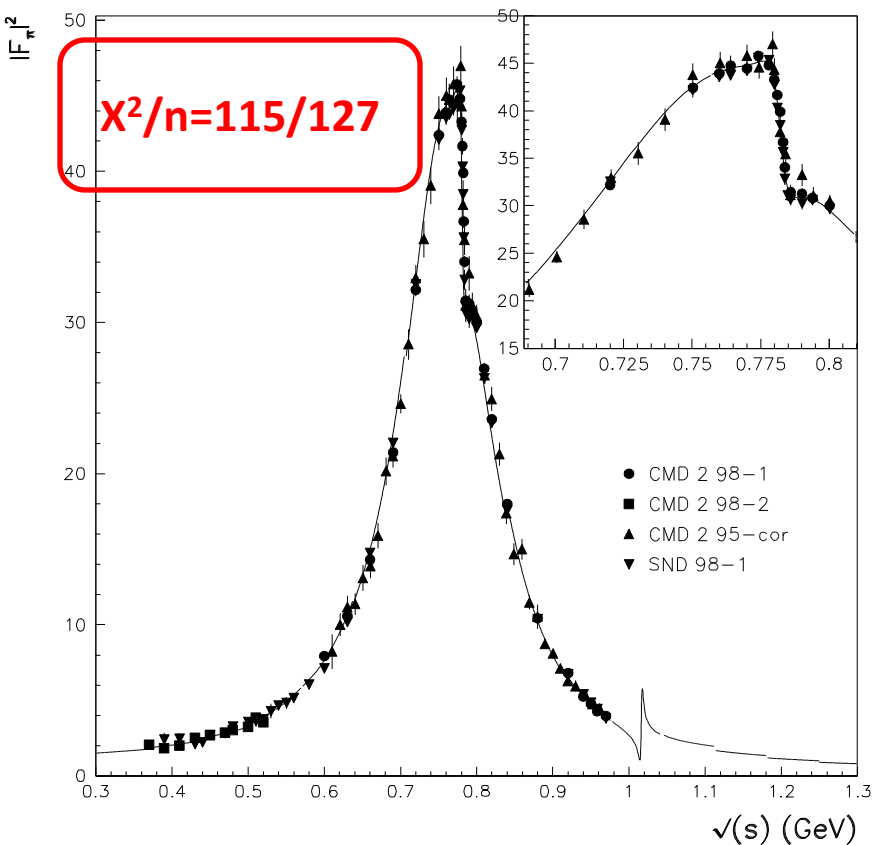
Absolute Magnitude of Spectra

- Absolute Magnitude of spectra for $e^+e^- \rightarrow \pi\pi / \pi\gamma / \eta\gamma / \pi\pi\pi$ & $\tau \rightarrow \pi\pi \nu_\tau$ essentially determined by g
- Absolute Magnitude of Experimental spectra determined by g **AND (Exp) scale uncertainties**
- **Simultaneous treatment of ALL data may allow to fit g and get/check the (best) scales**
(scales constrained by exp. Information)

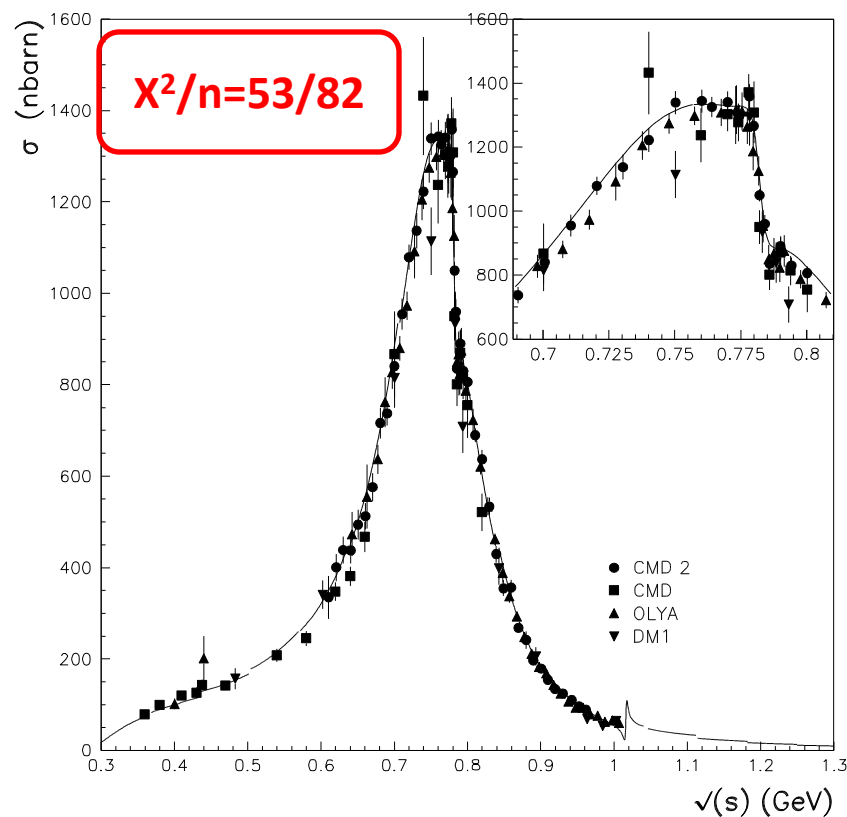
Global Fit : $e^+e^- \rightarrow \pi^+ \pi^-$ (NSK)

M. Benayoun *et al.* EPJ C 55 (2008) 139

FF's New Data

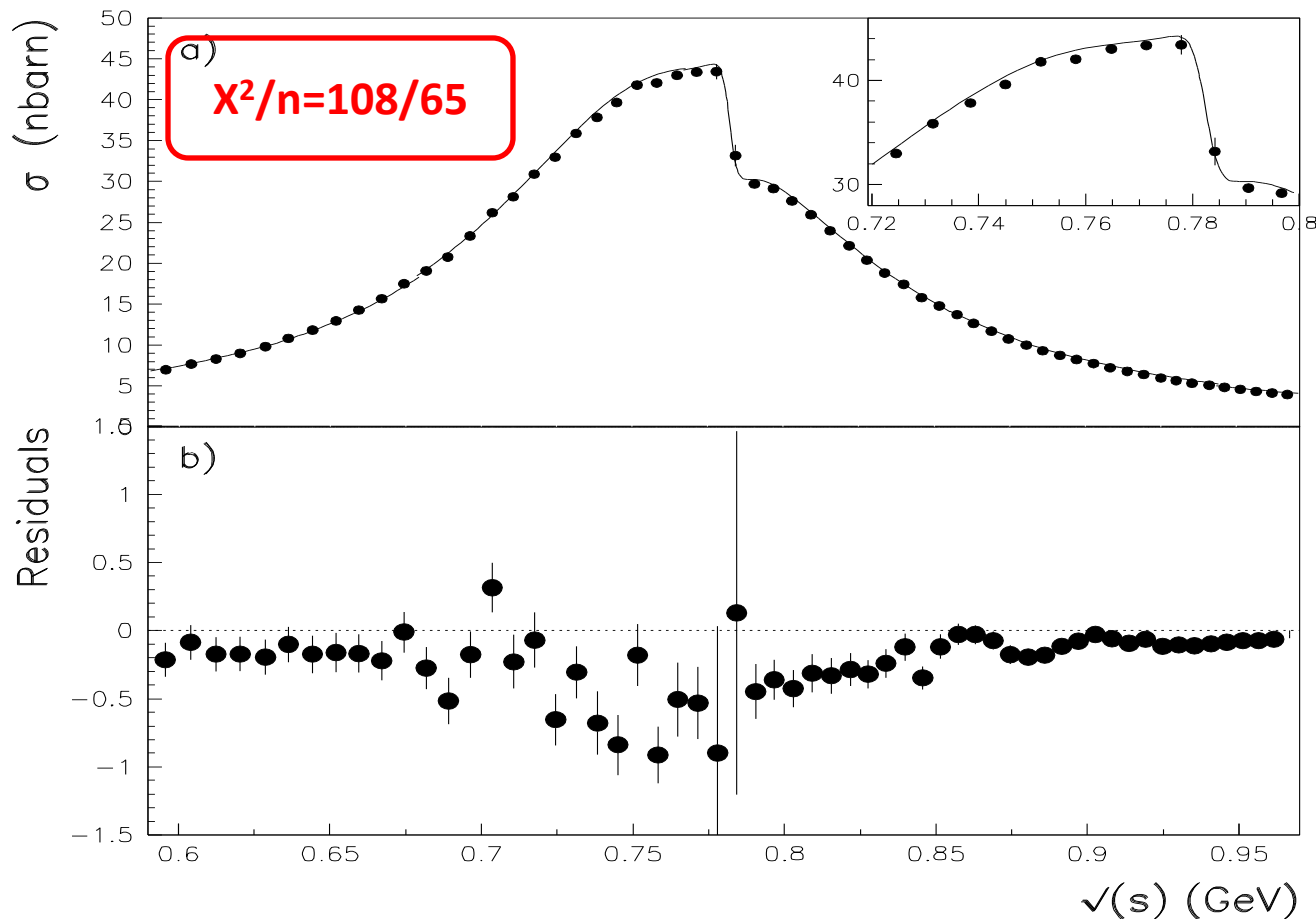


Cross Sections Old Data



Global Fit : $e^+e^- \rightarrow \pi^+ \pi^-$ (KLOE)

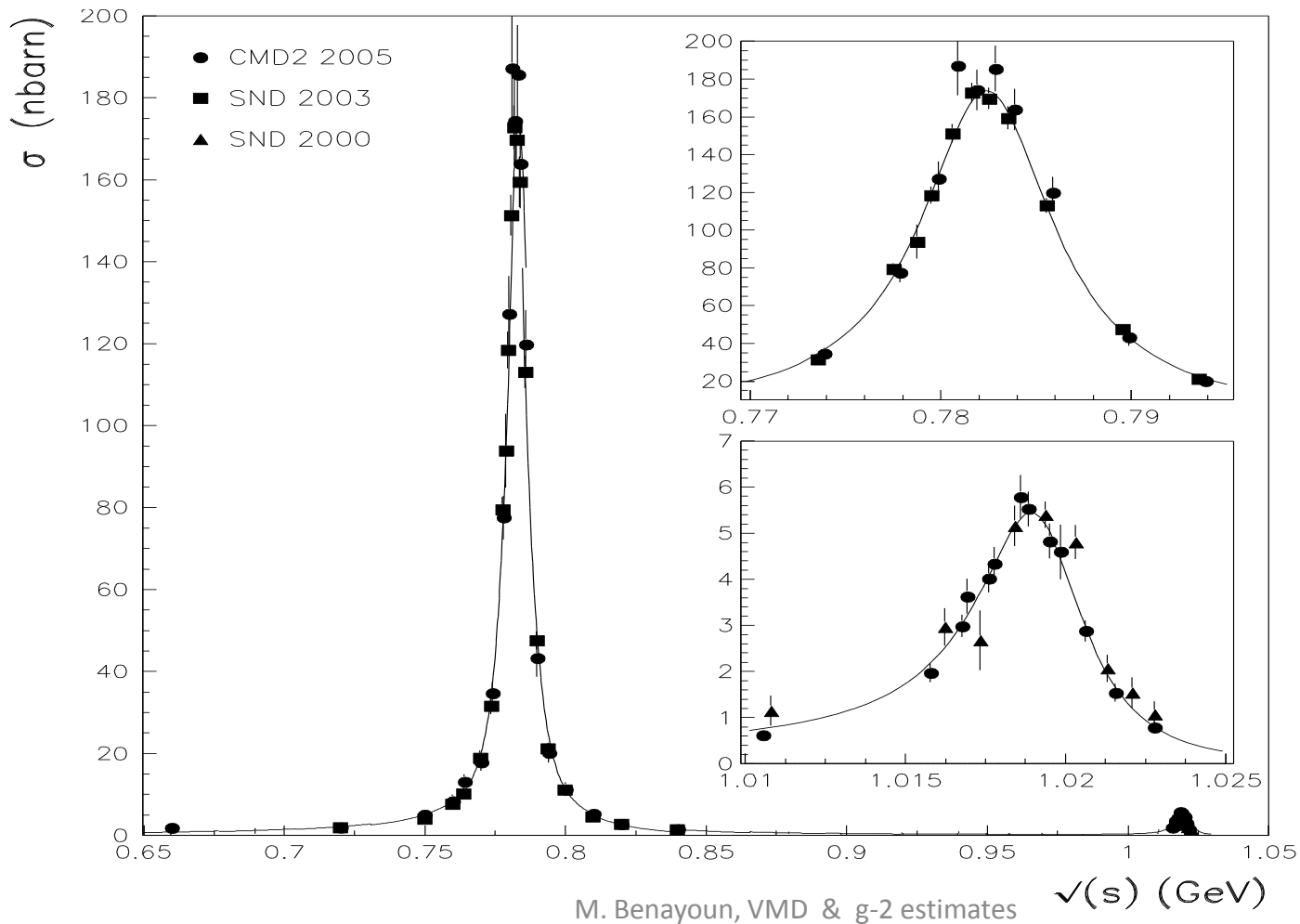
M. Benayoun *et al.* ArXiv 0907.4047



Rescaled

Global Fit : $e^+e^- \rightarrow \pi^0 \gamma$ (NSK)

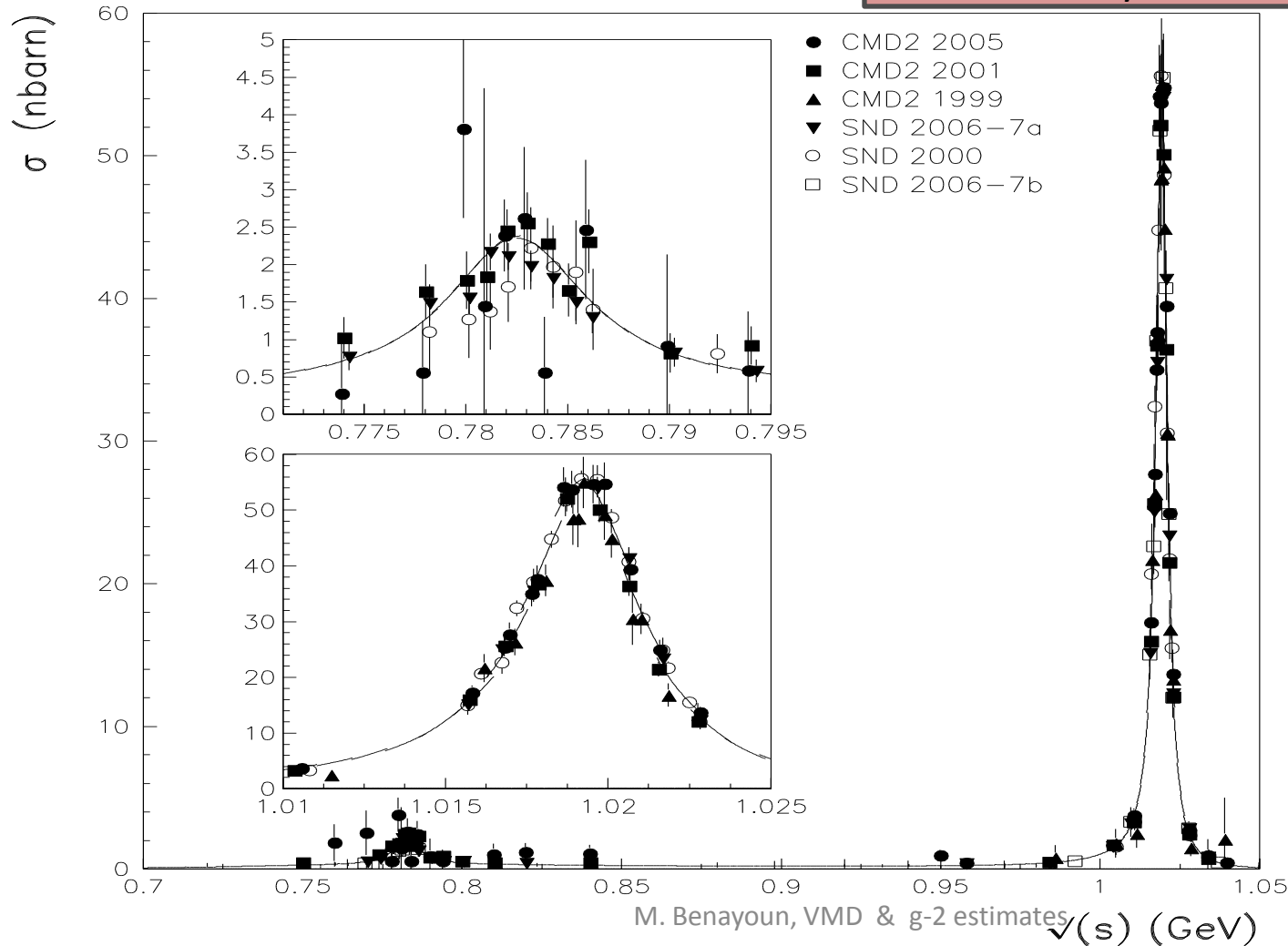
M. Benayoun *et al.* ArXiv 0907.4047



$\chi^2/n=66/86$

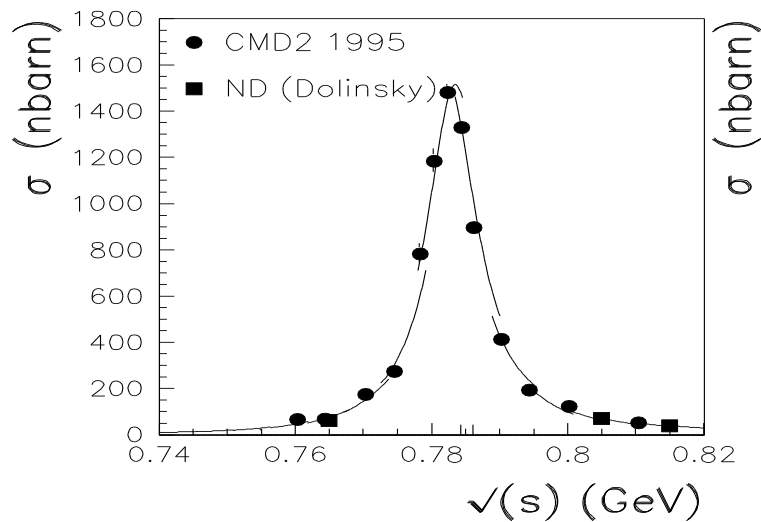
Global Fit : $e^+e^- \rightarrow \eta\gamma$ (NSK)

M. Benayoun *et al.* ArXiv 0907.4047

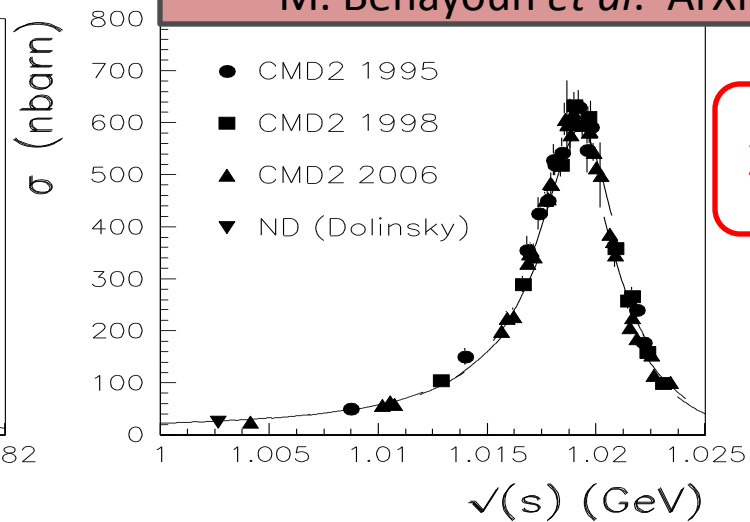


$\chi^2/n=135/182$

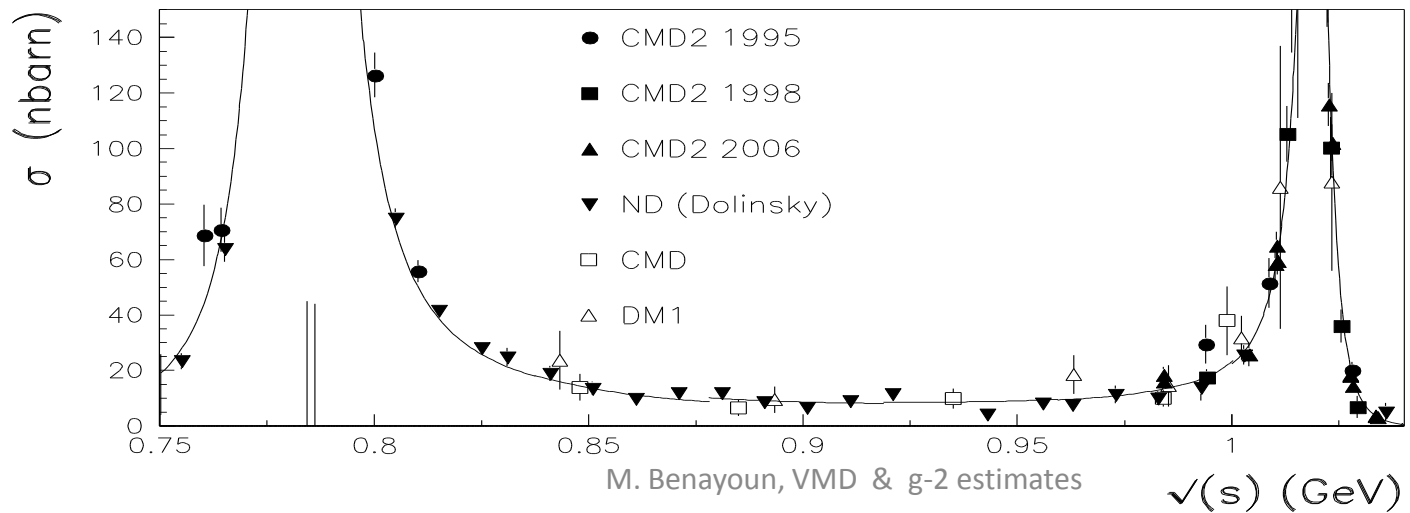
Global Fit : $e^+e^- \rightarrow \pi^+ \pi^- \pi^0$ (NSK)



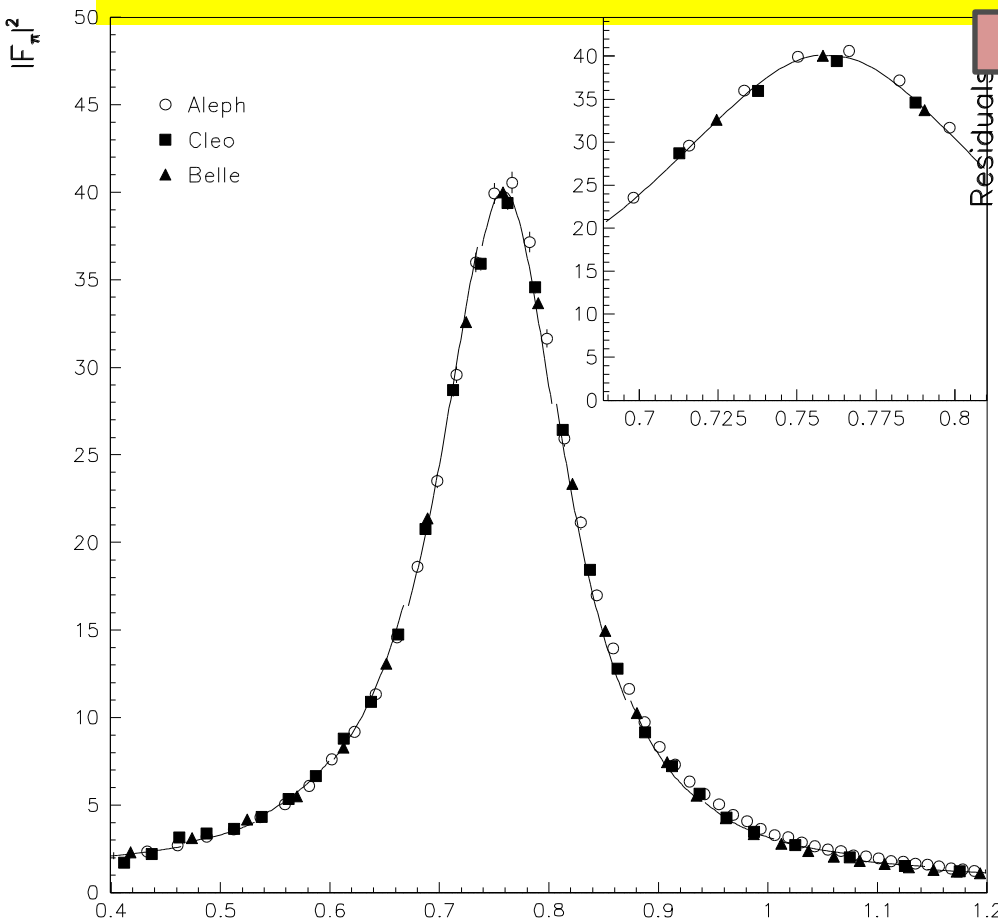
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$\chi^2/n=140/126$

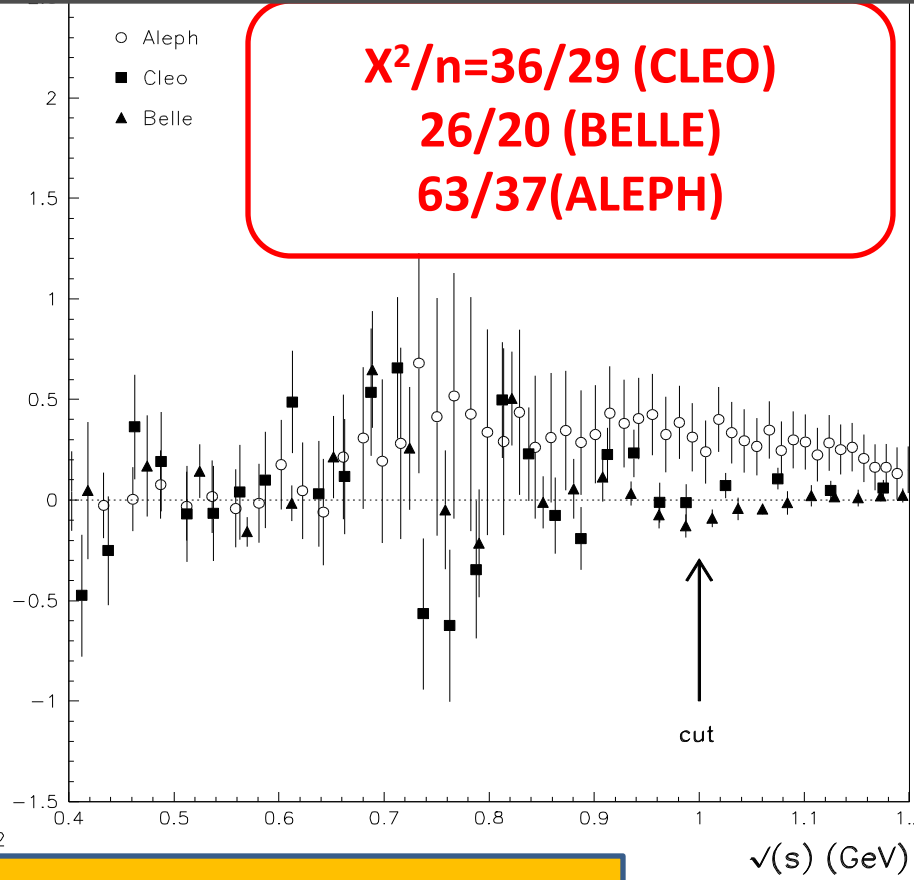


Global Fit : $\tau \rightarrow \pi^c \pi^0$ (CLEO/BELLE/ALEPH)



M. Benayoun *et al.* ArXiv 0907.5603

**$\chi^2/n=36/29$ (CLEO)
 $26/20$ (BELLE)
 $63/37$ (ALEPH)**

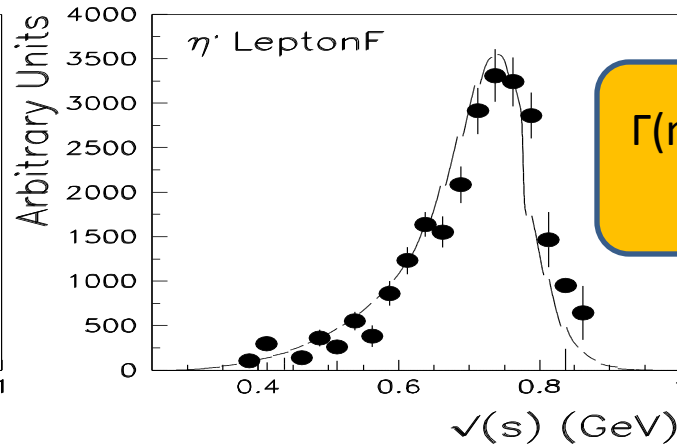
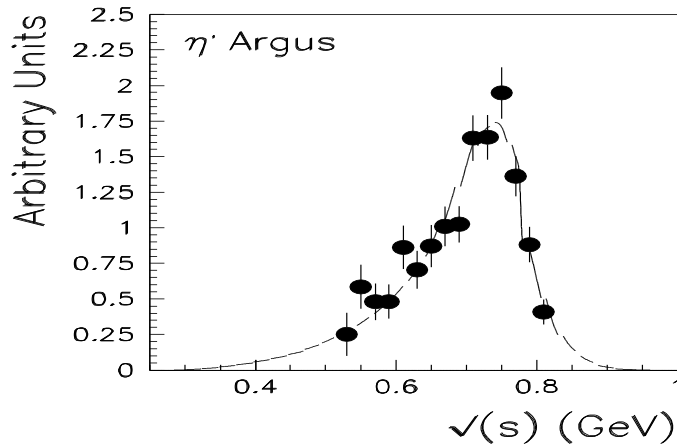


BELLE rescaling (%) :: Fit : $-4.84^{+1.37}_{-0.92}$ Exp: $-2. \pm 1. \pm 4.$

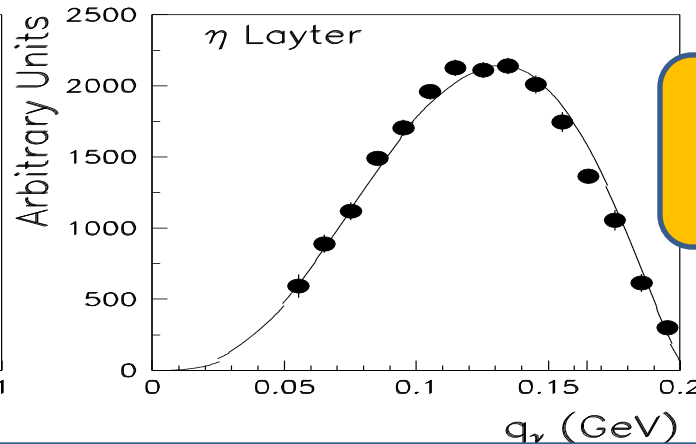
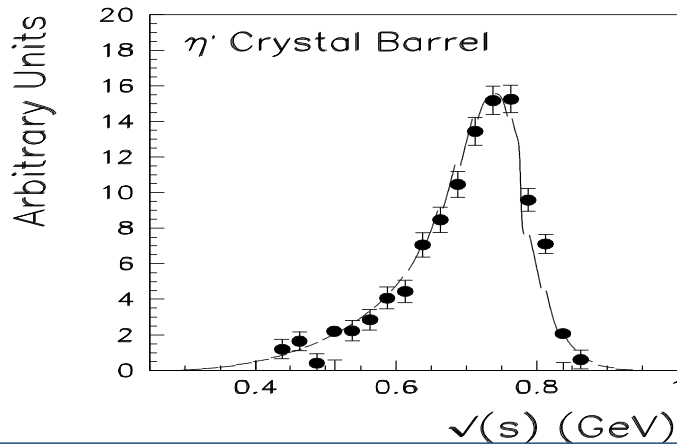
M. Fujikawa *et al.* Phys. Rev. D78 (2008) 072006

SIDE RESULT : Prediction for $\eta/\eta' \rightarrow \pi\pi \gamma$

M. Benayoun *et al.* ArXiv 0907.4047



$\Gamma(\eta' \rightarrow \pi\pi \gamma) = 53.11 \pm 1.47$
PDG : 60 ± 5 keV



$\Gamma(\eta \rightarrow \pi\pi \gamma) = 55.82 \pm 0.83$
PDG : 60 ± 4 eV

Lineshapes and yields : tests for g value and ρ^0 lineshape

Contribution of $\pi\pi$ to $g-2$ ($e^+e^- \rightarrow \pi^+\pi^-$)

Integrated from 0.630 to 0.958 GeV/c ($\times 10^{-10}$)

Data Set	Exp. Value	Reconstructed	χ^2/dof	Prob.
CMD2 (1995)	$362.1 \pm 2.4 \pm 2.2$	362.9 (+3.1/-4.5)_exp	41/42	51%
CMD2 (1998)	$361.5 \pm 1.7 \pm 2.9$	$362.2 \pm 2.1_{\text{exp}}$	38/38	49 %
SND (1998)	$361.0 \pm 1.2 \pm 4.7$	$361.0 \pm 2.1_{\text{exp}}$	26/44	99%
« new » NSK	$360.24 \pm 3.02_{\text{exp}}$	$361.7 \pm 1.3_{\text{exp}}$	126/126	48%
« new » NSK+KLOE	$358.51 \pm 2.41_{\text{exp}}$	$362.1 \pm 1.1_{\text{exp}}$	255/188	--

$e^+e^- \rightarrow \pi^+\pi^-$ & $V\pi\gamma$ & $P\gamma\gamma$

M. Davier *et al* arXiv 0906.5443 (2009)

All FSR Corrected

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++ old timelike		361.1 ± 1.3	178/208	94%
++ (π^0/η) γ		361.2 ± 1.3	372/468	>99%
++($\pi^+ \pi^- \pi^0$)		361.2 ± 1.3	523/597	>99%

M. Davier *et al* arXiv 0906.5443 (2009)
 M. Benayoun, VMD & $g-2$ estimates

$\pi\pi$: FSR Corrected

Contribution of $\pi\pi$ to $g-2$ (II)

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Adding τ data to e^+e^- Xsections

L.O. Contribution of $\pi\pi$ to g-2

Integrated from 0.630 to 0.958 GeV/c ($\times 10^{-10}$)

FSR Corrected

M. Benayoun *et al.* arXiv 0907.5603

Data Sets	a_μ from fit solution ($\times 10^{-10}$)	χ^2 /dof	Probability
All e^+e^- Data (NSK)	$361.2 \pm 1.3_{\text{tot}}$	523/597	99%
+BELLE + CLEO	$362.1 \pm 1.2_{\text{tot}}$	593/646	94 %
+BELLE + CLEO + KLOE	$361.7 \pm 1.1_{\text{tot}}$	729/709	30%
+ ALEPH	$367.2 \pm 1.0_{\text{tot}}$	613/634	72%
+ALEPH + KLOE	$366.7 \pm 0.9_{\text{tot}}$	364/697	4%
+ ALEPH+ BELLE + CLEO	$367.8 \pm 1.0_{\text{tot}}$	681/683	51%

NSK($\pi\pi$) +KLOE+BaBar : $360.8 \pm 2.0_{\text{tot}}$

// BaBar : $365.2 \pm 2.7_{\text{tot}}$

M. Davier *et al.* arXiv 0908.4300

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M. Davier *et al.* arXiv 0908.4300

Adding τ data to e^+e^- Xsections

L.O. Contribution of $\pi\pi$ to $g-2$
 Integrated from 0.630 to 0.958 GeV/c ($\times 10^{-10}$)

FSR Corrected

M. Benayoun *et al.* arXiv 0907.5603

Data Sets	a_μ from fit solution ($\times 10^{-10}$)	χ^2 / dof	Probability
All e^+e^- Data (NSK)	$361.2 \pm 1.3_{\text{tot}}$	523/597	99%
+BELLE + CLEO	$362.1 \pm 1.2_{\text{tot}}$	593/646	94%
+BELLE + CLEO + KLOE	$361.7 \pm 1.1_{\text{tot}}$	729/709	30%
+ ALEPH	$367.2 \pm 1.0_{\text{tot}}$	613/634	72%
+ALEPH + KLOE	$366.7 \pm 0.9_{\text{tot}}$	364/697	4%
+ ALEPH+ BELLE + CLEO	$367.8 \pm 1.0_{\text{tot}}$	681/683	51%

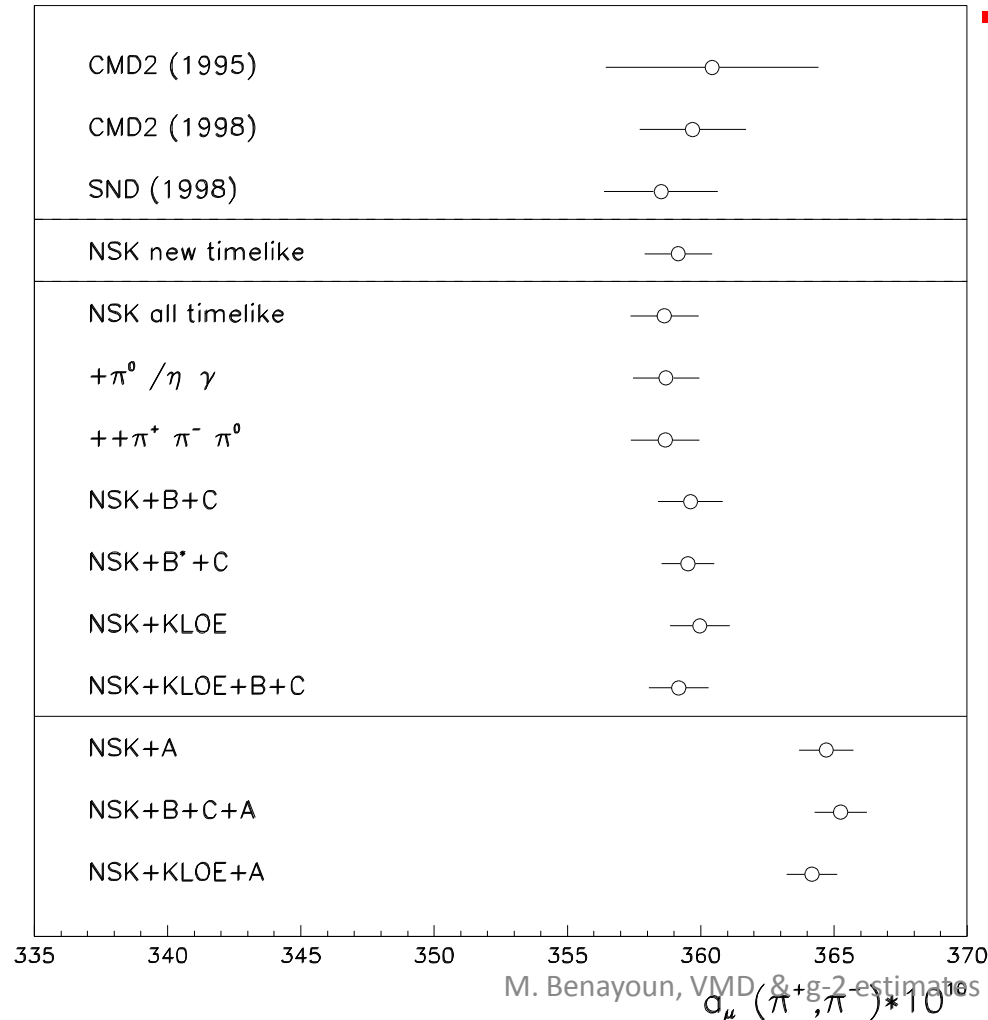
NSK($\pi\pi$) +KLOE+BaBar : $360.8 \pm 2.0_{\text{tot}}$ // BaBar : $365.2 \pm 2.7_{\text{tot}}$

M. Davier *et al.* arXiv 0908.4300

Contribution of $\pi\pi$ to $g-2$

Integrated from 0.630 to 0.958 GeV/c ($\times 10^{-10}$)

$\pi\pi$: NOT FSR Corrected

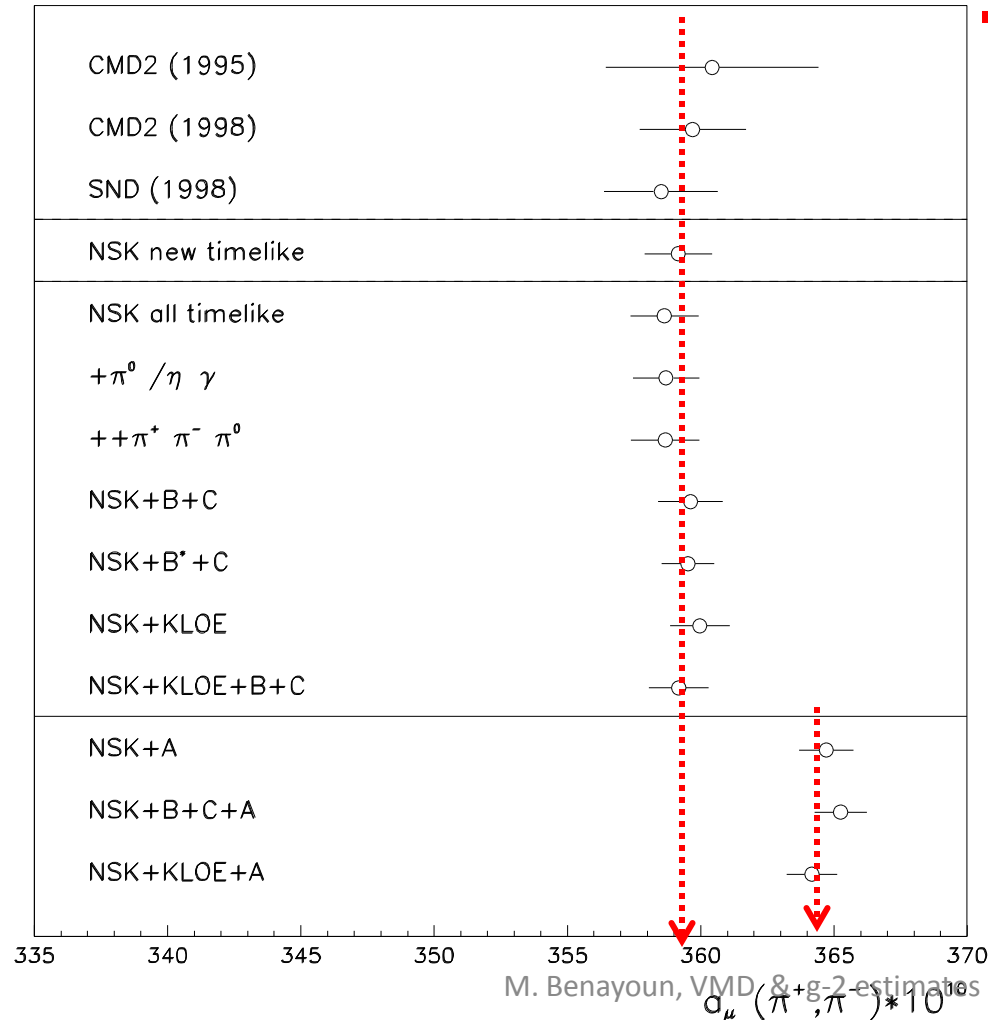


A=ALEPH
B=BELLE
C=CLEO

Contribution of $\pi\pi$ to $g-2$

Integrated from 0.630 to 0.958 GeV/c ($\times 10^{-10}$)

$\pi\pi$: NOT FSR Corrected



A=ALEPH
B=BELLE
C=CLEO

Conclusion and Outlook

Do VMD physics correlations improve $g-2$? **YES**

- Better systematics in $e^+e^- \rightarrow \pi\gamma/\eta\gamma/\pi\pi\pi$ may improve more the $\pi\pi$ contribution to $g-2$
- **Conversely** : $e^+e^- \rightarrow \pi\pi$ may improve $\pi\gamma/\eta\gamma/\pi\pi\pi$
- Understanding the $e^+e^- \rightarrow K\bar{K}$ puzzle too! (ϕ region)
- Extend VMD

Backup Slides

VMD :: The HLS Framework

- **Hidden Local Symmetry Model (HLS) as a whole**

M.Bando, T. Kugo & K. Yamawaki Phys. Rep. 164 (1988) 217

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

M.Benayoun & H.O'Connell PR D 58 (1998) 074006

→ $\mathcal{L}_A, \mathcal{L}_V, \mathcal{L}_{YM}, \mathcal{L}_{anomalous}$

(π/K form factors, $VPPP, \gamma PPP, VP\gamma$ & $P\gamma\gamma$ couplings)

- **BKY-like flavor U(3)/SU(3) breaking mechanisms**

M.Bando, T. Kugo & K. Yamawaki Phys. Rep. 164 (1988) 217

A. Bramon, A. Grau & G. Pancheri PL B 345 (1995) 263


M. Benayoun *et al.* PR D 59 (1999) 114027

G. Morpurgo PR D 42 (1990) 1497

M. Benayoun, L. DelBuono & H. O'Connell EPJ C 17 (2000) 593

M. Benayoun *et al.* EPJ C 55 (2008) 139

The Hidden Local Symmetry Model : The Non-Anomalous Sector

- Define $\xi_{L/R} = e^{[\mp i P / f_\pi]}$  PS field matrix
- Define covariant derivatives $D_\mu \xi_L, D_\mu \xi_R$
- Then $L/R = D_\mu \xi_{L/R} \xi_{L/R}^\dagger$ and $L_{A/V} = -\frac{f_\pi^2}{4} \text{Tr}[L \mp R]^2$
- The Full HLS Lagrangian : $L_{HLS} = L_A + a L_V + L_{YM}$

Expanded form: M.Benayoun & H.O'Connell PR D 58 (1998) 074006

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} - ig V_\mu \xi_{L/R} + i \xi_{L/R} G_{L/R}$$


Universal Vector Coupling

The Covariant Derivatives

- Covariant derivatives \neq for left- right- ξ fields :

$$D_{\mu} \xi_{L/R} = \partial_{\mu} \xi_{L/R} - ig V_{\mu} \xi_{L/R} + i \xi_{L/R} G_{L/R}$$

- With :

$$G_R = eQA_{\mu} \quad , \quad G_L = eQA_{\mu} + \frac{g_2}{\sqrt{2}} (W_{\mu}^{+} T_{+} + W_{\mu}^{-} T_{-})$$

T_{\pm} is CKM matrix reduced to V_{us} and V_{ud} terms

Anomalous Annihilations/Decays

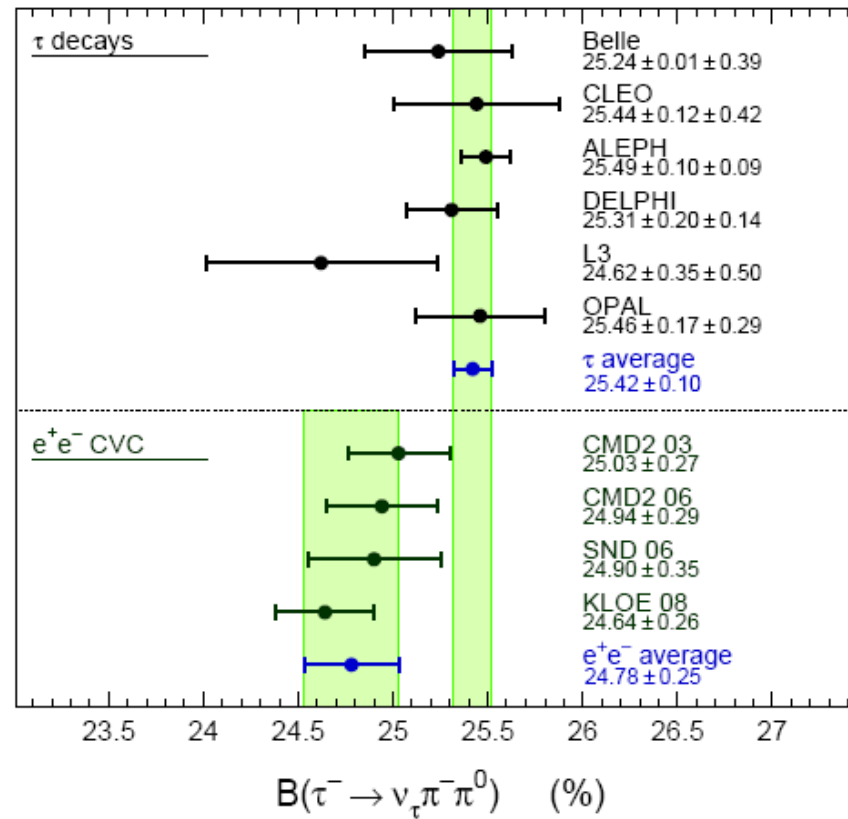
- Non-Anomalous annihilations : $e^+e^- \rightarrow \pi\pi / K\bar{K}$
or decays : $\tau \rightarrow \pi\pi \nu_\tau$
- Anomalous Processes : $e^+e^- \rightarrow \pi\gamma/\eta\gamma/\pi\pi\pi$
:: Other Effective Lagrangian pieces

$$L_{VVP} = -\frac{N_c g^2}{4\pi^2 f_\pi} c_3 \varepsilon^{\mu\nu\alpha\beta} \text{Tr} \left[\partial_\mu V_\nu \partial_\alpha V_\beta \right] \rightarrow L_{AVP}(g)$$

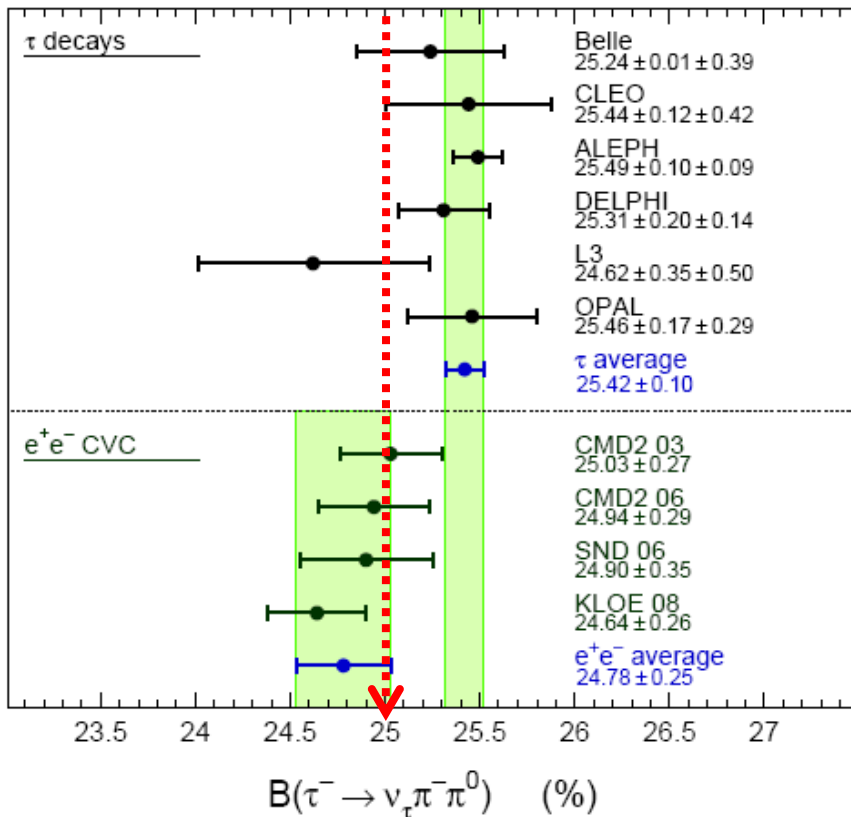
$$L_{VPPP} = -i \frac{N_c g}{4\pi^2 f_\pi^3} (c_1 - c_2 - c_3) \varepsilon^{\mu\nu\alpha\beta} \text{Tr} \left[V_\mu \partial_\nu P \partial_\alpha P \partial_\beta P \right]$$

$$+ (1 - c_4) L_{AAP}(e^2) + \left[1 - \frac{3}{4} (c_1 - c_2 + c_4) \right] L_{APPP}(e)$$

Br ($\tau \rightarrow \pi\pi\nu_\tau$)



$Br(\tau \rightarrow \pi\pi\nu_\tau)$



e^+e^- (incl KLOE): 24.78 ± 0.25
 e^+e^- (excl KLOE): 24.90 ± 0.28

T (incl. ALEPH) : 25.42 ± 0.10
 dist/ σ = 2.26

T (excl. ALEPH) : 25.30 ± 0.16
 dist/ σ = 1.63 (KLOE)
 dist/ σ = 1.25 (KLOE)

BELLE/ALEPH/CLEO

