New Results on the Hadronic Vacuum Polarization Contribution to the Muon g-2

Michel Davier (LAL – Orsay)

- the muon magnetic anomaly
- revisited τ spectral functions: Belle + updated corrections
- ee spectral functions after KLOE and BaBar
- combination of all ee data
- discussion and perspectives

Most recent data and analyses

- $\tau \to \pi\,\pi^0\,\nu_\tau$ data from Belle PRD 78 (2008) 072006

- $e^+ e^- \to \pi^+ \pi^-$ data
  - KLOE PLB 670 (2009) 285
  - BaBar arXiv:0908.3589v1

- updated τ-based analysis arXiv:0906.5443v2

  MD, A. Hoecker, G. Lopez Castro, B. Malaescu, X.H.Mo, G. Toledo Sanchez, P. Wang, C.Z. Yuan, Z. Zhang

- updated ee-based analysis arXiv:0908.4300v1

  MD, A. Hoecker, B. Malaescu, C.Z. Yuan, Z. Zhang

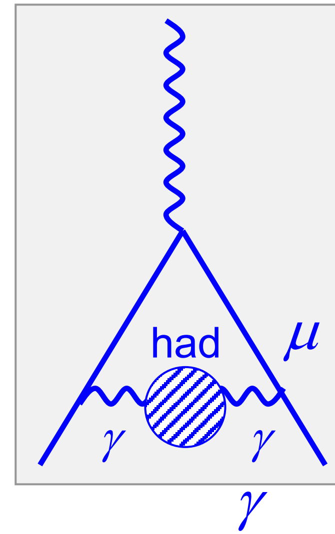
Hadronic Vacuum Polarization and Muon $(g-2)_\mu$

Dominant uncertainty from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on e+e−→hadrons cross section

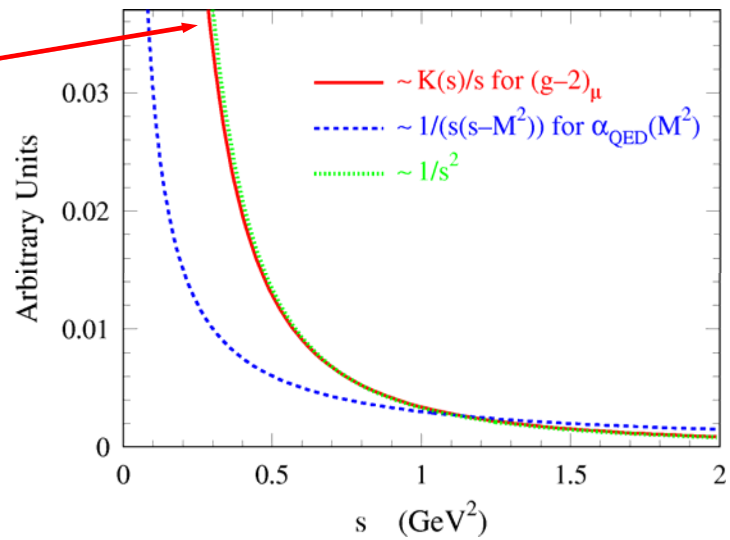
Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

$$12\pi \, \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0[e^+e^- \to \mathrm{hadrons}(\gamma)]}{\sigma_{pt}} \equiv R(s)$$



$$a_\mu^{\mathrm{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R(s)$$

Dispersion relation



The E-821 a_μ Measurement at BNL Updated

a_μ measured from a ratio of frequencies

$\omega_a = \omega_{precession} - \omega_{cyclotron}$ $\quad\quad$ $\omega_{precession} = \omega_L + \omega_T$ $\quad\quad$ $\omega_a = a_\mu \frac{eB}{m_\mu}$

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\widetilde{\omega}_p}{\omega_L/\widetilde{\omega}_p - \omega_a/\widetilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

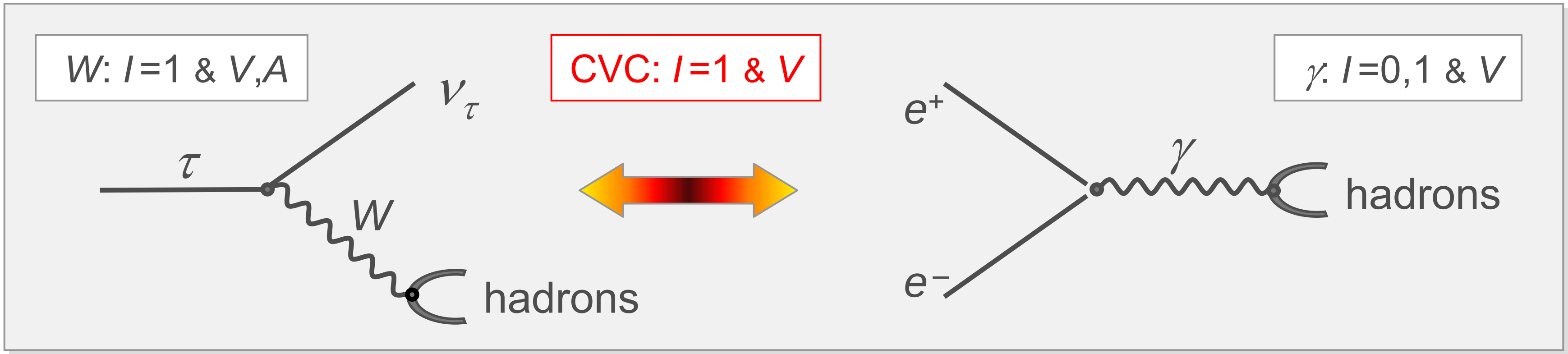
$\lambda = \omega_L / \omega_p = \mu_\mu / \mu_p$ from muonium hyperfine splitting

value used by E-821 3.18334539(10)
new value 3.183345137(85) Mohr et al., RMP 80 (2008) 633

$\Rightarrow$ change in a_μ $(+0.92\ 10^{-10})$
(see next review in RPP2009 (Hoecker-Marciano)

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 5.4 \pm 3.3)\ 10^{-10}$ updated
(± 6.3) (0.54 ppm)

The Role of τ Data through CVC – SU(2)



Hadronic physics factorizes (spectral Functions)

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{BR\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{BR\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \; \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \; \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions — mass spectrum — kinematic factor (PS)

SU(2) Breaking

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contrib.:

- Electroweak radiative corrections:
  - dominant contribution from short distance correction S_{EW}
  - subleading corrections (small)
  - long distance radiative correction $G_{\mathrm{EM}}(s)$

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02
Lopez Castro et al.' 06

- Charged/neutral mass splitting:

Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

  - $m_{\pi-} \neq m_{\pi 0}$ leads to phase space (cross sec.) and width (FF) corrections
  - ρ-ω mixing (EM $\omega \to \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho-} \neq m_{\rho 0}$ *** and $\Gamma_{\rho-} \neq \Gamma_{\rho 0}$ ***

Flores-Baez-Lopez Castro' 08
Davier et al.'09

- Electromagnetic decays: $\rho \to \pi\pi\gamma$***, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$
- Quark mass difference $m_u \neq m_d$ (negligible)

Situation at ICHEP'06 / 08

$a_\mu^{had}[ee] = (690.9 \pm 4.4) \times 10^{-10}$

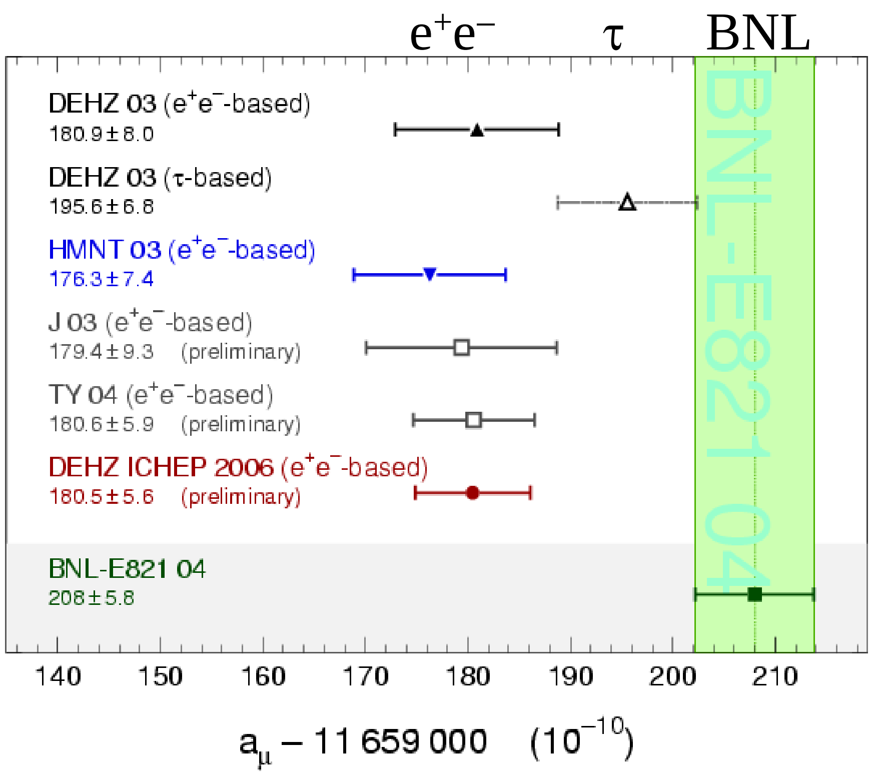
$a_\mu[ee] = (11\,659\,180.5 \pm 4.4_{had} \pm 3.5_{LBL} \pm 0.2_{QED+EW}) \times 10^{-10}$

Hadronic HO	$-(9.8 \pm 0.1) \times 10^{-10}$
Hadronic LBL	$+(12.0 \pm 3.5) \times 10^{-10}$
Electroweak	$(15.4 \pm 0.2) \times 10^{-10}$
QED	$(11\,658\,471.9 \pm 0.1) \times 10^{-10}$

Knecht-Nyffeler (2002), Melnikov-Vainhstein (2003)

Davier-Marciano (2004)

Kinoshita-Nio (2006)



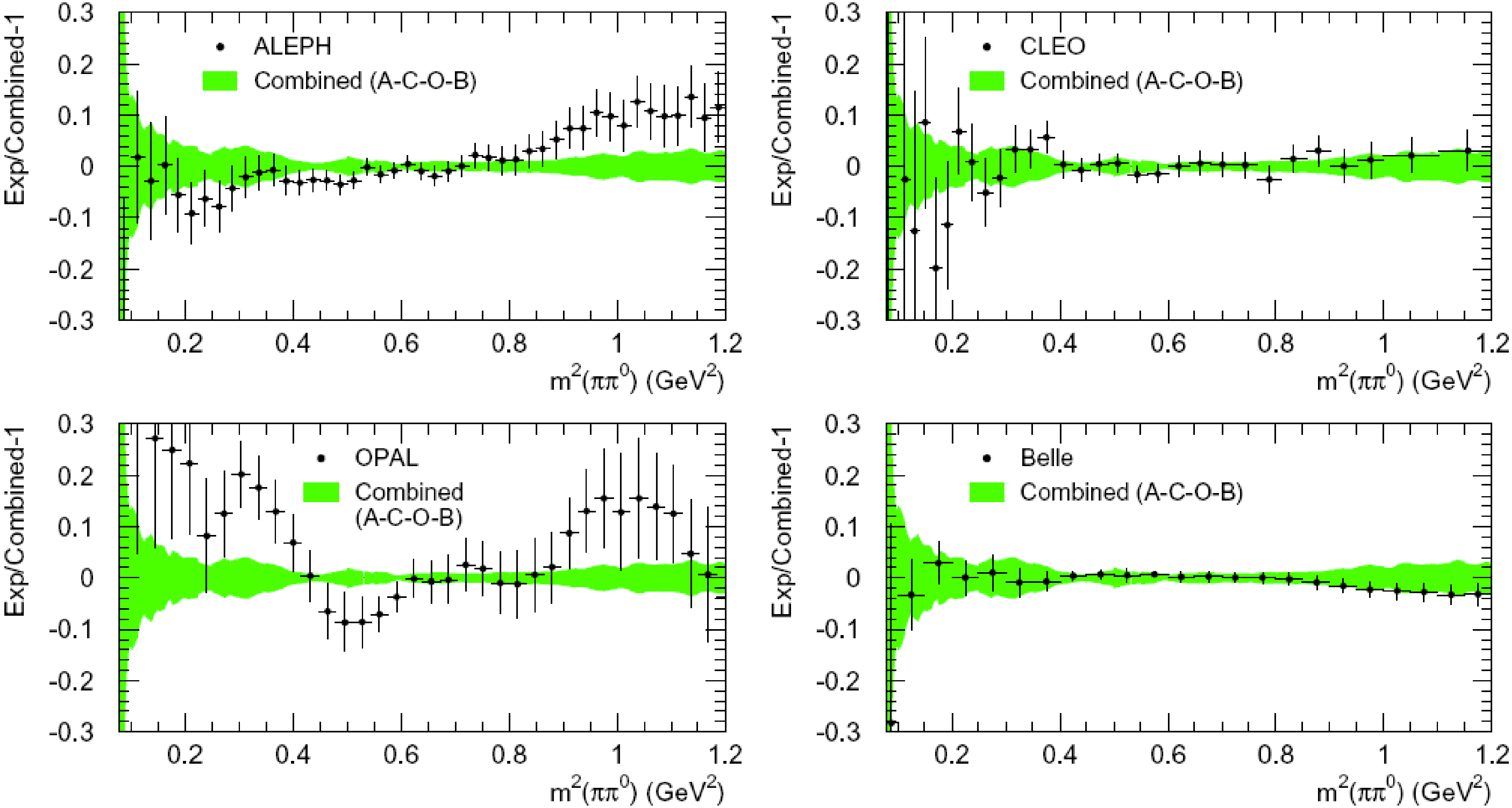
Observed Difference with BNL using e^+e^-:

$a_\mu[\exp] - a_\mu[SM] = (27.5 \pm 8.4) \times 10^{-10}$

➡ 3.3 „standard deviations"

But estimate using τ data consistent with E-821 !

Revisited Analysis using τ Data: including Belle



Test of the spectral function shapes from different experiments: WA BR used

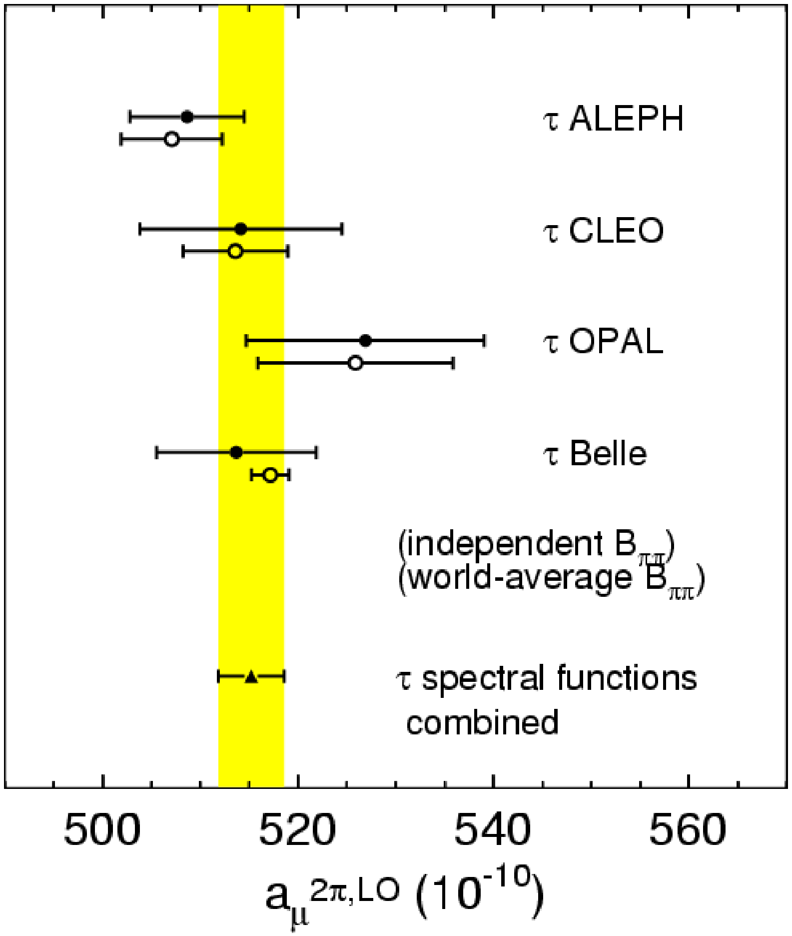
Revisited Analysis τ Data: new IB corrections

talk by G. Lopez Castro

Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model
$S_{\rm EW}$	-12.21 ± 0.15	
$G_{\rm EM}$	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	-16.07 ± 1.85	

disagreement with Maltman-Wolfe arXiv:0908.2391

Consistency of τ Data: Dispersion Integrals



- using BR from each experiment makes results independent from each other
- consistent results

- using WA BR checks consistency for the spectral function shapes

- WA BR + combined spectral function $\Rightarrow$

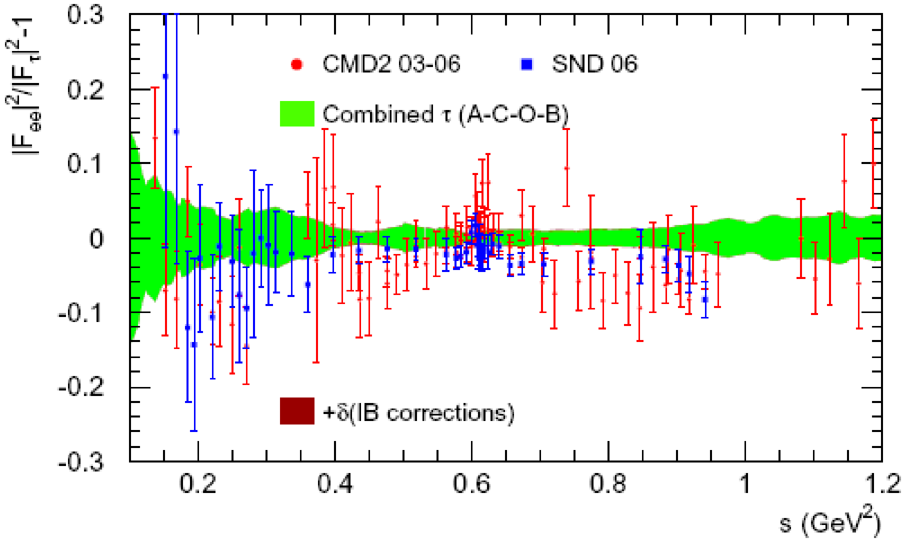
$a_\mu^{2\pi,LO}=(515.2\pm2.0_{exp}\pm0.9_{Be}\pm2.1_{B\pi\pi}\pm1.6_{IB})\ 10^{-10}$

- 0.7% precision

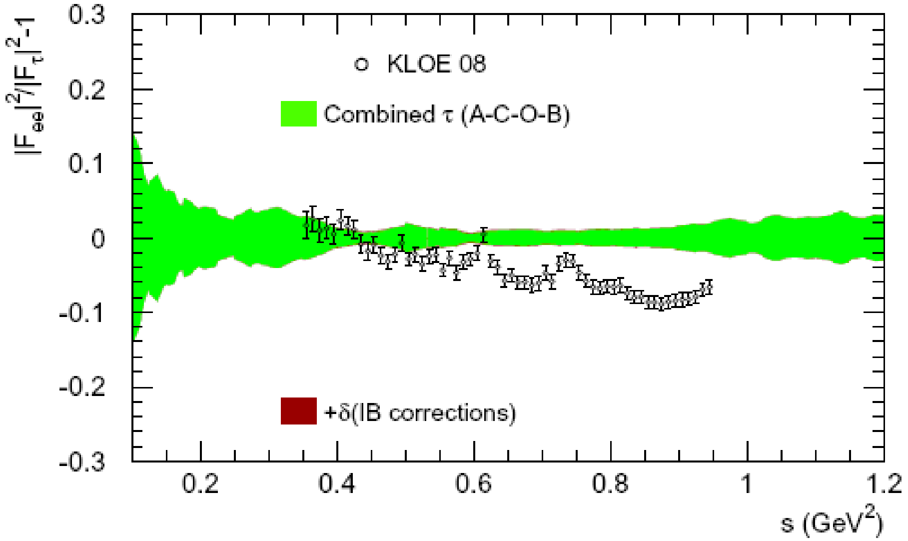
Comparison of ee and τ Data Revisited (1)

Relative comparison of IB-corrected τ and ee spectral functions (τ green band)

CMD-2, SND



KLOE



⇒ better agreement than before with CMD2-SND

⇒ strong disagreement with KLOE : slope…

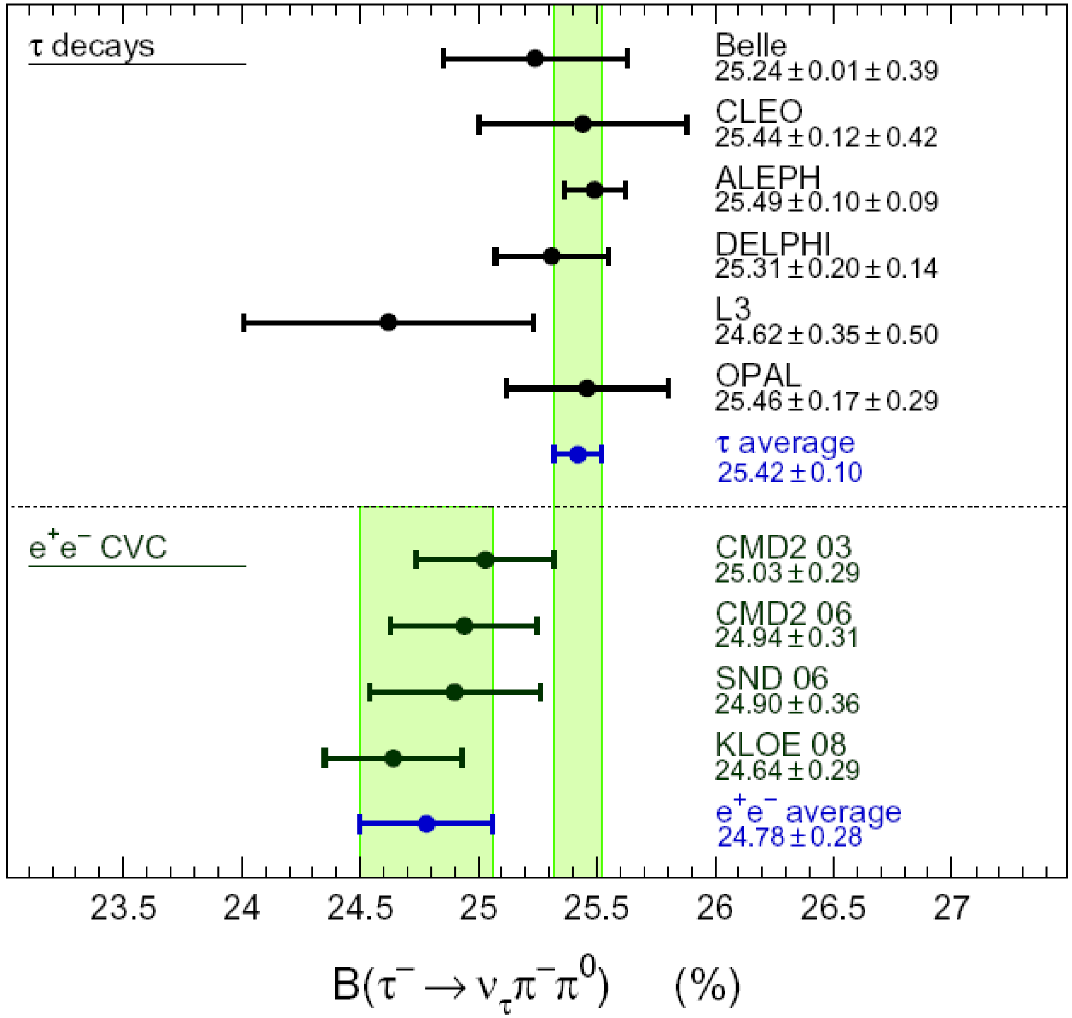
Comparison of ee and τ Data Revisited (2)

Global test of spectral functions:
prediction of τ BR using ee data

$$\mathcal{B}_X^{\mathrm{CVC}} = \frac{3}{2}\frac{\mathcal{B}_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\min}}^{m_\tau^2} ds\, s\, \sigma_{X^0}^I \left(1-\frac{s}{m_\tau^2}\right)^2 \left(1+\frac{2s}{m_\tau^2}\right)$$

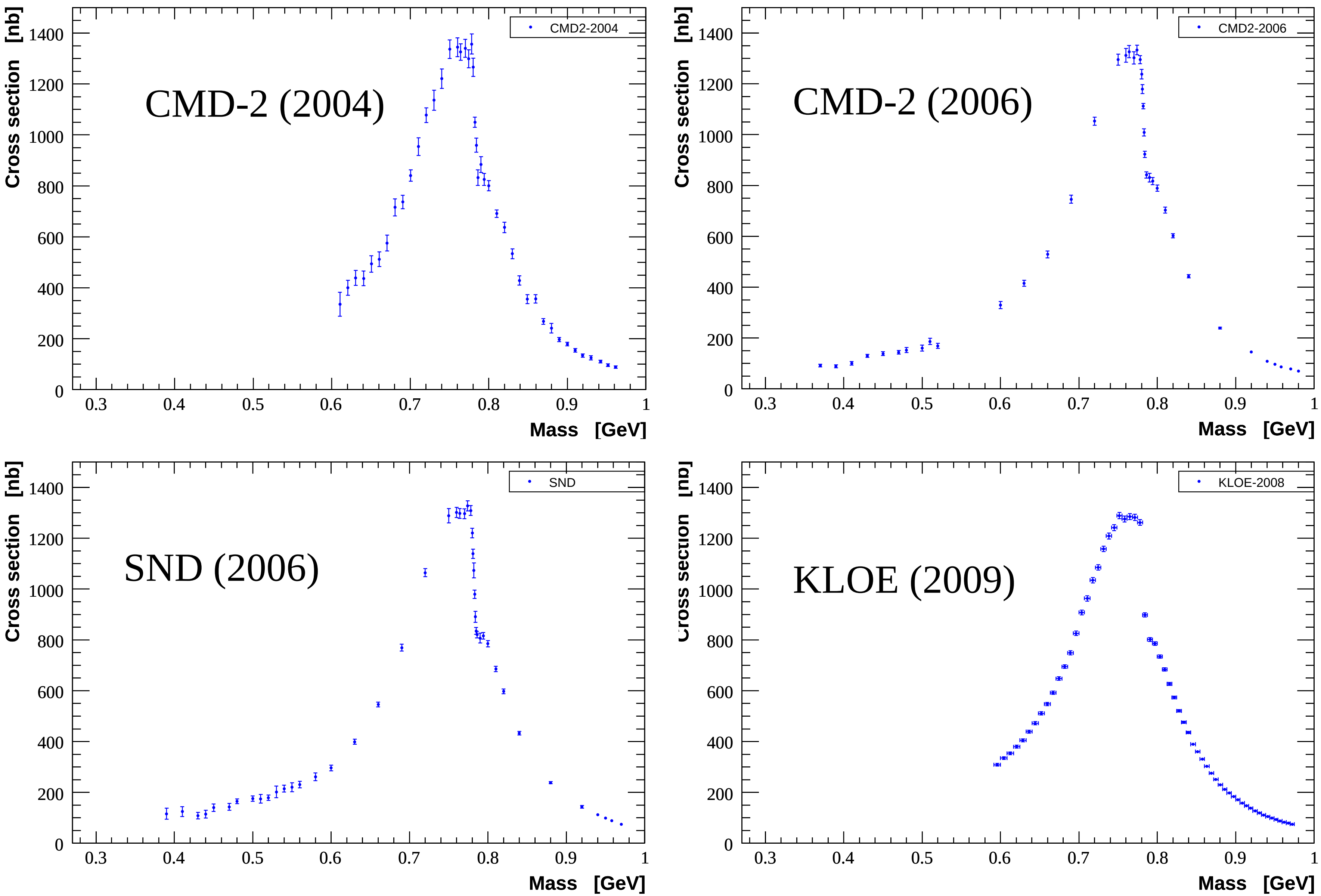
⇒ apply to $\pi\pi^0$ channel

IB corrections applied to ee data this time



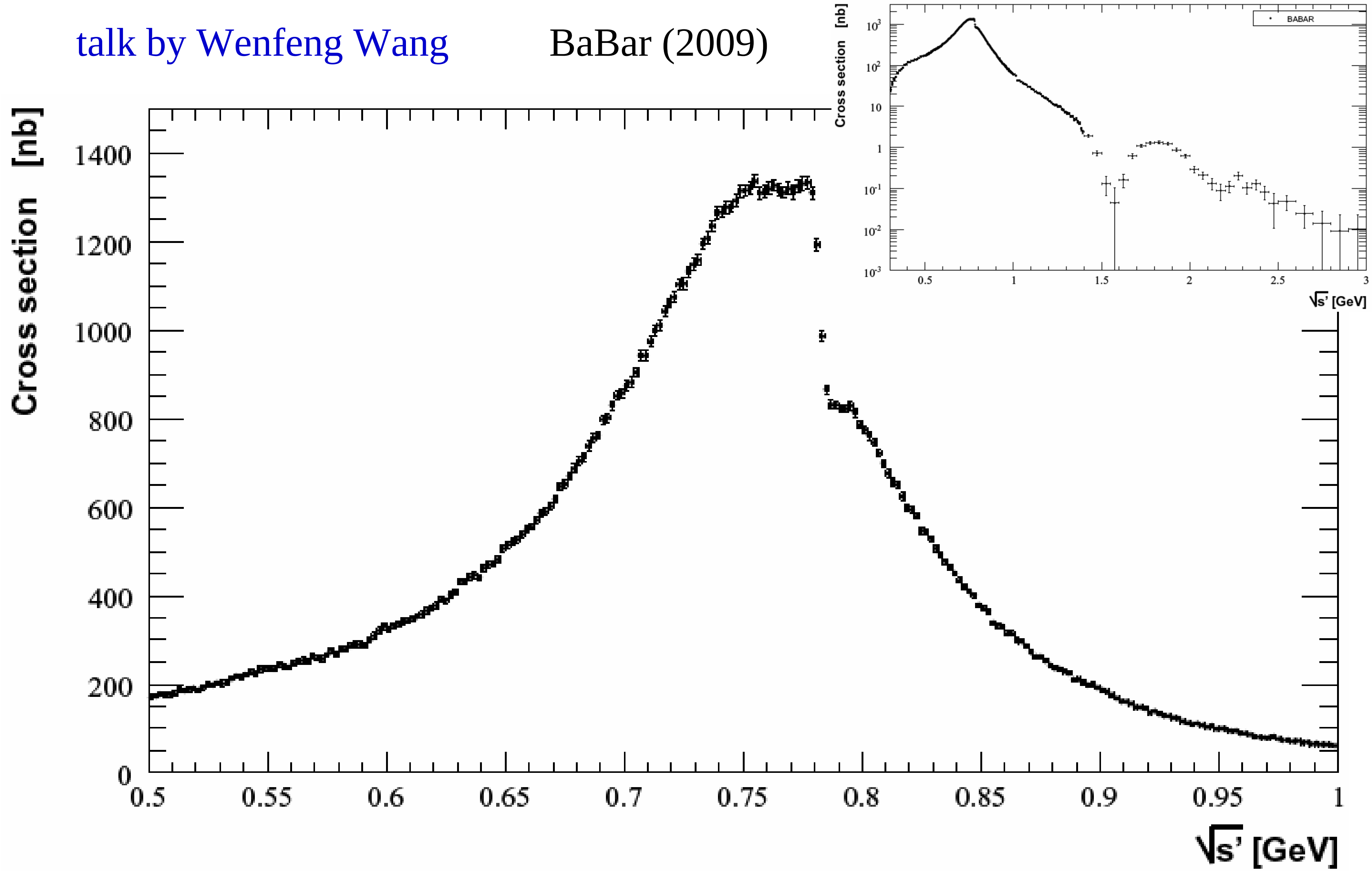
- data from CMD2-SND overconsistent ?
- fair agreement CMD2-SND with τ
- larger disagreement with KLOE

Data on $e^+e^- \to$ hadrons (1)



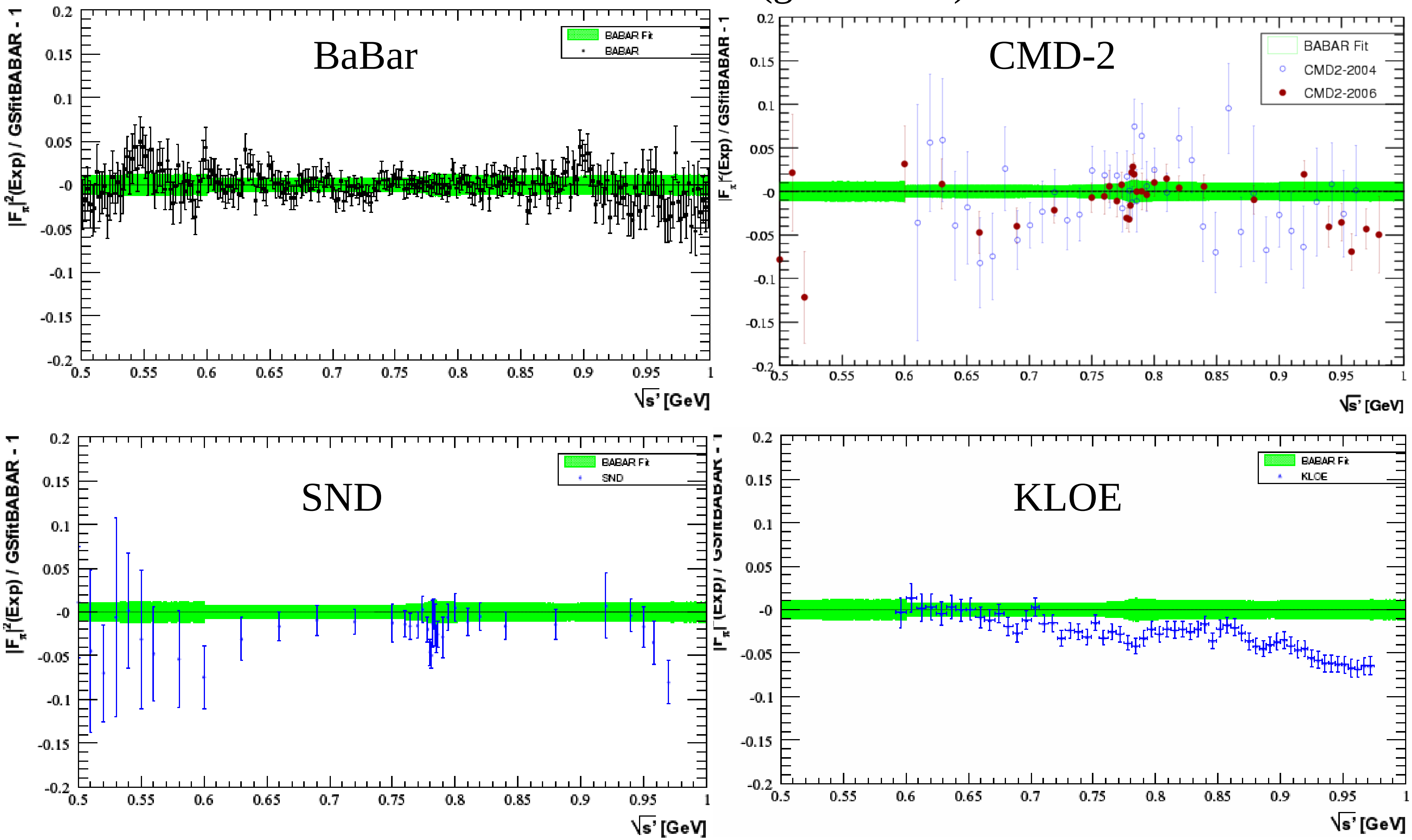
Data on $e^+e^- \to$ hadrons (2)

talk by Wenfeng Wang BaBar (2009)

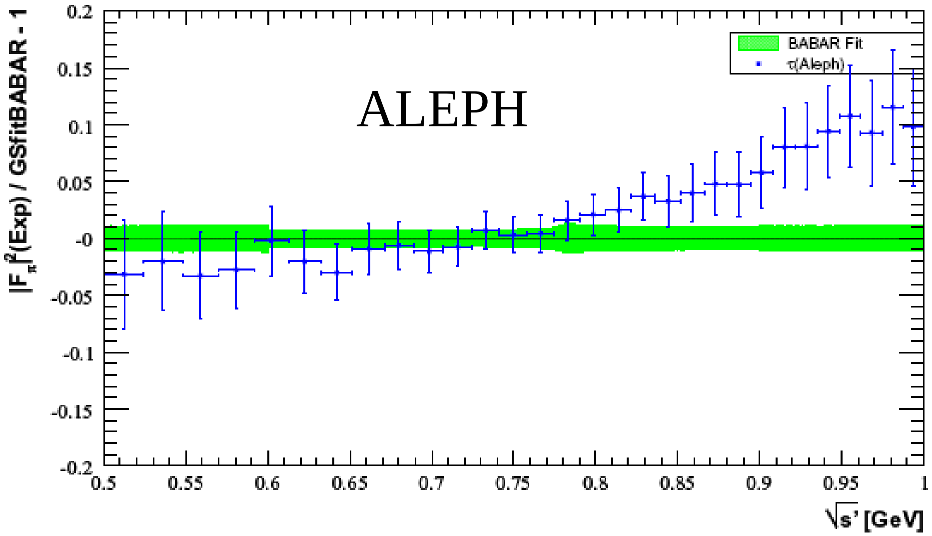


BaBar vs.other ee data (0.5-1.0 GeV)

direct relative comparison of cross sections with BaBar fit (stat + syst errors included)
(green band)



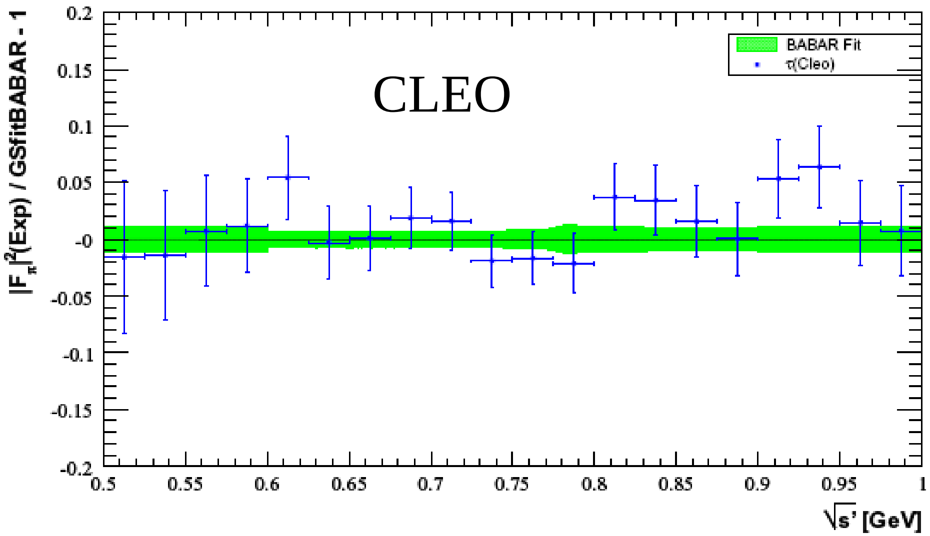
BaBar vs. IB-corrected τ data (0.5-1.0 GeV)

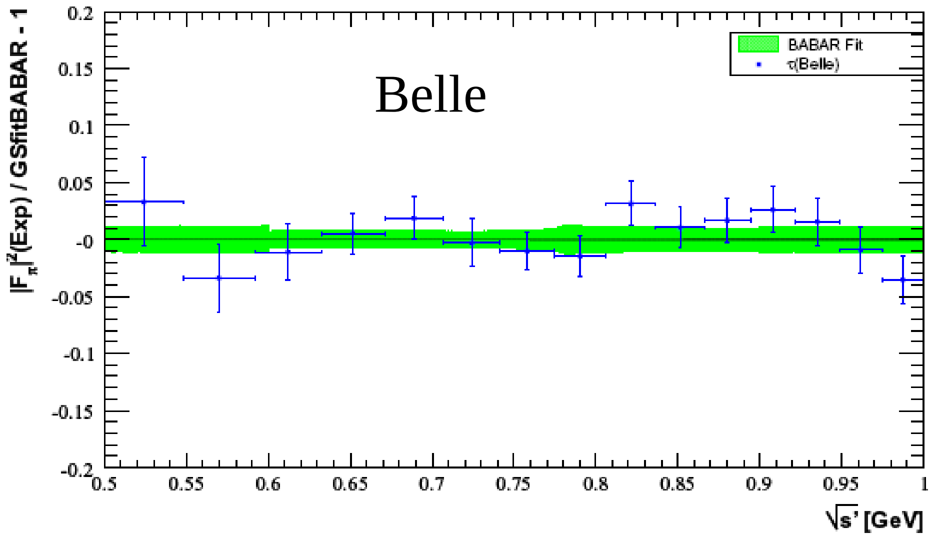


relative comparison w.r.t. BaBar of isospin-breaking corrected τ spectral functions

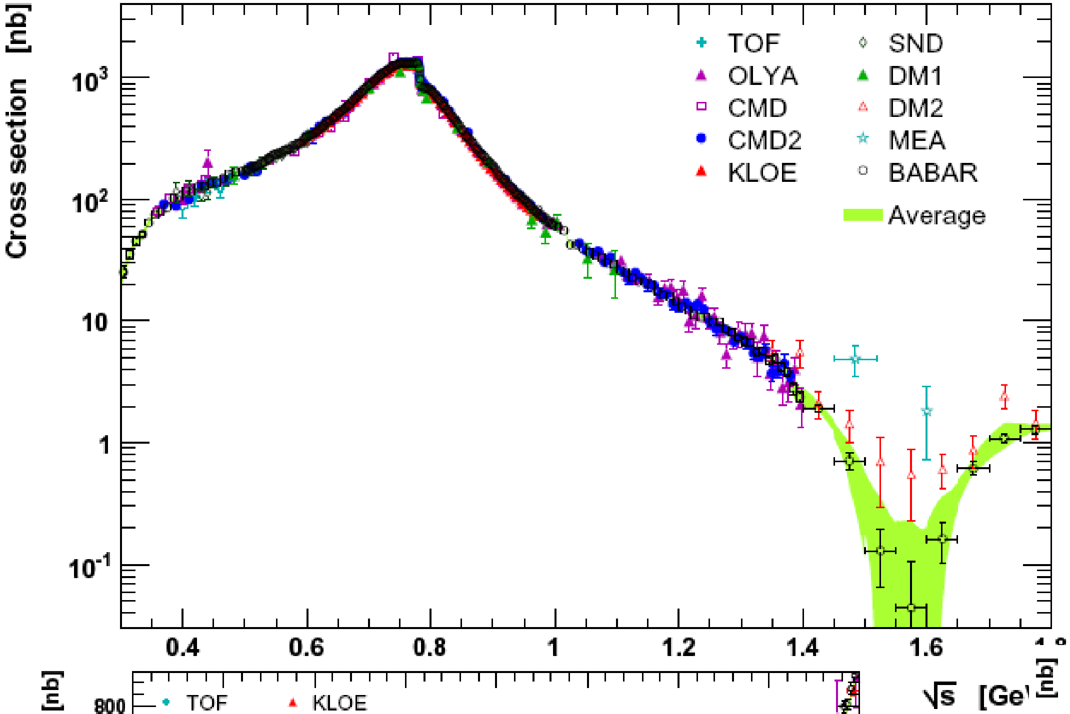
IB corrections: radiative corr., π masses, ρ-ω interference, ρ masses/widths

each τ data normalized to its own BR



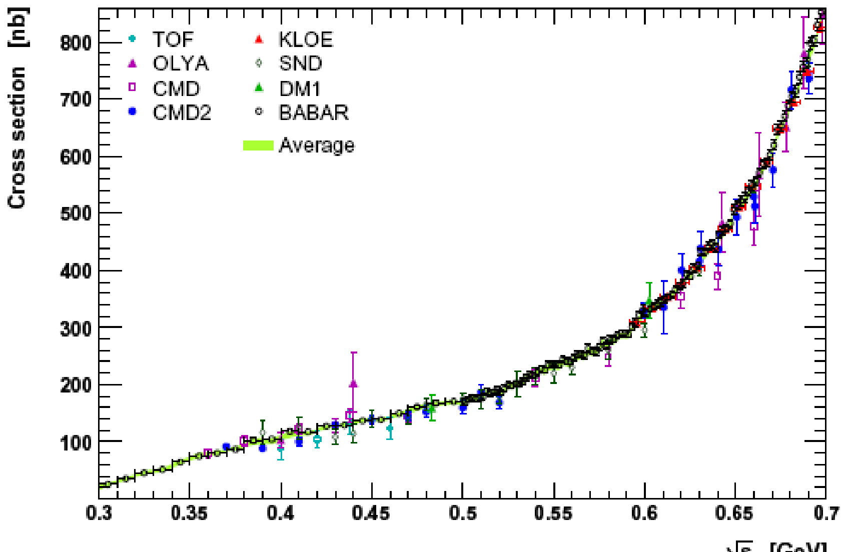


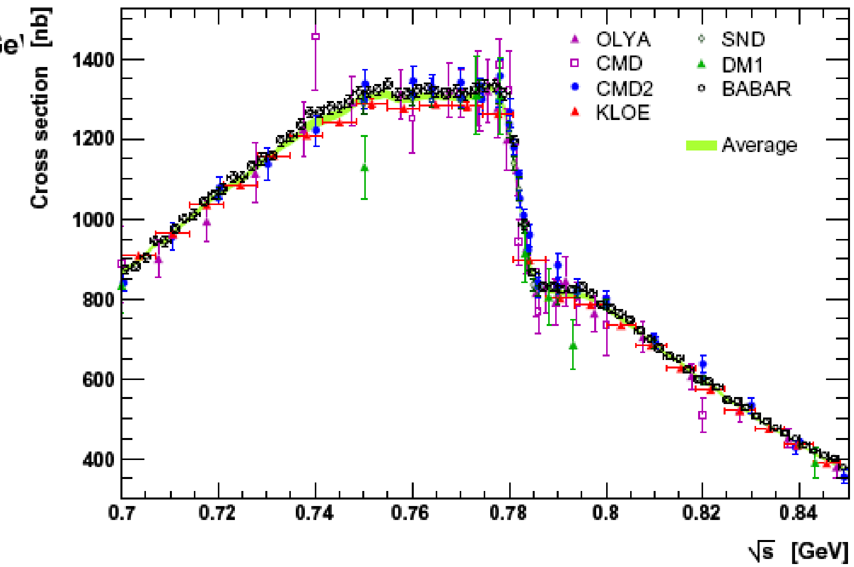
Combination of all e⁺e⁻ Data



arXiv: 0908.4300
MD-Hoecker-Malaescu-Yuan-Zhang

Improved procedure and software (HVPTools) for combining cross section data with arbitrary point spacing/binning

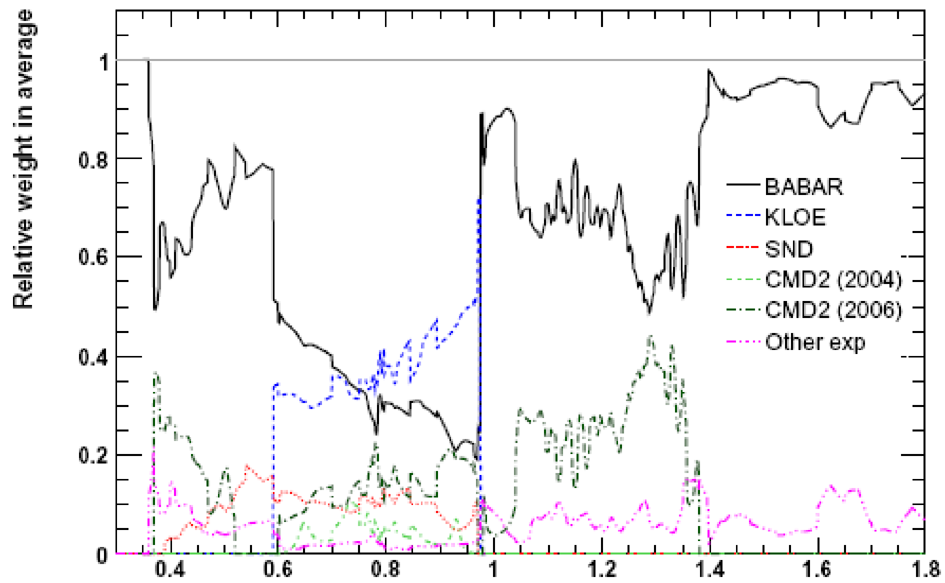


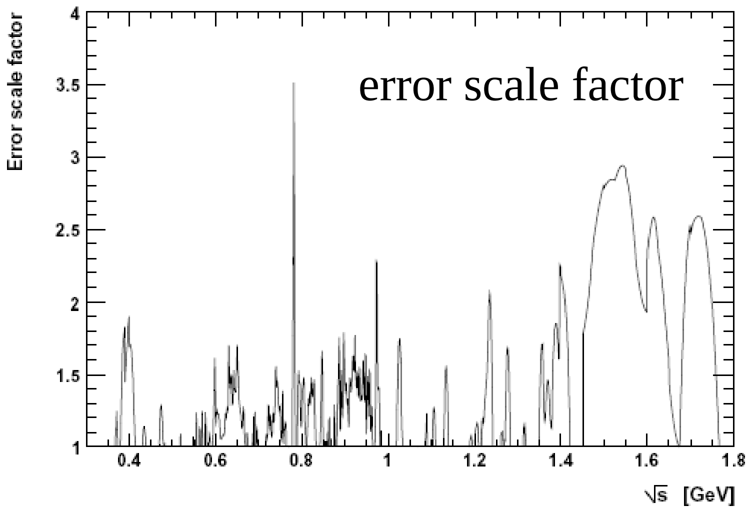


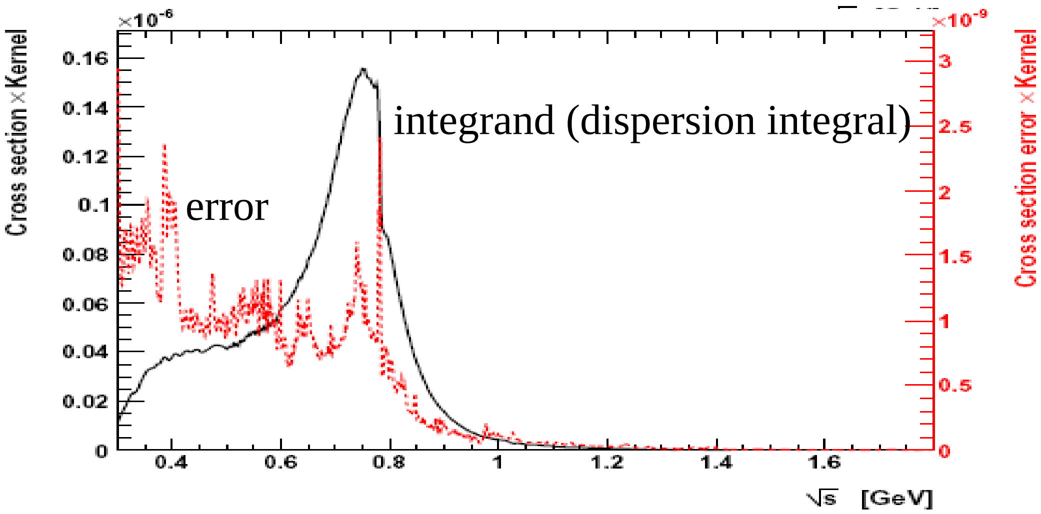
Obtaining the average cross section

- local weighted average performed
- full covariance matrices
- local χ^2 used for error rescaling
- average dominated by BaBar and KLOE, BaBar covering full range

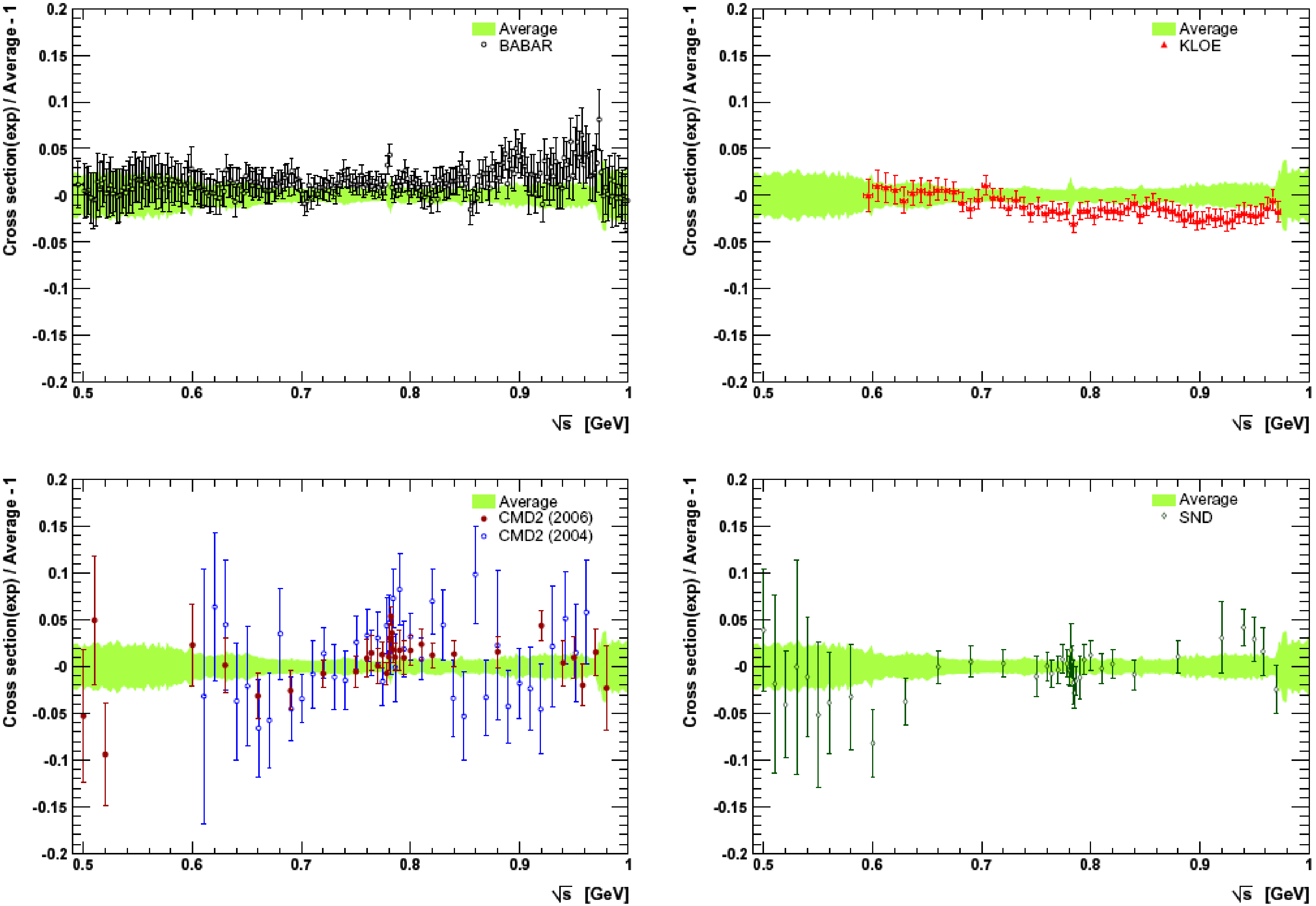
relative weights







Consistency of Experiments with Average



Computing $a_\mu^{\pi\pi}$ [$2m_\pi$,1.8 GeV]

Energy range (GeV)	Experiment	$a_\mu^{\rm had,LO}[\pi\pi]$ (10^{-10})
$2m_{\pi^\pm} - 0.3$	Combined e^+e^- (fit)	0.55 ± 0.01
$0.30 - 0.63$	Combined e^+e^-	$132.6 \pm 0.8 \pm 1.0$ $(1.3_{\rm tot})$
$0.63 - 0.958$	CMD2 03	$361.8 \pm 2.4 \pm 2.1$ $(3.2_{\rm tot})$
	CMD2 06	$360.2 \pm 1.8 \pm 2.8$ $(3.3_{\rm tot})$
	SND 06	$360.7 \pm 1.4 \pm 4.7$ $(4.9_{\rm tot})$
	KLOE 08	$356.8 \pm 0.4 \pm 3.1$ $(3.1_{\rm tot})$
	BABAR 09	$365.2 \pm 1.9 \pm 1.9$ $(2.7_{\rm tot})$
	Combined e^+e^-	$360.8 \pm 0.9 \pm 1.8$ $(2.0_{\rm tot})$
$0.958 - 1.8$	Combined e^+e^-	$14.4 \pm 0.1 \pm 0.1$ $(0.2_{\rm tot})$
Total	Combined e^+e^-	$508.4 \pm 1.3 \pm 2.6$ $(2.9_{\rm tot})$
Total	Combined τ [1]	$515.2 \pm 2.0_{\rm exp} \pm 2.2_{\mathcal{B}} \pm 1.6_{\rm IB}$ $(3.4_{\rm tot})$

Pre-BaBar combined ee	503.5 ± 3.5
BaBar	514.1 ± 3.8
Combined ee	508.4 ± 2.9
Combined τ	$515.2 \pm 3.0 \pm 1.6$ (3.4)

Other hadronic contributions

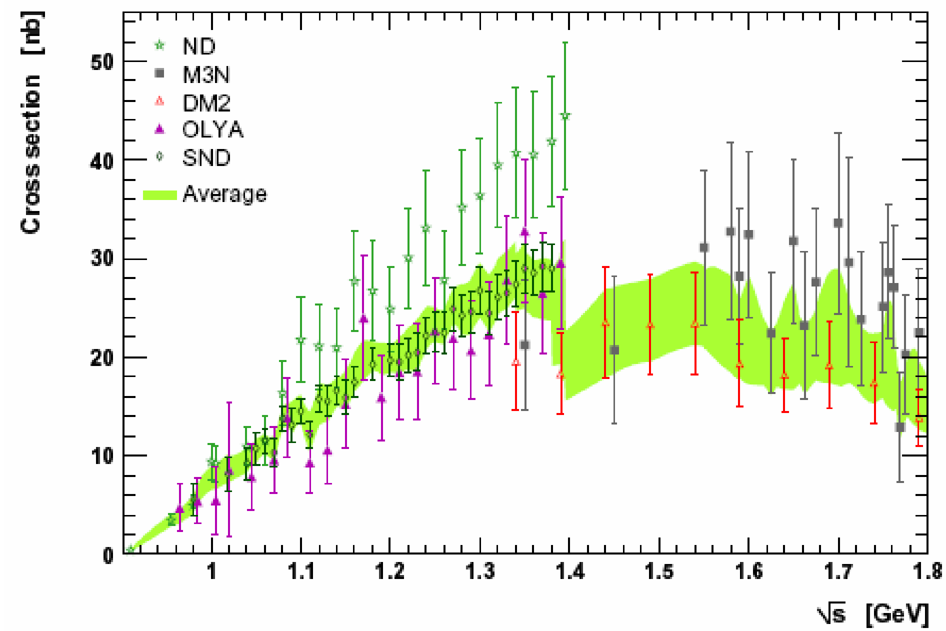
from MD-Eidelman-Hoecker-Zhang NP Proc. Suppl. 169 (2007) 288

Modes	Energy [GeV]	e^+e^-	τ
$\pi^+\pi^- 2\pi^0$	$2m_\pi$ – 1.8	$16.8 \pm 1.3 \pm 0.2_{rad}$	$21.4 \pm 1.3 \pm 0.6_{SU(2)}$
$2\pi^+ 2\pi^-$ (+BaBar)	$2m_\pi$ – 1.8	$13.1 \pm 0.4 \pm 0.0_{rad}$	$12.3 \pm 1.0 \pm 0.4_{SU(2)}$
ω (782)	0.3 – 0.81	$38.0 \pm 1.0 \pm 0.3_{rad}$	–
ϕ (1020)	1.0 – 1.055	$35.7 \pm 0.8 \pm 0.2_{rad}$	–
Other excl. (+BaBar)	$2m_\pi$ – 1.8	$24.3 \pm 1.3 \pm 0.2_{rad}$	–
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0.0_{rad}$	–
R [QCD]	1.8 – 3.7	$33.9 \pm 0.5_{theo}$	–
R [data]	3.7 – 5.0	$7.2 \pm 0.3 \pm 0.0_{rad}$	–
R [QCD]	5.0 – ∞	$9.9 \pm 0.2_{theo}$	–

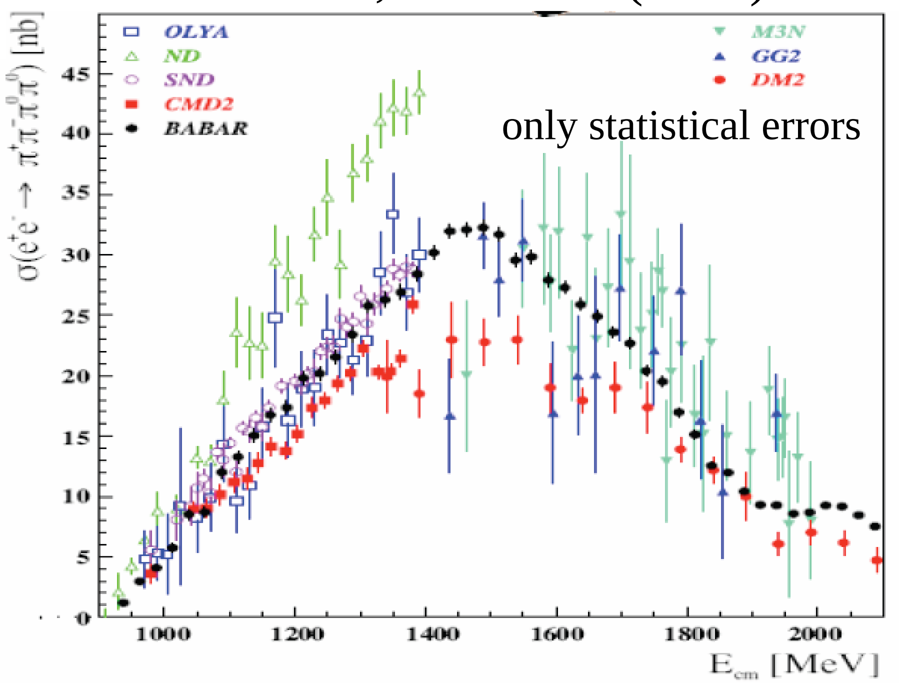
$\Rightarrow$ another large long-standing discrepancy in the $\pi^+ \pi^- 2\pi^0$ channel !

The Problematic $2\pi\ 2\pi^0$ Contribution

ee data used now (CMD2 discarded)

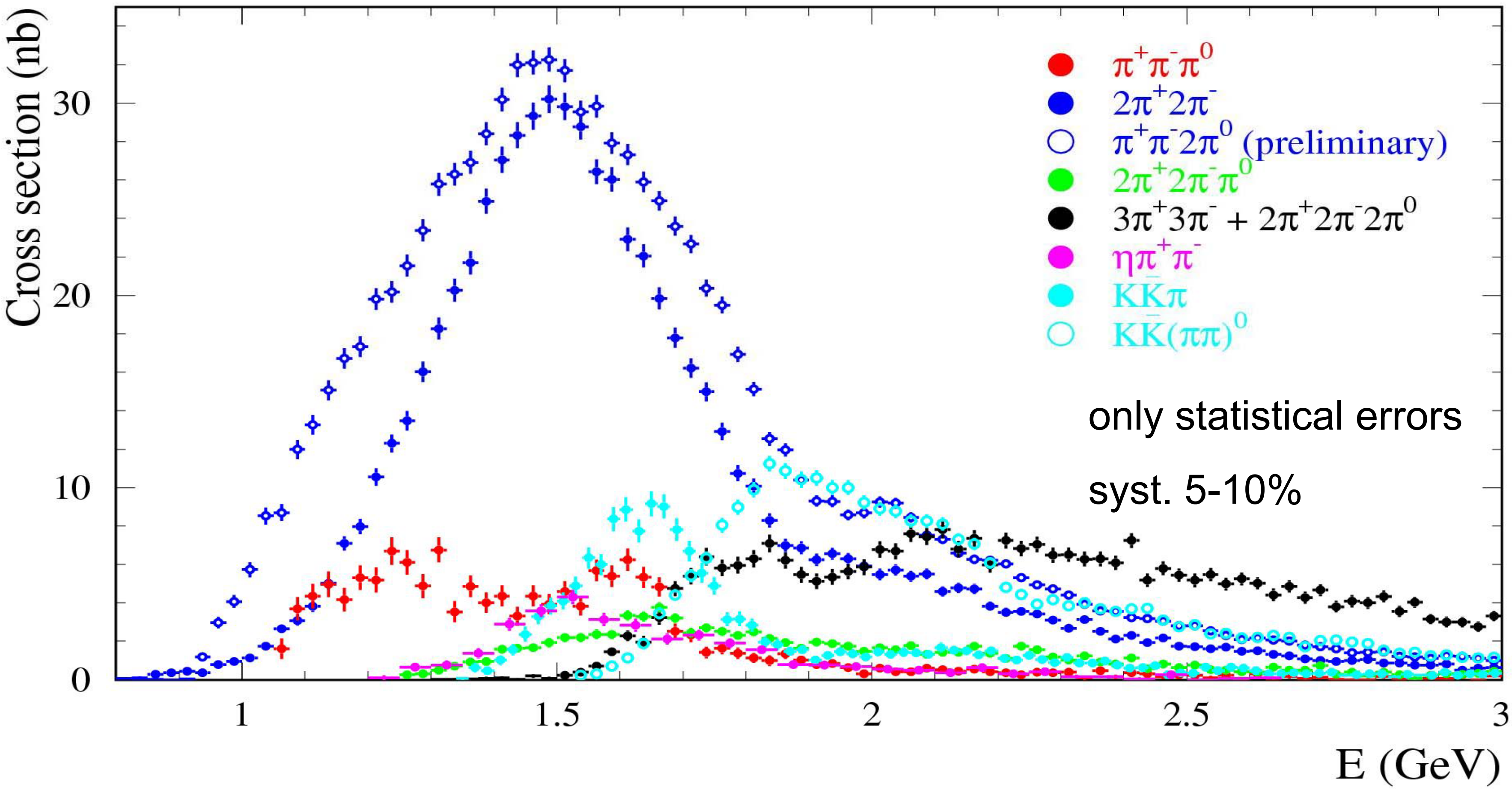


preliminary BaBar data:
A. Petzold, EPS-HEP (2007)



old contribution	16.8 ± 1.3	
update	17.6 ± 1.7	probably still underestimated (BaBar)
τ	21.4 ± 1.4	

BaBar Multi-hadronic Results



Still more channels under analysis: K^+K^-, $KK\pi\pi$ with K^0

Where are we?

- including BaBar 2π results in the e+e– combination + estimate of hadronic LBL contribution (Prades-de Rafael-Vainhstein, 2009) yields

$$a_\mu^{SM}[e+e-] = (11\,659\,183.4 \pm 4.1 \pm 2.6 \pm 0.2)\ 10^{-10}$$

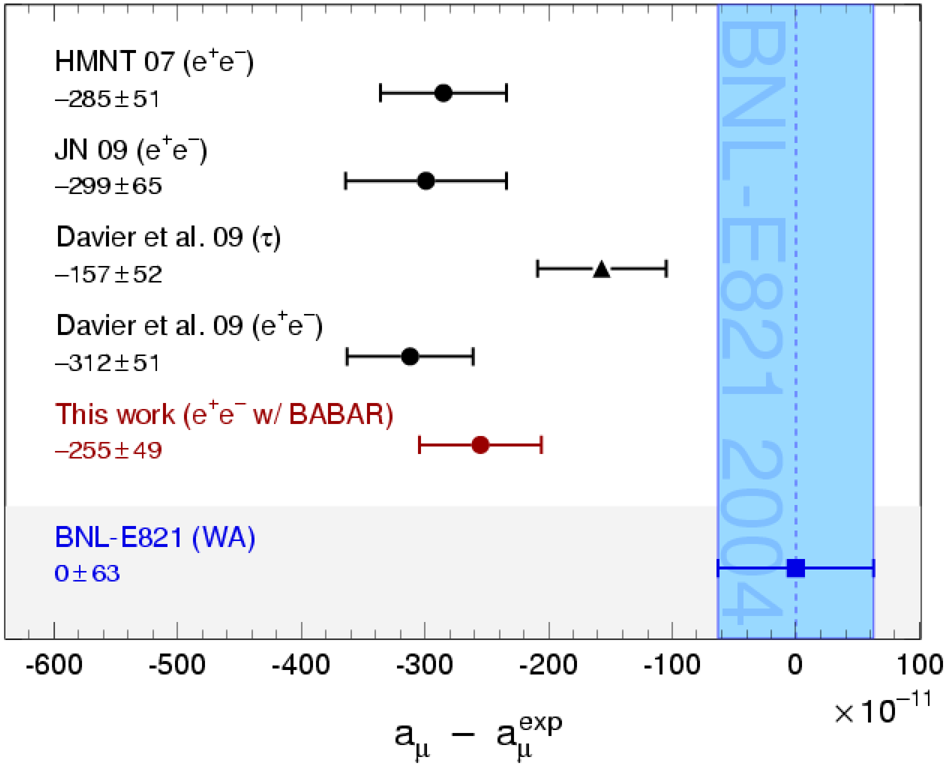
HVP LBL EW (±4.9)

- E-821 updated result 11 659 208.9 ± 6.3

- deviation (ee) 25.5 ± 8.0 (3.2 σ)

- updated τ analysis +Belle +revisited IB corrections

- deviation (τ) 15.7 ± 8.2 (1.9 σ)



Discussion

- BaBar 2π data complete and the most accurate, but expected gain in precision not fully realized because of discrepancy with KLOE
- however, previous τ/ee disagreement strongly reduced

  2.9σ (2006) $\rightarrow$ 2.4σ (τ update) $\rightarrow$ 1.5σ (including BaBar)
- a range of values for the deviation from the SM can be obtained, depending on the 2π data used:

BaBar	2.4σ
all ee	3.2σ
all ee –BaBar	3.7σ
all ee –KLOE	2.9σ
τ	1.9σ

- all approaches yield a deviation, but SM test limited by systematic effects not accounted for in the experimental analyses (ee) and/or the corrections to τ data

- at the moment some evidence for a deviation (~3σ), but not sufficient to establish a contribution from new physics (NP)
- however if NP is found at LHC, this deviation will constraint the NP phenomenology

Perspectives

- first priority is a clarification of the BaBar/KLOE discrepancy:
  - origin of the 'slope' (was very pronounced with the 2004 KLOE results, reduced now with the 2008 results)
  - normalization difference on ρ peak (most direct effect on a_μ)
  - Novosibirsk results in-between
  - slope also seen in KLOE/τ comparison; BaBar agrees with τ
- further checks of the KLOE results are possible: as method is based on MC simulation for ISR and additional ISR/ISR probabilities ⇒ long-awaited test with μμγ analysis
- contribution from multi-hadronic channels will continue to be updated with more results forthcoming from BaBar, particularly $2\pi\ 2\pi^0$
- new precise data expected from VEPP-2000 in Novosibirsk
- experimental error of E-821 direct a_μ measurement is a limitation, already now
  - ⇒ new proposal submitted to Fermilab to improve accuracy by a factor 4
  - ⇒ project at JPARC

Backup Slides

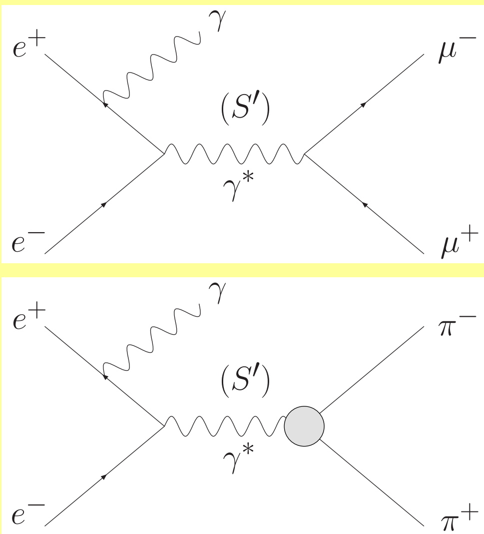
The Relevant Processes

$e^+ e^- \rightarrow \mu^+ \mu^- \gamma\ (\gamma)$ and $\pi^+ \pi^- \gamma\ (\gamma)$ measured simultaneously

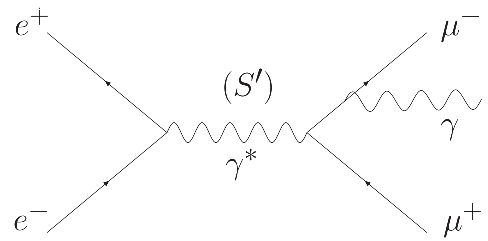
$$x = 2E^*_\gamma / \sqrt{s}$$

$$s' = s(1 - x)$$

ISR

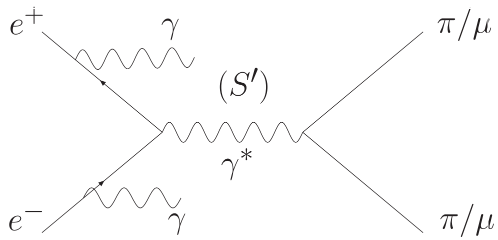


FSR

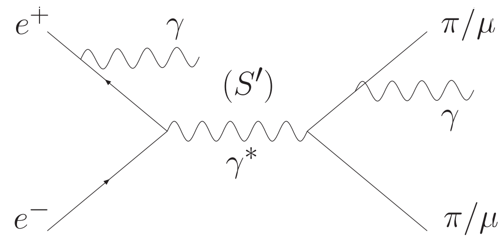


LO FSR negligible for ππ at s~(10.6 GeV)^2

ISR + add. ISR



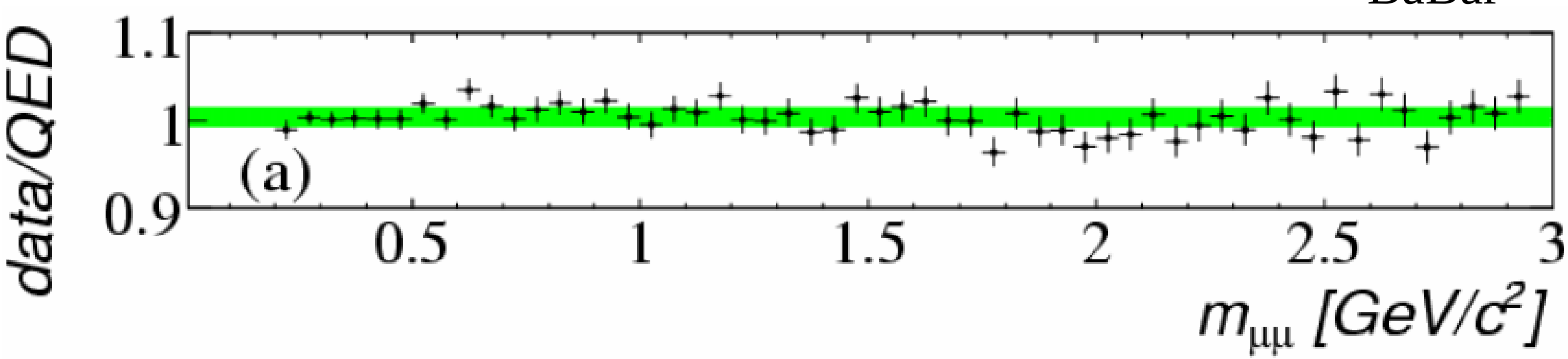
ISR + add. FSR



QED Test with $\mu\mu\gamma$ sample

- absolute comparison of $\mu\mu$ mass spectra in data and in simulation
- simulation corrected for data/MC efficiencies
- AfkQed corrected for incomplete NLO using Phokhara
- strong test (ISR probability drops out for $\pi\pi$)

BaBar



$$\frac{\sigma^{data}_{\mu\mu\gamma(\gamma)}}{\sigma^{NLO\ QED}_{\mu\mu\gamma(\gamma)}} = 1 + (4.0 \pm 1.9 \pm 5.5 \pm 9.4)\ 10^{-3}$$

(0.2 – 3 GeV)

ISR γ efficiency 3.4 syst.
trig/track/PID 4.0

BaBar ee luminosity

Obtaining the ππ(γ) cross section

$$\frac{dN_{\pi\pi\gamma(\gamma)}}{d\sqrt{s'}} = \frac{dL^{eff}_{ISR}}{d\sqrt{s'}} \, \varepsilon_{\pi\pi\gamma(\gamma)}(\sqrt{s'}) \, \sigma^0_{\pi\pi(\gamma)}(\sqrt{s'})$$

Unfolded spectrum

Acceptance from MC + data/MC corrections

Effective ISR luminosity from μμγ(γ) analysis (similar equation + QED)

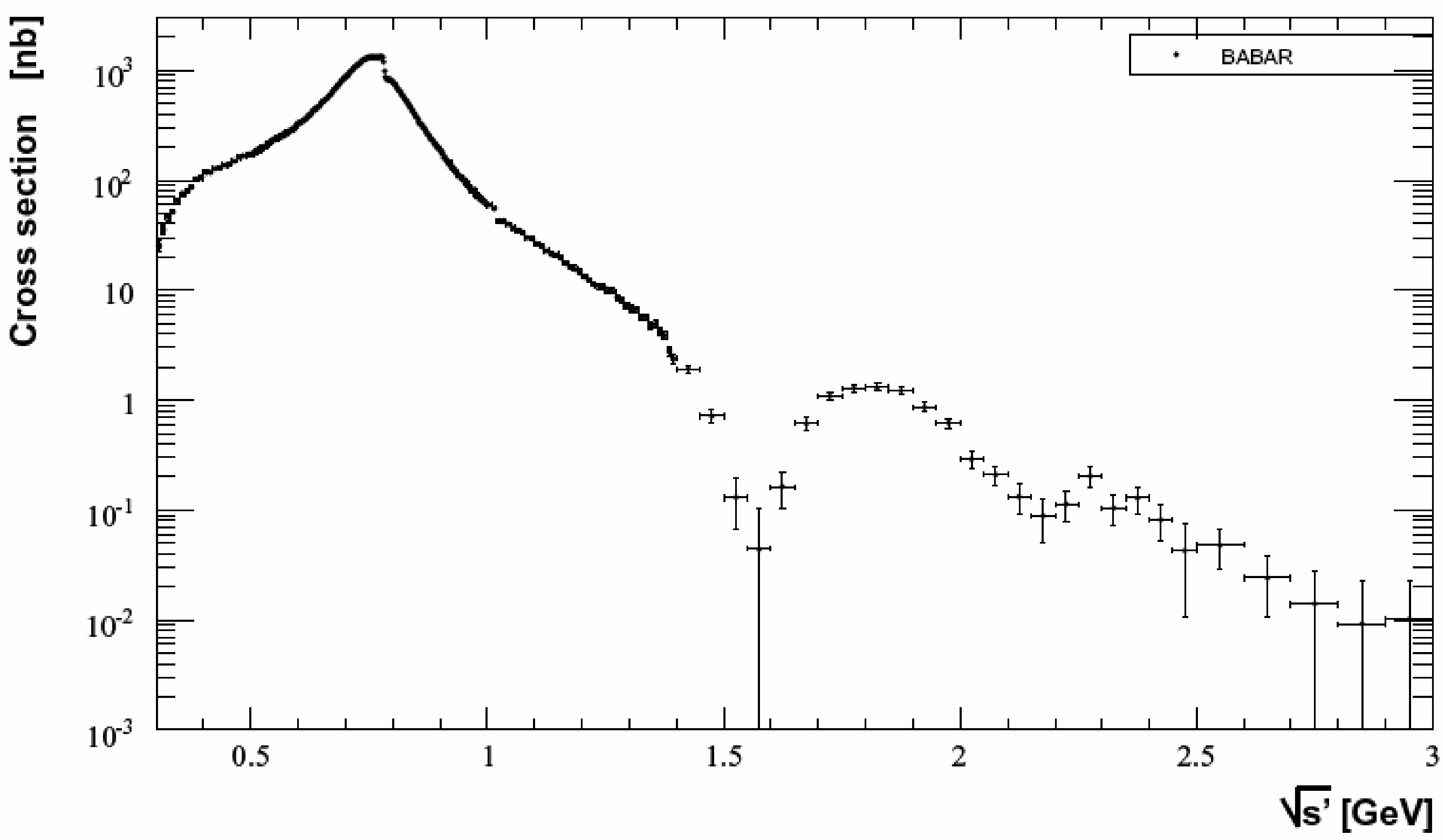
ππ mass spectrum unfolded (Malaescu arXiv:0907-3791) for detector response

Additional ISR almost cancels in the procedure (ππγ(γ) / μμγ(γ) ratio)
Correction (2.5 ±1.0) 10^{-3} ⇒ ππ cross section does not rely on accurate description of NLO in the MC generator

ISR luminosity from μμγγ in 50-MeV energy intervals
(small compared to variation of efficiency corrections)

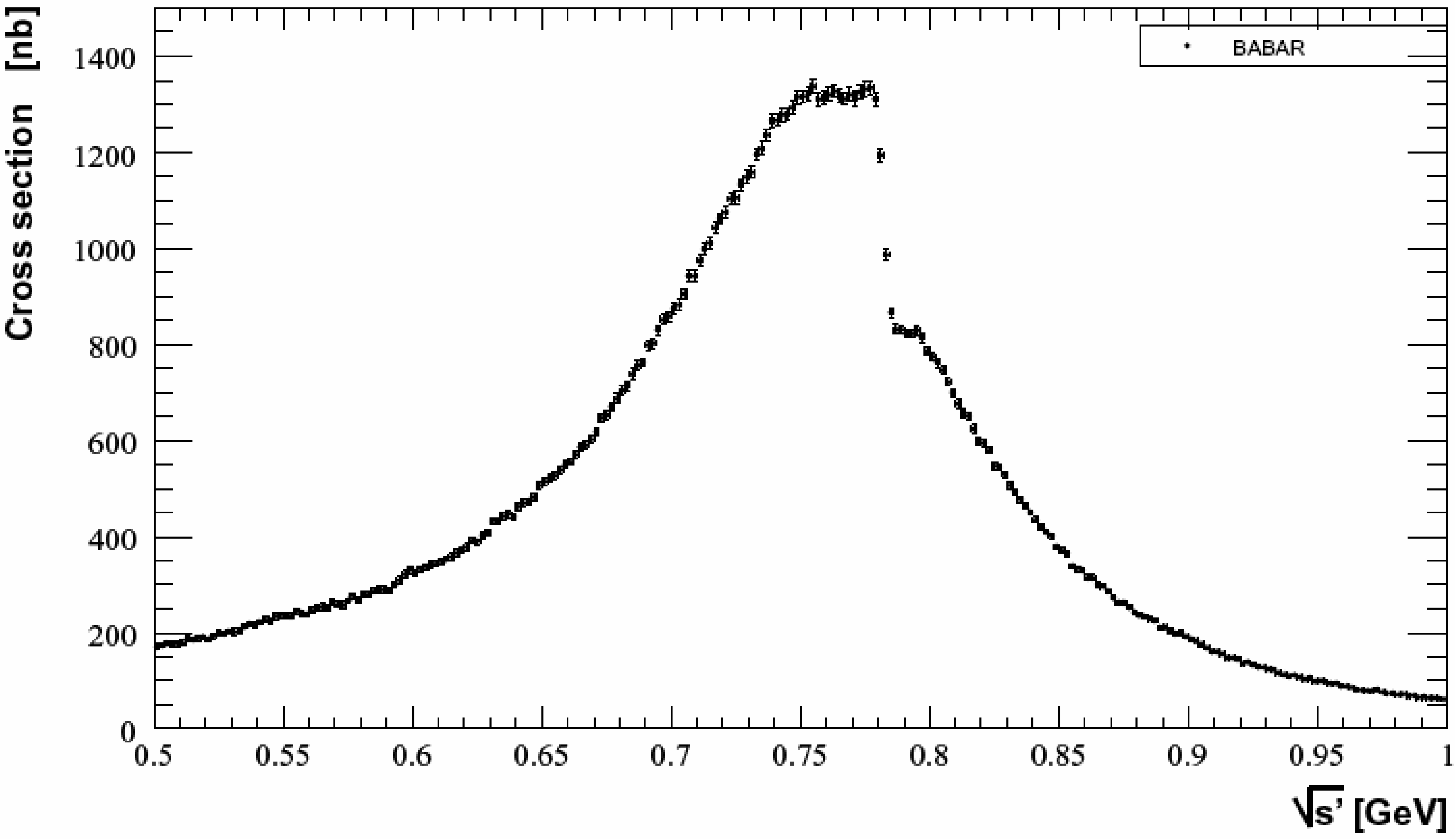
BaBar results (arXiv:0908.3589)

$e^+ e^- \to \pi^+ \pi^- (\gamma)$ bare (no VP) cross section diagonal errors stat+syst

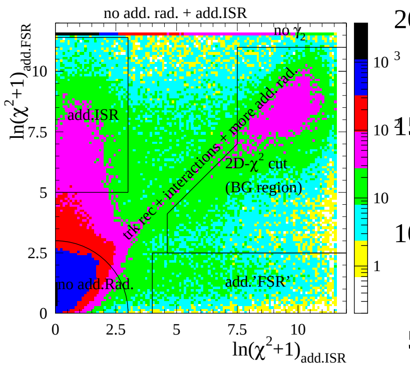


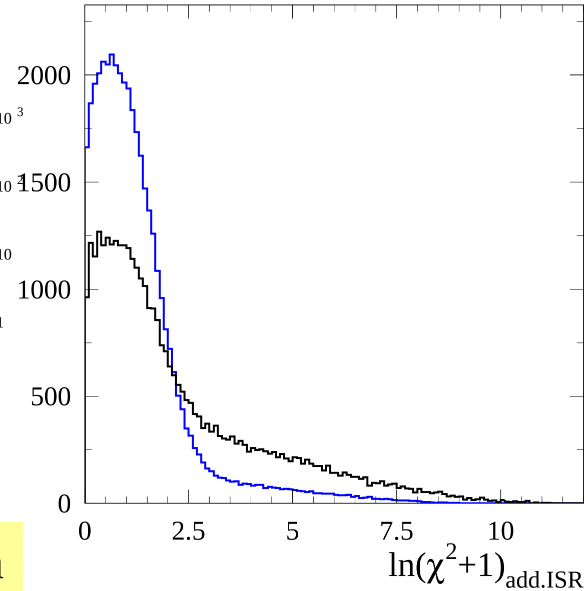
BaBar results in ρ region

2-MeV energy intervals

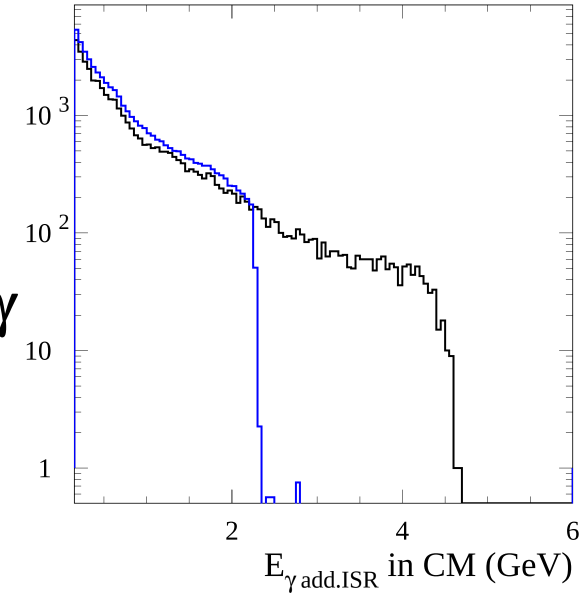


Additional ISR

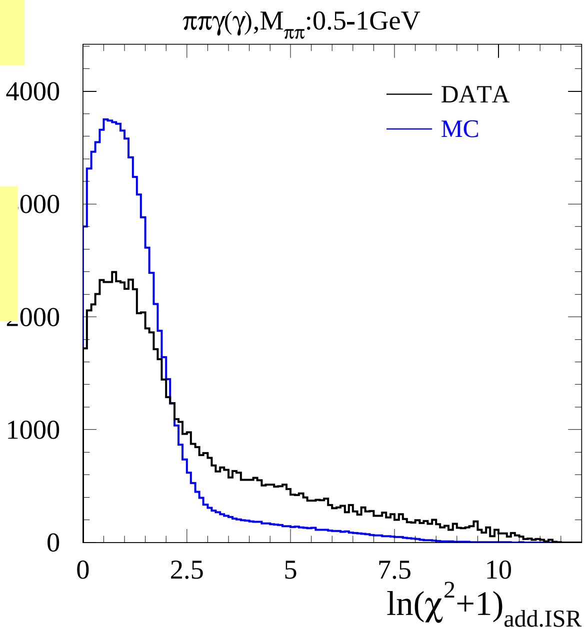




μμγγ

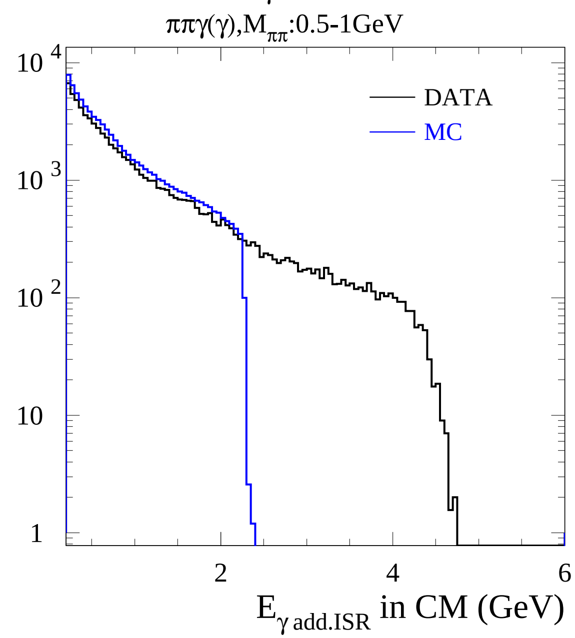


Angular distribution of add. ISR /beams!

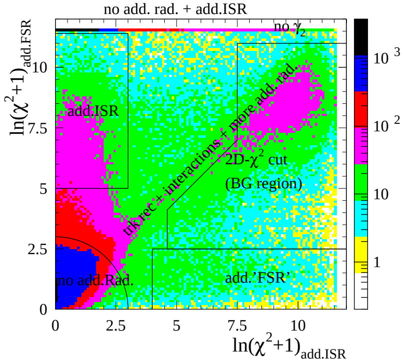


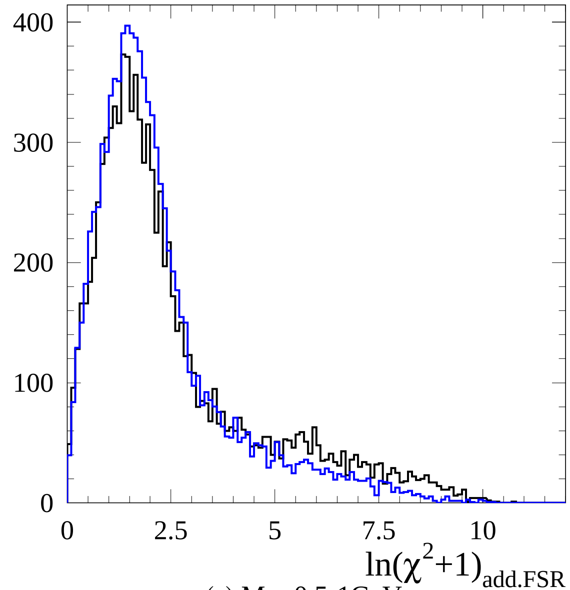
Energy cut-off for add. ISR in AfkQed

ππγγ



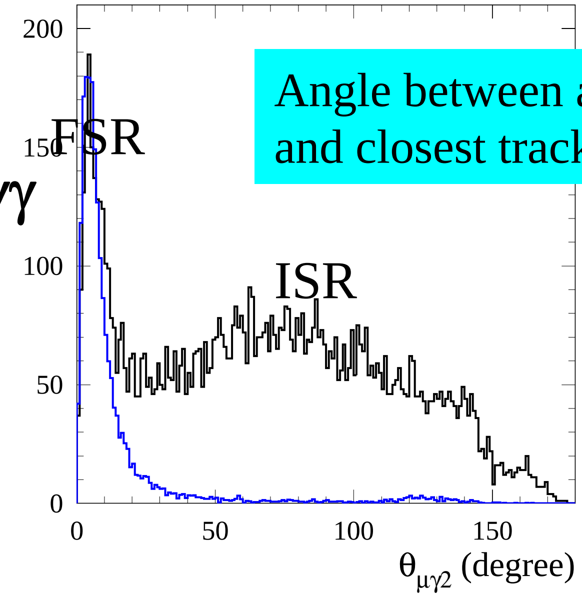
Additional FSR



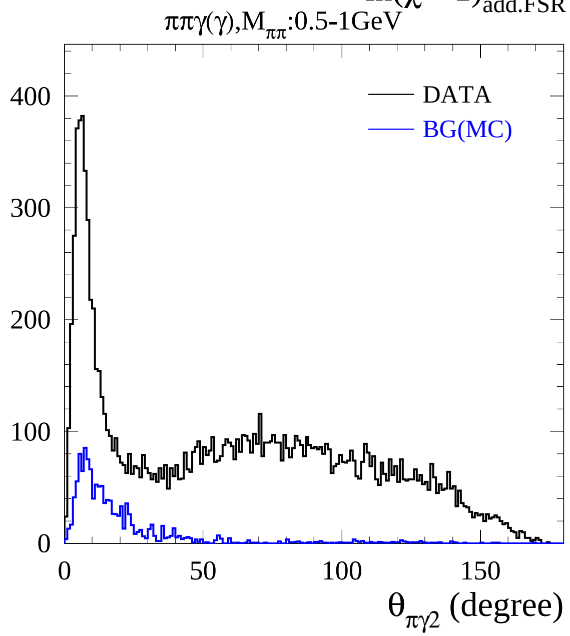


Angle between add γ and closest track

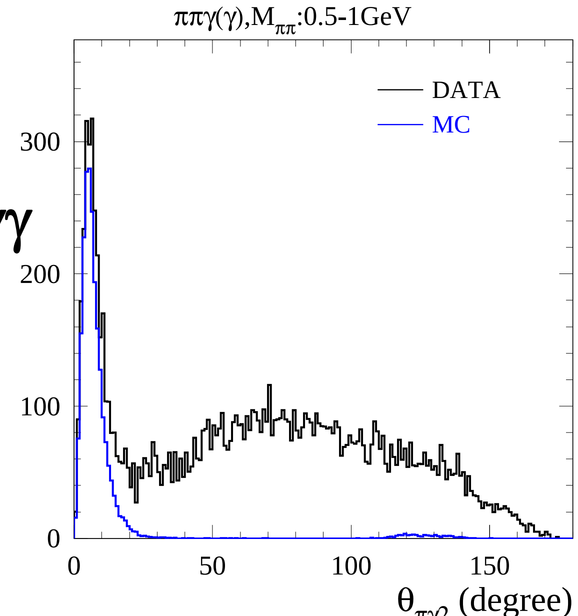
$\mu\mu\gamma\gamma$



Large-angle add.ISR in data ≠ AfkQed



$\pi\pi\gamma\gamma$



Evidence for FSR data ~ AfkQed

data/MC

$\mu\mu$ 0.96±0.06

$\pi\pi$ 1.21±0.05

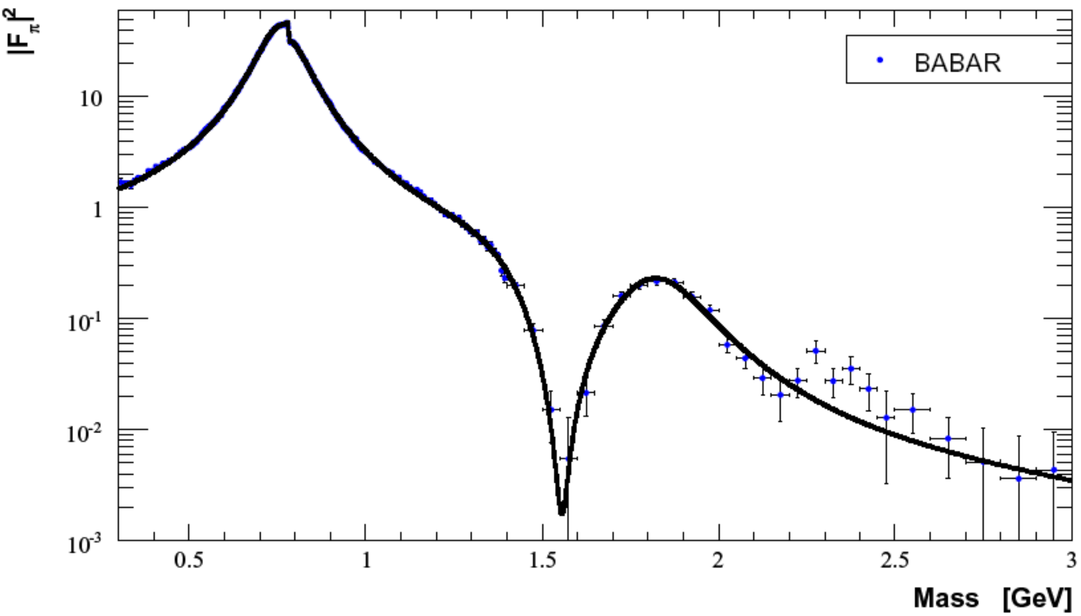
VDM Fit of the BaBar Pion Form Factor

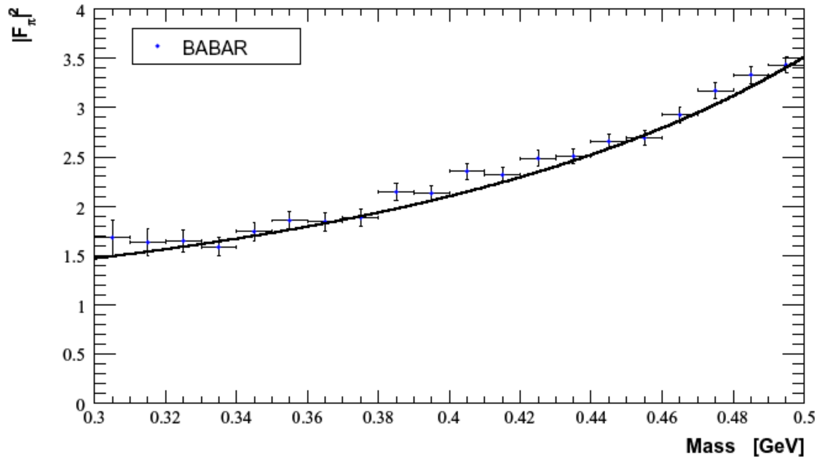
$$F_\pi(s) = \frac{BW_\rho^{GS}(s, m_\rho, \Gamma_\rho)\frac{1+\alpha BW_\omega^{KS}(s, m_\omega, \Gamma_\omega)}{1+\alpha} + \beta BW_{\rho'}^{GS}(s, m_{\rho'}, \Gamma_{\rho'}) + \gamma BW_{\rho''}^{GS}(s, m_{\rho''}, \Gamma_{\rho''})}{1+\beta+\gamma}$$

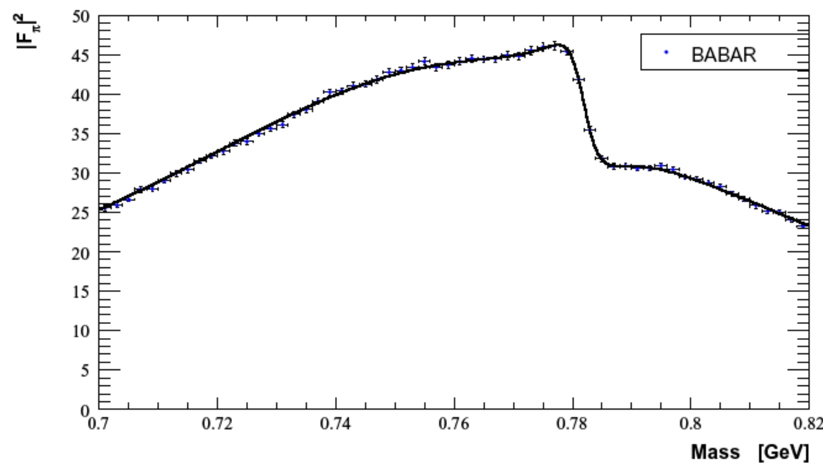
$$|F_\pi|^2(s') = \frac{3s'}{\pi\alpha^2(0)\beta_\pi^3}\,\sigma_{\pi\pi}(s')$$

$$\sigma_{\pi\pi}(s') = \frac{\sigma^0_{\pi\pi(\gamma)}(s')}{1+\frac{\alpha}{\pi}\eta(s')} \left(\frac{\alpha(s')}{\alpha(0)}\right)^2$$

add. FSR α Running (VP)





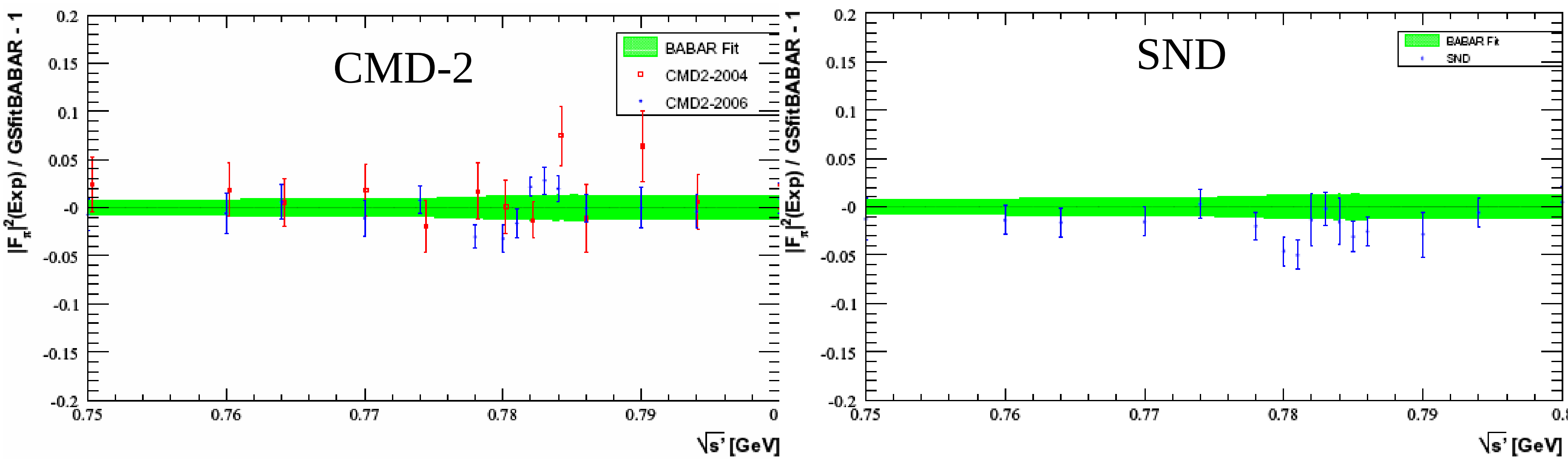


BaBar vs.other ee data ($\rho-\omega$ interference region)

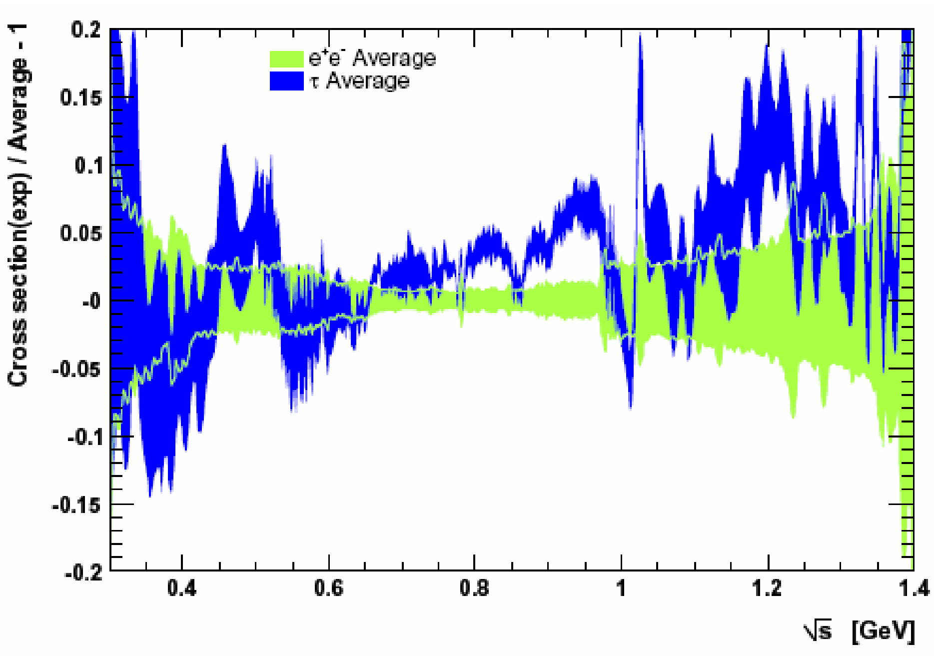
- mass calibration of BaBar checked with ISR-produced $J/\psi \rightarrow \mu\mu$
- expect $-(0.16 \pm 0.16)$ MeV at ρ peak
- ω mass determined through VDM mass fit

$$m_\omega^{fit} - m_\omega^{PDG} = -(0.12 \pm 0.29) \text{ MeV}$$

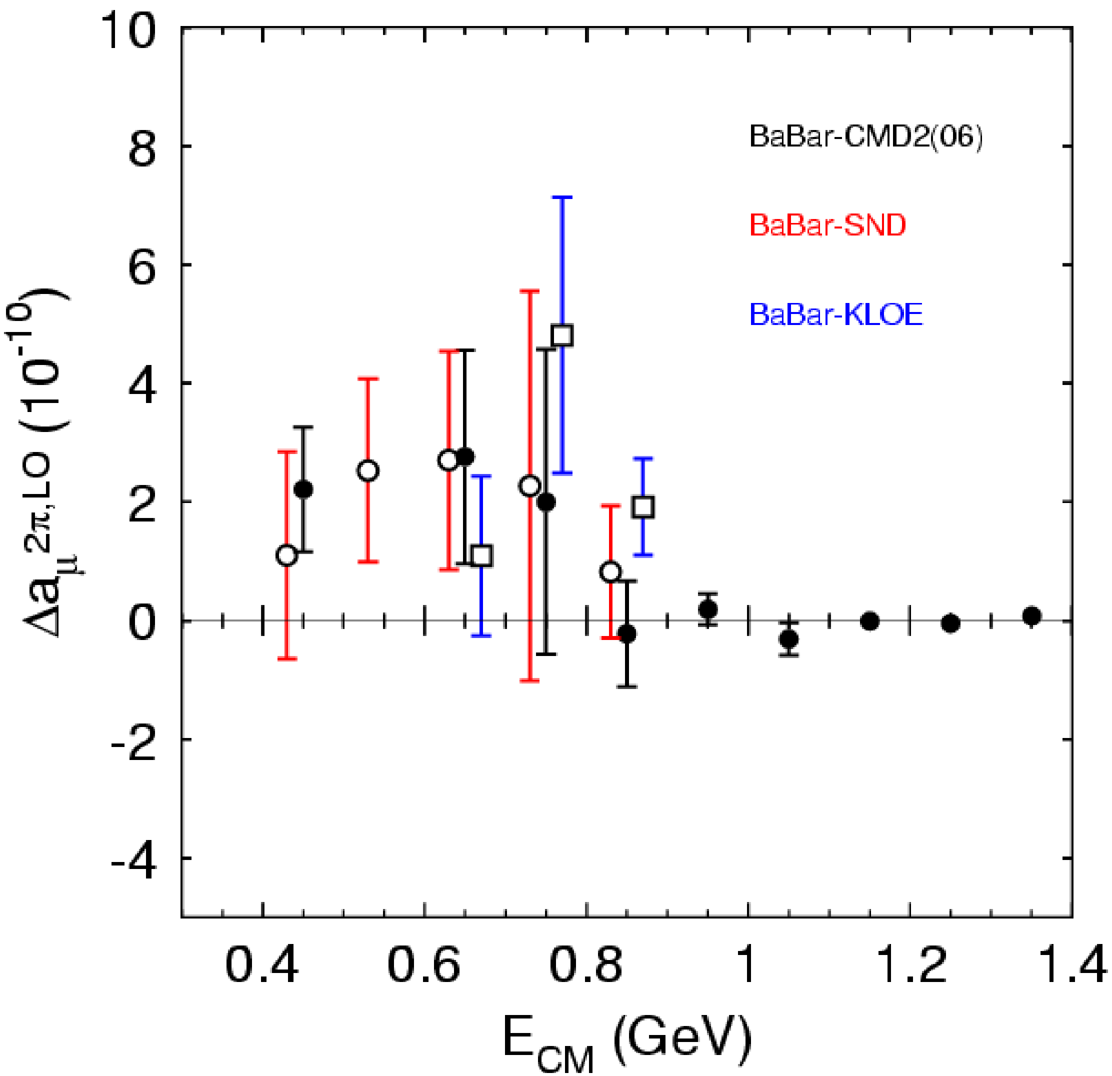
- Novosibirsk data precisely calibrated using resonant depolarization
- comparison BaBar/CMD-2/SND in ρ-ω interference region shows no evidence for a mass shift



Combined τ vs. Combined ee



Difference BaBar/Others in Broad Mass Bins



integrals in 0.1 GeV intervals

Computing $a_\mu^{\pi\pi}$

$$a_\mu^{\pi\pi(\gamma),LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\pi\pi(\gamma)}(s) \ ,$$

where $K(s)$ is the QED kernel,

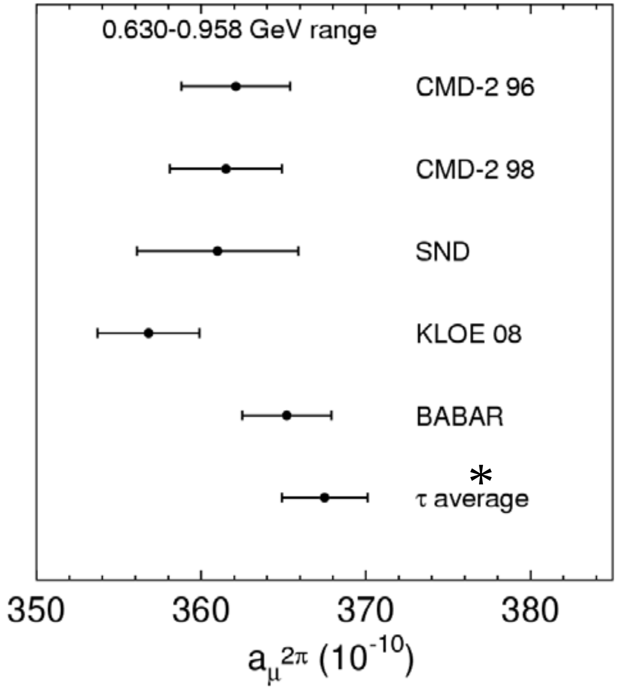
$$K(s) = x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right)\left[\ln(1+x) - x + \frac{x^2}{2}\right] + x^2\frac{1+x}{1-x}\ln x \ ,$$

with $x = (1-\beta_\mu)/(1+\beta_\mu)$ and $\beta_\mu = (1-4m_\mu^2/s)^{1/2}$.

$m_{\pi\pi}$ range (GeV)	$a_\mu^{\pi\pi(\gamma),LO}$ BABAR
0.28–0.30	$0.55 \pm 0.01 \pm 0.01$
0.30–0.50	$57.62 \pm 0.63 \pm 0.55$
0.50–1.00	$445.94 \pm 2.10 \pm 2.51$
1.00–1.80	$9.97 \pm 0.10 \pm 0.09$
0.28–1.80	$514.09 \pm 2.22 \pm 3.11$

$(\times 10^{-10})$

0.7% precision



0.28–1.8 (GeV)

BABAR 514.1 ± 3.8

previous e^+e^- combined 503.5 ± 3.5 *

τ combined 515.2 ± 3.5 *

* arXiv:0906-5443 MD et al.