Update of $g-2$ of the muon and $\Delta\alpha$

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I. Recent developments in $g-2$

- New exclusive data from CMD-2, SND, KLOE, BaBar, inclusive measurements from BES, CLEO.
- 2π: KLOE 2008 analysis (brand new results $\rightarrow$ S Mueller).
- Inclusive vs. sum of exclusive data below 2 GeV ($\rightarrow$ BaBar RadRet analyses; new 2π: M Davier)
- New (prelim.) HLMNT compilation; comparison

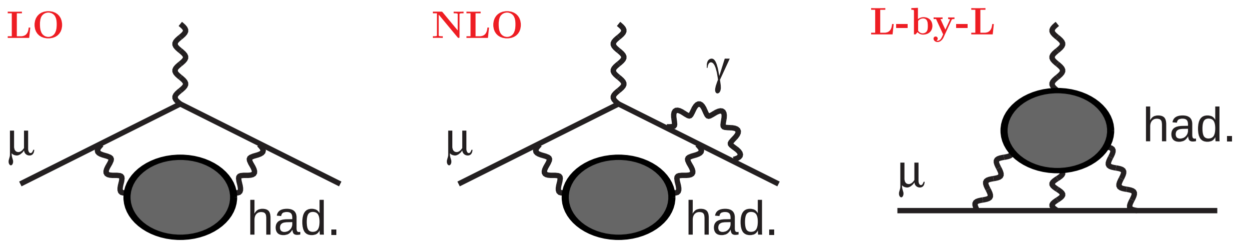
II. $\Delta\alpha(q^2)$: Vacuum Polarisation in the space- and time-like region. $\alpha(M_Z^2)$

III. Outlook

I. Recent developments in $g-2$

- QED: Predictions consolidated, further work (numerical five-loop) ongoing, big surprises very unprobable, error formidably small: $a_\mu^{\rm QED} = 116584718.08(15)\cdot 10^{-11}$ ✓
- EW: reliable two-loop predictions, accuracy fully sufficient: $a_\mu^{EW} = (154 \pm 2)\cdot 10^{-11}$ ✓
- Hadronic contributions: uncertainties completely dominate $\Delta a_\mu^{\rm SM}$!

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$



▶ Hadronic contributions from low γ virtualities not calculable with perturbative QCD

▶ For VacPol contributions use of *dispersion relations*, with exp. data for $\sigma^0_{\rm had}(s)$:

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

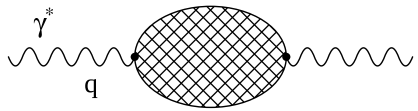
- Light-by-Light: [see talk A Nyffeler]

  – No dispersion relation for L-by-L. *First Principles* calculations from lattice QCD are underway by two groups: QCDSF and T Blum *et al.* Both approaches promising but at an early stage and no results yet.

  – Convergence of different recent model calculations. Below we will use the recent compilation from J Prades, E de Rafael, A Vainshtein: $a_\mu^{\mathrm{L-by-L}} = (10.5 \pm 2.6) \cdot 10^{-10}$

▶ Hadronic Vacuum Polarisation contributions: $\sigma^0_{\rm had}(s)$

- For low energies, need to sum ~ 24 exclusive channels
- $1.43 - 2$ GeV: Sum exclusive channels or use (old) inclusive data?
- above 2 GeV: inclusive data *and/or* use of perturbative QCD.
  [HLMNT use data driven approach and pQCD only from above $\sim$ 11.09 GeV.]
- In each channel: Data combination from many experiments, non-trivial w.r.t. error analysis/correlations/different energy ranges. [HLMNT use non-linear $\chi^2_{\rm min}$ fit.]
- Note: σ^0 must be the *undressed* hadronic cross section (i.e. photon VP *subtracted* [$\cdot \left(\frac{\alpha}{\alpha(q^2)}\right)^2$], otherwise double-counting with $a_\mu^{\rm had,NLO}$)



- but must *include final state photon radiation*.

⇝ Uncertainty in treatment of radiative corrections, especially for older data sets!
Assign additional error
HLMNT: $\delta a_\mu^{\rm had,VP+FSR} \simeq 1.8 \times 10^{-10}$ $[\sim 10 \cdot \Delta a_\mu^{\rm EW}]$

► Most important changes in input data since HMNT 2006

– CMD-2: K^+K^-, $2\pi^+2\pi^-\pi^0$, $2\pi^+2\pi^-2\pi^0$

– SND: K^+K^-, $K_S^0K_L^0$

– KLOE: $\pi^+\pi^-(\gamma)$, $\omega\pi^0$

– BaBar: $K^+K^-\pi^0$, $K_S^0\pi K$, $2\pi^+2\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-\eta$, $2\pi^+2\pi^-2\pi^0$

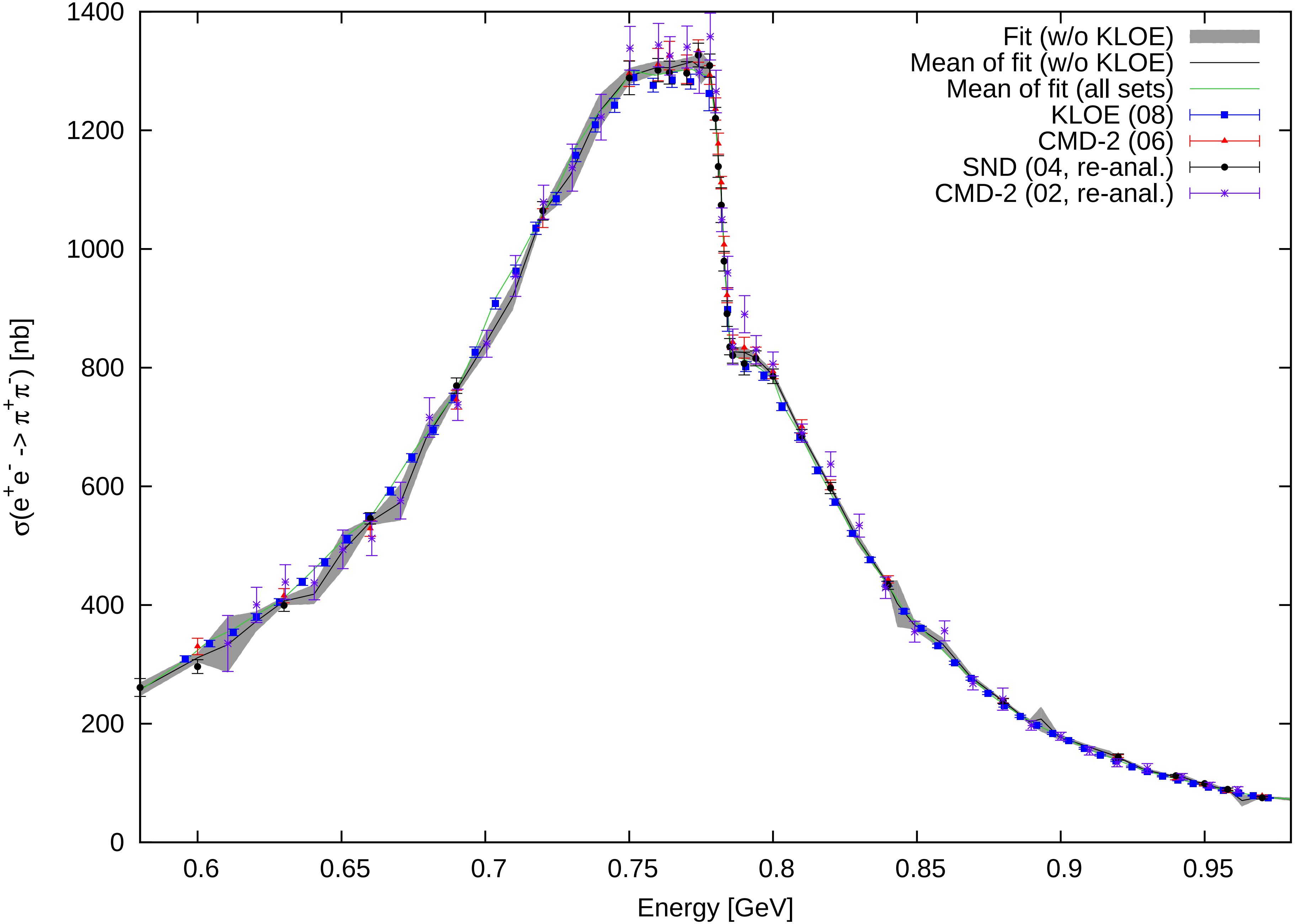
– BES: inclusive R

– CLEO: inclusive R

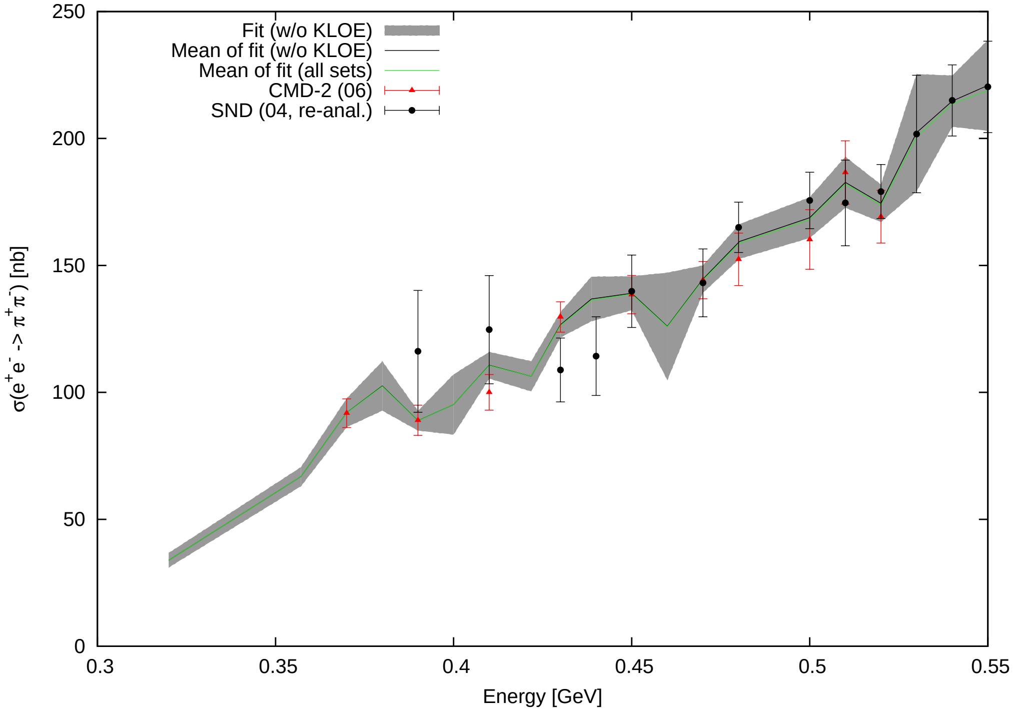
- In principle inclusion of new data in updated analysis straightforward.

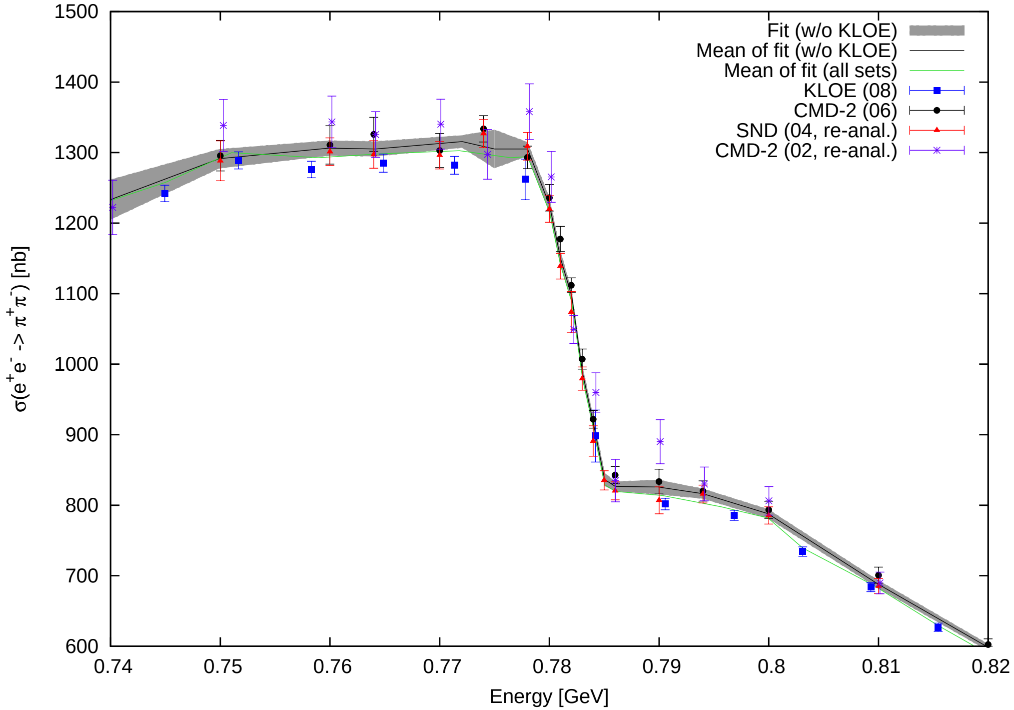
  Concentrate on two cases where not: most important $\mathbf{2\pi}$ and the $\mathbf{1.43 - 2}$ GeV region.

► The most important 2π channel: The overall picture looks very good



Zoom in low energy and peak and $\rho-\omega$ interference region



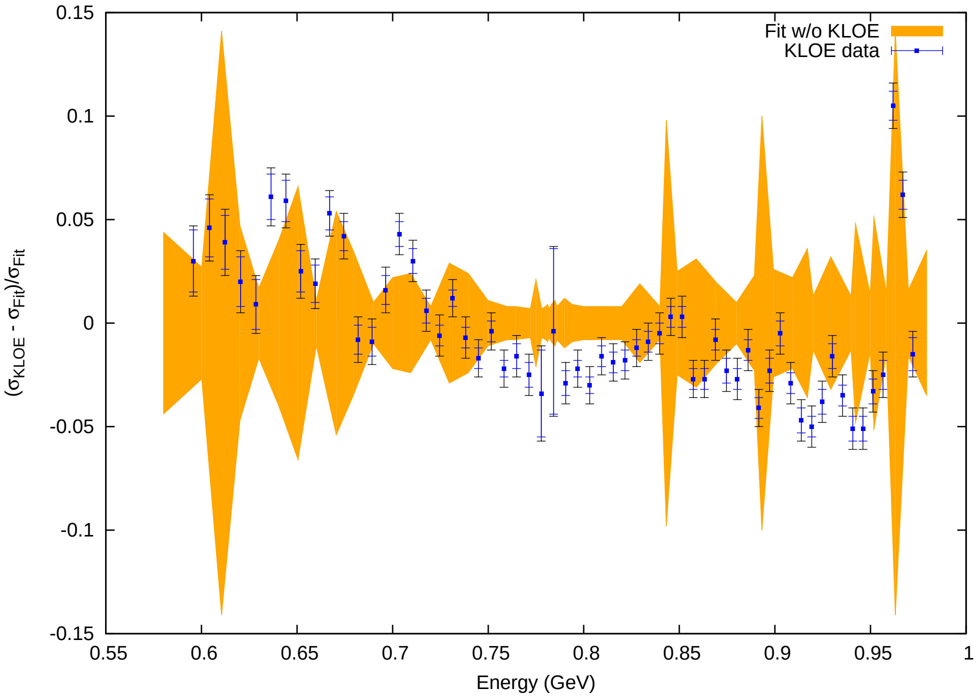


- Very good agreement between data from CMD-2 and SND, fully consistent with earlier data.
- Low energy points crucial for recent improvement of $a_\mu^{\pi\pi}$.
- $g-2$ integral over KLOE data agrees extremely well with the corresponding integral over all other sets:
  KLOE alone: $a_\mu^{\pi\pi} = (384.16 \pm 3.47) \cdot 10^{-10}$, all data without KLOE: $(384.12 \pm 2.51) \cdot 10^{-10}$.

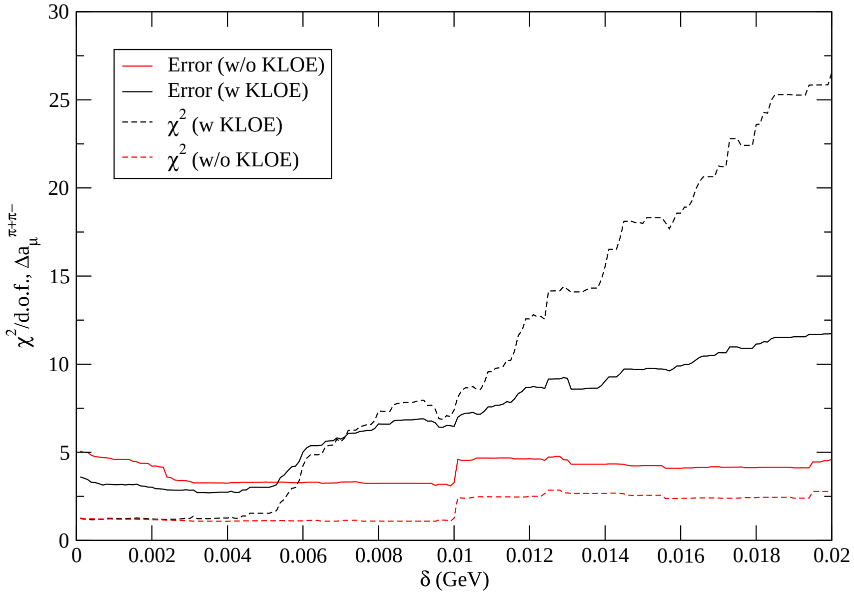
$\longrightarrow$ *However:* some differences in shape prevent good point-by-point combination:

KLOE 2008 $\pi\pi(\gamma)$ radiative return result compared to combination of all data w/out KLOE:

Normalised difference of cross sections



Error Δa_μ and $\chi^2_{\min}/\mathrm{dof}$ as a function of the clustering (~bin) size δ



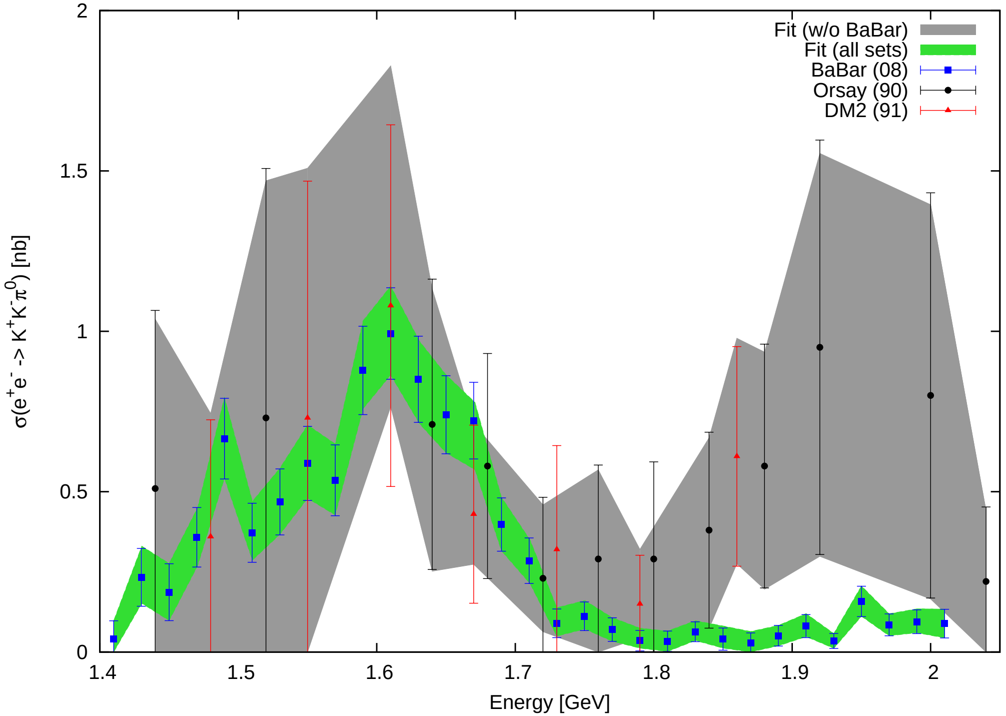
- Tension with other data; less stable fit (w.r.t. variation of cluster size) if KLOE combined pointwise.
- Low KLOE points would force renormalisation within systematic uncertainty, artificially pulling up the fit.

⇝ Combination of the KLOE data after integration only (same treatment as in HMNT 2006 compilation):

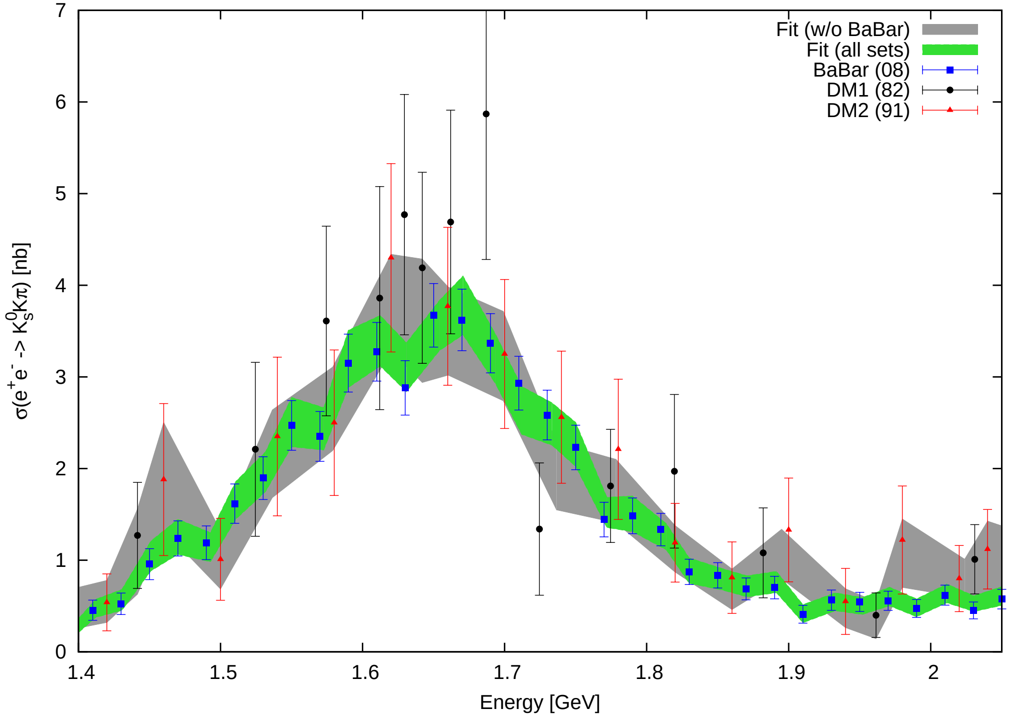
$$a_\mu^{\pi\pi}(0.596\,\mathrm{GeV} < \sqrt{s} < 0.972\,\mathrm{GeV}) = (384.13 \pm 2.03) \cdot 10^{-10}$$

Region below 2 GeV: influence of recent BaBar Radiative Return analyses

$K^+K^-\pi^-$ channel



$K_S^0K\pi$ channel

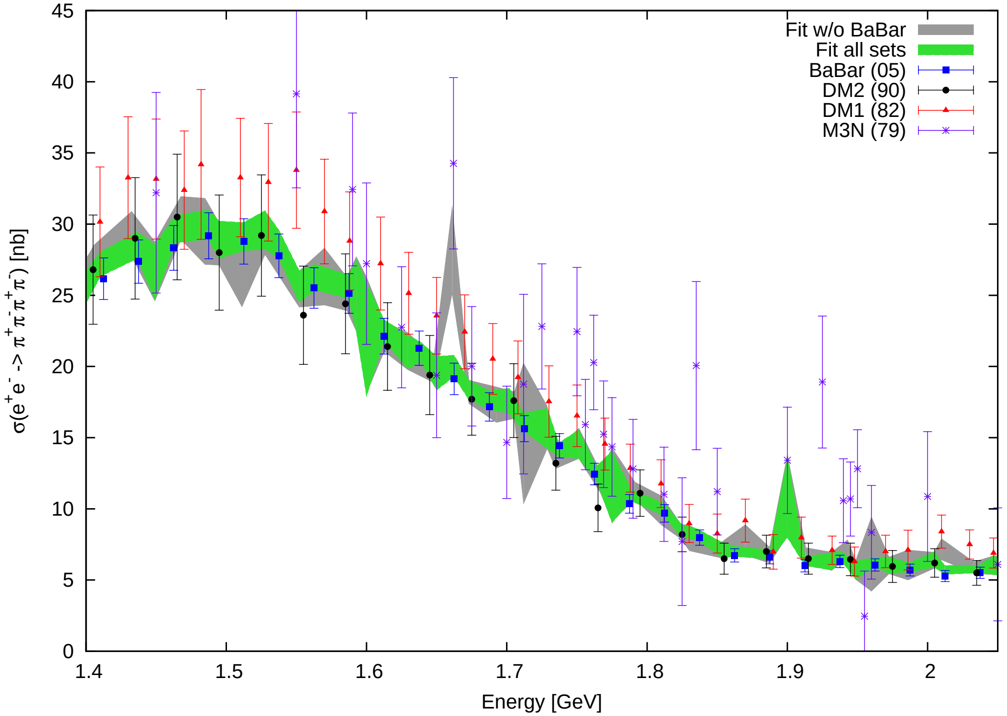


⟶ Important improvements over earlier data compilations.

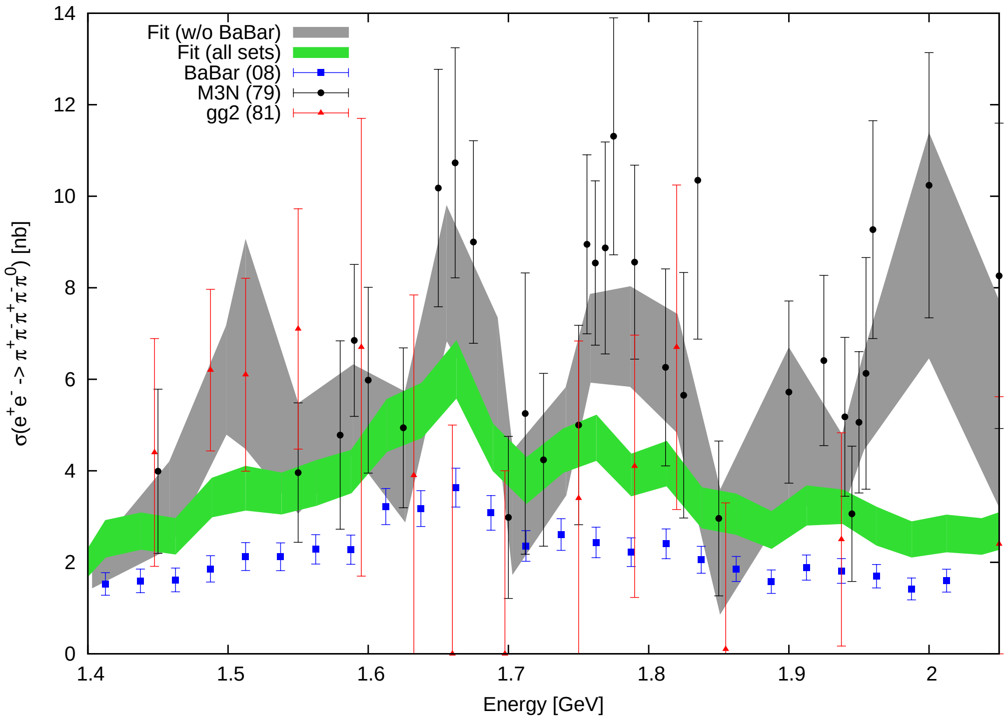
BaBar Radiative Return data lower than less precise older data in most channels.

Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd)

$2\pi^-2\pi^-$ channel



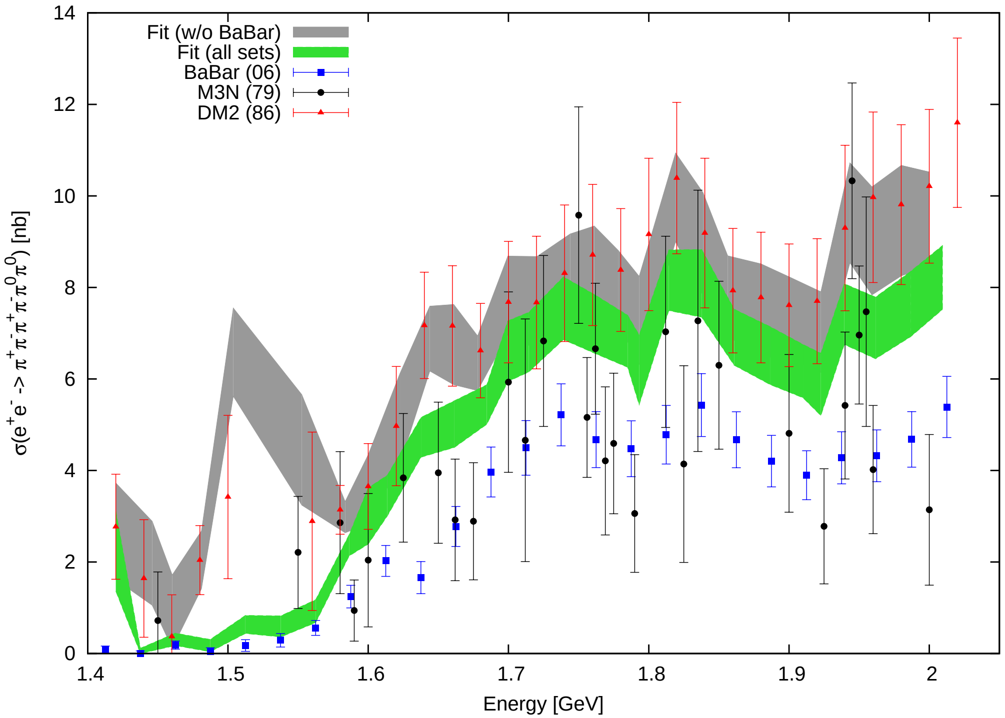
$2\pi^+2\pi^-\pi^0$ channel



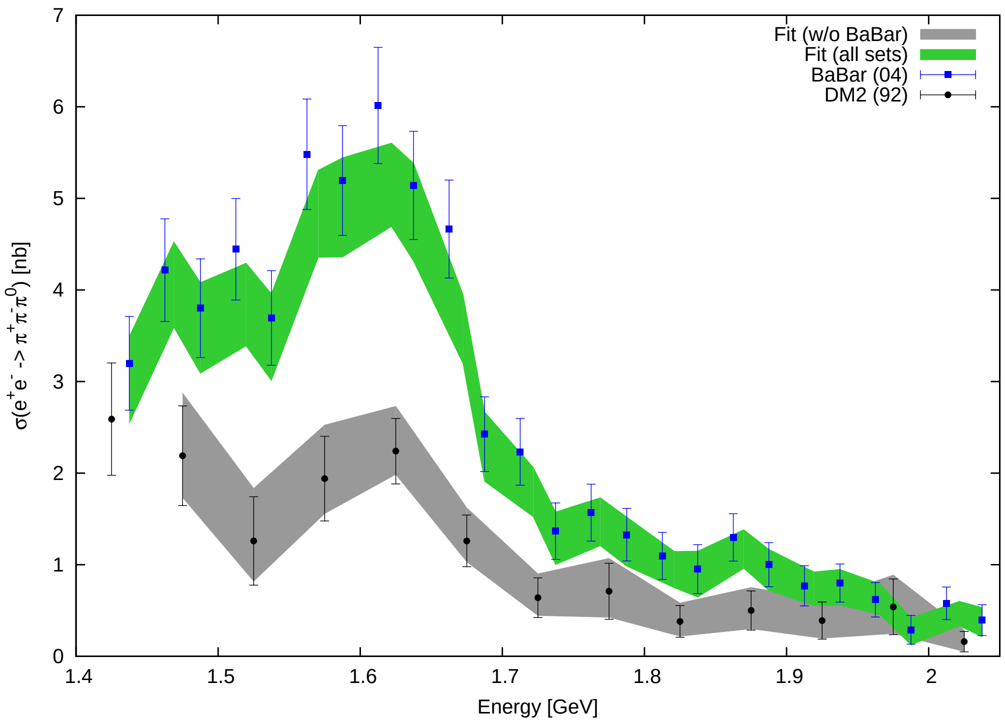
$\longrightarrow$ BaBar lower in $2\pi^+2\pi^-\pi^0$ channel $\rightsquigarrow$ errors for $g-2$ scaled up by $\sqrt{\chi^2_{\min}/\mathrm{dof}} = 1.29$.

► Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd 2)

$2\pi^-2\pi^-2\pi^0$ channel



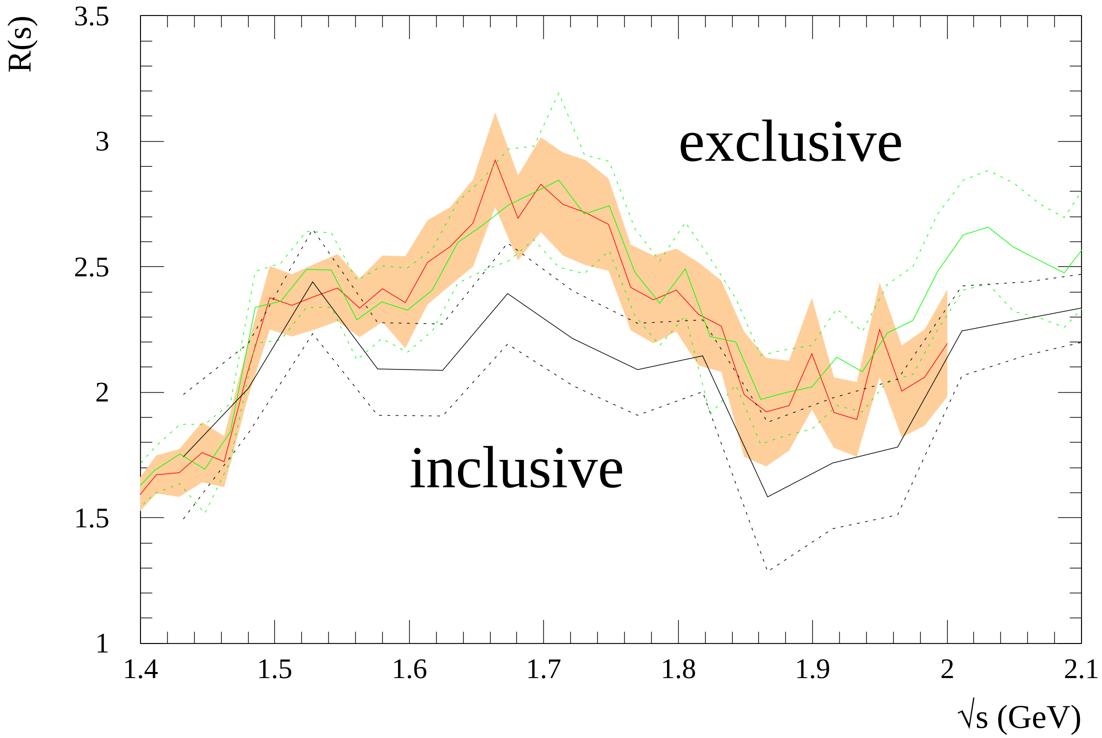
$\pi^+\pi^-\pi^0$ channel



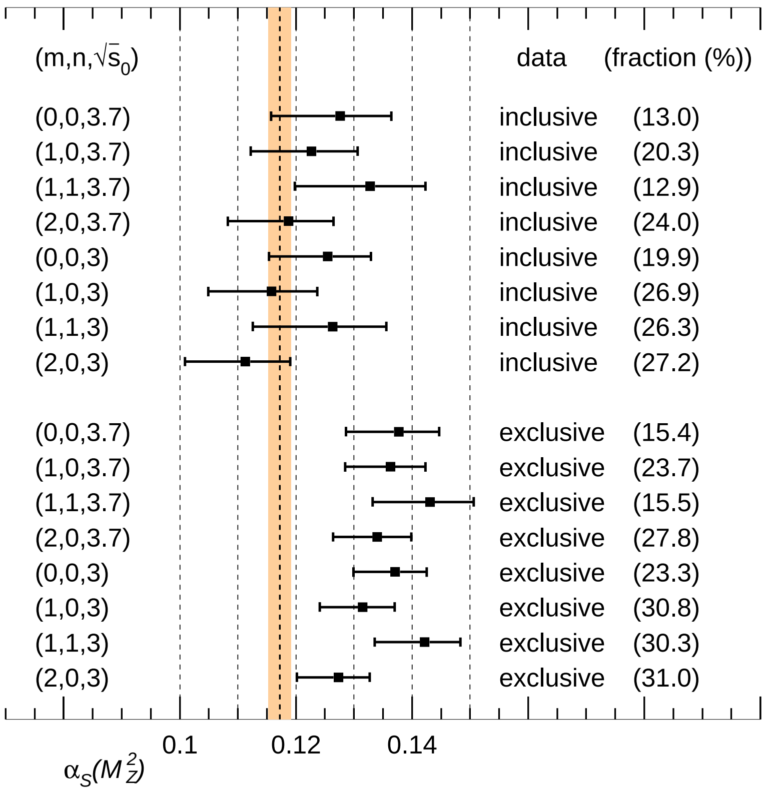
$\longrightarrow$ Again 'bad' $\chi^2_{\rm min}/{\rm dof}$ of 2.7 and 2.9. Data not really compatible, inflate error.

Region below 2 GeV: *inclusive* vs. *Sum over exclusive*

Data Green: old analysis, red/orange: new

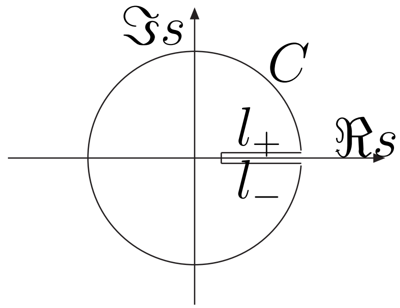


Sum rules 'determining' α_S:



- Shape similar, but normalisation different.
- Question of completeness/quality of sum of exclusive data vs. reliability/systematics of old inclusive data ($\gamma\gamma 2$, MEA, M3N, BBbar)
- HMNT up to now have used incl. data, in line with sum-rule analysis.

Check against perturbative QCD: QCD $\sum$-rule analysis



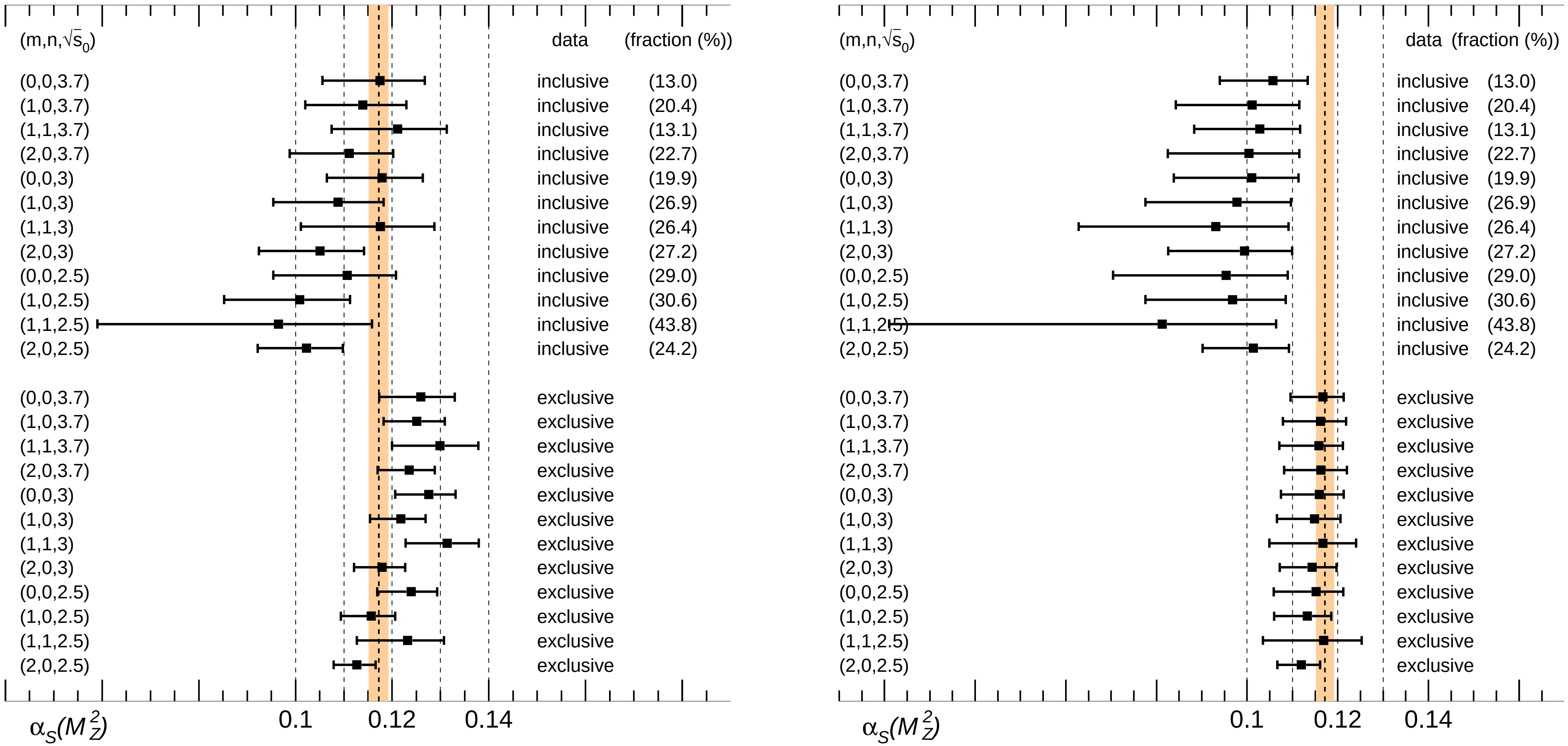
- Evaluate QCD $\sum$-rules of the form:

$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) f(s) = \int_C {\rm d}s\, D(s) g(s)\,, \qquad \text{with} \quad D(s) \equiv -12\pi^2 s \frac{\rm d}{{\rm d}s}\left(\frac{\Pi(s)}{s}\right)$$

- The Adler D function is calculable in pQCD: $D(s) = D_0(s) + D_{\rm m}(s) + D_{\rm np}(s)$
- Take $f(s) = (1 - s/s_0)^m (s/s_0)^n$ to maximize sensitivity to the required region, $g(s)$ follows.
- Choose s_0 below the open charm threshold ($n_f = 3$ for pQCD).
- For $m = 1$, $n = 0$ one gets e.g.

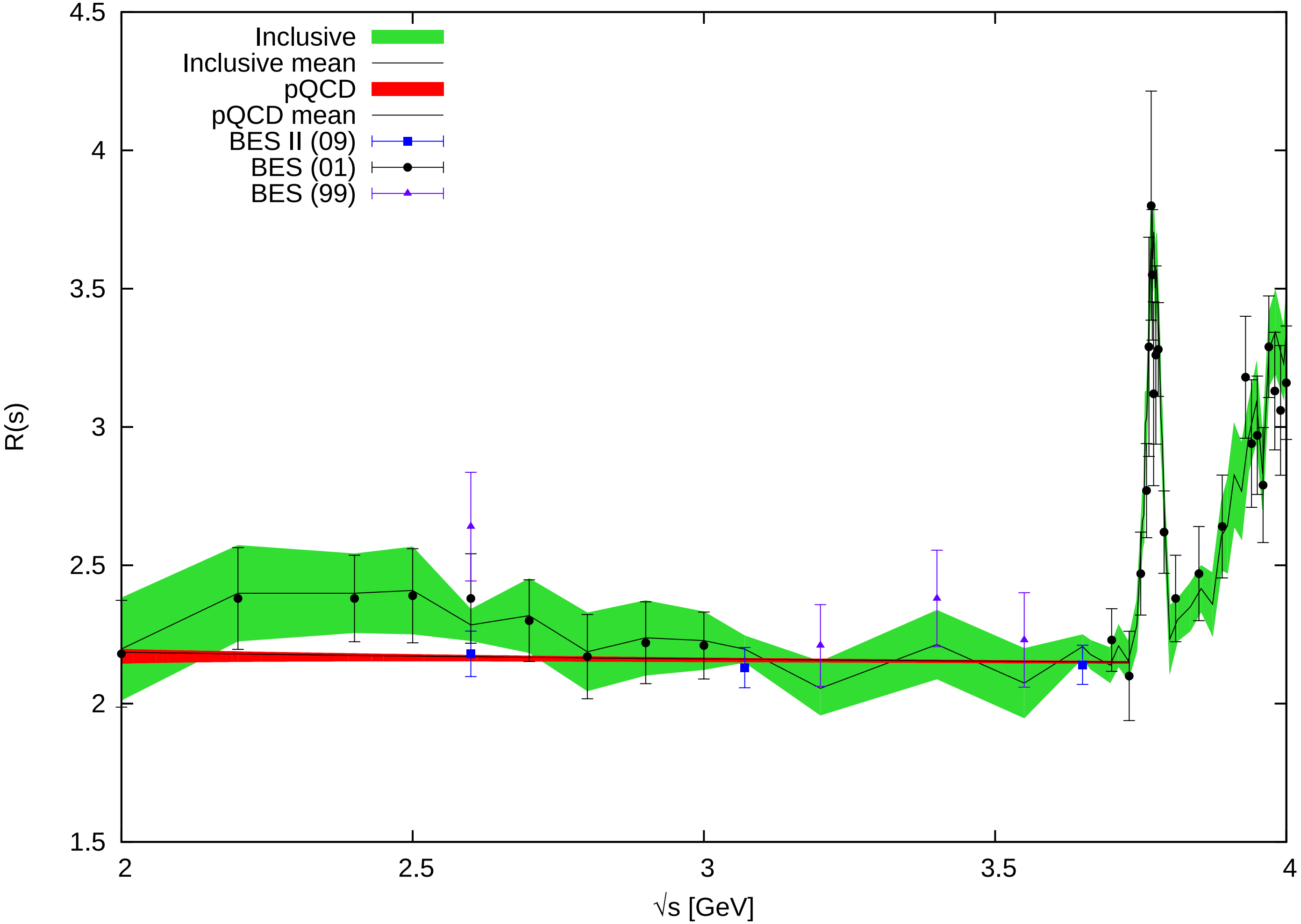
$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) \left(1 - \frac{s}{s_0}\right) = \frac{i}{2\pi} \int_C {\rm d}s \left(-\frac{s}{2s_0} + 1 - \frac{s_0}{2s}\right) D(s)\,.$$

▶ New Sum rule analyses: R: data only pQCD for 2 GeV $< \sqrt{s} < \sqrt{s_0}$ →



- Changes in data have changed the picture → *Sum over exclusive* in line with QCD.
- Still rely on isospin relations for missing channels! [Large error from $K\bar{K}\pi\pi$!]
- For new HLMNT: combine *inclusive* and *Sum over exclusive*.

Perturbative QCD vs. inclusive data above 2 GeV (below charm threshold)



- R_{uds} from pQCD mostly below data in region above 2 GeV
- Latest BES data agree very well with pQCD
- shift downwards relevant for $g-2$ and $\Delta\alpha$

- **Results of new HLMNT compilation (prelim.)** ('Unblinding' only this Monday.. :)

  - Accidental cancellation of mean value shifts between different energy regions (compared to HMNT 2006 analysis, units of 10^{-10}):
    - low energy exclusive channels, $0.32 - 1.43$ GeV: -0.76
    - inclusive–exclusive region, $1.43 - 2$ GeV: $+$2.10
    - higher energy inclusive, $2 - 11.09$ GeV: -1.35

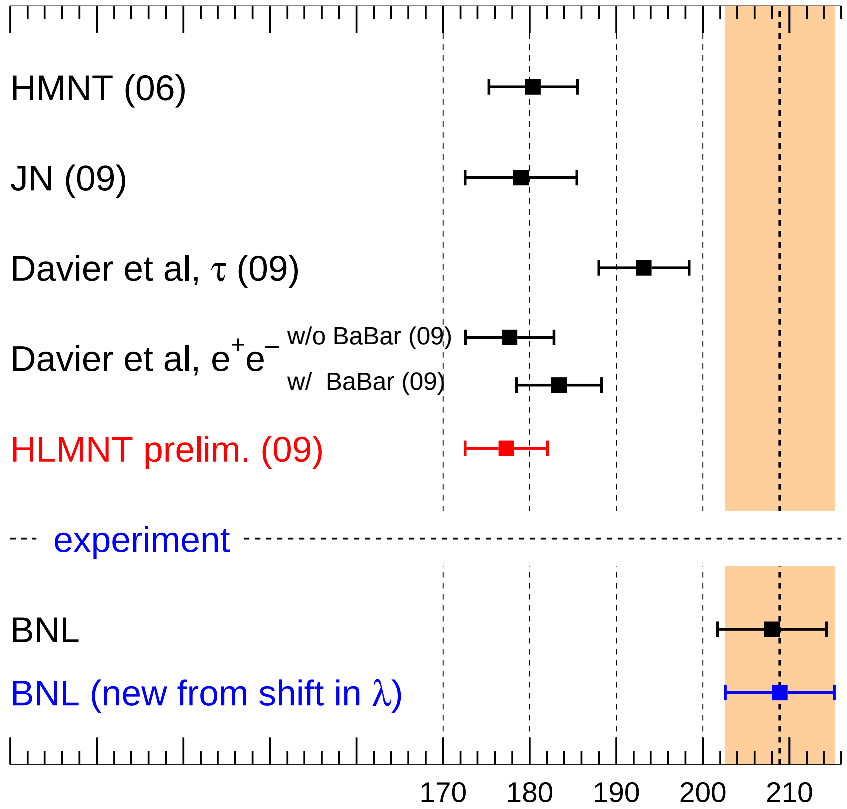
► $a_\mu(\mathrm{LO,had}) = 689.41 \pm 3.61_{\mathrm{exp}} \pm 1.82_{\mathrm{rad}}$

► $a_\mu(\mathrm{NLO,had}) = -9.79 \pm 0.06_{\mathrm{exp}} \pm 0.03_{\mathrm{rad}}$

[Was $a_\mu(\mathrm{NLO,had}) = -9.79 \pm 0.09_{\mathrm{exp}} \pm 0.03_{\mathrm{rad}}$.]

Various choices w.r.t. data, compilation, τ (?!), L-by-L: $a_\mu^{\rm SM}$ always stays $< a_\mu^{\rm EXP}$

$a_\mu^{\rm SM}$ compared to BNL world av.



$a_\mu^{\rm SM} \times 10^{10} - 11659000$

Davier et al.: $1.8/3.9/3.1\,\sigma$

JN 09: $3.2\,\sigma$ [179.0 ± 6.5]

Recent changes

TH: Improved LO hadronic (from e^+e^-):

[New data from CMD-2, SND, KLOE, BaBar, CLEO, BES. Combination of excl. (BaBar RadRet) and incl. data below 2 GeV.]

$(6894 \pm 46) \cdot 10^{-11} \longrightarrow (6894 \pm 40) \cdot 10^{-11}$

TH: Use of recent L-by-L compilation [PdeRV]: $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$

EXP: Small shift of BNL's value due to CODATA's shift of muon to proton magn. moment ratio:

Was $a_\mu = 116\,592\,080(63) \times 10^{-11}$

$\rightarrow a_\mu = 116\,592\,089(63) \times 10^{-11}$ $(0.5 ppm)$

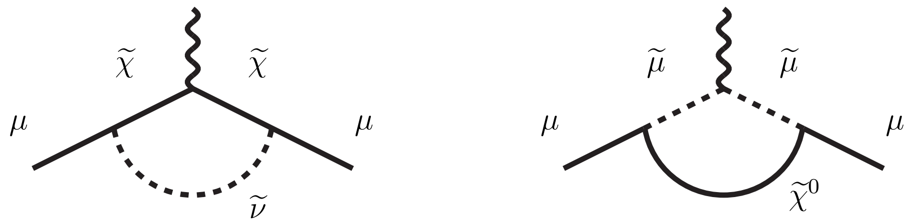
▶ With this input HLMNT get:

$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (31.6 \pm 7.9) \cdot 10^{-10}$, $\sim \mathbf{4.0\sigma}$

SUSY contributions in a_μ ?

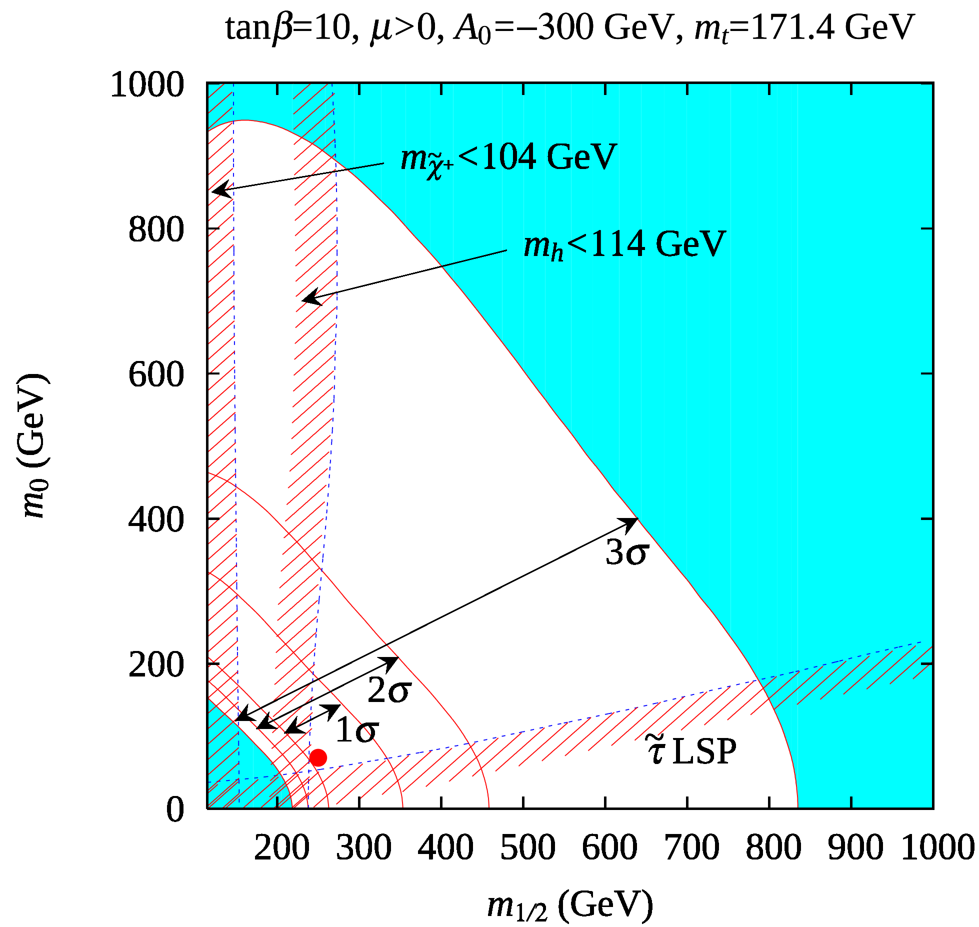
$$a_\mu^{\text{SUSY,1-loop}} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \tan\beta \, \text{sign}(\mu) \frac{m_\mu^2}{M_{\text{SUSY}}^2}$$

They mainly come from:



SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$, but

- no chargino at LEP
- so far no light Higgs
- $\tilde{\tau}$ prob. not LSP
- $+$ limits from direct searches
- SPS 1a′ in 1σ band from $g-2$



II. $\Delta\alpha(q^2)$: Vacuum Polarisation in the space- and time-like

- Why Vacuum Polarisation / running α corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

— Corrections for data used as input for $g-2$: 'undressed' $\sigma^0_{\rm had}$

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\,\sigma^0_{\rm had}(s)K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

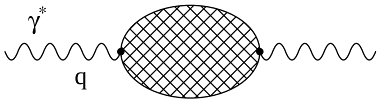
— Determination of α_s and quark masses from total hadronic cross section $R_{\rm had}$ at low energies and of resonance parameters.

— Part of higher order corrections in Bhabha scattering important for precise Luminosity determination.

— $\alpha(M_Z^2)$ a fundamental parameter at the Z scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits/constrain new physics.

$\longrightarrow$ Ingredient in MC generators for many processes.

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathrm{QED}}$:

$\Pi \; =$ 

Full photon propagator $\sim \; 1 \, + \, \Pi \, + \, \Pi \cdot \Pi \, + \, \Pi \cdot \Pi \cdot \Pi \, + \, \ldots$

$$\rightsquigarrow \qquad \alpha(q^2) \; = \; \frac{\alpha}{1 - \mathrm{Re}\Pi(q^2)} \; = \; \alpha \, / \, \left(1 - \Delta\alpha_{\mathrm{lep}}(q^2) - \Delta\alpha_{\mathrm{had}}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) \; = \; -\frac{q^2}{4\pi^2\alpha} \, \mathrm{P} \int_{m_\pi^2}^{\infty} \frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s - q^2} \, , \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{|1 - \Pi|^2}$$

[$\rightarrow \sigma^0$ requires 'undressing', e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the $|$full photon propagator$|^2$, i.e. $|$infinite sum$|^2$.
$\rightarrow$ To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Comparison of different compilations

- Timelike $\alpha(s)$ from Fred Jegerlehner's (2003 routine as available from his web-page)

$$\alpha(s = E^2) \;=\; \alpha \,/\, \Big(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\Big)$$

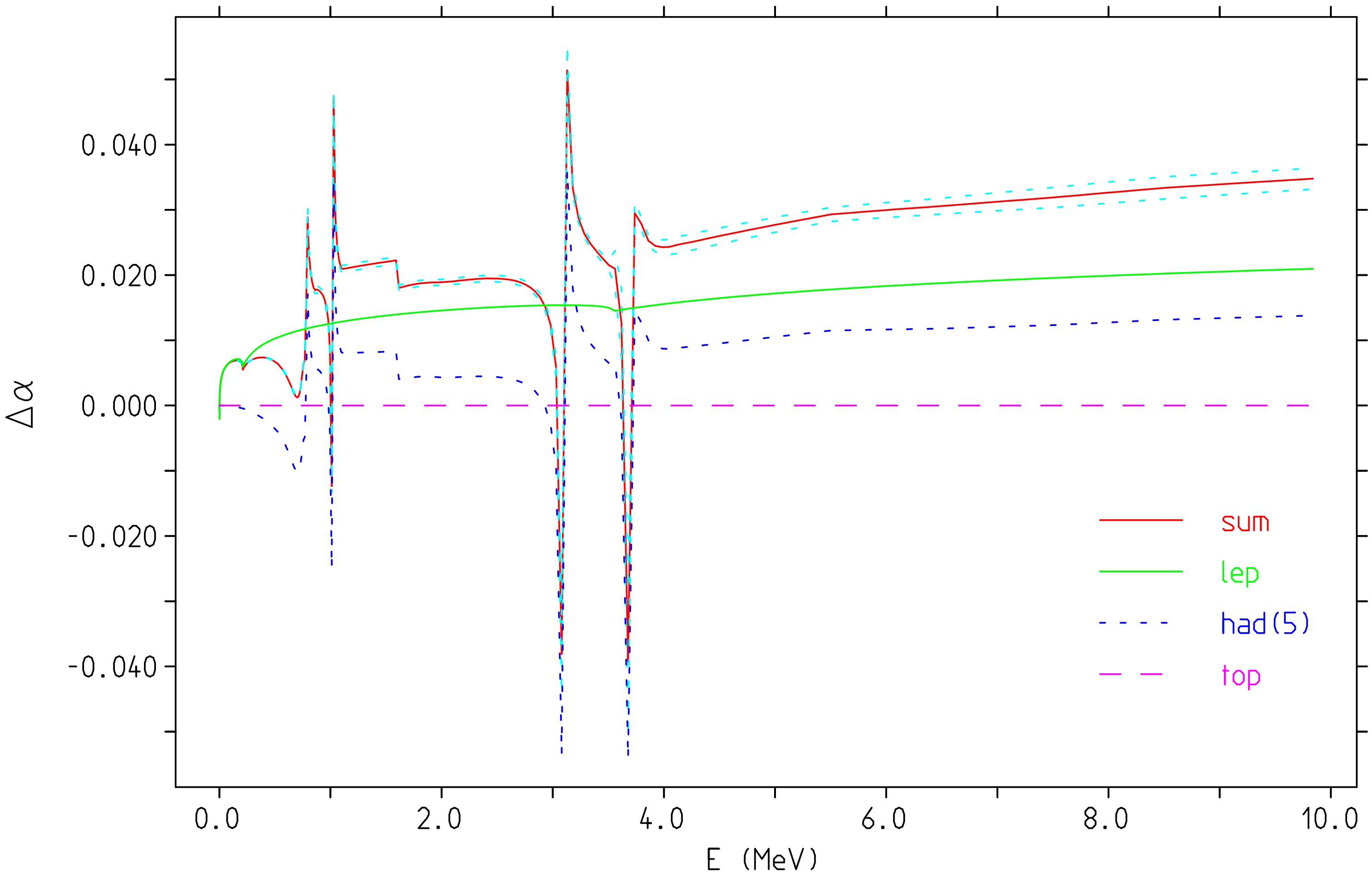
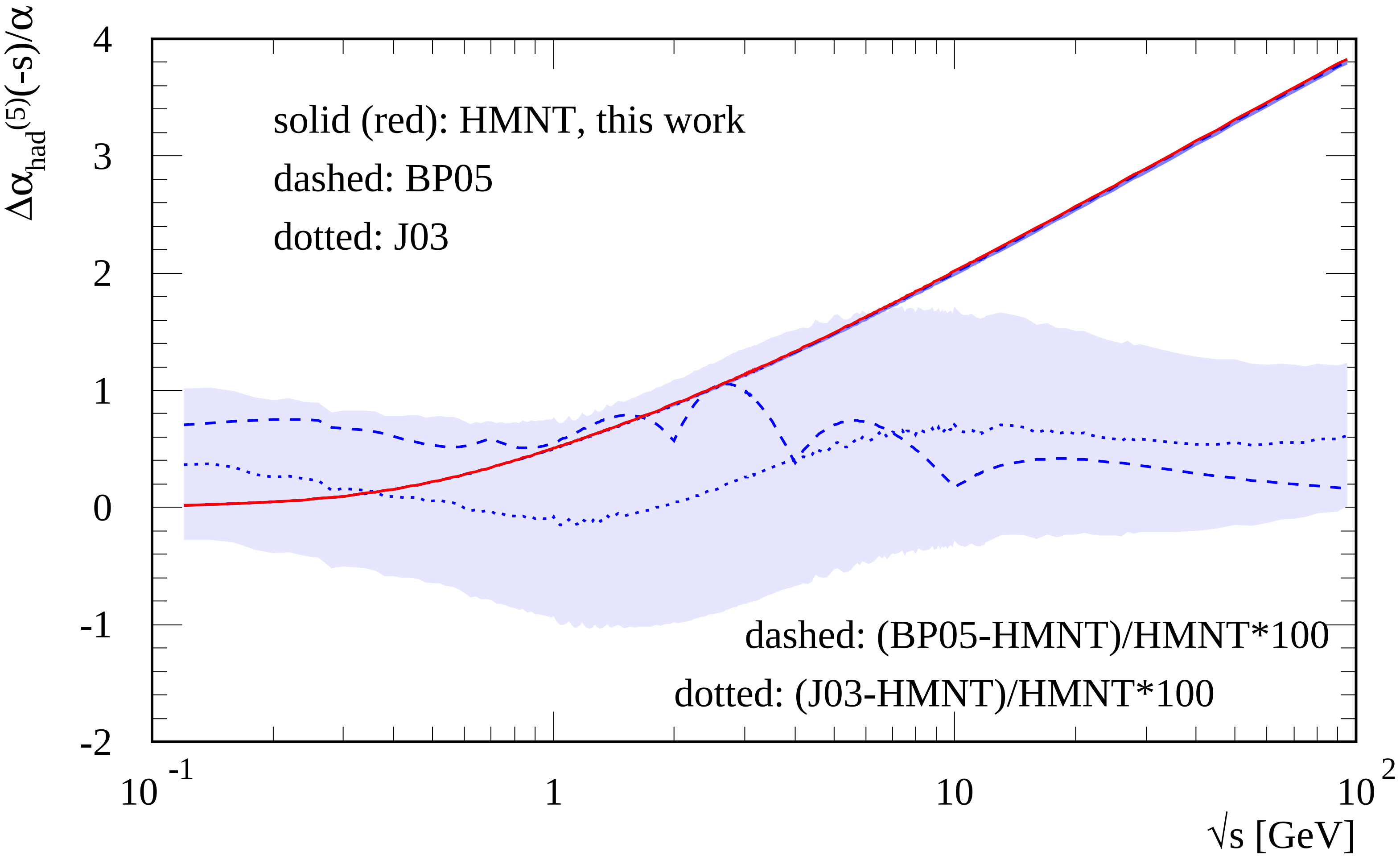


Figure from Fred Jegerlehner

- HMNT's evaluation of $\alpha_{\rm QED}(q^2)$ compared to other parametrisations:

Spacelike $\Delta\alpha^{(5)}_{\rm had}(-s)/\alpha$ (smooth $\alpha(q^2 < 0)$)

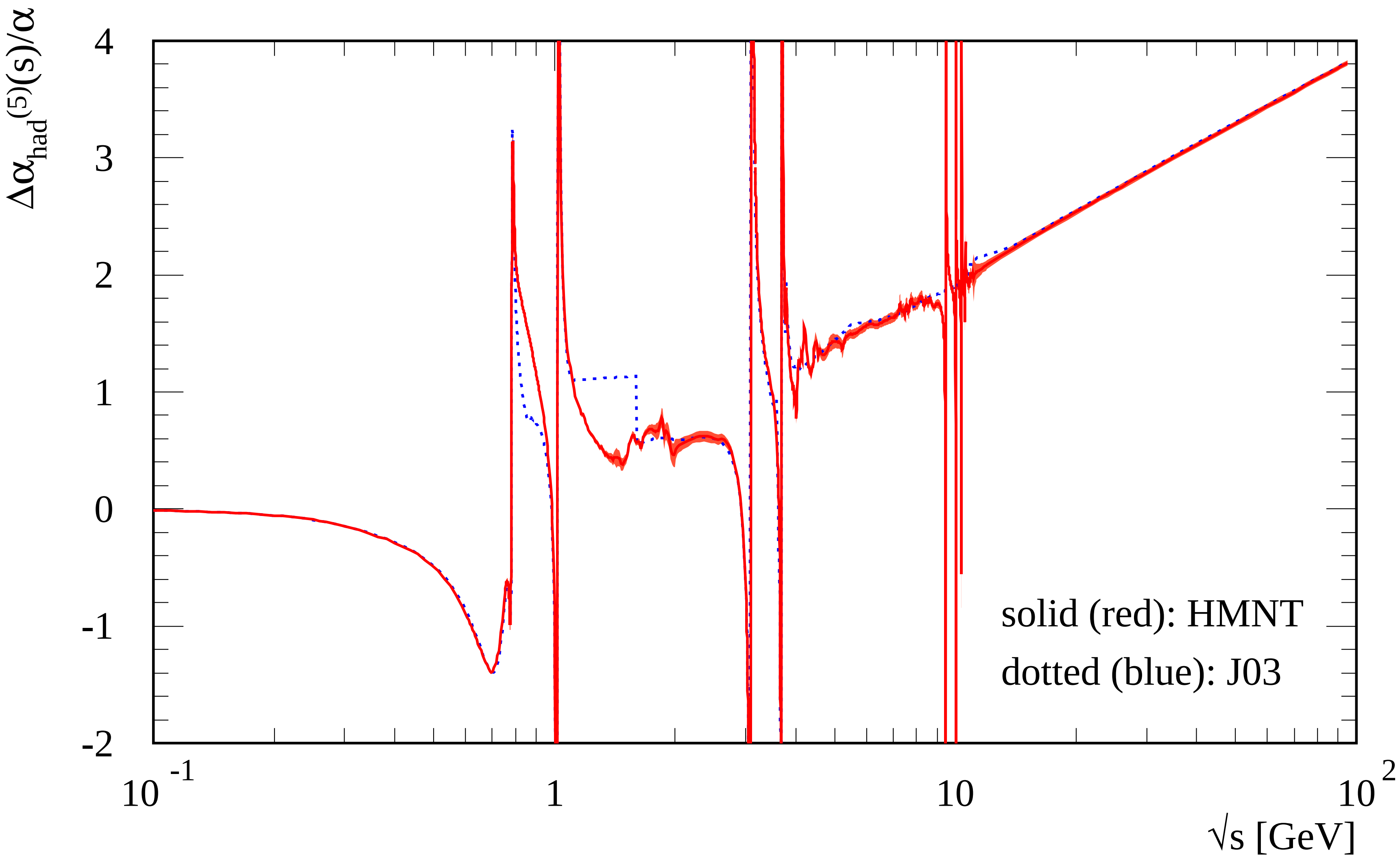


– Differences between parametrisations clearly visible but within error band (of HMNT)

– Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.

– What is in your MC?

Timelike $\alpha(s = q^2 > 0)$ follows resonance structure:



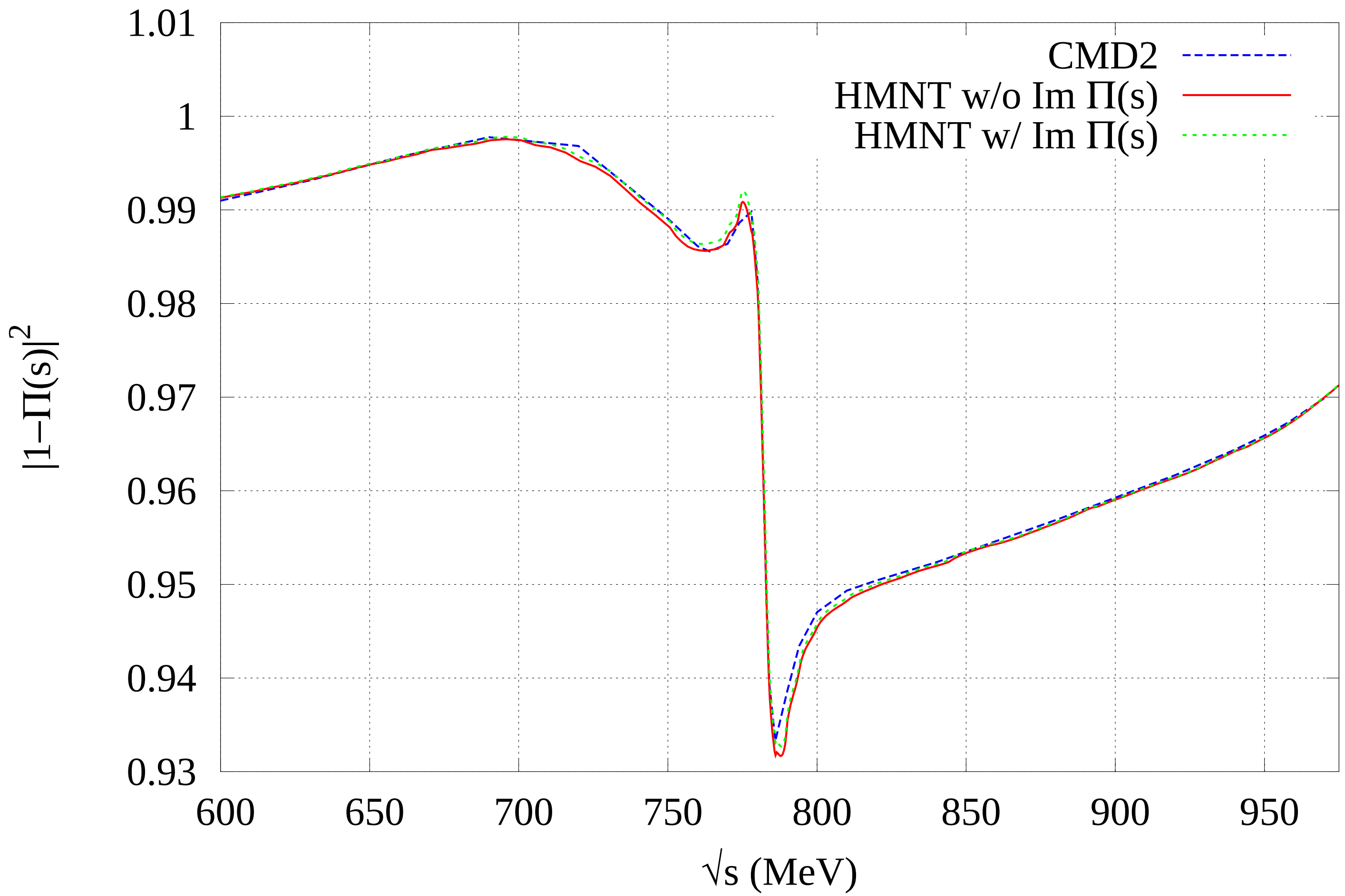
— Step below just a feature of unfortunate grid.

— Difference below 1 GeV not expected from data.

[Comparisons with other parametrisations confirm HMNT.]

- HMNT compared to Novosibirsk's new parametrisation $\longrightarrow$ G Fedotovich's talk

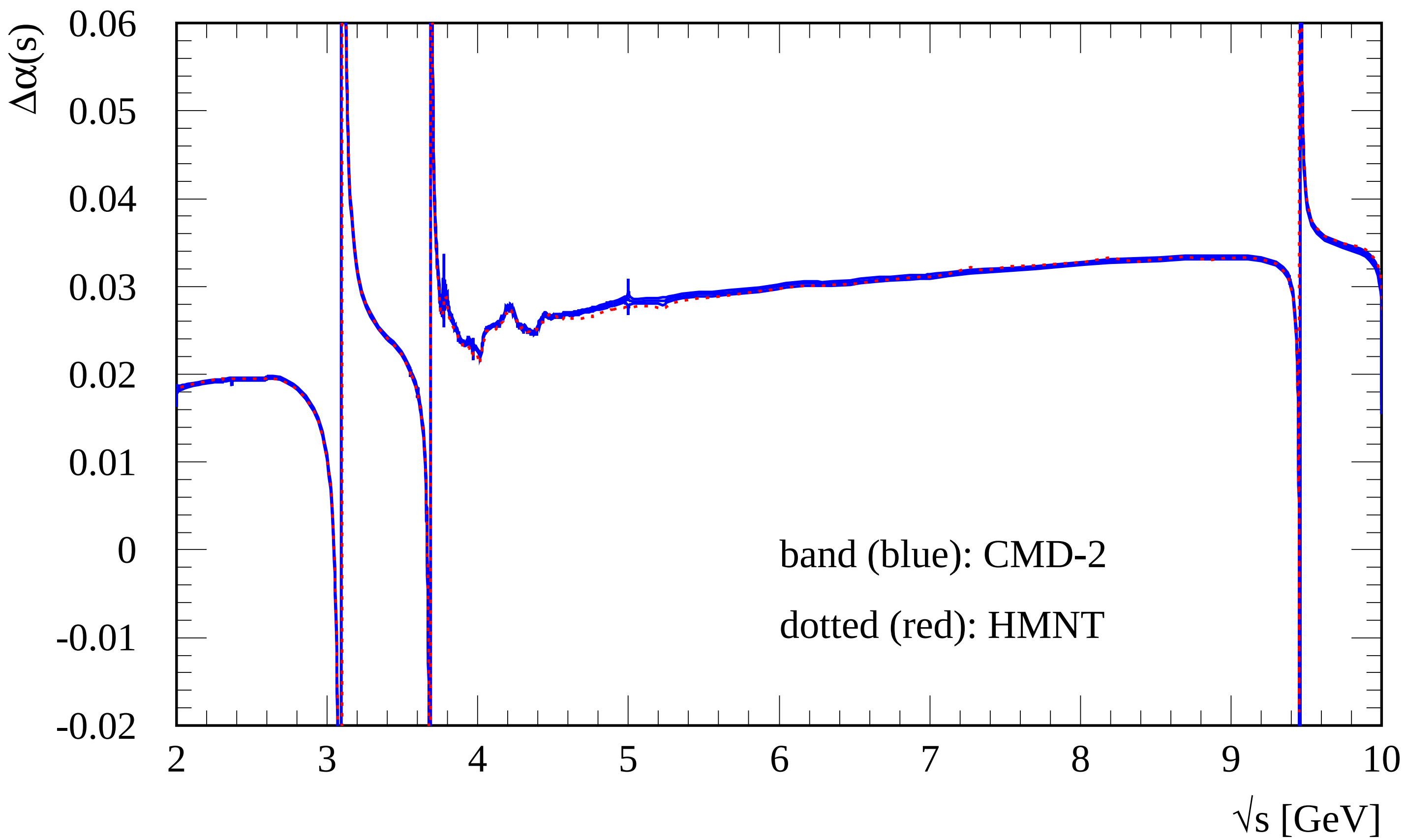
Timelike $|1-\Pi(s)|^2 \;\sim\; (\alpha(s)/\alpha)^2$ in ρ central energy region: A relevant correction!



(Different sign and prefactor, $-e^2$, used for Π by HMNT.)

$\longrightarrow$ Small but visible differences, as expected from independent compilations.

- HMNT compared to Novosibirsk   Timelike, $\Delta\alpha(q^2)$



$\longrightarrow$ Differences of about one per-mille in the 'undressing' factor, up to -3/+5 per-mille in the $\rho - \omega$ interference regime, but likely to cancel at least partly in applications.

$\longrightarrow$ As expected small contributions from $\mathrm{Im}\Pi$.

- **What about $\Delta\alpha(M_Z^2)$?** $\rightarrow$ M Passera's talk

$\longrightarrow$ With the same data compilation of $\sigma^0_{\rm had}$ as for $g-2$ HLMNT find:

$\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02760 \pm 0.00015}$ (HLMNT 09 prelim.)

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.947 \pm 0.020$ [HMNT '06: $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$]

Earlier compilations:

Group	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '08	0.027594 ± 0.000219	data driven/pQCD
($M_0 = 2.5$ GeV)	0.027515 ± 0.000149	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

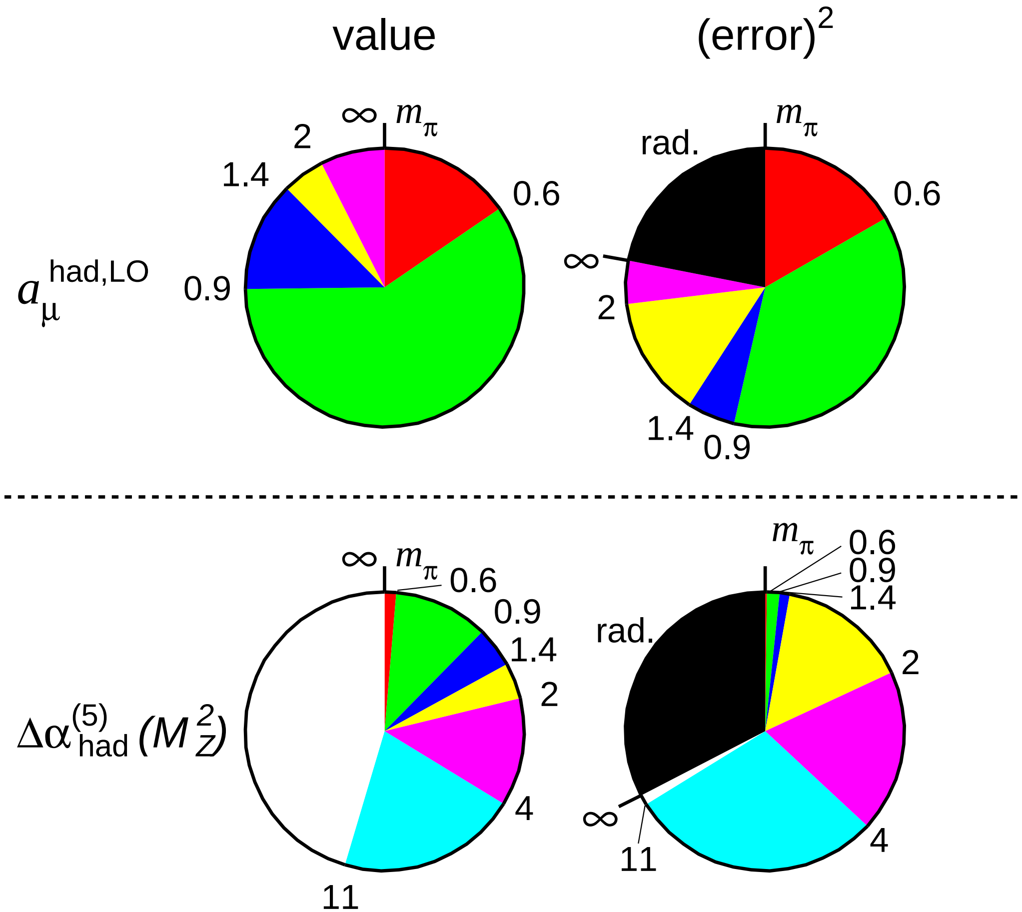
Adler function: $D(-s) \;=\; \frac{3\pi}{\alpha} s \frac{\rm d}{{\rm d}s} \Delta\alpha(s) \;=\; -(12\pi^2) s \frac{{\rm d}\Pi(s)}{{\rm d}s}$

allows use of pQCD and minimizes dependence on data.

III. Outlook

Where is improvement needed most urgently?

New pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2: enjoy!



Critical regions:

$\rightarrow a_\mu$:

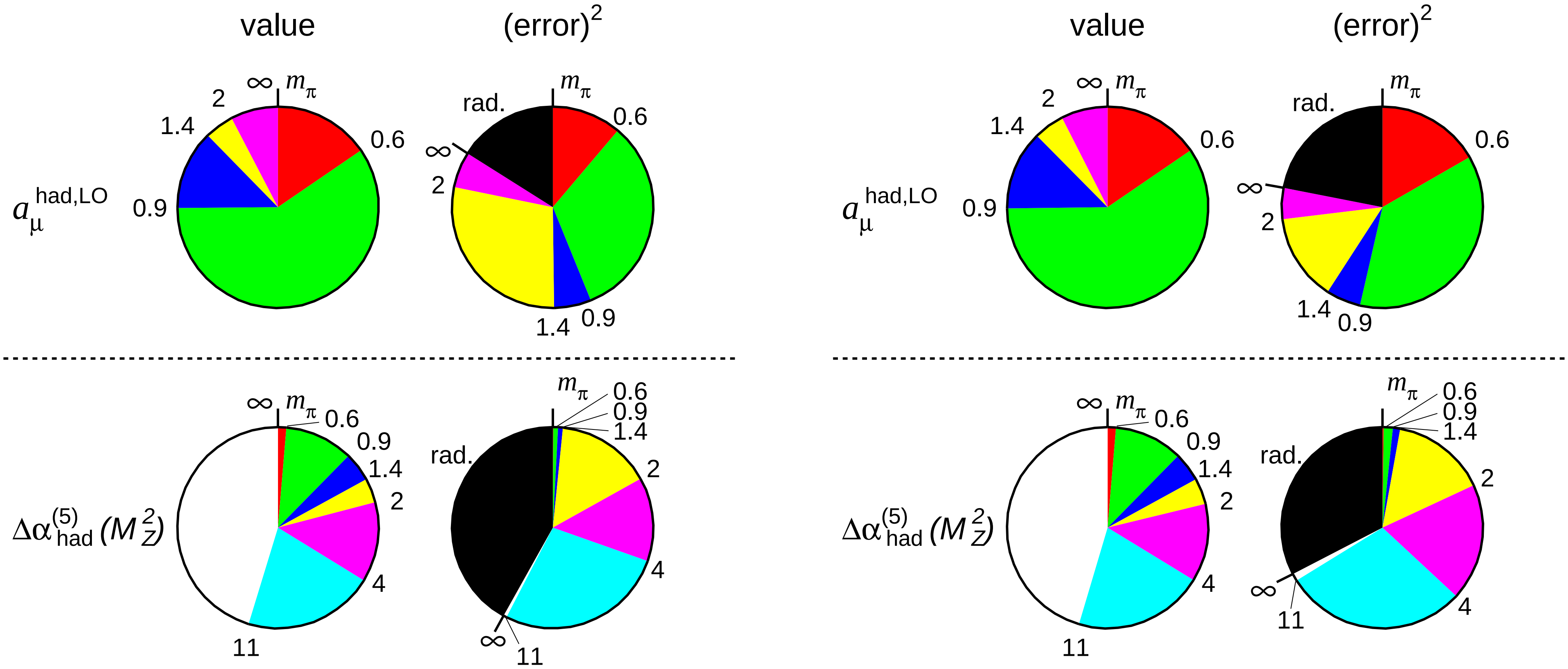
$1.4 - 2$ GeV significantly improved;
now ρ central and low needed, and
still more in region below 2 GeV.

$\rightarrow \alpha(M_Z)$:

inclusive data improved/replaced,
hence *better* control of radcors!

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2:

HMNT 06: HLMNT 09 (prelim.):



The biggest changes are from the incl–excl region and the inclusive data at higher energies.

Prospects for further improvements of $g-2$ and $\Delta\alpha$ through better data:

- Already more Radiative Return from KLOE !
  $\longrightarrow$ check 2π down to threshold and hopefully combine to squeeze error.

- BaBar already very successful with RadRet for higher multiplicity channels
  $\hookrightarrow$ critical region $1.4\ldots2$ GeV should improve further.
  $\longrightarrow$ $\pi\pi\gamma$!?!

- More opportunities for BELLE.

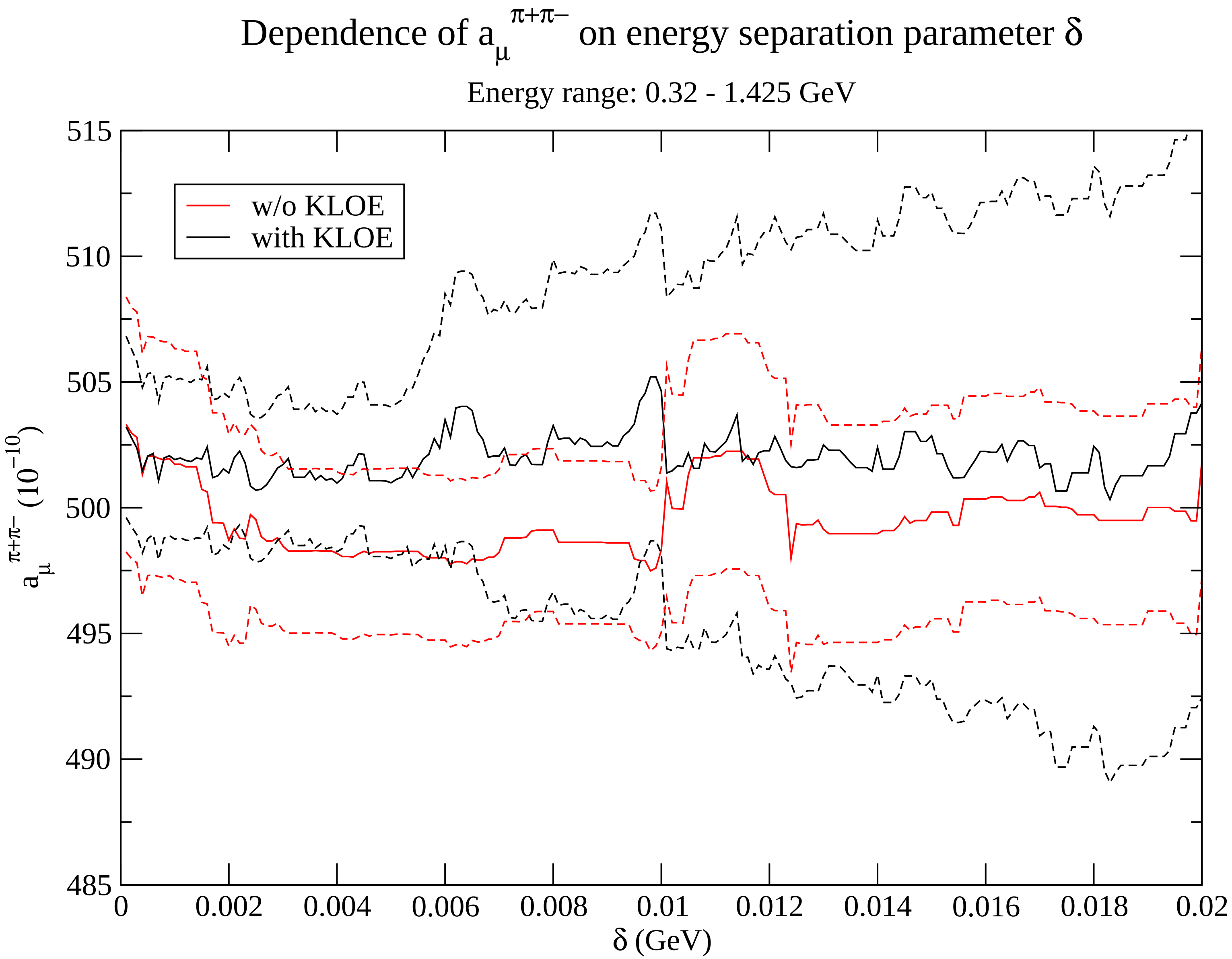
- With the upcoming VEPP-2000 in Novosibirsk, and hopefully further flavour factories, significant improvements foreseen.

- At higher energies, most relevant for $\Delta\alpha(M_Z^2)$, more analyses from BaBar and BES, and soon BES-II !

♣ With a new $g-2$ experiment, New Physics, if there, will be in reach.

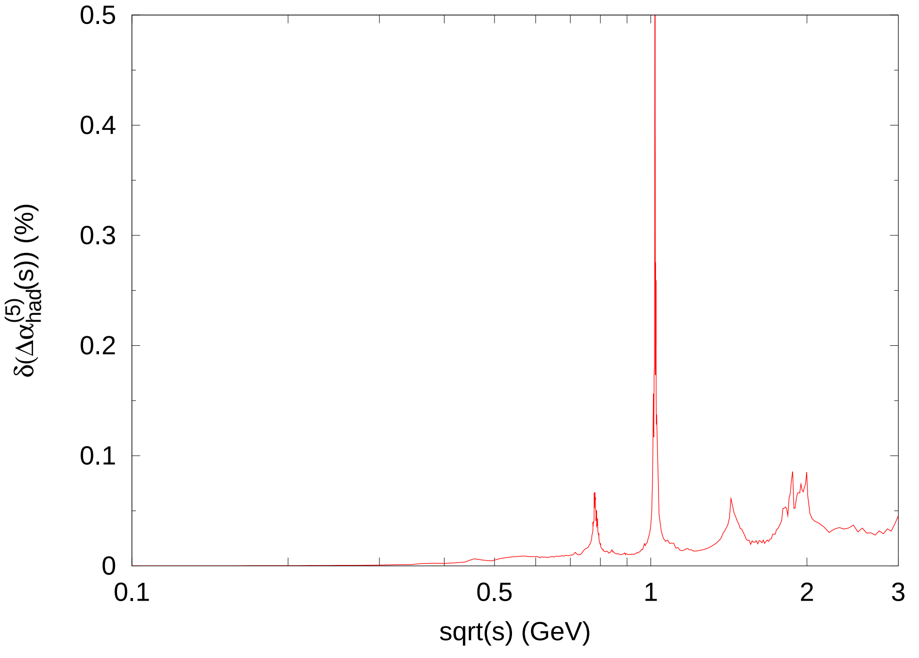
Extras:

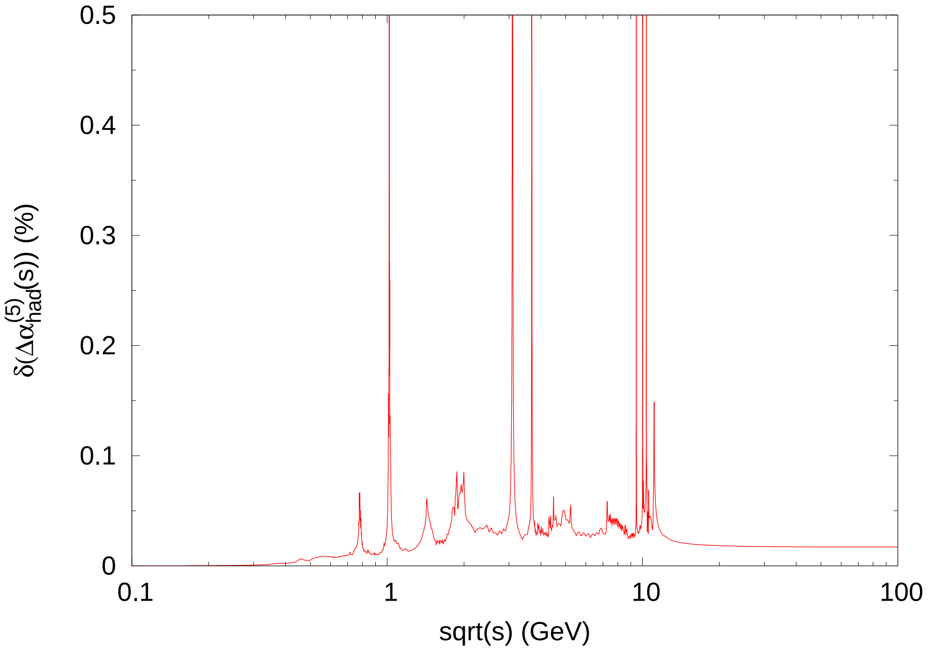
▶ Effect of KLOE $\pi\pi$ data if fitted pointswise:



$\delta\left(\Delta\alpha_{\rm had}^{(5)}(s)\right)$ of HMNT compilation

Error of VP in the timelike regime at low and higher energies:





$\longrightarrow$ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.