News of PHOTOS MC for $\gamma* \rightarrow \pi^-\pi^+(\gamma)$ and $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu_e(\gamma)$

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PHIPSI09 16 Oct 2009, IHEP Beijing

Outline

- Brief Introduction to PHOTOS
- $\gamma^* \to \pi^+\pi^-(\gamma)$
- $K^\pm \to \pi^+\pi^- e^\pm \nu_e(\gamma)$
- Summary

Brief Introduction to PHOTOS

- PHOTOS is used to simulate effect of QED in decays
- PHOTOS can be combined with other main process, generators
- Virtual QED correction $\propto |M_{\mathrm{Born}}|^2 \times \delta_{\mathrm{virt}}$
- In soft and collinear limits, $|M|^2 = |M_{\mathrm{Born}}|^2 \times \delta_{\mathrm{soft/collinear}}$
- Similar factorization is done for phase space too, but it is exact and full phase space is covered
- Process independent kernel in PHOTOS is used: Born matrix element times a factor

Brief Introduction to PHOTOS

- In PHOTOS, all processes can be simulated using PHOTOS kernel and exact matrix element
  $wt = \frac{|M_{\rm exact}|^2}{|M_{\rm kernel}|^2}$
- Results were compared process after process. We found PHOTOS kernel is a very good approximation
- QED, scalar QED decaying processes in PHOTOS
  - $Z \ \ (\gamma^*, H) \to \mu^+\mu^-(\gamma)$
  - $B^0 \to K^+K^-, \pi^+\pi^-, K^+\pi^-(\gamma)$, $B^\pm \to K^\pm K^0, \pi^\pm\pi^0(\gamma)$
  - $W^\pm \to l^\pm \nu_l(\gamma)$
  - $\gamma^* \to \pi^-\pi^+(\gamma)$
  - $K^\pm \to \pi^+\pi^-e^\pm\nu_e$ at work

$\gamma^* \to \pi^+\pi^-(\gamma)$

In PHOTOS, kernel for scalars is obtained from B decays
$B_0(P) \to \pi^\pm(q_1)K^\mp(q_2)\gamma(k,\epsilon)$

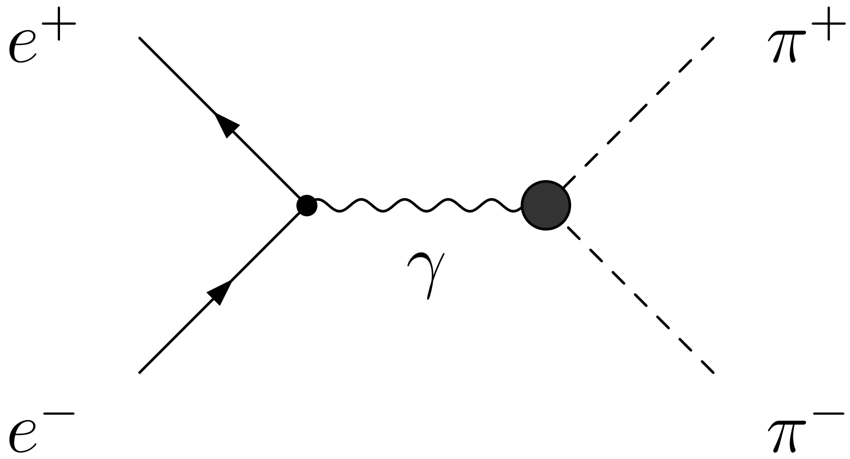
$$|M|^2_{\mathrm{PHOTOS}} = 4\pi\alpha|M_{Born}|^2 \left(Q_1\frac{q_1\cdot\epsilon}{q_1\cdot k} - Q_2\frac{q_2\cdot\epsilon}{q_2\cdot k}\right)^2$$

$Q_1,\ Q_2$ are the charges of final masons

Since spin structure of $e^+e^- \to \gamma^* \to \pi^+\pi^-\gamma$ is different from B decays, $\sum_{\lambda,\epsilon}|M|^2(\gamma^* \to \pi^+\pi^-\gamma)$ is different from kernel used in standard PHOTOS for scalars!

$\gamma^* \to \pi^+\pi^-(\gamma)$

$$e^+(p_1,\lambda_1)e^-(p_2,\lambda_2) \to \gamma^* \to \pi^+(q_1)\pi^-(q_2)$$



$$M_{born} = V_\mu H_0^\mu$$
$$V_\mu = \imath e \bar{u}(p_1,\lambda_1)\gamma_\mu v(p_2,\lambda_2)$$
$$H_0^\mu = \frac{eF_{2\pi}(S)}{S}(q_1 - q_2)^\mu$$

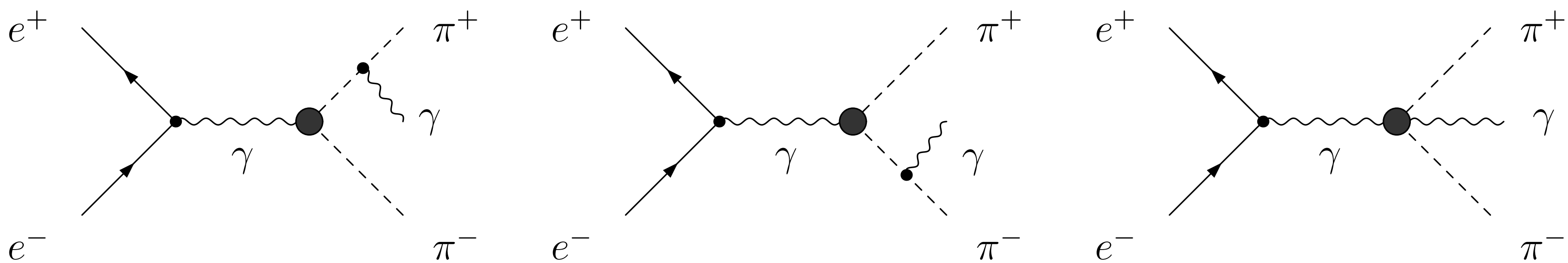
$$\sum_\lambda |M_{Born}|^2(S,T,U) = \frac{8(4\pi\alpha)^2 F_{2\pi}^2(S)}{S^2}\left(TU - m_\pi^2 S\right) \propto q^2 \sin\theta_B^2$$

$$S = 2p_1 \cdot p_2, \quad T = 2p_1 \cdot q_1, \quad U = 2p_1 \cdot q_2$$

q is the length of $\vec{q_1}$, $\theta_B = \angle p_1 q_1$

$\gamma^* \to \pi^+\pi^-\gamma$

$$e^+(p_1,\lambda_1)e^-(p_2,\lambda_2) \to \gamma^* \to \pi^+(q_1)\pi^-(q_2)\gamma(k,\epsilon)$$



$$M = V_\mu H^\mu \quad , \quad V_\mu = \imath e \bar{u}(p_1,\lambda_1)\gamma_\mu v(p_2,\lambda_2)$$

$$H^\mu = \frac{e^2 F_{2\pi}(S)}{S}\left\{(q_1+k-q_2)^\mu \frac{q_1\cdot\epsilon}{q_1\cdot k} + (q_2+k-q_1)^\mu \frac{q_2\cdot\epsilon}{q_2\cdot k} - 2\epsilon^\mu\right\}$$

Rewrite H^μ into two gauge invariant parts $H^\mu = H_I^\mu + H_{II}^\mu$

$$H_I^\mu = \frac{e^2 F_{2\pi}(S)}{S}\left((q_1-q_2)^\mu + k^\mu \frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}\right)\left(\frac{q_1\cdot\epsilon}{q_1\cdot k} - \frac{q_2\cdot\epsilon}{q_2\cdot k}\right)$$

$H_I^\mu \to \sqrt{4\pi\alpha} H_0^\mu \left(\frac{q_1\cdot\epsilon}{q_1\cdot k} - \frac{q_2\cdot\epsilon}{q_2\cdot k}\right)$ for soft and collinear limits

$\gamma^* \to \pi^+\pi^-(\gamma)$

$$H^\mu_{II} = \frac{e^2 F_{2\pi}(S)}{S}\left(k^\mu\left(\frac{q_1\cdot\epsilon}{q_1\cdot k}+\frac{q_2\cdot\epsilon}{q_2\cdot k}\right) - 2\epsilon^\mu - k^\mu \frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}\left(\frac{q_1\cdot\epsilon}{q_1\cdot k}-\frac{q_2\cdot\epsilon}{q_2\cdot k}\right)\right)$$

$$= \frac{2e^2 F_{2\pi}(S)}{S}\left(\frac{k^\mu}{q_2\cdot k + q_1\cdot k}(q_1\cdot\epsilon + q_2\cdot\epsilon) - \epsilon^{*\mu}\right)$$

free of soft and collinear and singularities!

Similar factor like $k^\mu \frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}$ in QCD amplitude, see A. van Hameren and Z. Was, Eur.Phys.J.C61:33-49,2009

$$A_I = \sum_{\lambda,\epsilon} |V_\mu H^\mu_I|^2$$

$$A'_I = A_I \frac{S - 4m_\pi^2}{|\vec{q}_1 - \vec{q}_2 + \vec{k}\frac{q_2\cdot k - q_1\cdot k}{q_2\cdot k + q_1\cdot k}|^2} = 4\pi\alpha \sum_\lambda |M_{Born}|^2 \sum_\epsilon \left(\frac{q_1\cdot\epsilon}{q_1\cdot k} - \frac{q_2\cdot\epsilon}{q_2\cdot k}\right)^2$$

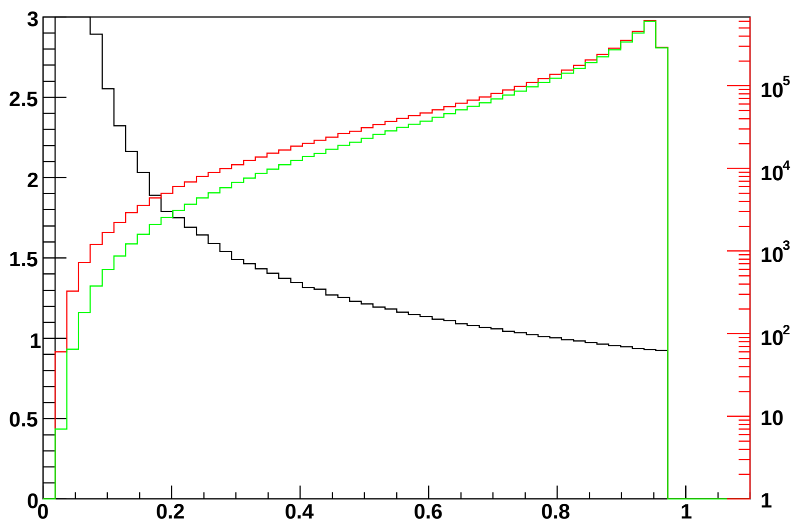
$$\sum_{\lambda,\epsilon} |M|^2 = A'_I + A_{\mathrm{remain}}$$

$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

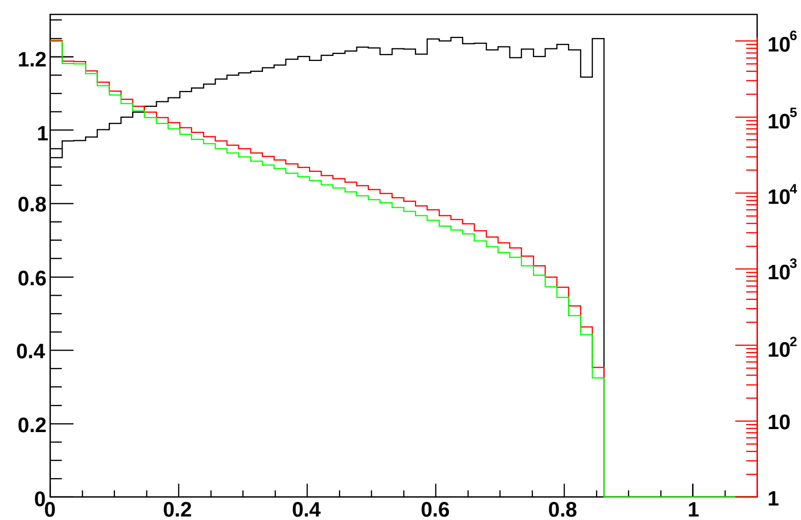
Comparison of A_I (green line) with kernel for scalars in PHOTOS (red line) at $\sqrt{S} = 2$GeV, black line denotes the ratio

Fraction of events with hard photons:

$3.8329 \pm 0.0020\%$, $4.2279 \pm 0.0021\%$



$M^2_{\pi+\pi-}/S$

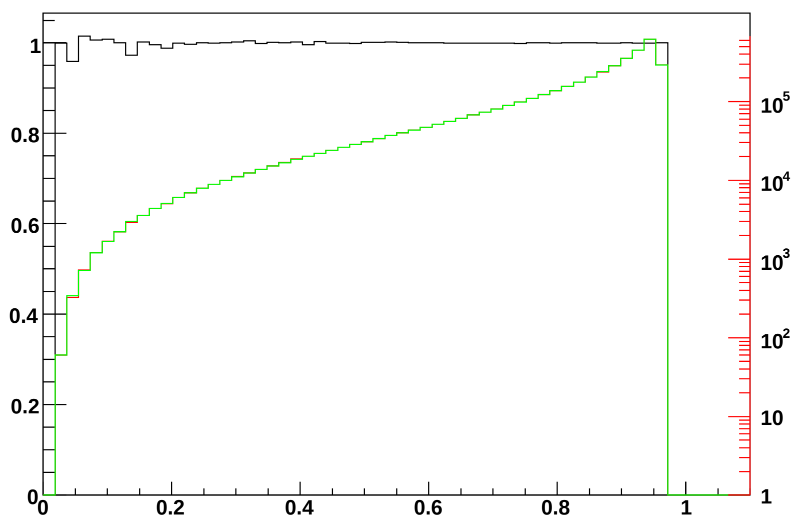


$M^2_{\pi+\gamma}/S$

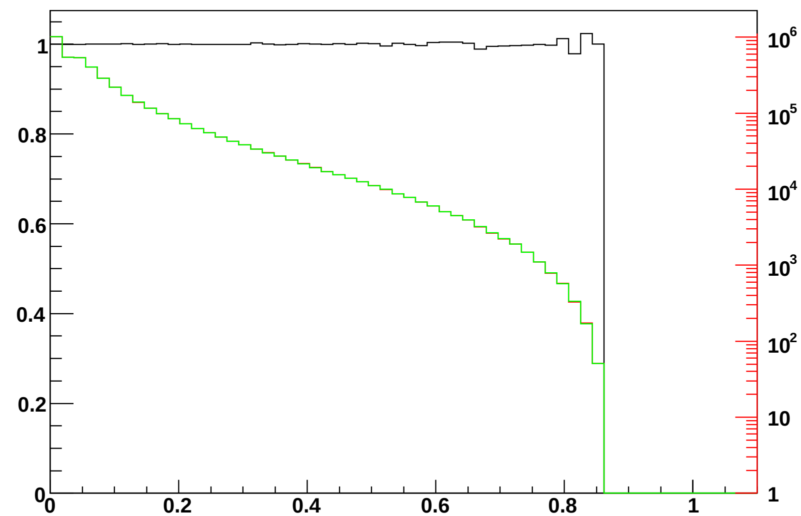
$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

Comparison of A'_I(green line) with kernel for scalars in PHOTOS (red line) at $\sqrt{S} = 2$GeV, black line denotes the ratio

Fraction of events with hard photons:
$4.2278 \pm 0.0021\%$, $4.2279 \pm 0.0021\%$. Very good agreement



$M^2_{\pi+\pi-}/S$



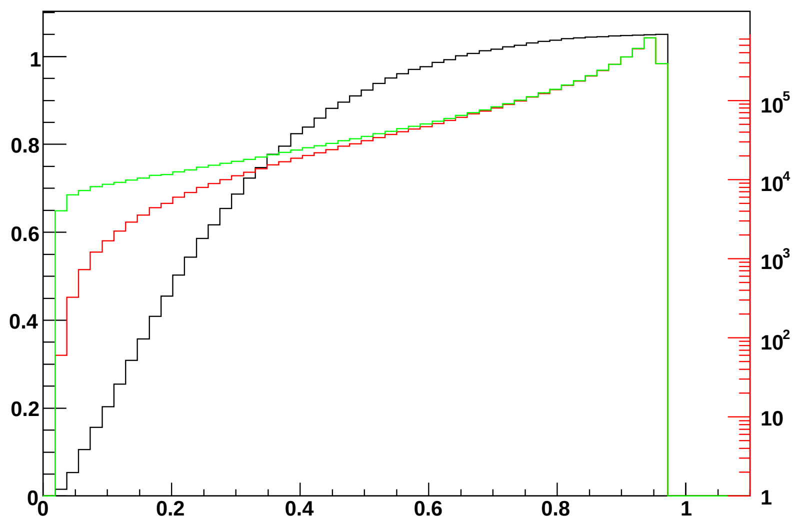
$M^2_{\pi+\gamma}/S$

$\gamma^* \to \pi^+\pi^-(\gamma)$ **Numerical Results**

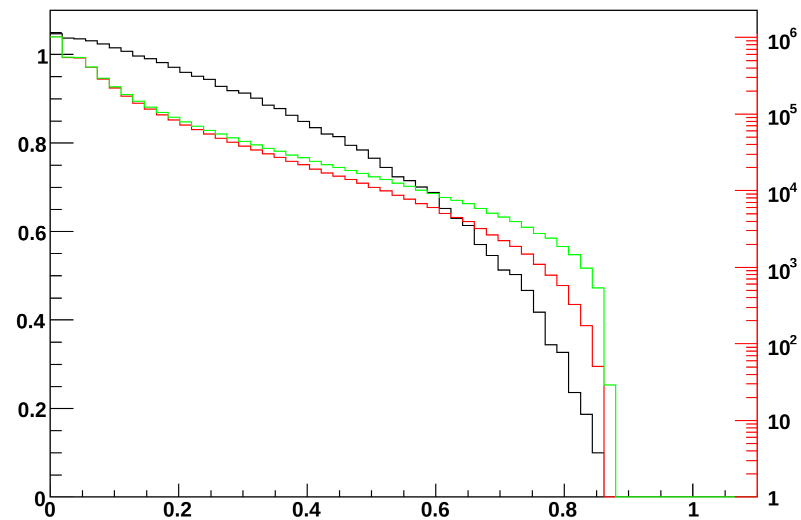
Comparison of complete amplitude (green line) with kernel for scalars in PHOTOS (red line) at $\sqrt{S} = 2$GeV, black line denote the ratio

Fraction of events with hard photons:

$4.4320 \pm 0.0021\%$, $4.2279 \pm 0.0021\%$



$M^2_{\pi+\pi-}/S$

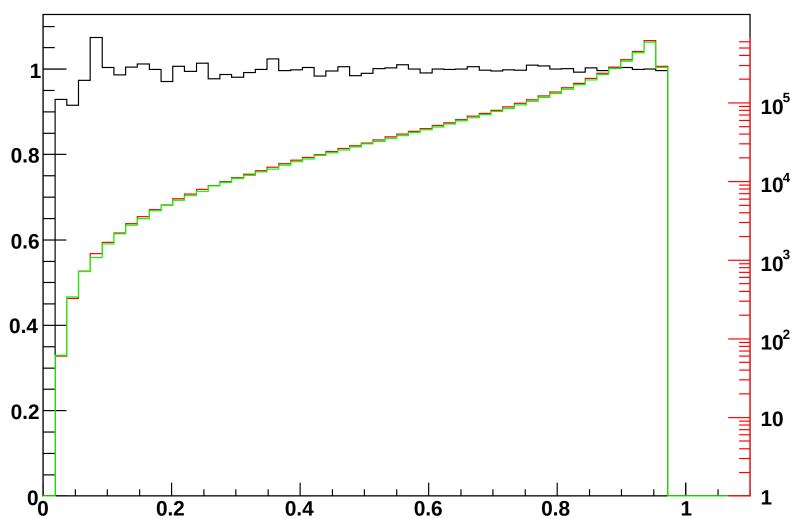


$M^2_{\pi+\gamma}/S$

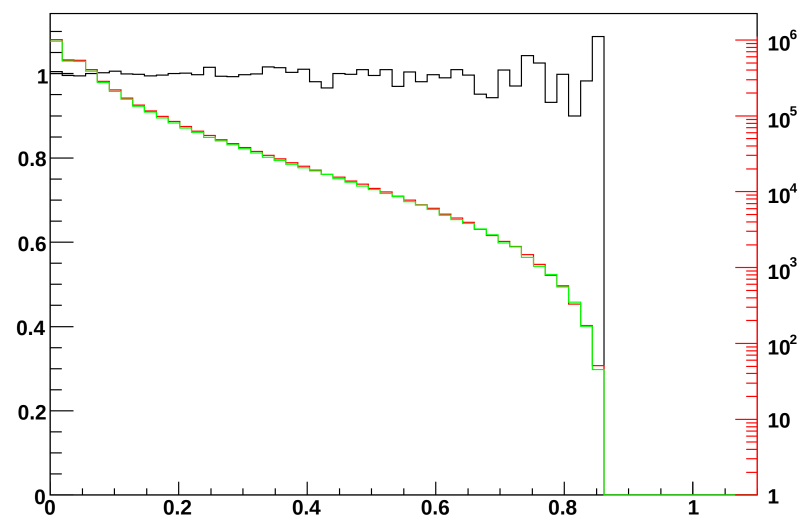
$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

Comparison of kernel for scalars in the case of one photon emission (red line) with the case where multi-photon (green line) emitted at $\sqrt{S} = 2$GeV, black line denotes the ratio

Fraction of events with hard photons:
$4.2279 \pm 0.0021\%$, $4.1377 \pm 0.0020\%$



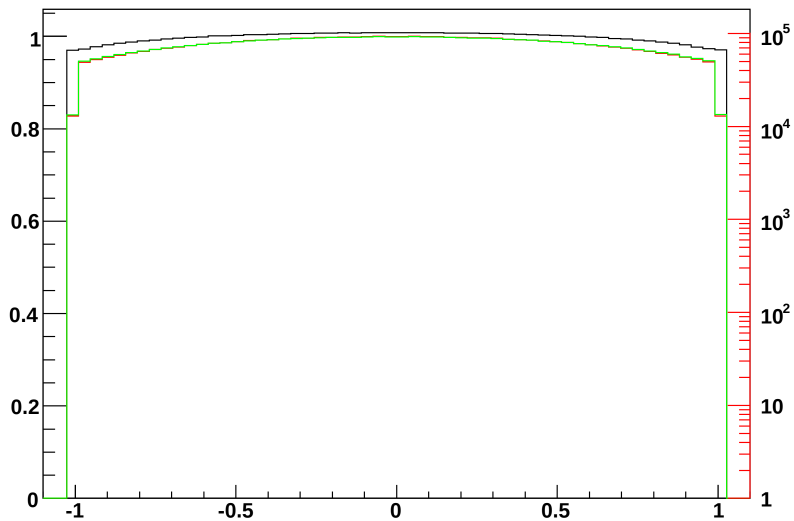
$M^2_{\pi^+\pi^-}/S$



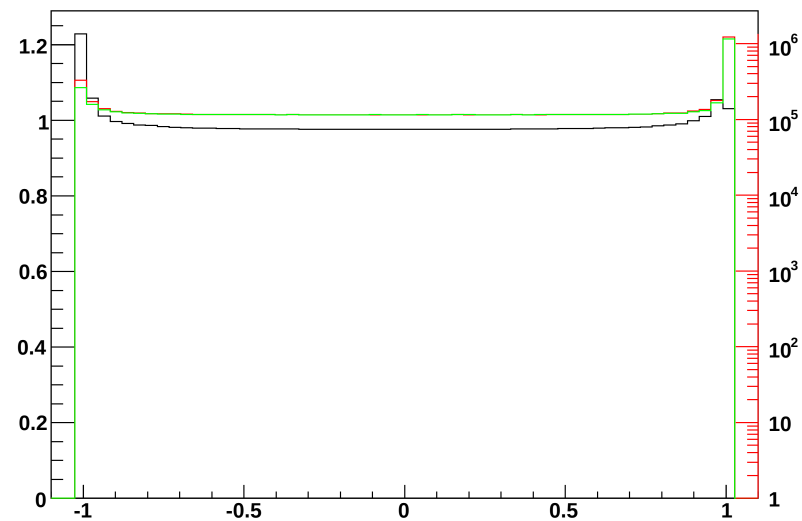
$M^2_{\pi^+\gamma}/S$

$\gamma^* \to \pi^+\pi^-(\gamma)$ Numerical Results

Comparison of A_I'(green) with kernel for scalars in PHOTOS (red) at $\sqrt{S} = 2$GeV, black line denotes the ratio



θ_γ distribution

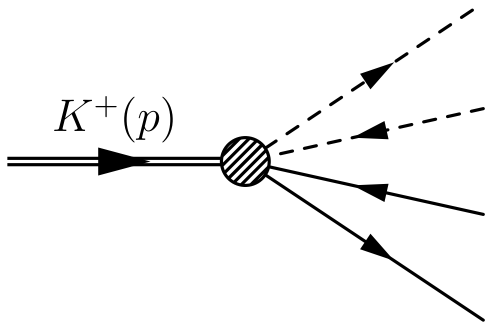


θ_{π^+} distribution

A little difference for θ_γ and θ_{π^+} distribution, angles are respect to the beam direction

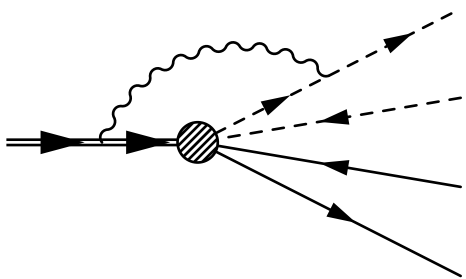
$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu_e(\gamma)$

Born

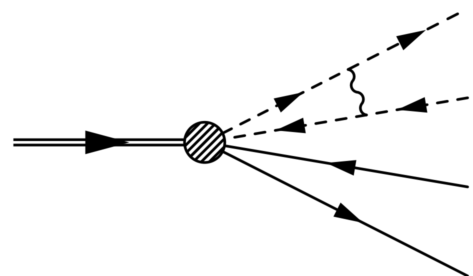


$$s_\pi = (q_+ + q_-)^2,$$
$$s_e = (p_e + p_\nu)^2,$$
$$\phi, \theta_\pi, \theta_e,$$

Virtual



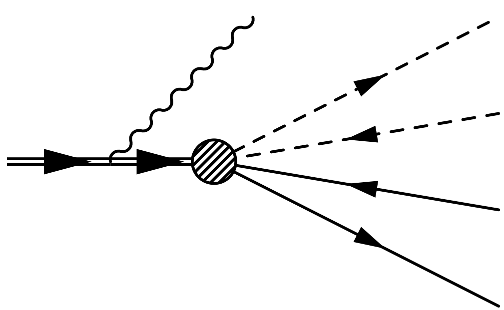
$a.)$



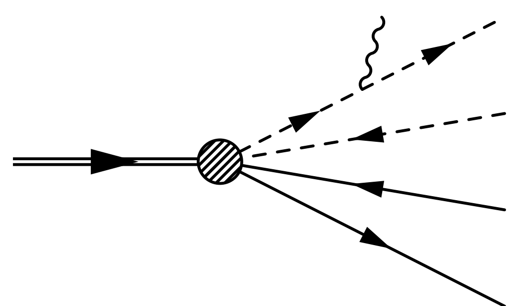
$b.)$

$\cdots +$ CTs

Real



$a.)$



$b.)$

$\cdots$

$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e(\gamma)$

Analytical Results

$$\frac{d\Gamma_{\rm Born+virt+soft}}{d\Omega} = \frac{d\Gamma_{\rm Born}}{d\Omega}\left(1 + \sigma P_\delta + \frac{\pi\alpha(1+\beta^2)}{2\beta} + \frac{\alpha}{\pi}K_{vs}\right)$$

$\sigma = \frac{\alpha}{2\pi}(2\rho - 1), \quad \rho = \ln\frac{2E_e}{m_e}$

$P_\delta = 2\ln\frac{\Delta\epsilon}{E_e} + \frac{3}{2}, \quad \Delta\epsilon$ soft-photon energy cutoff

K_{vs} depends on masses, kinematics and $\Delta\epsilon$, $\beta = \sqrt{1 - \frac{4m_\pi^2}{s_\pi}}$

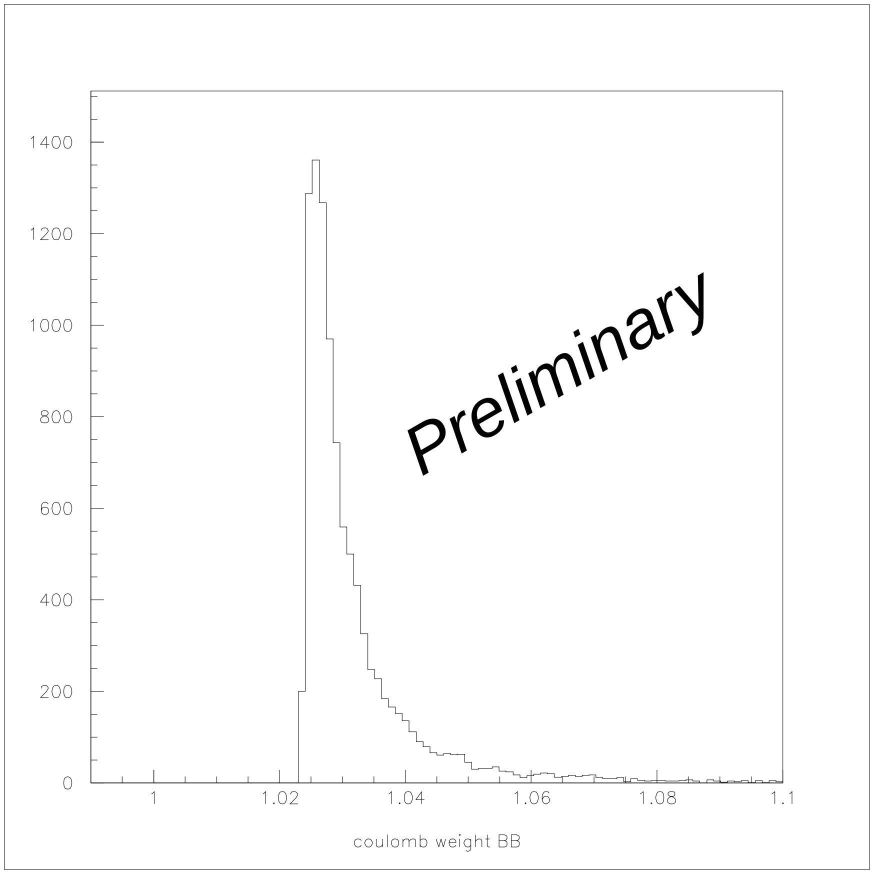
Add hard Photon

$$\frac{d\Gamma_{\rm Born+virt+real}}{d\Omega} = \frac{d\Gamma_{\rm Born}}{d\Omega}\left(1 + \frac{\pi\alpha(1+\beta^2)}{2\beta} + \frac{\alpha}{\pi}K\right)$$

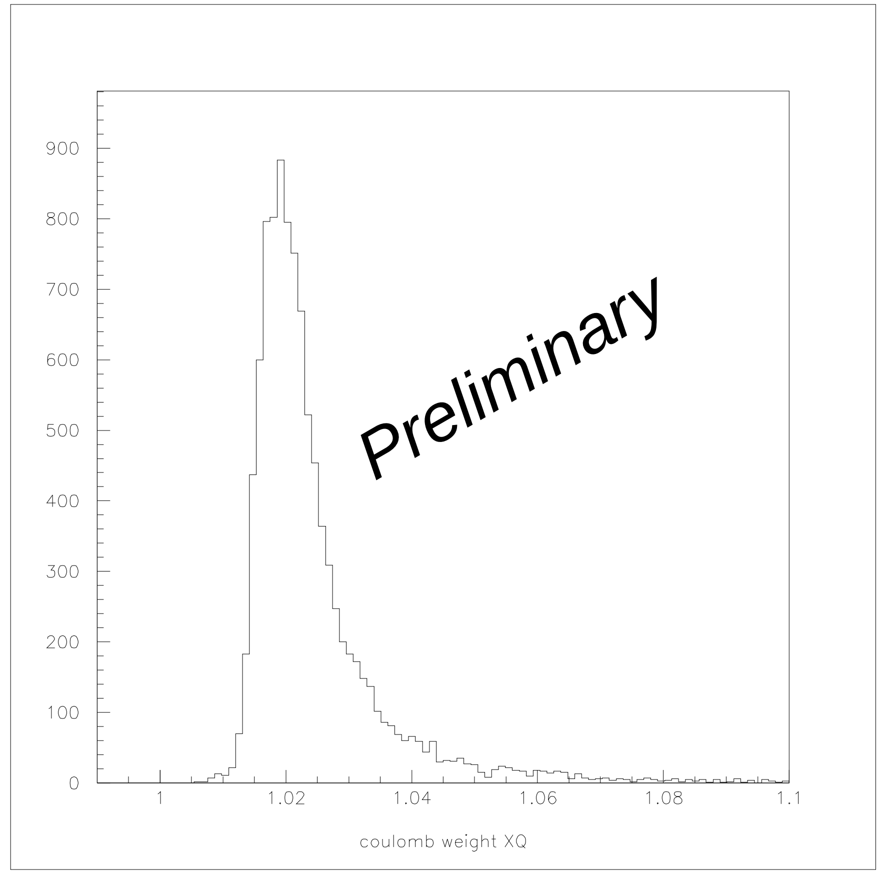
$K = K_{vs} + K_{hard}$, independent on $\Delta\epsilon$

$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e(\gamma)$

Comparison of my radiative correction with Coulomb correction from NA48



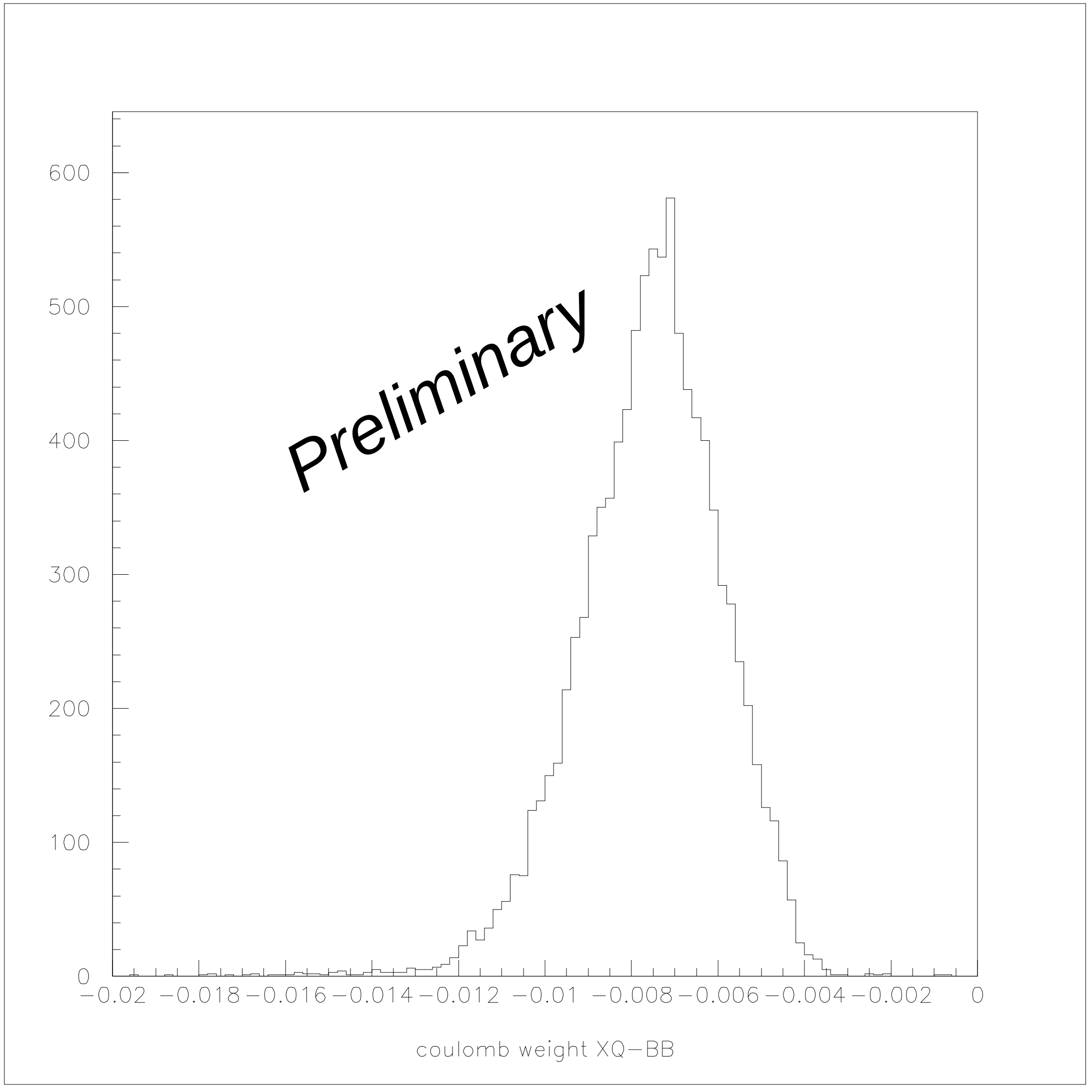
Coulomb correction from NA48



My Radiative correction

$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu_e(\gamma)$

Difference between radiative correction and Coulomb correction from NA48



Summary

- The cross section of $\gamma^* \to \pi^+\pi^-(\gamma)$ can be separated into an eikonal part and a remaining part using principle of gauge invariance. The eikonal part is identical to the kernel used in standard PHOTOS
- With PHOTOS $\gamma^* \to \pi^+\pi^-(\gamma)$ can be simulated using exact matrix element. Results were compared with kernel for scalars in PHOTOS. The approximated, easy to use version is correct up to 0.2% level.
- Multi-photon emission can be simulated too
- Analogies with QCD amplitudes are visible
- $K^\pm \to \pi^+\pi^- e^\pm \nu_e(\gamma)$ is calculated analytically, we compare the radiative correction with Coulomb correction from NA48, find a very good agreement
- We will implement the radiative correction for K_{e4} decay into PHOTOS