The muon g-2 and the bounds on the Higgs mass

Massimo Passera
INFN Padova

PHIPSI09
Institute of High Energy Physics, Beijing
October 13-16 2009

Work in collaboration with W.J. Marciano & A. Sirlin
PRD78 (2008) 013009 [updated]

$a_\mu^{SM} - a_\mu^{EXP}$

a_μ^{SM}: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857408\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+ \; 24.05050959\ (42)\ (\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr,Taylor & Newell '08

$+ \; 130.805\ (8)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

$+ \; 663\ (20)\ (\alpha/\pi)^5$ In progress

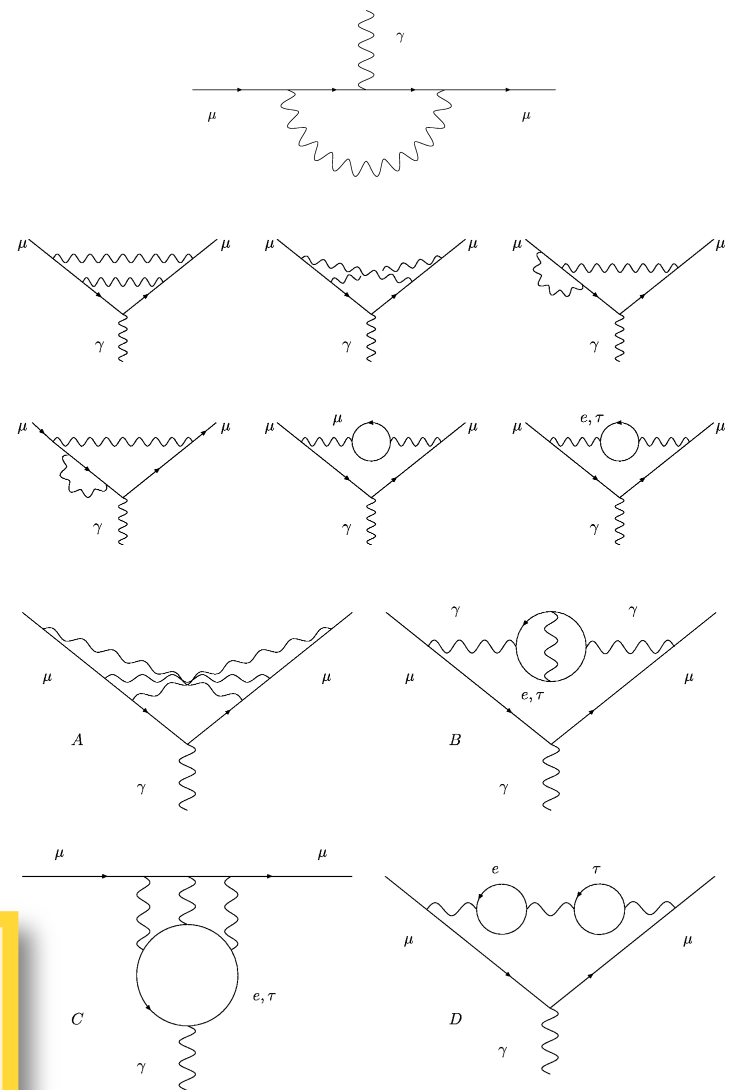
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.

Adding up, we get:

$a_\mu^{QED} = 116584718.08\ (14)(04) \times 10^{-11}$

from coeffs, mainly from 5-loop unc ⮠ ⮡ from new $\delta\alpha$('08)

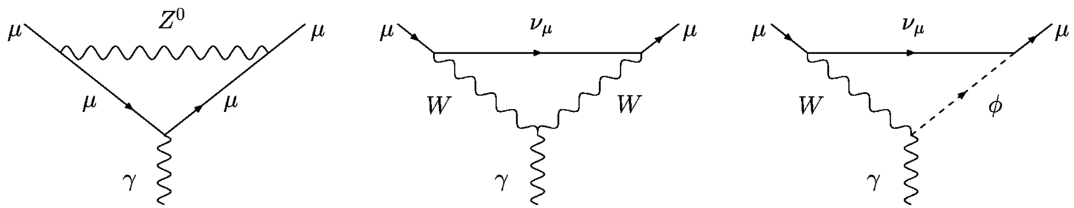
with $\alpha = 1/137.035999084(51)$ [0.37 ppb]



…

a_μ^{SM}: the Electroweak contribution

- ## One-loop term:



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M^2_{Z,W,H}}\right)\right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

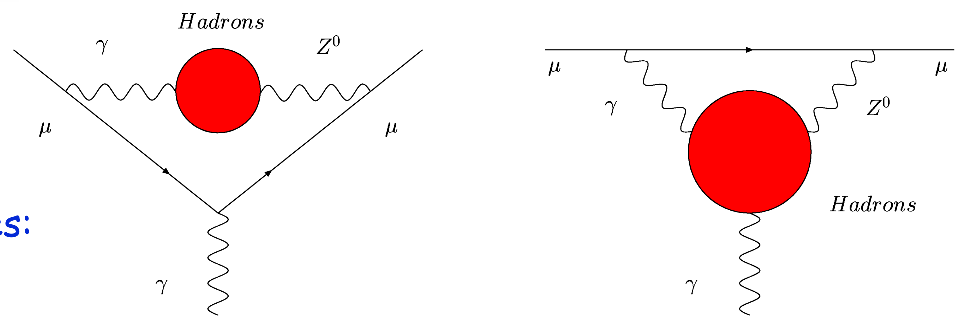
- ## One-loop plus higher-order terms:

a_μ^{EW} = 154 (2) (1) × 10^{-11}

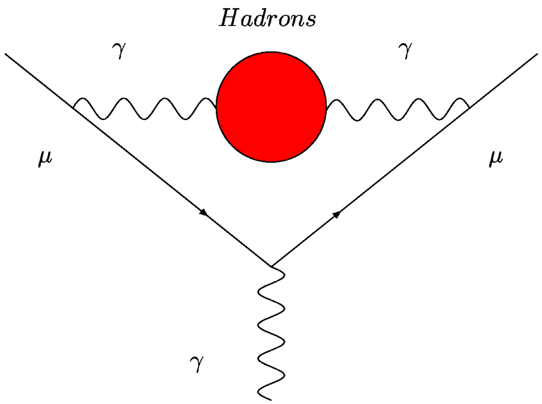
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrassi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

Higgs mass variation, M_{top} error, 3-loop nonleading logs

Hadronic loop uncertainties:



a_μ^{SM}: the hadronic leading-order (HLO) contribution



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^\infty ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^\infty \frac{ds}{s} K(s)\, R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}$$

Bouchiat & Michel 1961;
Gourdin & de Rafael 1969

$a_\mu^{\rm HLO}$ = 6909 $(39)_{\rm exp}$ $(19)_{\rm rad}$ $(7)_{\rm qcd}$ × 10^{-11} S. Eidelman, ICHEP06; M. Davier, TAU06

= 6894 $(42)_{\rm exp}$ $(18)_{\rm rad}$ × 10^{-11} Hagiwara, Martin, Nomura, Teubner, PLB649(2007)173

= 6903 $(53)_{\rm tot}$ × 10^{-11} F. Jegerlehner, A. Nyffeler, arXiv:0902.3360

= 6955 $(40)_{\rm exp}$ $(7)_{\rm qcd}$ × 10^{-11} Davier et al, arXiv:0908.4300 (includes new BaBar 2π)

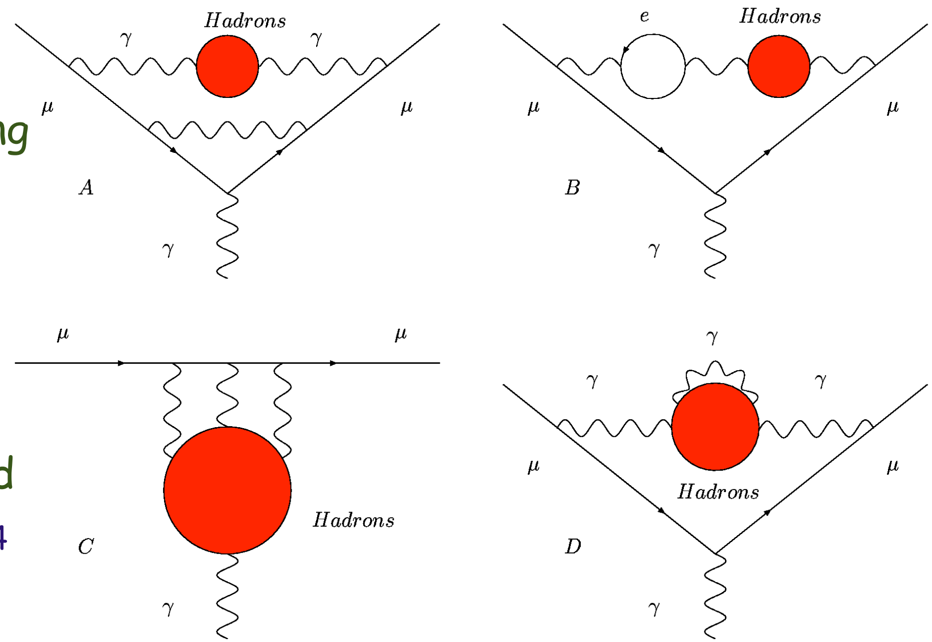
= 6894 $(36)_{\rm exp}$ $(18)_{\rm rad}$ × 10^{-11} HLMNT, October 09, Preliminary

= 7053 $(39)_{\rm exp}$ $(7)_{\rm rad}$$(7)_{\rm qcd}$$(19)_{\rm IB}$ × 10^{-11} Davier et al, arXiv:0906.5443v2 (tau)

a_μ^{SM}: the hadronic higher-order (HHO) contributions

Vacuum Polarization

$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:



$$a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. '03 & '06

Shifts by ~ -3×10^{-11} if tau data are used instead of the e^+e^- ones   Davier & Marciano '04

Light-by-Light

Recent values of the contribution of the hadronic LBL diagrams:

$a_\mu^{HHO}(lbl)$	=	$+105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
	=	$+116\ (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

This contribution will likely become the ultimate limitation of the SM prediction.

The muon g-2: Standard Model vs. Experiment

- Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072
with latest value of $\lambda = \mu_\mu/\mu_p$ (Codata '06)

	$a_\mu^{SM} \times 10^{11}$	$(\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}) \times 10^{11}$	σ
[1]	116 591 773 (53)	316 (82)	3.8
[2]	116 591 782 (59)	307 (86)	3.6
[3]	116 591 834 (49)	255 (80)	3.2
[4]	116 591 773 (48)	316 (79)	4.0
[5]	116 591 929 (52)	160 (82)	2.0

with a_μ^{HHO}(lbl) = 105 (26) $\times 10^{-11}$

[1] HMNT06, PLB649 (2007) 173.
[2] F. Jegerlehner and A. Nyffeler, arXiv:0902.3360.
[3] Davier et al, arXiv:0908.4300 August 2009 (includes BaBar)
[4] Hagiwara, Liao, Martin, Nomura, Teubner, Oct '09 (preliminary)
[5] Davier et al, arXiv:0906.5443v2 August 2009 (τ data).

- The theoretical error is now about the same as the exp. one.

$\Delta\alpha_{had}^{(5)}(M_Z^2)$ & M_H

The Hadronic Contribution to $\alpha(M_Z^2)$...

- The (light quarks part of the) hadronic contribution to the effective fine-structure constant at the scale M_z^2 is given by:

$$\Delta\alpha_{\rm had}^{(5)}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2}\, P\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{M_Z^2 - s}$$

- Progress due to significant improvement of data (in particular BES):

$\Delta\alpha_{had}^{(5)}(M_z)$ =	0.02800 (70)	Eidelman, Jegerlehner '95
	0.02775 (17)	Kuhn, Steinhauser 1998
	0.02749 (12)	Troconiz, Yndurain 2005
	0.02758 (35)	Burkhardt, Pietrzyk 2005
	0.02768 (22)	HMNT 2006
	0.02761 (23)	F. Jegerlehner 2008
	0.02760 (15)	HLMNT, Oct 2009, Prelim

… and the EW Bounds on the SM Higgs mass

- The dependence of SM predictions on the Higgs mass, via loops, provides a powerful tool to set bounds on its value.
- Comparing the theoretical predictions of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$
  [convenient formulae in terms of M_H, M_{top}, $\Delta\alpha_{had}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$ by Degrassi, Gambino, MP, Sirlin '98; Degrassi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06]

with

$M_W = 80.399\ (23)$ GeV [LEP+Tevatron, Aug' 09]

$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23153\ (16)$ [LEP+SLC]

and

$\Delta\alpha_{had}^{(5)}(M_Z) = 0.02760\ (15)$ [HLMNT Oct '09 Prelim]

$M_{top} = 173.1\ (1.3)$ GeV [CDF-D0, Mar '09]

$\alpha_s(M_Z) = 0.118\ (2)$ [PDG '08]

we get

$M_H = 96^{+32}_{-25}$ GeV & $M_H < 153$ GeV 95%CL

- The value of $\Delta\alpha_{had}^{(5)}(M_Z)$ is a key input of these EW fits…

The a_μ - M_H connection

How do we explain Δa_μ?

- Δa_μ can be explained in many ways: errors in HHO-LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or New Physics.
- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

a_μ^{HLO}: $$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\; s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)}$: $$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\varepsilon>0$), in the range:

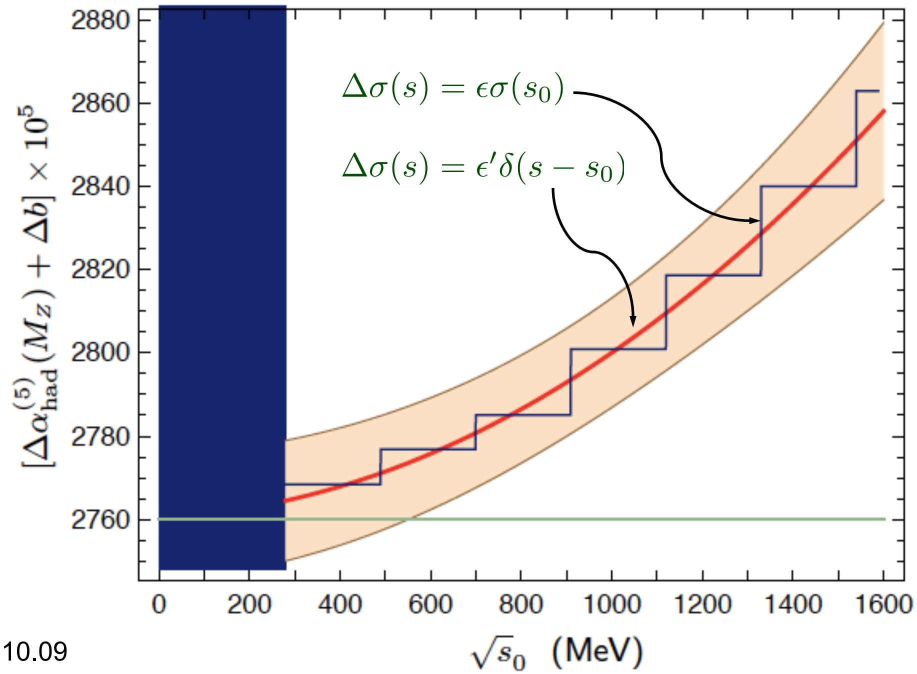
$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow$$

Shifts of a_μ^{HLO} and $\Delta\alpha_{had}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ is adjusted to bridge the g-2 discrepancy, the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ increases by:

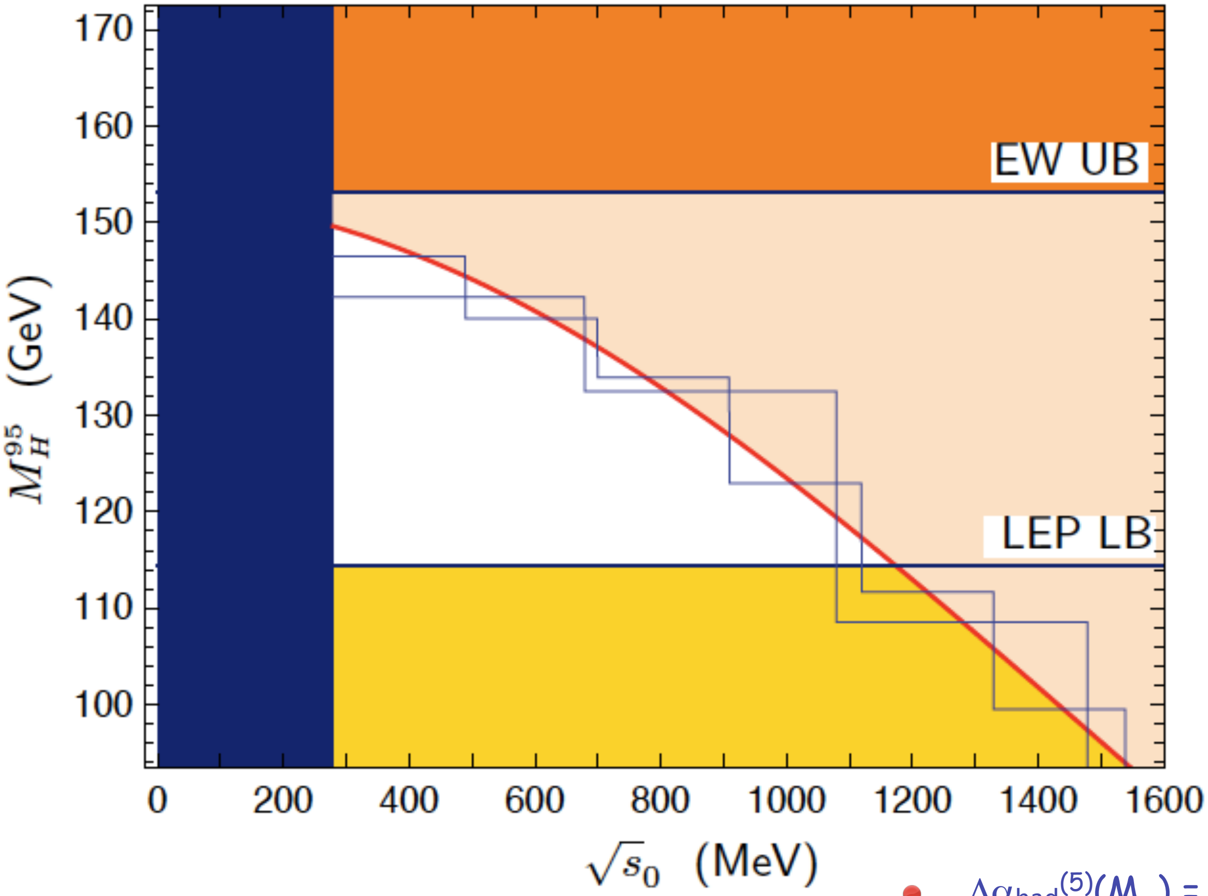
$$\Delta b(\sqrt{s_0},\delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2)\,\sigma(t^2)\,t\,dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2)\,\sigma(t^2)\,t\,dt}$$

- Adding this shift to $\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02760(15) [HLMNT09 Prelim] , with Δa_μ= 316(79) x 10^{-11} [HLMNT09 prelim], we obtain:



The muon g-2: connection with the SM Higgs mass

- How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?

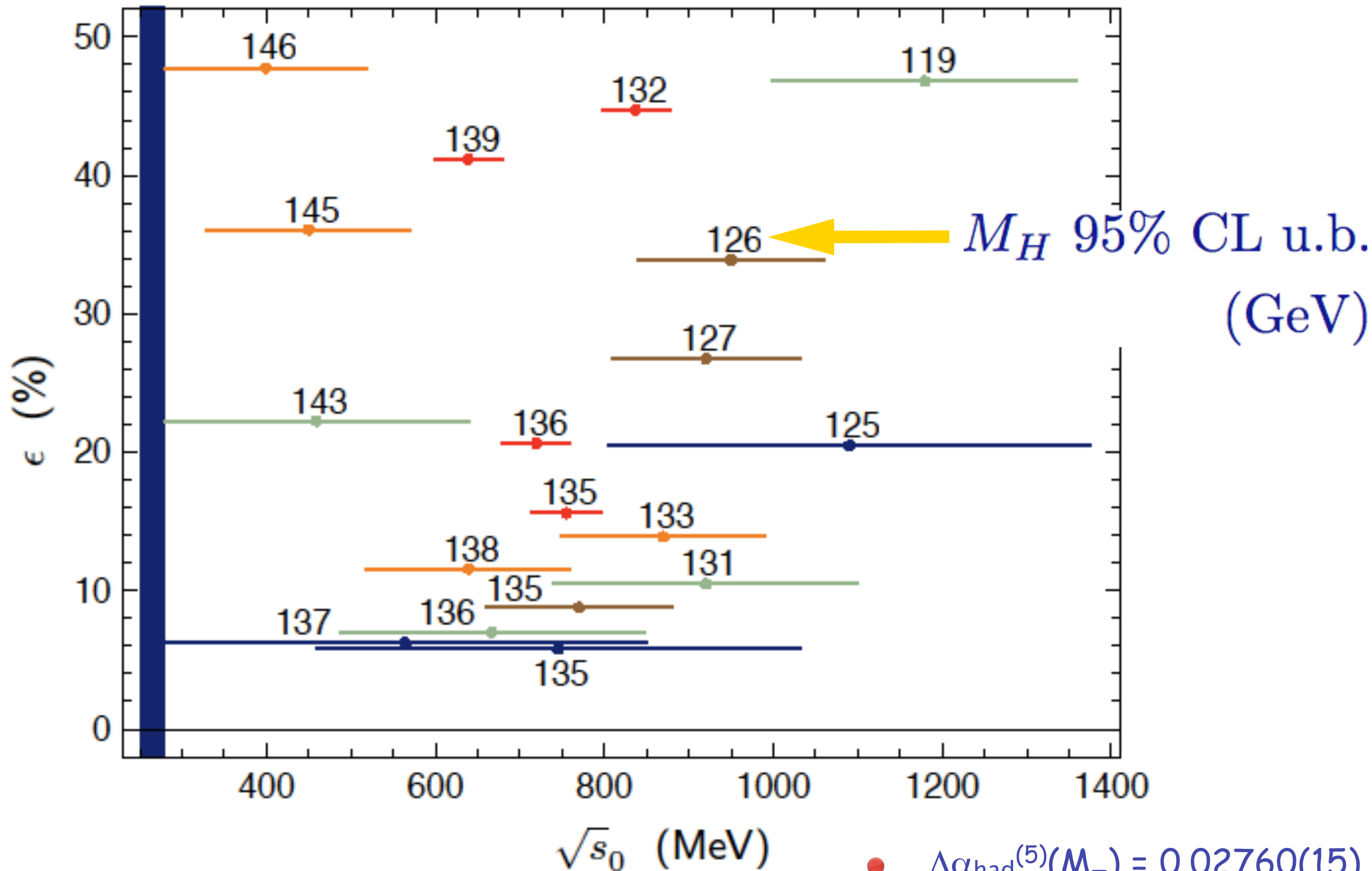


- $\Delta\alpha_{had}^{(5)}(M_Z) = 0.02760(15)$
- $\Delta a_\mu = 316(79) \times 10^{-11}$ HLMNT09 Prel.

The muon g-2: connection with the SM Higgs mass (2)

- The LEP direct-search lower bound is M_H^{LB} = 114.4 GeV (95%CL).
- The hypothetical shifts $\Delta\sigma = \varepsilon\sigma(s)$ that bridge the muon g-2 discrepancy conflict with the LEP lower limit when $\sqrt{s_0}$ > ~1.2GeV (for bin widths δ up to several hundreds of MeV).
- While the use of tau data in the calculation of a_μ^{HLO} reduces the muon g-2 discrepancy, it increases the value of $\Delta\alpha_{had}^{(5)}(M_Z)$, lowering the M_H upper bound (tension with the M_H lower bound).
- In a scenario where tau data agree with e^+e^- ones below ~1GeV (after isospin viol. effects & vector meson mixings -- see Benayoun's talk), we could assume that Δa_μ is bridged by hypothetical errors above ~1GeV. If so, M_H^{UB} falls below M_H^{LB} !!
- Scenarios where Δa_μ is accommodated without affecting M_H^{UB} are possible, but considerably more unlikely.

- How realistic are these shifts Δσ(s) when compared with the quoted exp. uncertainties? Study the ratio ε=Δσ(s)/σ(s):



- $\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02760(15)
- Δa_μ = 316(79) x 10^{-11} HLMNT09 Prel.

How realistic are these shifts $\Delta\sigma(s)$? (2)

- The minimum ε is ~ +4.6%. It occurs if σ is multiplied by $(1+\varepsilon)$ in the whole integration region (!), leading to M_H^{UB} ~ 75 GeV (!!)

- As the quoted exp. uncertainty of $\sigma(s)$ below 1 GeV is ~ a few per cent (or less), the possibility to explain the muon g-2 with these shifts $\Delta\sigma(s)$ appears to be unlikely.

- If, however, we allow variations of $\sigma(s)$ up to ~6% (7%), M_H^{UB} is reduced to less than ~138 GeV (139 GeV). E.g., the ~6% shift in [0.6, 1.2] GeV, required to fix Δa_μ, lowers M_H^{UB} to 133 GeV. Some tension with the M_H >~120GeV "vacuum stability" bound.

- Reminder: the above M_H upper bounds, like the LEP-EWWG ones, depend on the value of $\sin^2\theta_{\rm eff}^{\rm lept}$. They also depend on M_t & its unc. δM_t: We prepared simple formulae to translate easily M_H upper bounds discussed above into new values corresponding to M_t & δM_t inputs different from those employed here.

Conclusions

- Δa_μ can be due to New Physics, or to problems in a_μ^{SM} (or a_μ^{EXP}). Can it be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An increase $\Delta\sigma(s)$ could bridge Δa_μ, leading however to a decrease on the EW upper bound on the SM Higgs mass M_H .
- By means of a detailed analysis we conclude that solving Δa_μ via an increase of $\sigma(s)$ is unlikely in view of current experimental error estimates.
- However, if this turns out to be the solution, then the M_H upper bound drops to about 135 GeV which, in conjunction with the LEP 114 GeV direct lower limit, leaves a rather narrow window for M_H.

The End