Recent progress on isospin breaking corrections and their impact on the muon $g-2$ value*

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Abstract We describe some recent results on isospin breaking corrections which are of relevance for predictions of the leading order hadronic contribution to the muon anomalous magnetic moment $a_\mu^{\rm had,LO}$ when using τ lepton data. When these corrections are applied to the new combined data on the $\pi^\pm\pi^0$ spectral function, the prediction for $a_\mu^{\rm had,LO}$ based on τ lepton data gets closer to the one obtained using e^+e^- data.

Key words isospin breaking, tau decays, muon magnetic moment, radiative corrections

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1 Introduction

Currently, the accuracy of the Standard Model prediction of the muon anomalous magnetic moment $a_\mu = (g-2)/2$ is limited by the uncertainties of hadronic contributions [1, 2]. The dominant term in the leading order hadronic contribution $a_\mu^{\rm had,LO}$ and an important part of its associated uncertainty is provided by the $\pi\pi$ spectral function, which can be measured in e^+e^- annihilations and in τ lepton decays (more details about their current status are given in the accompanying contribution by Michel Davier [2]). Owing to the isotopic properties of the electromagnetic and $\Delta S=0$ weak vector currents, the so-called Conserved Vector Current (CVC) hypothesis, the spectral functions themselves and their contributions to $a_\mu^{\rm had,LO}$ must be the same after isospin breaking (IB) corrections are appropriately applied to input data [3].

In recent years, a comparison of e^+e^- and τ based measurements of the $\pi\pi$ spectral functions in the timelike region, have shown large discrepancies for center of mass energies above the $\rho(770)$ resonance peak, beyond the size expected for IB corrections [4, 5]. The predictions for $a_\mu^{\rm had,LO}$ based on these two sets of data have been in disagreement by more than 2σ [5, 6]. Moreover, the branching fraction for $\tau\to\pi\pi\nu$ predicted from e^+e^- data corrected by IB effects was underestimated by more than 4σ with respect to the average of direct measurements [4, 6]. Given this 'e^+e^- vs τ discrepancy'[2)], it is believed that τ decay data does not provide at present a reliable determination of $a_\mu^{\rm had,LO}$ (currently, other useful contributions from τ decay data involve only the 2π and 4π channels [2]). Even though unidentified errors may be affecting e^+e^- and/or τ decay data, understanding IB corrections becomes crucial to gain confidence about this important 2π contribution and, when consistency is achieved, to have a more precise prediction of $a_\mu^{\rm had,LO}$ from combined results.

In this contribution we summarize some recent results on the isospin breaking corrections that are relevant for understanding such discrepancies. As it is discussed in [2, 7], their application to the evaluation of $a_\mu^{\rm had,LO}$ from τ data leads to a value [7] that is closer to e^+e^--based calculations. The prediction of the $\tau\to\pi\pi\nu$ branching fraction based on the IB corrected e^+e^- data also shows a reduced discrepancy with respect to results of direct measurements [2, 7]. A preliminary version of this work appeared in Ref. [8]. In this new version we include a discussion of the complete set of IB corrections and we address some points that were unclear in some of our previous reports.

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2) Another reason is that e^+e^- data is more directly related to $a_\mu^{\rm had,LO}$ through the dispersion integral than τ decay data.

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2 IB corrections to the $a_\mu^{\mathrm{had,LO}}$ dispersion integral from τ data

The leading order hadronic contribution $a_\mu^{\mathrm{had,LO}}$ can be evaluated by using a combination of experimental data and perturbative QCD for the hadronic vacuum polarization (HVP) function of the photon. At low energies, where QCD does not provide a reliable calculation of Green functions, the HVP can be constructed as a sum over exclusive hadronic channels measured in $\mathrm{e^+e^-}$ annihilation. The dispersion integral relating each exclusive $\mathrm{e^+e^-} \to \mathrm{X^0}$ channel to $a_\mu^{\mathrm{had,LO}}$ is:

$$a_\mu^{\mathrm{had,LO}}\ [\mathrm{X^0,e^+e^-}] = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{K(s)}{s} R^{(0)}_{\mathrm{X^0}}(s)\ , \qquad (1)$$

where $R^{(0)}_{\mathrm{X^0}}(s)$ is the ratio of hadronic $\mathrm{X^0}$ to pointlike $\mu^+\mu^-$ bare cross sections [1, 2] in $\mathrm{e^+e^-}$ annihilation at a center of mass energy $\sqrt{s}$. The behavior of the QED kernel $K(s) \sim 1/s$ [9], enhances the low-energy contributions to $a_\mu^{\mathrm{had,LO}}$ in such a way that 91% of it comes from the energy region below 1.8 GeV and 73% is due to the $\pi\pi$ channel. Further details can be found in [2].

If isospin were an exact symmetry, we would be able to use in (1) the spectral functions measured in $\tau^- \to \mathrm{X^-}\nu$ decays, where $\mathrm{X^-}$ is the $(I=1, I_3=-1)$ isotopic partner of the $\mathrm{X^0}$ state. We can define an isotopic analogue of the ratio $R^{(0)}_{\mathrm{X^0}}(s)$ as follows (this quantity is related to the usual spectral function [2, 7] by $R^{(0)}_{\mathrm{X^-}}(s) = 3v_{\mathrm{X^-}}(s)$):

$$\frac{R^{(0)}_{X^-}(s)}{3} = \frac{m_\tau^2}{6|V_{\mathrm{ud}}|^2}\frac{\mathcal{B}_{\mathrm{X^-}}}{\mathcal{B}_{\mathrm{e}}}\frac{1}{N_{\mathrm{X}}}\frac{\mathrm{d}N_{\mathrm{X}}}{\mathrm{d}s}\times \left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}. \qquad (2)$$

In Eq. (2), $(1/N_{\mathrm{X}})\mathrm{d}N_{\mathrm{X}}/\mathrm{d}s$ is the normalized invariant mass spectrum of the hadronic final state, and $\mathcal{B}_{X^-}$ denotes the branching fraction of $\tau \to \mathrm{X^-}(\gamma)\nu_\tau$ (throughout this paper, final state photon radiation is implied for τ branching fractions). For numerical purposes [7], we use for the τ lepton mass the value $m_\tau = (1776.84 \pm 0.17)\,\mathrm{MeV}$ [10], and for the CKM matrix element $|V_{\mathrm{ud}}| = 0.97418 \pm 0.00019$ [11], which assumes CKM unitarity. For the electron branching fraction we use $\mathcal{B}_{\mathrm{e}} = (17.818 \pm 0.032)\%$, obtained [12] supposing lepton universality.

In the presence of IB effects, the spectral function (2) in τ decays must be corrected by the factor $R_{\mathrm{IB}}(s)/S_{\mathrm{EW}}$, in order to be used into the dispersion integral (1). Therefore, it becomes convenient to introduce the shift in the dispersion integral:

$$\Delta a_\mu^{\mathrm{had,LO}}\ [\mathrm{X^-},\tau] = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{K(s)}{s} R^{(0)}_{\mathrm{X^-}}(s) \times \left[\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}} - 1\right] \qquad (3)$$

produced by the IB corrections. The short-distance electroweak radiative effects encoded in S_{EW}, which includes the re-summation of terms of $O(\alpha^{\mathrm{n}}\ln^{\mathrm{n}}(m_{\mathrm{Z}}))$ and of $O(\alpha\alpha_{\mathrm{s}}^{\mathrm{n}}\ln^{\mathrm{n}}(m_{\mathrm{Z}}))$, lead to the correction $S_{\mathrm{EW}} = 1.0235 \pm 0.0003$ [4, 13–15]; the quoted uncertainty is attributed to neglected corrections of $O(\alpha\alpha_{\mathrm{s}}/\pi)$ [15]. This term provides the largest of IB effects in $a_\mu^{\mathrm{had,LO}}\ [\mathrm{X^-},\tau]$, as it can be seen in Table 1. The remaining IB effects included in $R_{\mathrm{IB}}(s)$ are discussed below.

Hereafter we focus on the $\mathrm{X^-} = \pi^-\pi^0$ channel of τ lepton decays. Beyond its rather large contribution to $a_\mu^{\mathrm{had,LO}}$, the precision attained in the measurement of the muon anomalous magnetic moment requires that the $\pi\pi$ contribution be evaluated below the 1% accuracy, making crucial the reliable computation of IB corrections [3, 4]. The s-dependent IB correction introduced in (3) is defined as:

$$R_{\mathrm{IB}}(s) = \frac{\mathrm{FSR}(s)}{G_{\mathrm{EM}}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2. \qquad (4)$$

The subscripts $i = 0, -$ refer to the electric charge of the 2π system produced in $\mathrm{e^+e^-}$ annihilation and in τ^- lepton decays, respectively. Each of the factors in $R_{\mathrm{IB}}(s)$ becomes unity in the limit of isospin symmetry, thus also $R_{\mathrm{IB}}(s) = 1$ in this limit.

In the Standard Model of quarks and leptons interactions, isospin symmetry is broken by the mass difference of u and d quarks, and by the effects of electromagnetic interactions. At the hadron level, the IB effects introduce some model dependence: hadronic matrix elements that are related by isospin symmetry, get modified in the presence of IB effects by photonic interactions and by the mass and width splitting of hadrons involved. Therefore, the usual procedure to test isospin symmetry predictions consist in comparing ‘bare’ hadronic matrix elements obtained from experimental data by removing the effects of IB corrections.

In the following we consider each of the energy-dependent factors that enter in $R_{\mathrm{IB}}(s)$ and quantify their effects in $\Delta a_\mu^{\mathrm{had,LO}}\ [\pi\pi,\tau]$. The corresponding corrections induced in the branching fraction of $\tau \to \pi\pi\nu$, which is an independent test of these IB corrections, can be found in Refs. [2, 7].

2.1 FSR and phase space corrections

The final state photonic corrections to $e^+e^- \to \pi^+\pi^-$, FSR(s), and the ratio of pion velocities $\beta_0(s)/\beta_-(s)$, are the best known corrections to be considered in Eq. (4). The FSR correction is computed using scalar QED and its expression is known analytically [16]. From Fig. 1 we observe that the effects of these two corrections are important close to threshold and they vanish rapidly for increasing values of s. The phase-space factor is very accurate as it depends only on the pion masses. Instead, we have attributed a $\pm 10\%$ uncertainty (see Table 1) to the contribution of FSR in $\Delta a_\mu^{\rm had,LO}$ [$\pi\pi,\tau$] to account for possible deviations from scalar QED. As it has been pointed out in Ref. [2], KLOE [17] and BABAR [18] measurements of $\pi^+\pi^-\gamma(\gamma)$ in electron-positron collisions support the validity of this hypothesis within the uncertainties quoted above.

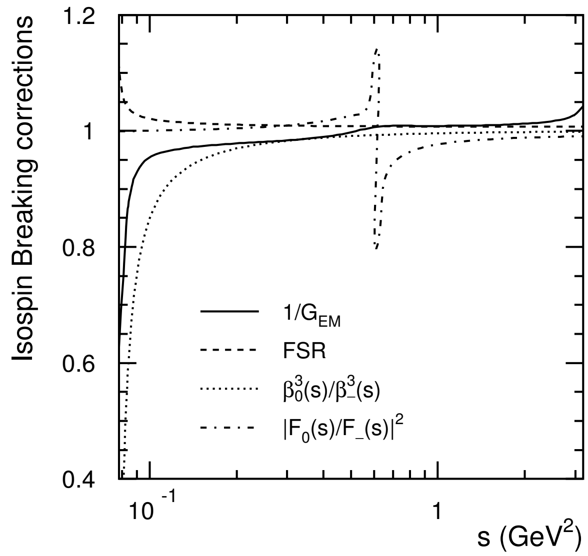

Fig. 1. Energy-dependent IB corrections contained in $R_{\rm IB}(s)$, Eq. (4).

Table 1. Contributions to $a_\mu^{\rm had,LO}$ [$\pi\pi,\tau$] from the IB corrections discussed in section 2 and Ref. [7]. The twofold corrections in the second column correspond to results obtained using the GS [24] and KS [25] parametrization of pion form factors, respectively. For comparison, the last column, denoted as DEHZ03, contains the results of Ref. [4].

source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model	DEHZ03
$S_{\rm EW}$	-12.21 ± 0.15		-12.1 ± 0.3
$G_{\rm EM}$	-1.92 ± 0.90		-1.0
FSR	$+4.67\pm0.47$		$+4.5$
ρ-ω interference	$+2.80\pm0.19$	$+2.80\pm0.15$	$+3.5\pm0.6$
$m_{\pi^\pm}-m_{\pi^0}$ effect on σ	-7.88		-7.0
$m_{\pi^\pm}-m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$	$+4.2$
$m_{\rho^\pm}-m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$	0.0 ± 2.0
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64	-1.4 ± 1.2
total	-16.07 ± 1.22	-16.70 ± 1.23	-9.3 ± 2.4
	-16.07 ± 1.85		

2.2 Long-distance correction

The definition of the long-distance photonic correction $G_{\rm EM}$ to the photon-inclusive hadronic spectrum in $\tau\to\pi\pi\nu$ decay can be found elsewhere [19, 20]. The virtual + real soft-photon corrections (which gives an infrared-convergent result and that we have named model-independent corrections in previous works [8, 20]) of Refs. [19, 20] are very similar numerically, despite the different pion-form factors used in both cases (resonance chiral model [21] and vector meson dominance [22] model, respectively). This is an expected behavior since $G_{\rm EM}$ is defined from the ratio of radiatively-corrected and tree-level 2π spectra [8, 20], thus the form factor dependences largely cancel.

The main difference between the calculations of Refs. [19] and [20] stems from the regular part (which is infrared finite, and we call model-dependent contributions in previous works) of real photon emission, and it can be traced back to the model-dependent contribution to $\tau\to\pi\pi\nu\gamma$ involving the $\rho\omega\pi$ vertex [20]. In practice, most of the experiments remove from their $\pi^-\pi^0$ spectrum, the events associated to the decay chain $\tau^-\to\pi^-\omega(\to\pi^0\gamma)\nu$, leaving the interferences of this with other τ lepton radiative amplitudes in the their $\pi\pi$ invariant mass distributions [7]. In order to remain consistent, we also removed from our $G_{\rm EM}$ correction the square of the radiative amplitude involving the $\rho\omega\pi$ vertex [7]. The resulting long-distance correction $\hat{G}_{\rm EM}$ gets closer to the one reported in Ref. [19]. The inverse of $\hat{G}_{\rm EM}$ is plotted as a solid line in Fig. 1 and its effect in the dispersion integral (3) is shown in the second row of Table 1. We have taken the difference between the effects of our $\hat{G}_{\rm EM}$ in $\Delta a_{\mu}^{\rm had,LO}$ $[\pi\pi,\tau]$ and the one of reference [19] (third column in Table 1) as an estimate of the uncertainty associated to model-dependence of the long-distance correction.

2.3 IB effects in pion form factors

The last IB correction factor in Eq. (4), the ratio of the electromagnetic to the weak pion form factors, involves two sources of IB: (a) a term that mixes the $I=1$ and 0 components of the electromagnetic current which is driven by the ρ-ω mixing and, (b) the mass and width difference of ρ vector mesons which affect only the $I=1$ component of the form factors. We discuss this contribution in more detail since it represents the main change in $\Delta a_{\mu}^{\rm had,LO}[\pi\pi,\tau]$ with respect to previous evaluations of IB corrections.

Under the above considerations, the pion form factors can be written as [7, 8]

$$F_0(s) = f_{\rho^0}(s)\left[1+\delta_{\rho\omega}\frac{s}{m_\omega^2-s-{\rm i}m_\omega\Gamma_\omega(s)}\right], \quad (5)$$

$$F_-(s) = f_{\rho^-}(s), \quad (6)$$

where the complex parameter $\delta_{\rho\omega}$ represents the strength of the $\rho-\omega$ mixing, and $f_{\rho^{0,-}}(s)$ denote[1)] the $I=1$ parts of the pion form factors which are dominated, below $\sqrt{s}\leqslant 1$ GeV, by the $\rho(770)$ vector meson.

There are different parametrizations of the form factors $f_{\rho^{0,-}}(s)$ in the literature which are inspired by different models [23] of the ρ meson propagator. However, one would expect that their ratio in Eq. (4) is relatively less sensitive to a particular model. Just for comparison, we adopt two commonly used phenomenological formulae: the Gounaris-Sakurai (GS) [24] and the Kühn-Santamaria (KS) [25] parametrizations. Consequently, the corrections induced in $\Delta a_{\mu}^{\rm had,LO}$ $[\pi\pi,\tau]$ by the IB parameters in the pion form factors are quoted as two separate values in the second column of Table 1.

In the following we discuss the different sources of IB in formulae (5) and (6):

1) Strength of ρ-ω mixing: to obtain $\delta_{\rho\omega}$ we have fitted [7] Eq. (5) with the GS and KS parametrizations to the e^+e^- data in the full energy range available and we have included the effects of higher $I=1$ resonances in $F_0(s)$. This approach differs from other recent determinations of the ρ-ω mixing strength which obtain $\delta_{\rho\omega}$ from fits to e^+e^- data below 1 GeV for a wider class of $F_0(s)$ models [23]. As a result of our fits reported in Ref. [7], we get for the strength and phases of the ρ-ω mixing parameter: $|\delta_{\rho\omega}^{\rm GS}|=(2.00\pm0.06)\times10^{-3}$, $\arg(\delta_{\rho\omega}^{\rm GS})=(11.6\pm1.8)^\circ$, and $|\delta_{\rho\omega}^{\rm KS}|=(1.87\pm0.06)\times10^{-3}$, $\arg(\delta_{\rho\omega}^{\rm KS})=(13.2\pm1.7)^\circ$. In both cases, GS and KS, we use an energy-dependent absorptive part of the ρ meson propagator given by $-{\rm i}\sqrt{s}\,\Gamma_{\rho^{0,-}}(s)$. Contrary to claims raised in a recent paper [23], we do not find a strong model-dependence of the $\delta_{\rho\omega}$ mixing parameter.

2) Width difference of $\rho^\pm$-ρ^0 mesons The energy-dependent decay widths of neutral and charged ρ mesons cannot be measured in an independent way with the accuracy required to estimate their effects in Eq. (3). Thus, the width difference $\Delta\Gamma_\rho=\Gamma_{\rho^\pm}-\Gamma_{\rho^0}$ must be computed from the total widths which are defined as a sum over their exclusive decay channels [26]. A simple counting of decay channels of charged and neutral ρ mesons give [26]

$$\begin{aligned}\Delta\Gamma_\rho = &\ \Gamma[\rho^\pm\to\pi^\pm\pi^0(\gamma)]- \\ &\Gamma[\rho^0\to\pi^+\pi^-(\gamma)]-0.08\ {\rm MeV},\end{aligned} \quad (7)$$

where the first two terms include the photon inclusive rates into two pions $(\pi\pi(\gamma_{\rm soft})+\pi\pi\gamma)$. The last numerical term in Eq. (7) accounts for the rather small difference of the remaining decay widths [10] $(\pi\gamma,\eta\gamma,l^+l^-,\cdots)$.

The 2π photon inclusive decay rates, first line in (7), were calculated including the virtual plus real photon radiative corrections in Ref. [26]. We include its energy-dependence in the following form:

$$\Delta\Gamma_{\pi\pi(\gamma)}=\frac{g_{\rho\pi\pi}^2\sqrt{s}}{48\pi}\left[\beta_-^3(s)(1+\delta_-)-\beta_0^3(s)(1+\delta_0)\right], \quad (8)$$

where $g_{\rho\pi\pi}$ denotes the $\rho\pi\pi$ coupling and $\delta_{-,0}$ contains the effects of photonic radiative corrections with real photons of all energies. Eq. (8) gives $\Delta\Gamma_{\pi\pi(\gamma)}=(-0.76\pm0.08)$ MeV at $\sqrt{s}=m_\rho$, which can be compared with a previous estimate, $\Delta\Gamma[\rho\to\pi\pi(\gamma)]=(+0.49\pm0.58)$ MeV [3], which was obtained by including only the effects of hard real photon emission [27]. The $\pm10\%$ uncertainty added to our result for the width difference is estimated from the difference observed between our predicted branching fraction for $\rho^0\to\pi^+\pi^-\gamma$ [26] and its measured value [10].

As it can be seen from a comparison of the second and third columns in Table 1, the width difference (which we call '$\pi\pi\gamma$, electrom. decays') in-

1) These form factors are normalized to unity when $s=0$.

duces the biggest change in $a_\mu^{\rm had,LO}$ compared to results of previous estimates [3]. Just to emphasize the origin of this important change, in Fig. 2 we compare the ratio of $I=1$ components of our form factors [7, 8] and the ones used in previous calculations [4], for three different values of the mass difference: $m_{\rho^+}-m_{\rho^0}=(+1,0,-1)$ MeV. The clear difference near the ρ resonance peak, produces the large change in the IB effects due to $\pi\pi\gamma$ electromagnetic decays.

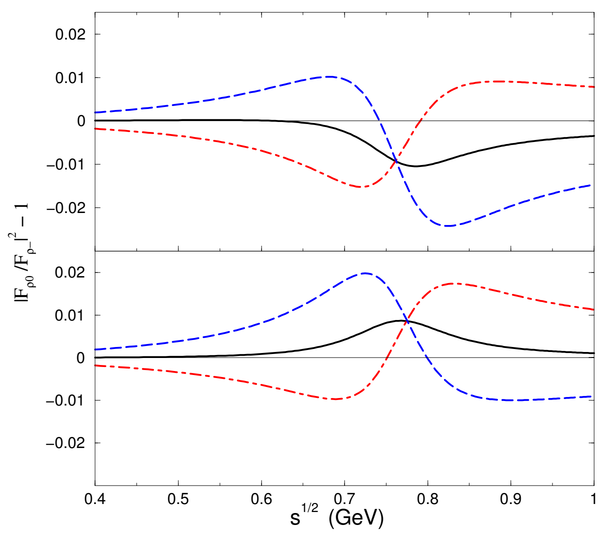

Fig. 2. Comparison of the ratio of $I=1$ components of pion form factors: (upper plot) our results [26], and (lower plot) the results of Ref. [4]. The dashed, solid and dashed-dotted lines corresponds to $m_{\rho^\pm}-m_{\rho^0}=(+1,0,-1)$ MeV.

3) Mass difference of $\rho^\pm$-ρ^0 mesons. In previous analysis of the IB effects on $\Delta a_\mu^{\rm had,LO}$ [4] it was assumed that the charged and neutral ρ mesons were degenerated, namely $\delta m_\rho \equiv m_{\rho^\pm}-m_{\rho^0}=(0\pm1)$ MeV. An IB effect in the ρ meson mass difference arises from the self-energy contribution generated by the $\rho^0-\gamma$ mixing term, which affects only the neutral ρ meson mass by $m_{\rho^0}-m_{\rho^0_{\rm bare}}\approx 3\Gamma(\rho^0\to e^+e^-)/(2\alpha)=1.45$ MeV [7]. When we remove this IB effect from the value $m_{\rho^\pm}-m_{\rho^0}=(-0.4\pm0.9)$ MeV measured by KLOE [28] from the Dalitz plot analysis of $\phi\to\pi^+\pi^-\pi^0$, we find $\delta m_\rho=(1.0\pm0.9)$ MeV [7]. We use this mass splitting in our evaluations shown in Table 1.

The IB effect produced in $a_\mu^{\rm had,LO}$ $[\pi\pi,\tau]$ by the ρ mass difference is shown in Table 1 for the KS and GS parametrizations. As it can be observed, this effect gives a rather small contribution.

In Fig. 3 we plot separately the IB corrections in the ratio of the $I=1$ components of the pion form factors discussed in subsection 2.3, while their combined effects including the $\rho-\omega$ mixing term is represented with a dashed-dotted line in Fig. 1. The IB effects that stems from the ratio of pion form factors are important close to the resonance peak, making $a_\mu^{\rm had,LO}$ $[\pi\pi,\tau]$ particularly sensitive to these corrections. The effect of the pion mass difference that affects the ρ meson decay widths is shown in the 6th. row of Table 1 and with a solid line in Fig. 3. Interestingly, the effects of the pion mass difference and of the $\pi\pi\gamma$ electromagnetic decays partly cancel each other as it can be observed from Table 1 and Fig. 3.

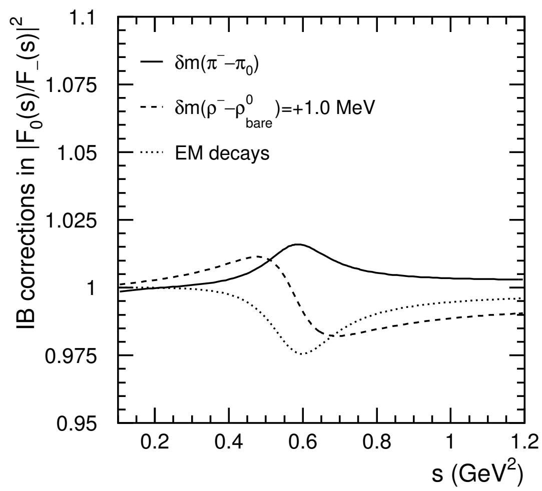

Fig. 3. Contributions of mass and width differences to the ratio of $I=1$ components of form factors.

3 Conclusions

Isospin breaking (IB) corrections are of great relevance to improve the accuracy and gain confidence on the Standard Model prediction of the leading order hadronic contribution $a_\mu^{\rm had,LO}$ to the muon anomalous magnetic moment. These corrections are also important in view of future and more precise measurements of the muon anomalous magnetic moment [29]. We have presented in this work, a summary of some recent results about these IB corrections.

As it can be concluded from our results summarized in Table 1, the new IB corrections produce the change $\Delta a_\mu^{\rm had,LO}$ $[\pi\pi,\tau]=(-16.07\pm1.85)\times10^{-10}$ which is larger by -6.8×10^{-10} units when compared to results used previously in [4]. The main change in the new corrections is due to the effect of the $\rho^\pm$-ρ^0 width difference [26], which quantifies an important IB correction near the resonance of the $\pi\pi$ system. We have calculated, in two commonly used phenomenological models [24, 25], the effects of IB corrections that are important around the ρ resonance region, and have found that the model-dependence of pion form factors is not very important.

The new IB corrections get closer the results of $a_\mu^{\rm had,LO}$ $[\pi\pi]$ based on e^+e^- and τ lepton data (see [2, 7]). These corrections also affect the prediction of the $\tau \to \pi\pi\nu$ branching fraction obtained from e^+e^- data via the isospin symmetry. As it was discussed in [2, 7], the large discrepancy observed in previous comparisons of CVC predictions and direct measurements of this observable [6] is also reduced to an acceptable level after the new IB corrections are applied. It is very appealing that the new IB corrections reduce simultaneously the different manifestations of the so-called e^+e^- vs. τ lepton discrepancy.

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