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Muon $g-2$ discrepancy: new physics or a relatively light Higgs? *

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Abstract After a brief review of the muon $g-2$ status, we discuss hypothetical errors in the Standard Model prediction that might explain the present discrepancy with the experimental value. None of them seems likely. In particular, a hypothetical increase of the hadroproduction cross section in low-energy e^+e^- collisions could bridge the muon $g-2$ discrepancy, but it is shown to be unlikely in view of current experimental error estimates. If, nonetheless, this turns out to be the explanation of the discrepancy, then the 95% CL upper bound on the Higgs boson mass is reduced to about 135 GeV which, in conjunction with the experimental 114.4 GeV 95% CL lower bound, leaves a narrow window for the mass of this fundamental particle.

Key words muon anomalous magnetic moment, Standard Model Higgs boson

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1 Introduction status of a_μ

The anomalous magnetic moment of the muon, a_μ, is one of the most interesting observables in particle physics. Indeed, as each sector of the Standard Model (SM) contributes in a significant way to its theoretical prediction, the precise a_μ measurement by the E821 experiment at Brookhaven [1, 2] allows us to test the entire SM and scrutinize viable "new physics" appendages to this theory [3, 4].

The SM prediction of the muon $g-2$ is conveniently split into QED, electroweak (EW) and hadronic (leading- and higher-order) contributions: $a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HLO} + a_\mu^{\rm HHO}$. The QED prediction, computed up to four (and estimated at five) loops, currently stands at $a_\mu^{\rm QED} = 116584718.08(15) \times 10^{-11}$ [5], while the EW effects provide $a_\mu^{\rm EW} = 154(2) \times 10^{-11}$ [6]. The latest calculations of the hadronic leading-order contribution, via the hadronic e^+e^- annihilation data, are in agreement: $a_\mu^{\rm HLO} = 6894(40)\times10^{-11}$ [7] (this preliminary result, presented at this workshop, updates the value $6894(46)\times10^{-11}$ of Ref. [8]) and $6903(53)\times10^{-11}$ [9]. These determinations include the 2008 $e^+e^- \to \pi^+\pi^-(\gamma)$ cross section data from KLOE [10] (see also [11]). A somewhat larger value, $6955(41)\times10^{-11}$ [12], was recently obtained including also the 2009 $\pi^+\pi^-(\gamma)$ data of BaBar [13].

The higher-order hadronic term is further divided into two parts: $a_\mu^{\rm HHO} = a_\mu^{\rm HHO}({\rm vp}) + a_\mu^{\rm HHO}({\rm lbl})$. The first one, $-98\,(1)\times10^{-11}$ [8], is the $O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions [14]. The second term, also of $O(\alpha^3)$, is the hadronic light-by-light contribution; as it cannot be determined from data, its evaluation relies on specific models. The latest determinations of this term, $116(39)\times10^{-11}$ [9, 15] and $105(26)\times10^{-11}$ [16], are in very good agreement. If we add the latter to $a_\mu^{\rm HLO}$, for example the value of Ref. [7], and the rest of the SM contributions, we obtain $a_\mu^{\rm SM} = 116591773(48)\times10^{-11}$. The difference with the experimental value $a_\mu^{\rm EXP} = 116592089(63)\times10^{-11}$ [2] (note the tiny shift upwards, with respect to the value reported in [1], due to the updated value of the muon-proton magnetic moment ratio [17]) is $\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = +316(79)\times10^{-11}$, i.e., 4.0σ (all errors were added in quadrature). Slightly smaller discrepancies are found employing the $a_\mu^{\rm HLO}$ values reported in [12] (which also includes the re-

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cent $\pi^+\pi^-(\gamma)$ data of BaBar) and [9]: 3.2σ and 3.6σ, respectively. We will use the $a_\mu^{\rm HLO}$ value of Ref. [7] (which also provides the hadronic contribution to the effective fine-structure constant later required for our analysis), but we expect that a consistent inclusion of the recent $\pi^+\pi^-(\gamma)$ BaBar data would not change our basic conclusions. For reviews of a_μ see Refs. [7, 9, 18].

The term $a_\mu^{\rm HLO}$ can alternatively be computed incorporating hadronic τ-decay data, related to those of hadroproduction in e^+e^- collisions via isospin symmetry [19]. The long-standing difference between the e^+e^-- and τ-based determinations of $a_\mu^{\rm HLO}$ [20] has been recently somewhat lessened by a re-analysis [21] where the isospin-breaking corrections [22] were revisited taking advantage of more accurate data and new theoretical investigations (recent $\tau^- \to \pi^-\pi^0\nu_\tau$ data from the Belle experiment [23] were also included). In spite of this, the τ-based value remains higher than the e^+e^--based one, leading to a smaller (1.9σ) difference Δa_μ. On the other hand, recent analyses of the pion form factor claim that the τ and e^+e^- data are consistent after isospin violation effects and vector meson mixings are considered, further confirming the e^+e^--based discrepancy [24].

The 3–4σ discrepancy between the theoretical prediction and the experimental value of the muon $g-2$ can be explained in several ways. It could be due, at least in part, to an error in the determination of the hadronic light-by-light contribution. However, if this were the only cause of the discrepancy, $a_\mu^{\rm HHO}(\rm lbl)$ would have to move up by many standard deviations (roughly ten) to fix it. Although the errors assigned to $a_\mu^{\rm HHO}(\rm lbl)$ are only educated guesses, this solution seems unlikely, at least as the dominant one.

Another possibility is to explain the discrepancy Δa_μ via the QED, EW and hadronic higher-order vacuum polarization contributions; this looks very improbable, as one can immediately conclude inspecting their values and uncertainties reported above. If we assume that the $g-2$ experiment E821 is correct, we are left with two options: possible contributions of physics beyond the SM, or an erroneous determination of the leading-order hadronic contribution $a_\mu^{\rm HLO}$ (or both). The first of these two explanations has been extensively discussed in the literature; updating Ref. [25] we will study whether the second one is realistic or not, and analyze its implications for the EW bounds on the mass of the Higgs boson.

2 Connection with the Higgs mass

The hadronic leading-order contribution $a_\mu^{\rm HLO}$ can be computed via the dispersion integral [26]

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} {\rm d}s\, K(s)\,\sigma(s), \qquad (1)$$

where $\sigma(s)$ is the total cross section for e^+e^- annihilation into any hadronic state, with vacuum polarization and initial state QED corrections subtracted off (for a detailed discussion of these radiative corrections and the precision of the Monte Carlo generators used to analyze the hadronic cross section measurements see [27]), and s is the squared momentum transfer. The well-known kernel function $K(s)$ (see [28]) is positive definite, decreases monotonically for increasing s and, for large s, behaves as $m_\mu^2/(3s)$ to a good approximation. About 90% of the total contribution to $a_\mu^{\rm HLO}$ is accumulated at center-of-mass energies $\sqrt{s}$ below 1.8 GeV and roughly three-fourths of $a_\mu^{\rm HLO}$ is covered by the two-pion final state which is dominated by the $\rho(770)$ resonance [12]. Exclusive low-energy e^+e^- cross sections were measured at colliders in Frascati, Novosibirsk, Orsay, and Stanford, while at higher energies the total cross section was determined inclusively.

Let's now assume that the discrepancy $\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = +316(79)\times 10^{-11}$, is due to – and only to – hypothetical errors in $\sigma(s)$, and let us increase this cross section in order to raise $a_\mu^{\rm HLO}$, thus reducing Δa_μ. This simple assumption leads to interesting consequences. An upward shift of the hadronic cross section also induces an increase of the value of the hadronic contribution to the effective fine-structure constant at M_Z [29],

$$\Delta\alpha_{\rm had}^{(5)}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2}\, P\int_{4m_\pi^2}^{\infty} {\rm d}s\, \frac{\sigma(s)}{M_Z^2 - s} \qquad (2)$$

(P stands for Cauchy's principal value). This integral is similar to the one we encountered in Eq. (1) for $a_\mu^{\rm HLO}$. There, however, the weight function in the integrand gives a stronger weight to low-energy data. Let us define

$$a_i = \int_{4m_\pi^2}^{s_u} {\rm d}s\, f_i(s)\,\sigma(s) \qquad (3)$$

($i=1,2$), where the upper limit of integration is $s_u < M_Z^2$, and the kernels are $f_1(s) = K(s)/(4\pi^3)$ and $f_2(s) = [M_Z^2/(M_Z^2 - s)]/(4\alpha\pi^2)$. The integrals a_i with $i=1,2$ provide the contributions to $a_\mu^{\rm HLO}$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, respectively, from $4m_\pi^2$ up to s_u (see Eqs. (1,2)). An increase of the cross section $\sigma(s)$ of the form

$$\Delta\sigma(s) = \epsilon\sigma(s) \qquad (4)$$

in the energy range $\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$, where ϵ and δ are positive constants and $2m_\pi + \delta/2 <$

$\sqrt{s}_0 < \sqrt{s}_u - \delta/2$, increases a_1 by $\Delta a_1(\sqrt{s}_0,\delta,\epsilon) = \epsilon\int_{\sqrt{s}_0-\delta/2}^{\sqrt{s}_0+\delta/2} 2t\,\sigma(t^2)\,f_1(t^2)\,\mathrm{d}t$. If we assume that the muon $g-2$ discrepancy is entirely due to this increase in $\sigma(s)$, so that $\Delta a_1(\sqrt{s}_0,\delta,\epsilon) = \Delta a_\mu$, the parameter ϵ becomes

$$\epsilon = \frac{\Delta a_\mu}{\int_{\sqrt{s}_0-\delta/2}^{\sqrt{s}_0+\delta/2} 2t\,f_1(t^2)\,\sigma(t^2)\,\mathrm{d}t}, \quad (5)$$

and the corresponding increase in $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ is

$$\Delta a_2(\sqrt{s}_0,\delta) = \Delta a_\mu \frac{\int_{\sqrt{s}_0-\delta/2}^{\sqrt{s}_0+\delta/2} f_2(t^2)\,\sigma(t^2)\,t\,\mathrm{d}t}{\int_{\sqrt{s}_0-\delta/2}^{\sqrt{s}_0+\delta/2} f_1(t^2)\,\sigma(t^2)\,t\,\mathrm{d}t}. \quad (6)$$

The shifts $\Delta a_2(\sqrt{s}_0,\delta)$ were studied in Ref. [25] for several bin widths δ and central values $\sqrt{s}_0$.

The present global fit of the LEP Electroweak Working Group (EWWG) leads to the Higgs boson mass $M_H = 87^{+35}_{-26}$ GeV and the 95% confidence level (CL) upper bound $M_H^{UB} \simeq 157$ GeV [30]. This result is based on the recent preliminary top quark mass M_t=173.1(1.3) GeV [31] and the value $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$= 0.02758(35) [32]. The LEP direct-search 95% CL lower bound is $M_H^{LB} = 114.4$ GeV [33]. Although the global EW fit employs a large set of observables, M_H^{UB} is strongly driven by the comparison of the theoretical predictions of the W boson mass and the effective EW mixing angle $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ with their precisely measured values. Convenient formulae providing the M_W and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ SM predictions in terms of M_H, M_t, $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$, and $\alpha_s(M_Z)$, the strong coupling constant at the scale M_Z, are given in [34]. Combining these two predictions via a numerical χ^2-analysis and using the present world-average values M_W = 80.399(23) GeV [35], $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ = 0.23153(16) [36], M_t=173.1(1.3) GeV [31], $\alpha_s(M_Z)$=0.118(2) [37], and the determination $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z) = 0.02758(35)$ [32], we get $M_H = 92^{+37}_{-28}$ GeV and $M_H^{UB} = 158$ GeV. We see that indeed the M_H values obtained from the M_W and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ predictions are quite close to the results of the global analysis.

The M_H dependence of a_μ^{SM} is too weak to provide M_H bounds from the comparison with the measured value. On the other hand, $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ is one of the key inputs of the EW fits. For example, employing the latest (preliminary) value $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ = 0.02760(15) presented at this workshop [7] instead of 0.02758(35) [32], the M_H prediction derived from M_W and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ shifts to $M_H = 96^{+32}_{-25}$ GeV and $M_H^{UB} = 153$ GeV. To update the analysis of Ref. [25] we considered the new values of $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ obtained shifting 0.02760(15) [7] by $\Delta a_2(\sqrt{s}_0,\delta)$ (including their uncertainties, as discussed in [25]), and computed the corresponding new values of M_H^{UB} via the combined χ^2-analysis based on the M_W and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ inputs (for both $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ and a_μ^{HLO} we used the values reported in Ref. [7]). Our results show that an increase $\epsilon\sigma(s)$ of the hadronic cross section (in $\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2]$), adjusted to bridge the muon $g-2$ discrepancy Δa_μ, decreases M_H^{UB}, further restricting the already narrow allowed region for M_H. We conclude that these hypothetical shifts conflict with the lower limit M_H^{LB} when $\sqrt{s}_0 \gtrsim 1.2$ GeV, for values of δ up to several hundreds of MeV. In [25] we pointed out that there are more complex scenarios where it is possible to bridge the Δa_μ discrepancy without significantly affecting M_H^{UB}, but they are considerably more unlikely than those discussed above.

If τ data are used instead of e^+e^- ones in the calculation of the dispersive integral in Eq. (1), a_μ^{HLO} increases to $7053(45)\times10^{-11}$[21] and the discrepancy drops to $\Delta a_\mu = +157(82)\times10^{-11}$, i.e. 1.9σ. While using τ data reduces the Δa_μ discrepancy, it increases $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ by approximately 2×10^{-4} [1)], leading to a sharply lower M_H prediction [38]. Indeed, increasing the previously employed value $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z)$ = 0.02760(15) [7] by 2×10^{-4} and using the same above-discussed previous inputs of the χ^2-analysis, we find an M_H^{UB} value of only 138 GeV. If the remaining 1.9σ discrepancy Δa_μ is bridged by a further increase $\Delta\sigma(s) = \epsilon\sigma(s)$ of the hadronic cross section, M_H^{UB} decreases to even lower values, leading to a scenario in near conflict with M_H^{LB}.

Recent analyses of the pion form factor below 1 GeV claim that τ data are consistent with the e^+e^- ones after isospin violation effects and vector meson mixings are considered [24]. In this case one could use the e^+e^- data below ∼1 GeV, confirmed by the τ ones, and assume that Δa_μ is accommodated by hypothetical errors in the e^+e^- measurements occurring above ∼1 GeV, where disagreement persists between these two data sets. Our analysis shows that this assumption would lead to M_H^{UB} values inconsistent with M_H^{LB}.

In the above analysis, the hadronic cross section $\sigma(s)$ was shifted up by amounts $\Delta\sigma(s) = \epsilon\sigma(s)$ adjusted to bridge Δa_μ. Apart from the implications for M_H, these shifts may actually be inadmissibly large when compared with the quoted experimental uncertainties. Consider the parameter $\epsilon = \Delta\sigma(s)/\sigma(s)$.

1) This number represents our rough update of the value reported in Ref. [20].

Clearly, its value depends on the choice of the energy range $[\sqrt{s}_0-\delta/2, \sqrt{s}_0+\delta/2]$ where $\sigma(s)$ is increased and, for fixed $\sqrt{s}_0$, it decreases when δ increases. Its minimum value, $\sim 5\%$, occurs if $\sigma(s)$ is multiplied by $(1+\epsilon)$ in the whole integration region, from $2m_\pi$ to infinity. Such a shift would lead to $M_{\rm H}^{\rm UB}\sim 75$ GeV, well below $M_{\rm H}^{\rm LB}$. Higher values of ϵ are obtained for narrower energy bins, particularly if they do not include the ρ-ω resonance region. For example, a huge $\epsilon\sim 55\%$ increase is needed to accommodate Δa_μ with a shift of $\sigma(s)$ in the region from $2m_\pi$ up to 500 MeV (reducing $M_{\rm H}^{\rm UB}$ to 146 GeV), while an increase in a bin of the same size but centered at the ρ peak requires $\epsilon\sim 9\%$ (lowering $M_{\rm H}^{\rm UB}$ to 135 GeV). As the quoted experimental uncertainty of $\sigma(s)$ below 1 GeV is of the order of a few per cent (or less, in some specific energy regions), the possibility to explain Δa_μ with these shifts $\Delta\sigma(s)$ appears to be unlikely. Lower values of ϵ are obtained if the shifts occur in energy ranges centered around the ρ-ω resonances, but also this possibility looks unlikely, since it requires variations of $\sigma(s)$ of at least $\sim 6\%$. If, however, such shifts $\Delta\sigma(s)$ indeed turn out to be the solution of the Δa_μ discrepancy, then $M_{\rm H}^{\rm UB}$ is reduced to about 135 GeV.

It is interesting to note that in the scenario where Δa_μ is due to hypothetical errors in $\sigma(s)$, rather than "new physics", the reduced $M_{\rm H}^{\rm UB}\lesssim 135$ GeV induces some tension with the approximate 95% CL lower bound $M_{\rm H}\gtrsim 120$ GeV required to ensure vacuum stability under the assumption that the SM is valid up to the Planck scale [39] (note, however, that this lower bound somewhat decreases when the vacuum is allowed to be metastable, provided its lifetime is longer than the age of the universe [40]). Thus, one could argue that this tension is, on its own, suggestive of physics beyond the SM.

We remind the reader that the present values of $\sin^2\theta_{\rm eff}^{\rm lept}$ derived from the leptonic and hadronic observables are respectively $(\sin^2\theta_{\rm eff}^{\rm lept})_l=0.23113(21)$ and $(\sin^2\theta_{\rm eff}^{\rm lept})_h=0.23222(27)$ [36]. In Ref. [25] we pointed out that the use of either of these values as an input parameter leads to inconsistencies in the SM framework that already require the presence of "new physics". For this reason, we followed the standard practice of employing as input the world-average value for $\sin^2\theta_{\rm eff}^{\rm lept}$ determined in the SM global analysis. Since $M_{\rm H}^{\rm UB}$ also depends sensitively on $M_{\rm t}$, in [25] we provided simple formulae to obtain the new values derived from different $M_{\rm t}$ inputs.

A 3–4σ discrepancy between the theoretical prediction and the experimental value of the muon $g-2$ would have interesting implications if truly due to "new physics" (i.e. beyond the SM expectations). Supersymmetry provides a natural interpretation of this discrepancy (see Ref. [4] for a review). For illustration purposes, we assume a single mass $m_{\rm susy}$ for sleptons, sneutrinos and gauginos that enter the $a_\mu^{\rm susy}$ calculation. Then one finds [41] (including leading two-loop effects)

$$a_\mu^{\rm susy}\simeq {\rm sgn}(\mu)\times 130\times 10^{-11}\left(\frac{100\ {\rm GeV}}{m_{\rm susy}}\right)^2\tan\beta, \quad (7)$$

where ${\rm sgn}(\mu)=\pm$ is the sign of the μ term in supersymmetry models and $\tan\beta>3$–4 is the ratio of the two scalar vacuum expectation values, $\tan\beta=\langle\phi_2\rangle/\langle\phi_1\rangle$. The $\tan\beta$ factor is an important source of enhancement. As experimental constraints on the Higgs mass have increased, so has the lower bound on $\tan\beta$. With larger $\tan\beta$ now required, it appears inevitable that supersymmetric loops have a fairly major effect on the theoretical prediction of the muon $g-2$ if $m_{\rm susy}$ is not too large. In fact, equating (7) and the discrepancy Δa_μ, for example the value $\Delta a_\mu=+316(79)\times 10^{-11}$ obtained using the $a_\mu^{\rm HLO}$ determination of Ref. [7], one finds ${\rm sgn}(\mu)=+$ and

$$m_{\rm susy}\simeq 64^{+10}_{-7}\sqrt{\tan\beta}\ \ {\rm GeV}. \quad (8)$$

For $\tan\beta\sim 4$–50, these values are in keeping with mainstream supersymmetric expectations. Several alternative "new physics" explanations have also been suggested [3].

3 Conclusions

We examined a number of hypothetical errors in the SM prediction of the muon $g-2$ that could be responsible for the present 3–4σ discrepancy Δa_μ with the experimental value. None of them looks likely. In particular, updating Ref. [25] we showed how an increase $\Delta\sigma(s)=\epsilon\sigma(s)$ of the hadroproduction cross section in low-energy e^+e^- collisions could bridge Δa_μ. However, such increases lead to reduced $M_{\rm H}$ upper bounds – even lower than 114.4 GeV (the LEP lower bound) if they occur in energy regions centered above $\sim$ 1.2 GeV). Moreover, their amounts are generally very large when compared with the quoted experimental uncertainties, even if the latter were significantly underestimated. The possibility to bridge the muon $g-2$ discrepancy with shifts of the hadronic cross section therefore appears to be unlikely. If, nonetheless, this turns out to be the solution, then the 95% CL upper bound $M_{\rm H}^{\rm UB}$ drops to about 135 GeV.

If τ-decay data are used instead of e^+e^- ones in the calculation of $a_\mu^{\rm SM}$, the muon $g-2$ discrepancy decreases to $\sim 2\sigma$. While this reduces Δa_μ, it raises the value of $\Delta\alpha_{\rm had}^{(5)}(M_{\rm Z})$ leading to $M_{\rm H}^{\rm UB}$ =138 GeV, thus increasing the tension with the LEP lower bound and suggesting a near conflict with it should one try to overcome the full discrepancy. One could also consider a scenario, suggested by recent studies, where the τ data confirm the e^+e^- ones below ∼ 1 GeV, while a discrepancy between them persists at higher energies. If, in this case, Δa_μ is fixed by hypothetical errors in the e^+e^- measurements above ∼1 GeV, where the data sets disagree, one also finds values of $M_{\rm H}^{\rm UB}$ inconsistent with the LEP lower bound.

If the Δa_μ discrepancy is real, it points to "new physics", like low-energy supersymmetry where Δa_μ is reconciled by the additional contributions of supersymmetric partners and one expects $M_{\rm H} \lesssim 135$ GeV for the mass of the lightest scalar [42]. If, instead, the deviation is caused by an incorrect leading-order hadronic contribution, it leads to reduced $M_{\rm H}^{\rm UB}$ values. This reduction, together with the LEP lower bound, leaves a narrow window for the mass of this fundamental particle. Interestingly, it also raises the tension with the $M_{\rm H}$ lower bound derived in the SM from the vacuum stability requirement.

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