Status and accuracy of the Monte Carlo generators for luminosity measurements

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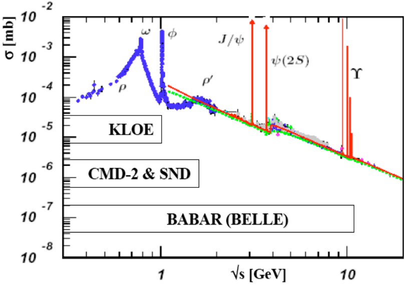
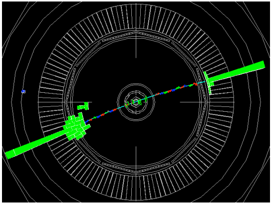
in collaboration with the BabaYaga@NLO authors

and with many thanks to the contributors of the Luminosity Section of the Report of the WG "Radiative Corrections & Monte Carlo Tools " [See talk by H. Czyz]

Why precision luminosity generators?





Bhabha tracks @ the B–factory PEP-II

- Precision measurements of the hadronic cross section at low energies require a precise knowledge of the e^+e^- collider luminosity $\mathcal{L}$

$$\int \mathcal{L}\, dt = N_{\rm obs}/\sigma_{\rm th}$$

★ Precise knowledge of the luminosity needs normalization processes with clean topology, high statistics and calculable with high theoretical accuracy → wide–angle QED processes $e^+e^- \to e^+e^-$ (Bhabha scattering), $e^+e^- \to \gamma\gamma$ and $e^+e^- \to \mu^+\mu^-$, with typical experimental errors in the range few $0.1\% \div \mathcal{O}(1\%)$

- High theoretical accuracy and comparison with data require precision Monte Carlo (MC) tools, including radiative corrections at the highest standard as possible

Typical theory of the MC generators

- ⋆ The most precise MC generators include exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections matched with higher–order (HO) leading logarithmic (LL) contributions + vacuum polarization, using a data based routine [Jegerlehner, HMNT,...] for the calculation of the non–perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ contribution
- ⋆ The methods used to account for multiple photon corrections are the (LEP/SLC borrowed) analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in the generators BabaYaga/BabaYaga@NLO] is a MC solution of the QED DGLAP equation for the electron SF $D(x,Q^2)$

$$D(x,Q^2) \;=\; \Pi(Q^2)\sum_{n=0}^{\infty} \int \frac{\delta(x-x_1\cdots x_n)}{n!} \prod_{i=0}^{n}\left[\frac{\alpha}{2\pi}P(x_i)\;L\;dx_i\right]$$

  ⋆ $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi}LI_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x)dx$
  $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale

- The LL accuracy can be improved by matching NLO & HO corrections
  G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227 & Phys. Lett. **B663** (2008) 209

$$d\sigma^{\infty}_{\rm matched} = F_{SV}\;\Pi(Q^2,\varepsilon)\;\sum_{n=0}^{\infty}\frac{1}{n!}\;\left(\prod_{i=0}^{n}F_{H,i}\right)\;|\mathcal{M}_{n,LL}|^2\;d\Phi_n$$

- ⋆ $[\sigma^{\infty}_{\rm matched}]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm exact}$, avoiding double counting and preserving exponentiation of $\alpha^n L^n, n \geq 2$ leading logs
- ⋆ theoretical error shifted to $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections

Status of the luminosity generators

Generator	Processes	Theory	Accuracy	Web address
BHAGENF/BKQED	$e^+e^-/\gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$	1%	www.lnf.infn.it/~graziano/bhagenf/bhabha.html
BabaYaga v3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	Parton Shower	$\sim 0.5\%$	www.pv.infn.it/~hepcomplex/babayaga.html
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + PS	$\sim 0.1\%$	www.pv.infn.it/~hepcomplex/babayaga.html
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	0.5%(LEP1)	placzek.home.cern.ch/placzek/bhwide
MCGPJ	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + SF	$< 0.2\%$	cmd.inp.nsk.su/~sibid

Sources of (possible) differences and theoretical uncertainty

- ⋆ "Technical precision": due to different details in the implementation of the *same* radiative corrections [e.g. different scales in higher–order collinear logs]. It can be estimated through *tuned comparisons* between the predictions of the different generators
- ⋆ Theoretical accuracy: due to *approximate or partially included* pieces of radiative corrections [e.g. exact NNLO photonic or pair corrections]. It can be evaluated through explicit comparisons with the exact perturbative calculations, if available

- At $\mathcal{O}(\alpha^2)$, infrared–enhanced photonic $\mathcal{O}(\alpha^2 L)$ most important NNLO sub–leading corrections taken into account through factorization of $\mathcal{O}(\alpha L) \times \mathcal{O}(\alpha)_{\rm non-log}$ contributions

G. Montagna, O. Nicrosini and F. Piccinini, Phys. Lett. **B385** (1996) 348

Large–angle Bhabha: tuned comparisons at meson factories

Without vacuum polarization, to compare consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga people, Wang Ping and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

★ Agreement well below 0.1%! ★

At BaBar (cross sections in nb)

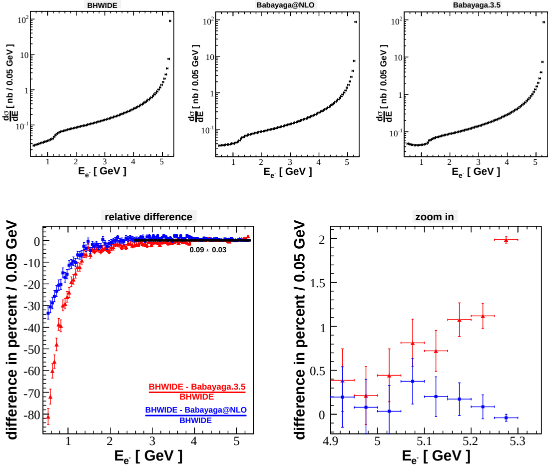
By A. Hafner and A. Denig

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

★ Agreement at the $\sim$ 0.1% level! ★

BabaYaga@NLO vs BHWIDE at BaBar

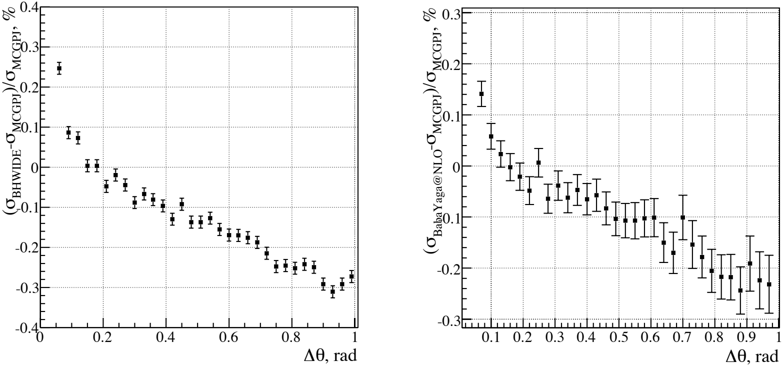
From the Luminosity Section of the WG Report "Radiative Corrections & MC Tools"
By A. Hafner and A. Denig, using realistic luminosity cuts @



- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for distributions. Larger differences correspond to very hard photon emission and do not influence noticeably the luminosity measurement

MCGPJ, BabaYaga@NLO and BHWIDE at VEPP–2M

From the Luminosity Section of the WG Report "Radiative Corrections & MC Tools"
By A. Sibidanov, with realistic selection cuts for luminosity @ CMD–2
Based on A.B. Arbuzov *et al.*, Eur. Phys. J. **C46** (2006) 689



- The three generators agree within 0.1% for the typical experimental acollinearity cut $\Delta\theta \sim 0.2 \div 0.3$ rad

★ Main conclusion from tuned comparisons: technical precision of the generators well under control, the small remaining differences being due to slightly different details in the calculation of the same theoretical ingredients [additive vs factorized formulations, different scales for higher–order leading log corrections]

The main question: how to establish the MC theoretical accuracy?

1. By comparing with the available NNLO calculations, thanks to the impressive progress in this area during the last few years
2. By estimating the size of partially accounted corrections, if exact or complete calculations are/were not yet available [e.g. as for pair corrections and one–loop corrections to $e^+e^- \to e^+e^-\gamma$ till some weeks ago! Update on new exact calculations and related comparisons in progress in the next slides]

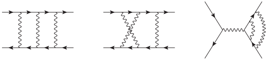
For example, by expanding the matched PS formula up to $\mathcal{O}(\alpha^2)$, the (approximate) BabaYaga@NLO NNLO cross section can be cast into the form

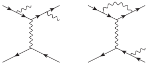
$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

- $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$ $\longrightarrow$ compared with the corresponding available NNLO QED calculation
- $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung $\longrightarrow$ presently estimated relying upon existing (partial) results
- $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung $\longrightarrow$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

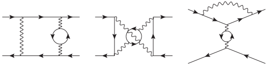
The recent progress in NNLO Bhabha calculations

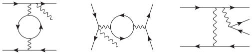
- **Photonic corrections** A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



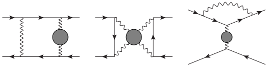


- **Electron loop corrections** R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280 / S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26



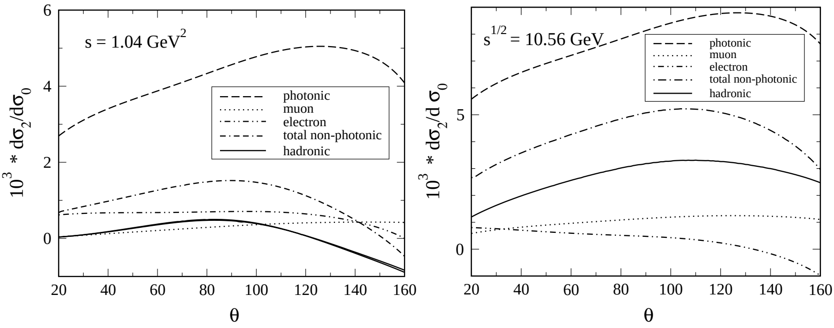


- **Heavy fermion and hadronic corrections** R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601 / S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602 / J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



NNLO QED corrections: typical size at the Φ and B factories

From the Luminosity Section of the WG Report "Radiative Corrections & MC Tools"
By NNLO groups



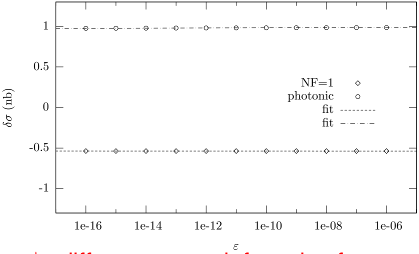
- NNLO QED corrections amount to some per mille and are dominated by photonic (dashed line) and electron loop (dashed–dotted) corrections
- The bulk [due to the reducible contributions] of such corrections is *effectively* incorporated in the most precise generators through the matching of NLO corrections with multiple photon contributions and the insertion of vacuum polarization in the $\mathcal{O}(\alpha)$ diagrams.
  To what extent?

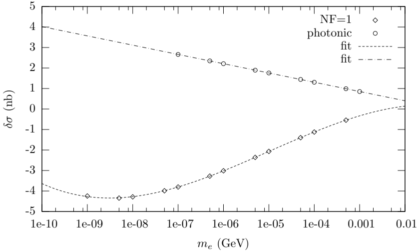
Comparison with NNLO calculation for $\sigma_{\rm SV}^{\alpha^2}$

Thanks to R. Bonciani, A. Ferroglia and A. Penin!
Using realistic cuts for luminosity @

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarization contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)





★ differences are infrared safe, as expected

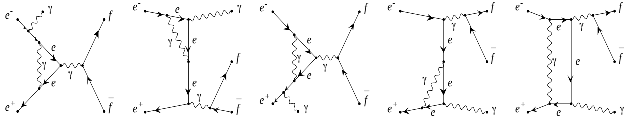
★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\text{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

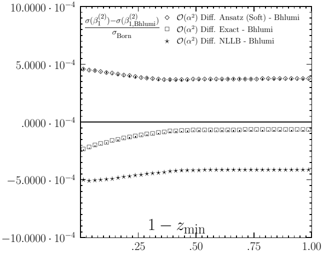
★ New! The exact perturbative calculation of $\sigma^{\alpha^2}_{\mathrm{SV,H}}$ for full $s+t$ Bhabha scattering appeared on the arXiv just a few weeks ago! ★

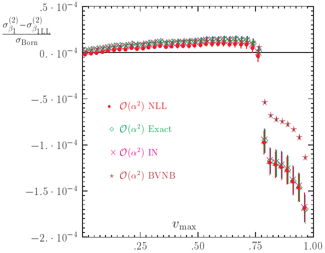
S. Actis, P. Mastrolia and G. Ossola, `arXiv:0909.1750 [hep-ph]`



- Using the results available for $t-$channel Bhabha scattering (left plot) and $s-$channel annihilation processes (right plot)

S. Jadach, M. Melles, B.F.L. Ward and S. Yost, PL **B377** (1996) 168 & PL **B450** (1999) 262
C. Glosser, S. Jadach, B.F.L. Ward and S. Yost, Phys. Lett. **B605** (2005) 123





the uncertainty of the most precise generators for one–loop corrections to single hard bremsstrahlung can be conservatively estimated to be $\sim 0.05\%$

A further important source of error: lepton and hadron pairs

New!: from the Luminosity Section of the WG Report "Radiative Corrections & MC Tools"

- A Desy–Zeuthen & Katowice collaboration [H. Czyz, J. Gluza, M. Gunia, T. Riemann and M. Worek] did a new, exact calculation of pair corrections, based on exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-, l = e, \mu, \tau), e^+e^-(\pi^+\pi^-)$
- Results: in comparison with the approximation of BabaYaga@NLO and using realistic KLOE and BaBar luminosity cuts (cross sections in nb)

Electron pair corrections				
	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
KLOE	529.469	-1.794	-1.570	0.04
BaBar	6.744	-0.008	-0.008	0.00
Muon pair corrections				
	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
KLOE	529.469	-0.241	-0.250	0.002
BaBar	6.744	-0.004	-0.003	0.015
Pion pair corrections				
	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
KLOE	529.469	-0.186	in progress	–
BaBar	6.744	-0.003	in progress	–

⋆ The uncertainty due to lepton and hadron pair corrections is at the level of a few units in 10^{-4} [further comparisons in progress] ⋆

Status of the MC theoretical accuracy

Main conclusion of the Luminosity Section of the WG Report "Radiative Corrections & MC Tools"

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

Source of error (%)	Φ–factories	$\sqrt{s}$ = 3.5 GeV	B–factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$ [conservative?]	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$ [in progress]	$\sim$0.05	$\sim$0.1[1]	$\sim$0.02[2]
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12÷0.14	0.18	0.11÷0.12
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07÷0.08	0.11	0.06÷0.07

- Comparisons with the Novosibirsk $\Delta\alpha^{(5)}_{\rm had}(q^2)$ parameterization routine and with the calculation by Actis *et al.* for $e^+e^-\gamma$ at one loop would put the evaluation of the $\lvert\delta^{\rm err}_{\rm VP}\rvert$ and $\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$ uncertainties on firmer grounds

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved about ten years ago for small–angle Bhabha luminosity monitoring at LEP/SLC

[1] Very preliminary, work in progress using realistic BES-III and CLEO-c luminosity cuts

[2] Preliminary and assuming BaBar cuts. Work in progress for BELLE event selection

Conclusions & perspectives

- Recent remarkable progress in reducing the theoretical error to the luminosity measurements at flavour factories down to $\sim$ 0.1%

★ Both exact $\mathcal{O}(\alpha)$ and multiple photon corrections are implemented in the most precise MC luminosity tools and are necessary ingredients for 0.1% theoretical accuracy [together with vacuum polarization]

★ At least three generators for large–angle Bhabha scattering (BabaYaga@NLO, BHWIDE, MCGPJ) agree within 0.1% for integrated cross sections and $\sim 1\%$ (or better) for distributions

- Precision generators also available for $\gamma\gamma$ production (BabaYaga@NLO, MCGPJ) and $\mu^+\mu^-, \mu^+\mu^-\gamma$ final states (BabaYaga@NLO, KKMC, MCGPJ)

★ NNLO QED calculations allow to assess the MC theoretical accuracy at the 0.1% level and, if necessary, to improve it below the one per mille

★ Possible and feasible improvements concern
  - Tuned comparisons: understanding of the (minor) residual differences between program predictions for large–angle Bhabha [if needed] and new comparisons for the $e^+e^- \to \gamma\gamma, \mu^+\mu^- [\mu^+\mu^-\gamma]$ processes
  - Theoretical accuracy: deeper analysis of the uncertainty due to pair corrections [in progress], one–loop corrections to $e^+e^- \to e^+e^-\gamma$ [started] and hadronic vacuum polarization

★ The present MC accuracy is robust and already sufficient for per mille luminosity measurements at meson factories ★

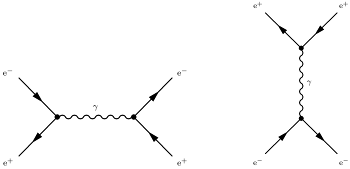
Backup Slides

The luminosity monitoring QED processes

Using wide angles selection cuts, with typical experimental errors in the range few $0.1\% \div \mathcal{O}(1\%)$ [e.g. $\delta\mathcal{L}_{\rm exp}/\mathcal{L}_{\rm exp} = 0.3\%$ for Bhabha @ KLOE]

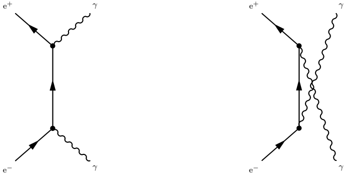
Homi J. Bhabha (1909-1966)
Proc. Roy. Soc. **A154** (1936) 195

- $e^+e^- \to e^+e^-$ (Bhabha scattering)
  [KLOE, CMD-2 and SND, BES, CLEO-c, BaBar]



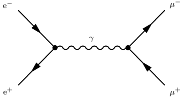
$$|M|^2 \propto \alpha^2 \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} + \frac{2u^2}{ts} \right)$$

- $e^+e^- \to \gamma\gamma$ [KLOE, CLEO-c, BaBar, BES-III]



$$|M|^2 \propto \alpha^2 \left(\frac{u}{t} + \frac{t}{u} \right)$$

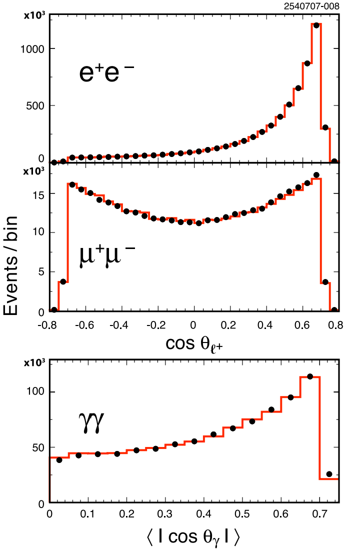
- $e^+e^- \to \mu^+\mu^-$ [CLEO-c, BaBar]



$$|M|^2 \propto \alpha^2 \frac{t^2+u^2}{s^2}$$

Experimental luminosity errors: from Φ to $B-$factories

S. Dobbs *et al.*, [CLEO–c Coll.], Phys. Rev. **D76** (2007) 112001

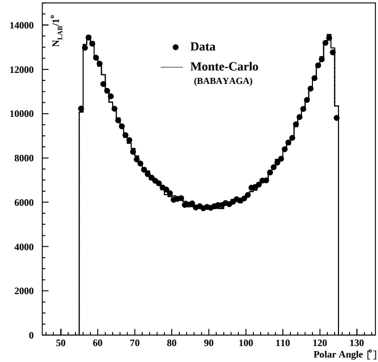


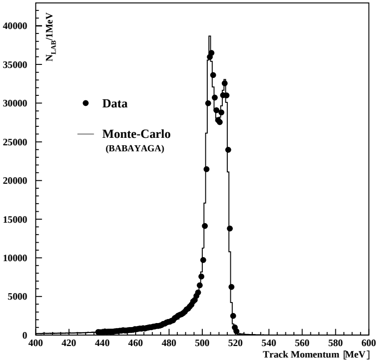
Using wide angles selection cuts

- Bhabha scattering
  - KLOE: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} = 0.3\%$
  - CLEO-c: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 1\%$
  - BES-III: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim \text{few } 0.1\%$
  - BaBar: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} = 0.7\%$
- $\gamma\gamma$ production
  - KLOE: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim \text{few } 0.1\%$
  - CLEO-c: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 1\%$
  - BaBar: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 1.5\%$
- $\mu^+\mu^-$ production
  - CLEO-c: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 0.8\%$
  - BaBar: $\frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \sim 0.5\%$

KLOE Bhabha data vs BabaYaga v3.5/BabaYaga@NLO

F. Aloisio *et al.*, [KLOE Coll.], Phys. Lett. **B606** (2005) 12





$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.5\%\,({\rm th\ BabaYaga\,v3.5}) = 0.6\%\ [\text{as of 2006}]$$

F. Ambrosino *et al.*, [KLOE Coll.], Eur. Phys. J. **C47** (2006) 589

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.1\%\,({\rm th\ BabaYaga@NLO}) = 0.3\%\ [\text{now!}]$$

F. Ambrosino *et al.*, [KLOE Coll.], `arXiv:0707.4078 [hep-ex]`

Matching NLO and higher–order corrections

C.M. Carloni Calame *et al.*, Nucl. Phys. **584** (2000) 459 & Nucl. Phys. Proc. Suppl. **131** (2004) 48
G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227 & Phys. Lett. **663** (2008) 209 [BabaYaga@NLO]

Exact NLO soft+virtual (SV) corrections and hard bremsstrahlung (H) matrix elements can be combined with the QED PS through a matching procedure

- $d\sigma_{LL}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$
- $d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{LL}^{SV}(\varepsilon) + d\sigma_{LL}^{H}(\varepsilon)$
- $d\sigma_{\rm exact}^{\alpha} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1}|^2 d\Phi_1 \equiv d\sigma_{\rm exact}^{SV}(\varepsilon) + d\sigma_{\rm exact}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_{\alpha} - C_{\alpha,LL})$ $\qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$

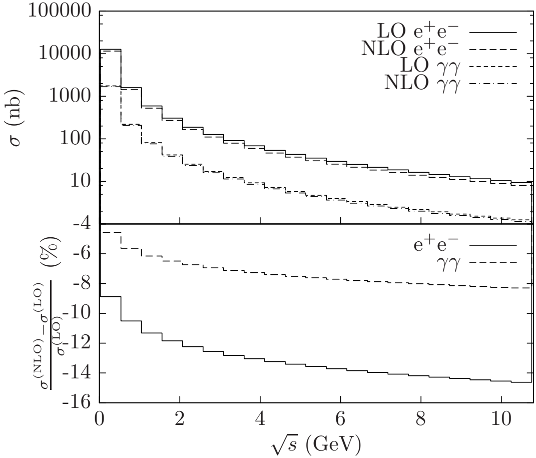
$$d\sigma_{\rm matched}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

in such a way that

- ⋆ $[\sigma_{\rm matched}^{\infty}]_{\mathcal{O}(\alpha)} = \sigma_{\rm exact}^{\alpha}$, avoiding double counting and preserving exponentiation of $\alpha^n L^n, n \geq 2$ leading logs
- ⋆ theoretical error shifted to $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections

NLO corrections to e^+e^- and two–photon production

Bhabha and $\gamma\gamma$ production cross section as a function of the c.m. energy



- ⋆ NLO corrections range from several per cent from $\Phi-$factories to about 10–15% at the $B-$factories
- The corrections to $\gamma\gamma$ production are about one half of those to Bhabha, for comparable cuts

Large–angle Bhabha: size of the radiative corrections

for bare (w/o photon recombination) $e^{\pm}$ final–states

Event selection criteria: for $\phi-$ and B–factories

- a $\sqrt{s} = 1.02\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\text{max}} = 10^\circ$
- b $\sqrt{s} = 1.02\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\text{max}} = 10^\circ$
- c $\sqrt{s} = 10\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 4\,\mathrm{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\text{max}} = 10^\circ$
- d $\sqrt{s} = 10\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 4\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\text{max}} = 10^\circ$

Relative corrections (in %)

setup	a.	b.	c.	d.
$\delta_\alpha^{\text{exact}}$	-10.00	-12.52	-12.00	-14.43
$\delta_\alpha^{\text{non-log}}$	-0.40	-0.65	-0.41	-0.70
δ_{HO}	0.39	0.93	0.80	1.64
$\delta_{\alpha^2 L}$	0.04	0.09	0.06	0.11
δ_{VP}	1.73	2.43	4.59	6.03

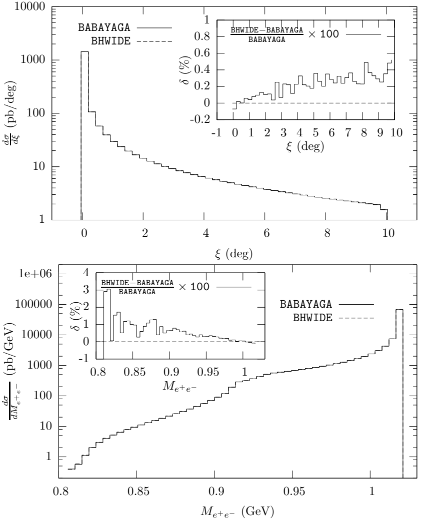
⋆ Both exact $\mathcal{O}(\alpha)$ and higher–order corrections (including vacuum polarization) necessary for 0.1% theoretical precision ⋆

- Vacuum polarization included in both lowest–order and NLO diagrams with $\Delta\alpha_{\text{had}}^{(5)}$ contribution through a parameterization routine (Jegerlehner, HMNT, ...), returning a data driven error estimate

F. Jegerlehner, Nucl. Phys. Proc. Suppl. **126/181-182** (2004/2008) 325/135
K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, PR **D69** (2004) 093003 and PL **B649** (2007) 173

BabaYaga@NLO vs BHWIDE at DAΦNE

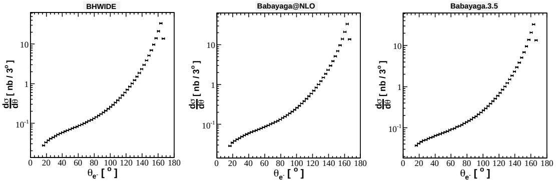
G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

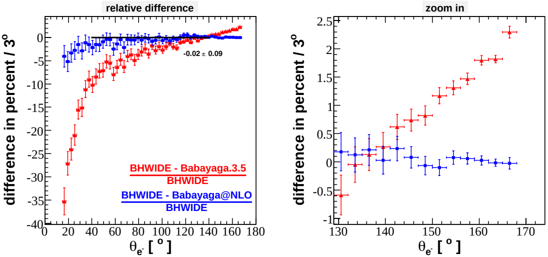


- Agreement for distributions within a few 0.1%, a few % only in the dynamically suppressed hard tails

BabaYaga@NLO vs BHWIDE at BaBar

By A. Hafner and A. Denig
with realistic selection cuts for luminosity at BaBar





- `BabaYaga@NLO` and `BHWIDE` well agree (at a few per mille level) also for distributions

BabaYaga@NLO vs BHWIDE at BaBar

By A. Hafner and A. Denig
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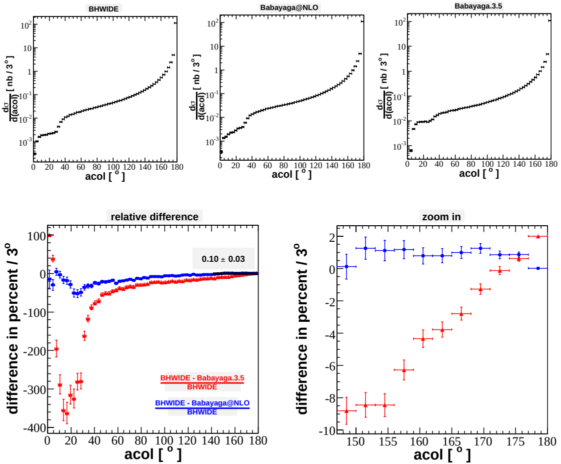


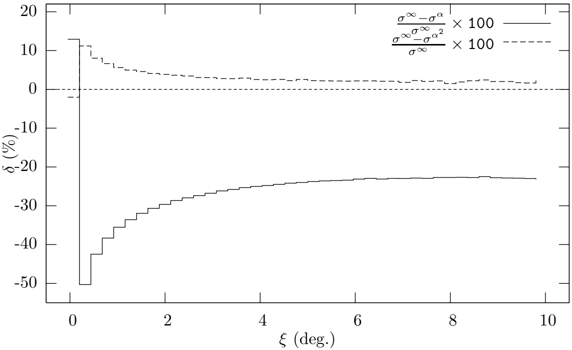
- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for distributions

Exponentiation beyond $\mathcal{O}(\alpha^2)$ in BabaYaga@NLO

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

Even with a complete two–loop generator at hand, resummation of leading logarithms beyond $\mathcal{O}(\alpha^2)$ could be neglected?

Bhabha cross section as a function of the acollinearity ξ @ DAΦNE



- Resummation beyond $\mathcal{O}(\alpha^2)$ important for precision predictions!

The $e^+e^- \to \gamma\gamma$ process: size of radiative corrections and accuracy

G. Balossini *et al.*, Phys. Lett. **B663** (2008) 209

Selection criteria – ϕ, $\tau-$charm and B factories

a) $\sqrt{s} = 1, 3, 10\,\mathrm{GeV}$, $E_{\mathrm{min}} = 0.3\sqrt{s}$, $\vartheta_\gamma^{\mathrm{min,max}} = 45° \div 135°$, $\xi_{\mathrm{max}} = 10°$

Cross sections (nb) & relative corrections (%)

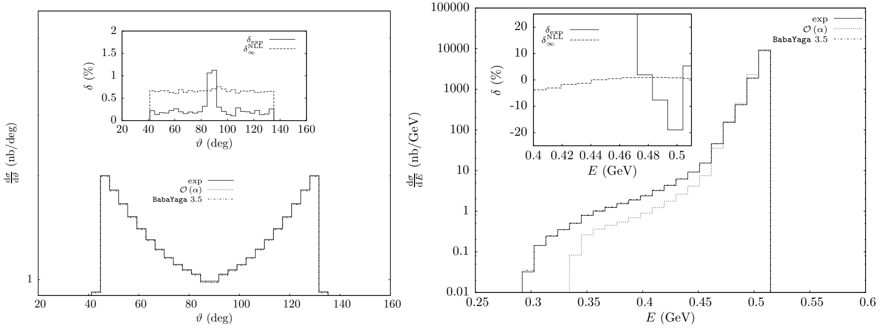
$\sqrt{s}$ (GeV)	1	3	10
σ_{Born}	137.53	15.281	1.3753
$\sigma_\alpha^{\mathrm{PS}}$	128.55	14.111	1.2529
σ_{NLO}	129.45	14.211	1.2620
$\sigma_{\mathrm{exp}}^{\mathrm{PS}}$	128.92	14.169	1.2597
$\sigma_{\mathrm{matched}}$	129.77	14.263	1.2685
δ_α	-5.87	-7.00	-8.24
δ_∞	-5.65	-6.66	-7.77
$\delta_\alpha^{\mathrm{non-log}}$	0.70	0.71	0.73
δ_{HO}	0.24	0.37	0.51

- Like for Bhabha, both exact $\mathcal{O}(\alpha)$ and higher–order corrections necessary for 0.1% theoretical precision in $\gamma\gamma$ production ★

★ Theoretical accuracy: $\sim 0.1\%$, also thanks to no contribution (and related $\Delta\alpha_{\mathrm{had}}^{(5)}$ uncertainty) due to vacuum polarization correction

$e^+e^- \to \gamma\gamma(n\gamma)$: distributions [for $\Phi-$factories]

Angular and energy distribution of the most energetic photon



⋆ Interplay of NLO and multiple photon corrections also necessary for precise simulations of $\gamma\gamma$ differential cross sections

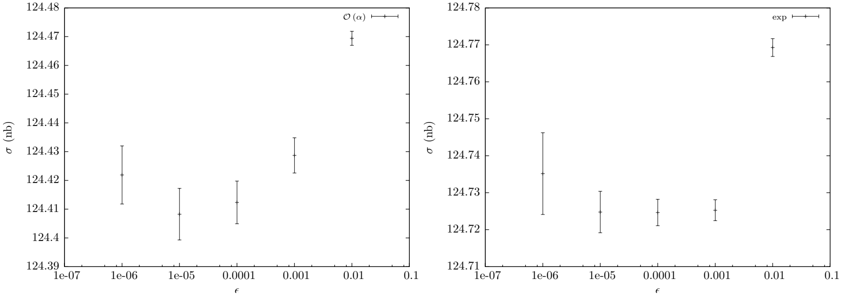
$e^+e^- \to \gamma\gamma(n\gamma)$ in BabaYaga@NLO: technical tests

- Perfect agreement with BKQED for the $\mathcal{O}(\alpha)$ [NLO] corrections to the inclusive $e^+e^- \to \gamma\gamma(\gamma)$ cross section

F.A. Berends and R. Kleiss, Nucl. Phys. **B186** (1981) 22

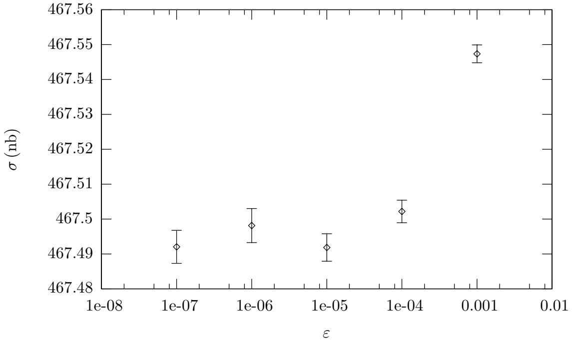
$\sqrt{s}$(GeV)	6	10	20
$\delta_T^{\mathrm{BKQED}}$(%)	13.8	15.3	17.4
$\delta_T^{\mathrm{BabaYaga@NLO}}$(%)	13.81(1)	15.30(1)	17.51(10)

- Successful independence from the soft–hard photon separator ϵ, in the numerical limit $\epsilon \to 0$



Technical test of BabaYaga@NLO: ϵ independence

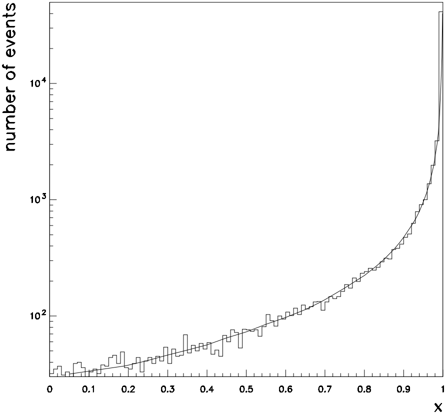
G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227



- Independence of the matched PS cross section from variations of the soft–hard separator ϵ successfully checked! [for large–angle Bhabha cross section @ DAΦNE]

Technical test of BabaYaga: $D(x, Q^2)$

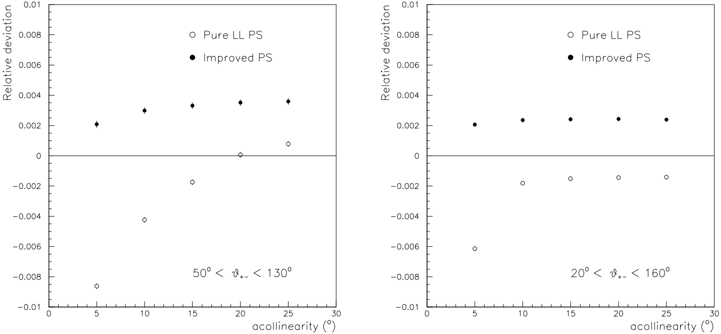
C.M. Carloni Calame *et al.*, Nucl. Phys. **B584** (2000) 459



- Parton Shower reconstruction (histogram) of the x distribution of the electron Structure Function $D(x, Q^2)$ (solid line)

Theoretical accuracy of BabaYaga v3.5

C.M. Carloni Calame, Phys. Lett. **B520** (2001) 16

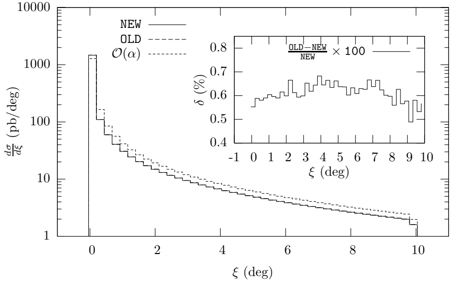


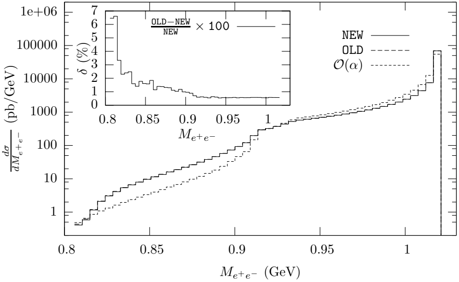
- Relative difference between the $\mathcal{O}(\alpha)$ BabaYaga predictions (original LL version and improved 3.5 version) and the exact $\mathcal{O}(\alpha)$ Bhabha cross section, as a function of the acollinearity cut, for two angular acceptances at $\sqrt{s} = 1$ GeV

BabaYaga@NLO vs BabaYaga v3.5 at DAΦNE

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

$\sqrt{s} = 1.02\,\mathrm{GeV},\ E^{\pm}_{\mathrm{min}} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\mathrm{max}} = 10^\circ$

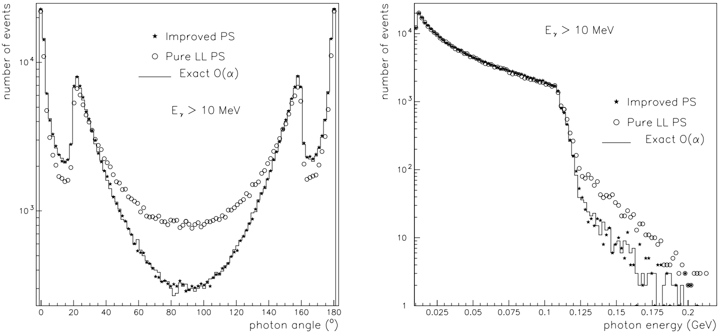




- BabaYaga@NLO differs from BabaYaga v3.5 at ~ 0.5 % level in the statistically dominant regions for luminosity monitoring at the Φ–factories, due to $\mathcal{O}(\alpha)$ non–log contributions
- Higher–order [beyond $\mathcal{O}(\alpha)$] leading log corrections amount to several per cent on distributions and are essential for precision luminosity studies

Improved PS algorithm in BabaYaga v3.5

C.M. Carloni Calame, Phys. Lett. **B520** (2001) 16

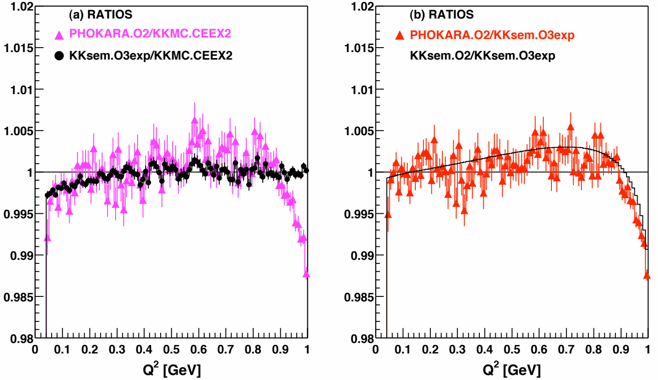


- Comparison between the $\mathcal{O}(\alpha)$ BabaYaga predictions (original LL version and improved 3.5 version) and the exact $\mathcal{O}(\alpha)$ matrix element for the angular and energy photon distributions

$e^+e^- \to \mu^+\mu^-\gamma(\gamma)$: KKMC vs PHOKHARA at Φ–factories

S. Jadach, Acta Phys. Pol. **B36** (2005) 2387

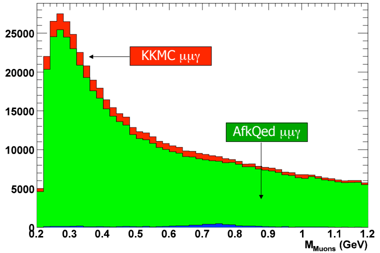
Including initial–state radiation only, both in the signal and radiative corrections



- Predictions of KKMC and PHOKHARA for the muon pair spectrum $d\sigma/dQ^2$ in $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$ at $\sqrt{s} =$ 1.02 GeV agree within 0.2% in the central region and differ at high Q^2 by $\sim 1\%$, probably because of lack of soft–photon exponentiation in PHOKHARA. Final–state radiation requires more tests.

$e^+e^- \to \mu^+\mu^-\gamma$: BabaYaga@NLO vs Dixon at B–factories

Some discrepancy at BaBar between KKMC and AfkQED for muons invariant mass [see talk by N. Berger @ EPS HEP 2007]



- Leading–order (w/o radiative corrections) predictions of `BabaYaga@NLO` and Dixon calculation, including both initial– and final–state radiation, at a B–factory $\sqrt{s} = 10.58\,\mathrm{GeV}$ with cuts: $M_{\mu\mu} \leq 2\,\mathrm{GeV}, \ |\cos\vartheta_\gamma| \leq 0.9$, no muon cuts

Thanks to Lance Dixon!

$M_{\mu\mu}$ (GeV)	σ_{LO} Dixon [pb]	σ_{LO} BabaYaga@NLO [pb]
$0.320 \div 0.480$	$2.88(1)$	$2.90(3)$
$0.480 \div 0.640$	$2.12(1)$	$2.11(1)$
$0.640 \div 0.800$	$1.66(1)$	$1.66(1)$
$0.800 \div 0.960$	$1.37(1)$	$1.37(1)$
$0.960 \div 1.120$	$1.17(1)$	$1.18(1)$

⋆ Excellent agreement! ⋆