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Final report on Weihai Summer School

During my three-year study as an undergraduate student, I focused mainly on the fundamental theories in physics, and had a limited understanding of what researchers' tasks are in high energy experiments. The lectures at Weihai inspired me, because I not only learnt about the physics new frontiers such as dark matter detection, but also gained an impression about accelerators. Here I would like to share some of my notes and comments on some of these topics.

Standard Model is by far the most precise physics theory. It contains two parts: non-Abelian Gauge Theory and Spontaneous Symmetry Breaking. Specifically, I want to go through Gauge theory, which refers to a field theory whose Lagrangian is invariant under a series of local group transformations. To understand what "invariant" means, we start with a global group transformation. First we consider the following expression:

$$\mathcal{L}_{global} = \partial_\mu \phi^\dagger \partial_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

Notice that $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ is a complex field. We can easily check that this field is invariant under a rotation transformation, either in a matrix form or if we can add a phase to ϕ:

Let $\phi' = e^{i\alpha}\,\phi$ where α is a constant, then $\phi'^\dagger = e^{-i\alpha}\,\phi^\dagger$; thus we have $\phi'^\dagger\phi'$=$\phi^\dagger\phi$ and $\partial_\mu \phi^{\dagger\prime} \partial_\mu \phi' = \partial_\mu \phi^\dagger \partial_\mu \phi$ since their phase terms cancel; in group theory this is a U(1) symmetry. U(n) stands for unitary groups in n dimensions. Equivalently, another way to look at this is that the Lagrangian (density) has an SO(2) (2-dimensional special orthogonal group) global symmetry.

Now consider locally, if the rotation depends on position, that is

$$\phi' = U \qquad U = e^{iY\theta(x)}$$

Then the derivative would obey chain rule, $\partial_\mu \phi' = U \partial_\mu \phi + \phi \partial_\mu U$. The failure of the derivative to commute with U introduces an additional term, which means this Lagrangian does not have a local symmetry. To modify, we introduce a new derivative operator D with the form:

$$D_\mu = \partial_\mu - igYA_\mu$$

Where g is called the coupling constant which reflects the strength of an interaction. It is also demanded by us that $A'_\mu = A_\mu + \frac{1}{g}\partial_\mu \theta$. This makes the transformation experienced by

$D_\mu\phi$ a phase transformation. Finally, we now have a locally gauge invariant Lagrangian:

$$\mathcal{L}_{local} = D_\mu\phi^\dagger D^\mu\phi - \frac{1}{4}F^{\mu\vartheta}F_{\mu\vartheta} - V$$

Notice the mass term $A^\mu A_\mu$ is not gauge invariant, thus its gauge particle is massless, giving a long range force. This classical gauge picture is almost complete, except the fact that this theorem needs modification if U in non-Abelian group. For instance, a rotation group in 3 dimensions is related to its order of transformation. Yang–Mills theory is a gauge theory based on the SU(N) group, and is essential to the unification of the weak forces as well as QCD.

In SU(2), specifically, U can be written as: $U = e^{i\theta^\alpha(x)t^\alpha}$, which is a general form for rotations. Starting with the invariant Lagrangian:

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x)$$

Again introduce gauge derivative:

$$D_\mu \equiv \partial_\mu - igA_{\mu\alpha}t^\alpha; F_{\mu\vartheta} = F^\alpha{}_{\mu\vartheta}t_\alpha$$

The new Lagrangian becomes invariant under gauge transformation:

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F_\alpha{}^{\mu\vartheta}F^\alpha{}_{\mu\vartheta}$$

Yang–Mills theory describes the behavior of elementary particles using these non-Abelian Lie groups. Thus it forms the basis of our understanding of the Standard Model of particle physics.

Except for a taste of the Standard Model, I learned what are the advantages and disadvantages of circular/linear colliders; I got to know what people are trying to look at beyond SM; I gained knowledge about data processing, and even wrote a program to build strategies in the classical card game “Black Jack”.

Being a leader of our group gave me the experience to organize teamwork and make plans. Most my teammates were shy and less talkative, but by the last day we already became good friends. Everybody was willing to contribute to our final presentation. Thus although most of us are first year and second years, we obtained the third place, which made me proud. So for me I benefitted from both in and outside the class, and I really appreciate those who provided this opportunity for us. Thank you!