Spontaneous Symmetry Breaking

----WHEP summer school summary

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Introduction

In WHEP summer school, I learned much knowledge of high energy physics. I attended many lectures of Professor Yang Jinmin about Quantum Field Theory. This makes me see the QFT in profound ways. So I spend some time on QFT and make a summary about the important section of QFT ---Spontaneous Symmetry Breaking.

Theory

Symmetry Breaking:

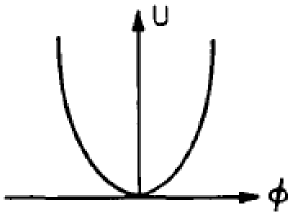
1. Explicit symmetry breaking:$L_{Sym} + L_{Breaking}$
2. spontaneous breaking:

   The action respects the symmetry, but its vacuum is not invariant under the symmetry.

   Thus, the interactions of the particles excited around this vacuum do not show the symmetry explicitly.

Example:

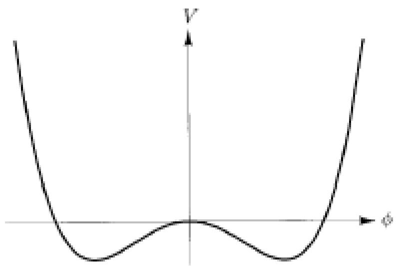
Consider classical ϕ^4 theory:



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Invariant under $\phi \to -\phi$.
Hamiltonian: $H = \int d^3x[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4]$.
$\varphi_0 = 0$ minimize the Hamiltonian.
Symmetry: $\phi \to -\phi$. $\varphi_c = \langle\phi\rangle = 0$ is invariant under $\phi \to -\phi$. This vacuum preserves the symmetry. The oscilations around this vacuum is discribed by the action with explicit symmetry.

If we change the sign before μ^2,



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Minimum energy configuration is uniform $\phi(x) = \phi_0$, minimizing the potential $V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$, $V'(\phi) = -m^2\phi + \frac{\lambda}{3!}\phi^3 = 0$ we find two minima

$$\phi_0 = \pm v = \pm\sqrt{\frac{6}{\lambda}}\mu \quad .$$

We can see that the symmetry of the Lagrangian $\phi \to -\phi$ interchanges the two minima. So Any one of the two vacuum is not invariant under the symmetry.

Define $\phi(x) = v + \sigma(x)$, so the Lagrangian can be expressed in the form:

$$
\begin{aligned}
L &= \frac{1}{2}(\partial_\mu(v+\sigma))^2 + \frac{1}{2}\mu^2(\mathrm{v}+\sigma)^2 - \frac{\lambda}{4!}(v+\sigma)^4 \\
&= \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\mu\sigma^3 - \frac{\lambda}{4!}\sigma^4
\end{aligned}
$$

This Lagrangian describes a simple scalar field of mass $\sqrt{2}\mu$, with σ^3 *and* σ^4 interactions. The $\phi \to -\phi$ symmetry is no longer apparent; it is hidden in the relations among the three coefficients---depending only on two parameters.

This is simplest example of spontaneous symmetry breaking. For other complex models, there will be many interesting phenomena.