Validation of hZZ Coupling Improvement from $\Sigma BR_i = 1$ Constraint

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Use formalism in Chapter 10, section 10.7.2

of "Probability and Statistics in Particle Physics" by A.G. Frodesen, O. Skjeggestad, H. Tofte

10.7.2 Linear LS model with linear constraints; Lagrangian multipliers

We will consider a general linear LS estimation problem with linear constraint equations, in the form

$$\left.\begin{aligned} \chi^2(\underline{\theta}) &= (\underline{y}-A\underline{\theta})^T V^{-1}(\underline{y}-A\underline{\theta}) = \text{minimum}, \\ B\underline{\theta} - \underline{b} &= \underline{0}. \end{aligned}\right\} \qquad (10.101)$$

$$C \equiv A^T V^{-1} A, \qquad V_B \equiv BC^{-1}B^T$$

$$V(\underline{\hat{\theta}}) = C^{-1} - (BC^{-1})^T V_B^{-1} (BC^{-1}).$$

$$\Theta_1 = \epsilon_Z \qquad \Theta_2 = \epsilon_b \qquad \Theta_3 = \epsilon_W \qquad \Theta_4 = \epsilon_\Pi$$

$$Y_1 = \sigma_{ZH} - g_{Z0}^2 F_0 \qquad Y_2 = \sigma_{ZH} \cdot B_b - \frac{g_{b0}^2 g_{Z0}^2}{\Gamma_0} G_b$$

$$Y_3 = \sigma_{ZH} \cdot B_W - \frac{g_{W0}^2 g_{Z0}^2}{\Gamma_0} G_W \qquad Y_4 = \sigma_{WH} \cdot B_b - \frac{g_{b0}^2 g_{W0}^2}{\Gamma_{T0}} F_1$$

$$\chi^2 = \frac{(Y_1 - 2\Theta_1)^2}{V_{11}} + \frac{(Y_2 - (2\Theta_1 + 2\Theta_2 - \Theta_4))^2}{V_{22}} + \frac{(Y_3 - (2\Theta_1 + 2\Theta_3 - \Theta_4))^2}{V_{33}}$$

$$+ \frac{(Y_4 - (2\Theta_2 + 2\Theta_3 - \Theta_4))^2}{V_{44}}$$

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & -1 \\ 2 & 0 & 2 & -1 \\ 0 & 2 & 2 & -1 \end{pmatrix}$$

$$\sigma_{zn}(1+2\epsilon_z) - \sigma_{zn}\cdot B_s(1+2\epsilon_b+2\epsilon_z-\epsilon_r) - \sigma_{zn}\cdot B_w(1+2\epsilon_w+2\epsilon_z-\epsilon_r) = 0$$

$$2(\sigma_{zn} - \sigma_{zn}\cdot B_s - \sigma_{zn}\cdot B_w)\theta_1 - 2\sigma_{zn}\cdot B_s\theta_2 - 2\sigma_{zn}\cdot B_w\theta_3$$

$$+ (\sigma_{zn}\cdot B_s + \sigma_{zn}\cdot B_w)\theta_4 - [\sigma_{zn}\cdot B_s + \sigma_{zn}\cdot B_w - \sigma_{zn}] = 0$$

using approx $\sigma_{zn} = \sigma_{zn}(B_s + B_w)$

$$B = (0, -2\sigma_{zn}B_s, -2\sigma_{zn}\cdot B_w, \sigma_{zn})$$

$$C = \begin{pmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{v_{11}} & & & \\ & \frac{1}{v_{22}} & & \\ & & \frac{1}{v_{33}} & \\ & & & \frac{1}{v_{44}} \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & -1 \\ 2 & 0 & 2 & -1 \\ 0 & 2 & 2 & -1 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{V_{11}} & & & \\ & \frac{1}{V_{22}} & & \\ & & \frac{1}{V_{33}} & \\ & & & \frac{1}{V_{44}} \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & -1 \\ 2 & 0 & 2 & -1 \\ 0 & 2 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{2}{V_{11}} & 0 & 0 & 0 \\ \frac{2}{V_{22}} & \frac{2}{V_{22}} & 0 & -\frac{1}{V_{22}} \\ \frac{2}{V_{33}} & 0 & \frac{2}{V_{33}} & -\frac{1}{V_{33}} \\ 0 & \frac{2}{V_{44}} & \frac{2}{V_{44}} & -\frac{1}{V_{44}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{4}{V_{11}} + \frac{4}{V_{22}} + \frac{4}{V_{33}} & \frac{4}{V_{22}} & \frac{4}{V_{33}} & -2\left(\frac{1}{V_{22}} + \frac{1}{V_{33}}\right) \\ \frac{4}{V_{22}} & 4\left(\frac{1}{V_{22}} + \frac{1}{V_{44}}\right) & \frac{4}{V_{44}} & -2\left(\frac{1}{V_{22}} + \frac{1}{V_{44}}\right) \\ \frac{4}{V_{33}} & \frac{4}{V_{44}} & 4\left(\frac{1}{V_{33}} + \frac{1}{V_{44}}\right) & -2\left(1/V_{33} + 1/V_{44}\right) \\ -2\left(\frac{1}{V_{22}} + \frac{1}{V_{33}}\right) & -2\left(\frac{1}{V_{22}} + \frac{1}{V_{44}}\right) & -2\left(\frac{1}{V_{33}} + \frac{1}{V_{44}}\right) & \frac{1}{V_{22}} + \frac{1}{V_{33}} + \frac{1}{V_{44}} \end{pmatrix}$$

$$\frac{1}{V_{11}}, \frac{1}{V_{22}} >> \frac{1}{V_{33}}, \frac{1}{V_{44}}$$

$$C = \begin{pmatrix} 4\left(\frac{1}{V_{11}} + \frac{1}{V_{22}}\right) & \frac{4}{V_{22}} & \varnothing & -\frac{2}{V_{22}} \\ \frac{4}{V_{22}} & \frac{4}{V_{22}} & \varnothing & -\frac{2}{V_{22}} \\ \varnothing & \varnothing & 4\left(\frac{1}{V_{33}} + \frac{1}{V_{44}}\right) & \varnothing \\ -\frac{2}{V_{22}} & -\frac{2}{V_{22}} & \varnothing & \frac{1}{V_{22}} \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{V_{11}} & & & \\ & \frac{1}{V_{22}} & & \\ & & \frac{1}{V_{33}} & \\ & & & \frac{1}{V_{44}} \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & -1 \\ 2 & 0 & 2 & -1 \\ 0 & 2 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{2}{V_{11}} & 0 & 0 & 0 \\ \frac{2}{V_{22}} & \frac{2}{V_{22}} & 0 & -\frac{1}{V_{22}} \\ \frac{2}{V_{33}} & 0 & \frac{2}{V_{33}} & -\frac{1}{V_{33}} \\ 0 & \frac{2}{V_{44}} & \frac{2}{V_{44}} & -\frac{1}{V_{44}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{4}{V_{11}} + \frac{4}{V_{22}} + \frac{4}{V_{33}} & \frac{4}{V_{22}} & \frac{4}{V_{33}} & -2\left(\frac{1}{V_{22}} + \frac{1}{V_{33}}\right) \\ \frac{4}{V_{22}} & 4\left(\frac{1}{V_{22}} + \frac{1}{V_{44}}\right) & \frac{4}{V_{44}} & -2\left(\frac{1}{V_{22}} + \frac{1}{V_{44}}\right) \\ \frac{4}{V_{33}} & \frac{4}{V_{44}} & 4\left(\frac{1}{V_{33}} + \frac{1}{V_{44}}\right) & -2\left(1/V_{33} + 1/V_{44}\right) \\ -2\left(\frac{1}{V_{22}} + \frac{1}{V_{33}}\right) & -2\left(\frac{1}{V_{22}} + \frac{1}{V_{44}}\right) & -2\left(\frac{1}{V_{33}} + \frac{1}{V_{44}}\right) & \frac{1}{V_{22}} + \frac{1}{V_{33}} + \frac{1}{V_{44}} \end{pmatrix}$$

$$\frac{1}{V_{11}}, \frac{1}{V_{22}} >> \frac{1}{V_{33}}, \frac{1}{V_{44}}$$

$$C = \begin{pmatrix} 4\left(\frac{1}{V_{11}} + \frac{1}{V_{22}}\right) & \frac{4}{V_{22}} & \emptyset & -\frac{2}{V_{22}} \\ \frac{4}{V_{22}} & \frac{4}{V_{22}} & \emptyset & -\frac{2}{V_{22}} \\ \emptyset & \emptyset & 4\left(\frac{1}{V_{33}} + \frac{1}{V_{44}}\right) & \emptyset \\ -\frac{2}{V_{22}} & -\frac{2}{V_{22}} & \emptyset & \frac{1}{V_{22}} \end{pmatrix}$$

$$C \equiv A^T V^{-1} A, \qquad V_B \equiv B C^{-1} B^T$$

$$V(\underline{\hat{\theta}}) = C^{-1} - (BC^{-1})^T V_B^{-1} (BC^{-1}).$$

Too messy to do these matrix inversions analytically.
Use Root linear algebra classes instead. We start with:

$V_{11} = (.0051)^2 \quad V_{22} = (.0028)^2 \quad V_{33} = (.015)^2$

$V_{44} = (.028)^2$

$\theta_1 = \epsilon_z \quad \theta_2 = \epsilon_b \quad \theta_3 = \epsilon_w \quad \theta_4 = \epsilon_n$

Perform coupling fit with $\sum_i BR_i = 1$ including $\Delta BR(H \to BSM)$

(the constraint $\sum_i BR_i = 1$ is model independent if $\Delta BR(H \to BSM)$ is included in the fit)

CEPC Higgs Coupling Precision assuming 5 ab^{-1}

$\frac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0}$	∞	8	4	2	1	0.1
ZZ	0.26%	0.24%	0.22%	0.19%	0.18%	0.17%
WW	1.2%	1.2%	1.2%	1.2%	1.2%	1.2%
bb	1.3%	1.3%	1.2%	1.2%	1.2%	1.2%
$\tau^+\tau^-$	1.4%	1.4%	1.4%	1.4%	1.3%	1.3%
gg	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
cc	1.7%	1.7%	1.7%	1.6%	1.6%	1.6%
$\gamma\gamma$	4.7%	4.7%	4.7%	4.7%	4.7%	4.7%
Γ_{tot}	2.8%	2.7%	2.5%	2.4%	2.3%	2.3%

sqrt(abs(CC_inv(0,*)))= 0.00255 0.00255 0.00255 0.0051
sqrt(abs(CC_inv(1,*)))= 0.00255 0.0160858 0.0142303 0.0230328
sqrt(abs(CC_inv(2,*)))= 0.00255 0.0142303 0.014299 0.0205409
sqrt(abs(CC_inv(3,*)))= 0.0051 0.0230328 0.0205409 0.0334795

sqrt(abs(VVth(0,*)))= 0.00176826 -0.00237357 0.00155386 -0.00253745
sqrt(abs(VVth(1,*)))= -0.00237357 0.014667 0.0137042 0.0203434
sqrt(abs(VVth(2,*)))= 0.00155386 0.0137042 0.0141249 0.0195611
sqrt(abs(VVth(3,*)))= -0.00253745 0.0203434 0.0195611 0.0284425