Modified Stochastic Branching Model for Supersymmetric Particle Branching process

Supervisor: Prof. Chan Aik Hui, Prof. Oh Choo Hiap
Master (Research) Zhang Yuanyuan

National University of Singapore

April 18, 2016

Overview

Introduction
- Particle Physics
- Multiplicity Distribution
- Hadronic Collisions

QCD jets as Branching Process
- Jet calculus to Branching Properties
- Quark and Gluon Initiated Jets

Supersymmetric Particle Branching
- Supersymmetry
- Pure SUSY Initiated Jets
- SUSY plus Ordinary Jets

Summary

Introduction

Particle Physics

- Theory : Standard Model
- Experiments : High energy particle collisions, Ultra high energy cosmic ray detections
- Comparison of observables from experiments and theory $\rightarrow$ validation or beyond Standard Model

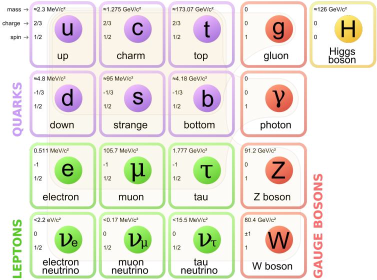

Figure: Elementary Particles

Multiplicity Distribution

- straightforward observable, the probability p_n for producing n particle in a collision process. Probability Distribution.
- contains information of dynamical mechanism and statistics of multi-particle production

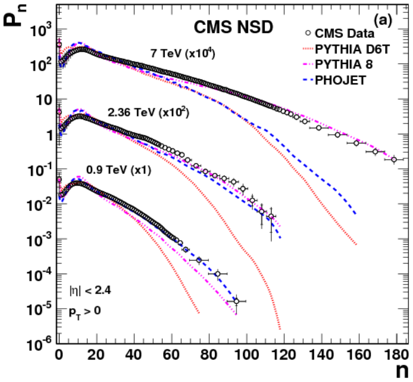

Figure: pp collision charged particle distributions at $\sqrt{s} = 0.9, 2.36, 7$TeV with CMS detector

Hadronic Collisions

Hadrons are made up of partons (quarks and gluons).

Collision Process

- Before Scattering: Initial state parton shower.
- Scattering: hard scattering + semi-hard, soft scattering (underlying events)
- After Scattering: final state parton shower and hadronization.

Approximation

- In high energy collision, ignore the initial state radiation
- The hadronization : Local Parton-Hadron Duality (LPHD) Specta of Partons before hadronization $\propto$ Spectra of Hadrons

QCD jets as Branching Process

Jet calculus to Branching Properties

Jets

High energy quark or gluon (carry color charge), create particles around them, form a spray of collimated(parallel) particles.

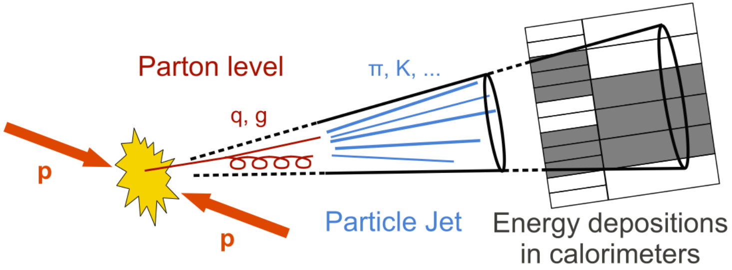

Figure: Jets

Jet calculus to Branching Properties

- [Konishi,1979] establish a Jet Calculus algorithm based on tree diagram understanding of partons evolution equation.
- Tree diagram, in the Leading Logarithm approximation (LLA), as branching process. Virtuality ($Q^2 = p^\mu p_\mu - m_0^2$) strongly ordered.

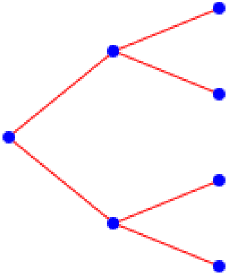

Figure: Tree Diagram

Branching Equation

[Konishi,1979] get two differential equation for single Quark- and Gluon- jets multiplicity distribution generating function G and Q.

$$\begin{cases} \dfrac{\partial G}{\partial Y} & = A(G^2 - G) + B(Q^2 - G) \\ \dfrac{\partial Q}{\partial Y} & = \tilde{A}(QG - Q) \end{cases}$$

- $G = \sum_{n,m=0}^{\infty} x_g^n x_q^m p_{g \to nm}$ the $p_{g \to nm}$ is the probability for one gluon branch into n gluons and m quarks, same for Q
- Three basic branching process $g \to g + g$, $q \to q + g$ and $g \to q + \bar{q}$, with branching probability $A, \tilde{A}$ and B respectively
- Evolution parameter Y act as "time parameter"

Generating function

The multiplicity distribution $p_n(t)$ can be collected into one expression, the generating function:

$$P(x,t) = p_0 + p_1 x + p_2 x^2 + \cdots + p_n x^n + \cdots = \sum_{n=0}^{\infty} p_n(t) x^n$$

One can get multiplicity distribution from GF by Taylor expansion.

Approximation for Branching Equation

- Following [Giovannini,2008] way to calculate branching probability $A = \dfrac{C_A}{\epsilon}$, $\tilde{A} = \dfrac{C_F}{\epsilon}$ and $B = \dfrac{N_f}{3}$.The ϵ small, $A = \dfrac{C_A}{\epsilon}, \tilde{A} = \dfrac{C_F}{\epsilon} > B = \dfrac{N_f}{3}$, omit process $B : g \to q + \bar{q}$. Only consider process $A : g \to g + g$ and $\tilde{A} : q \to q + g$.
- In $A : g \to g + g$ and $\tilde{A} : q \to q + g$, quark number do not change. Only consider gluon number n for multiplicity contribution.

Stochastic Branching Process

Here we introduce two branching process.

- Simple Birth process: For whole system, the probability of producing a new individual is $\lambda n \Delta t$, with current population n.
  Example $g \to g + g$ with $\lambda = A$.
- Poisson process: For whole system, the probability of producing a new individual is $\nu \Delta t$, this probability do not depend on current population n.
  Example $q \to q + g$ with $\nu = m\tilde{A}$ (m is the quark number), when only counting the gluon number, the quark can be seen as immigrants for gluon system.

Quark initiated and Gluon initiated Jets

- Quark initiated (initial number of quarks m, gluon 0) : Poisson process $\tilde{A} : q \to q + g$ plus simple birth process $A : g \to g + g$. Initial condition $P_Q(x, Y = 0) = p_0 = 1$

$$\frac{\partial P_Q(x,Y)}{\partial Y} = A(x^2 - x)\frac{\partial P_Q(x,Y)}{\partial x} + m\tilde{A}(x-1)P_Q(x,Y)$$

$$P_Q(x,Y) = \left[\frac{1}{(1-e^{AY})x + e^{AY}}\right]^k \quad ; \quad k = \frac{m\tilde{A}}{A}$$

- Gluon initiated (initial number of gluon a, quark 0): Simple birth process $A : g \to g + g$. Initial condition $P_G(x, Y = 0) = p_a x^a = x^a$

$$\frac{\partial P_G(x,Y)}{\partial Y} = A(x^2 - x)\frac{\partial P_G(x,Y)}{\partial x}$$

$$P_G(x,Y) = \left[\frac{x}{(1-e^{AY})x + e^{AY}}\right]^a$$

Generalized Multiplicity Distribution

[Chew, 1987] propose a general distribution for initial quark number m and gluon number a, initial condition $P_{\mathsf{GMD}}(x, Y=0) = p_a x^a = x^a$

$$\frac{\partial P_{\mathsf{GMD}}(x,Y)}{\partial Y} = A(x^2 - x)\frac{\partial P_{\mathsf{GMD}}(x,Y)}{\partial x} + m\tilde{A}(x-1)P_{\mathsf{GMD}}(x,Y)$$

The generating function is as follow

$$\begin{aligned} P_{\mathsf{GMD}}(x,Y) =& \frac{x^a}{[(1-e^{AY})x + e^{AY}]^{k+a}} \\ =& [\frac{1}{(1-e^{AY})x + e^{AY}}]^k \times \left[\frac{x}{(1-e^{AY})x + e^{AY}}\right]^a \\ =& P_Q(x,Y) \times P_G(x,Y) \end{aligned}$$

The generalized multiplicity distribution(GMD) is the convolution of gluon initiated jets MD and quark initiated jets MD.

Evolution Parameter Y

The evolution parameter act as "time parameter"

$$Y = \frac{1}{2\pi b} \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}$$

- The primary parton with large virtuality Q^2_{max} ($Y = Y_{\text{max}}$), branch out particles. The virtuality of particles decrease until reach hadronization threshold Q_0^2 ($Y = 0$). Notice $Y_{\text{max}} \to Y = 0$ with initial m quarks and a gluons.
- Prove system with initial m quarks and a gluons evolve from $Y_{\text{max}} \to Y = 0$, have same MD as system with same initial condition from $Y = 0 \to Y_{\text{max}}$
- Notice all initial quarks and gluons have same virtuality t_{max} or Y_{max} (total evolution time)

Supersymmetric Particle Branching

Supersymmetry

- New symmetry between fermions and bosons, all ordinary parton has its super-parterners, differ by half spin
- Spontaneous broken symmetry, mass difference between normal particle and SUSY
- R parity $R_P = (-1)^{3(B-L)+2s}$ conserve, process involve even number of SUSY particles $\rightarrow$ Lightest Supersymmetric Particle(LSP), can not decay

SUSY Branching Process Probability

- Following the same way [Giovannini,2008] of ordinary parton to calculate SUSY branching probability, involving squark $\tilde{q}$ and gluino $\tilde{g}$
- Approximation, only choose terms with $\frac{1}{\epsilon}$:

$$I = \frac{C_F}{\epsilon} : \tilde{q} \to \tilde{q}g \qquad J = \frac{C_A}{\epsilon} : \tilde{g} \to \tilde{g}g$$

SUSY branching	A_0^{ba}
$q \to \tilde{q} + \tilde{g}$	$C_F/2$
$\tilde{q} \to \tilde{q} + g$	C_F/ϵ
$\tilde{q} \to \tilde{q} + g$	C_F
$g \to \tilde{q} + \tilde{q}$	$N_f/3$
$g \to \tilde{g} + \tilde{g}$	$2C_A/3$
$\tilde{g} \to \tilde{g} + g$	C_A/ϵ
$\tilde{g} \to q + \tilde{q}$	$N_f/2$

Table: SUSY Branching Probability

Pure SUSY Initiated Jets Branching

- Three basic processes $I : \tilde{q} \to \tilde{q}g$, $J : \tilde{g} \to \tilde{g}g$ and $A : g \to gg$.
- The SUSY particle branching consist of two phases [Berezinsky,2001]:
  Phase One: SUSY particle start branching with high virtuality Q^2_{max} (or Y_{max}), until their virtuality reach the mass scale of SUSY particle $M^2_{\text{SUSY}}(Y = Y_{\text{SUSY}})$. The SUSY particles on-shell decay into ordinary parton and LSP $\tilde{\chi}^0_1$.

  $$\tilde{q} \to q + \tilde{\chi}^0_1 \quad \tilde{g} \to q + \bar{q} + \tilde{\chi}^0_1$$

  Phase Two: The ordinary partons decayed from SUSY particles continue branching until hadronization threshold (Q^2_0 and $Y = 0$).

Modified Stochastic Model for SUSY Branching

Two phase simple birth and Poisson process can describe SUSY parton branching, Poisson process parameter ν change.

- Phase One $[Y = 0, Y_1 = Y_{\text{max}} - Y_{\text{SUSY}}]$
  Initial squark $\tilde{q}$ number l, gluino $\tilde{g}$ number p (of the same Y_{max}), gluon number 0, Initial condition $P_{\text{SUSY}}(x, Y) = p_0 = 1$

$$\frac{\partial P_{\text{SUSY}}}{\partial Y} = A(x^2 - x)\frac{\partial P_{\text{SUSY}}}{\partial x} + \underline{(lI + pJ)}(x-1)P_{\text{SUSY}}$$

- Phase Two $[Y_1, Y_{\text{max}}]$
  The phase one equation gives generating function $P_{\text{SUSY}}(x, Y_1)$ at $Y = Y_1$, it is the initial condition for phase two. The l squarks and p gluinos change to $l + 2p$ quarks.

$$\frac{\partial P_{\text{SUSY}}}{\partial Y} = A(x^2 - x)\frac{\partial P_{\text{SUSY}}}{\partial x} + \underline{(l + 2p)\tilde{A}}(x-1)P_{\text{SUSY}}$$

Generating Function and Multiplicity Distribution for SUSY Jets

Solve the two phase equation, we get generating function

$$P_{\mathsf{SUSY}}(x,Y) = \frac{[(1-e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1-e^{\lambda Y})x + e^{\lambda Y}]^{k_1}}$$

with $\lambda = A$, $k_1 = \dfrac{lI + pJ}{A}$ and $k_2 = \dfrac{(l+2p)\tilde{A}}{A}$, and multiplicity distribution

$$p_{\mathsf{SUSY}}(n,Y) = \quad e^{\lambda(Y-Y_1)(k_1-k_2)}(1-e^{-\lambda Y})^n (e^{-\lambda Y})^{k_1} \frac{(n+k_1-1)!}{(k_1-1)!n!}$$
$$\times\, {}_2F_1(k_2-k_1, -n, 1-k_1-n, \frac{e^{\lambda Y} - e^{\lambda Y_1}}{e^{\lambda Y} - 1})$$

${}_2F_1(a,b,c,d)$ is the hypergeometric function.

SUSY plus Ordinary Jets

More general case, intial partons :SUSY and ordinary partons. The $I:\tilde{q}\to\tilde{q}g$, $J:\tilde{g}\to\tilde{g}g$, $\tilde{A}:q\to qg$ and $A:g\to gg$ four process happen.

- The $\tilde{q}$, $\tilde{g}$ and q(initial number l, p and m), undergoes a two phase Poisson plus simple birth process, its generating function

$$P_{\widetilde{Q}\widetilde{G}Q}(x,Y)=\frac{[(1-e^{\lambda(Y-Y_1)})x+e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1-e^{\lambda Y})x+e^{\lambda Y}]^{k_1}}$$

  with different $k_1=\dfrac{lI+pJ+m\tilde{A}}{A}$ and $k_2=\dfrac{(l+2p+m)\tilde{A}}{A}$.
- The gluon (initial number a) branching described by

$$P_G(x,Y)=\left[\frac{x}{(1-e^{AY})x+e^{AY}}\right]^a$$

Generating Function for SUSY plus Ordinary Jets

Following the GMD generating function composition:

$$P_{\mathsf{GMD}}(x, Y) = P_G(x, Y) \times P_Q(x, Y)$$

We know the generating function for mix initial partons (SUSY and ordinary partons)

$$P_{\mathsf{mix}}(x) = P_{\tilde{Q}\tilde{G}Q}(x, Y) \times P_G(x, Y)$$
$$= \frac{[(1 - e^{\lambda(Y - Y_1)})x + e^{\lambda(Y - Y_1)}]^{k_1 - k_2}}{[(1 - e^{\lambda Y})x + e^{\lambda Y}]^{k_1}} \times \left[\frac{x}{(1 - e^{AY})x + e^{AY}}\right]^a$$

due to the independence of all individual jets.

Multiplicity Distribution for SUSY plus Ordinary Jets

We can get the SUSY included multiplicity distribution (SIMD) by Taylor expansion of generating function

$$p_{\mathsf{SIMD}}(n,Y) = e^{\lambda(Y-Y_1)(k_1-k_2)}(e^{-\lambda Y})^{k_1+a}(1-e^{-\lambda Y})^{n-a}$$
$$\times \frac{(n+k_1-1)!}{(k_1+a-1)!(n-a)!}{}_2F_1[k_2-k_1,-n+a,1-k_1-n,\frac{e^{\lambda Y}-e^{\lambda Y_1}}{e^{\lambda Y}-1}]$$

Consistency check:

1. When initial quark number $m=0$ gluon number $a=0$, this distribution reduced to pure SUSY MD
2. When $Y=Y_1$ or $k_1=k_2=k$, which means no SUSY particle involved, this distribution reduced to GMD.

Data Fitting

Simply the expression for SIMD, with $m = e^{\lambda Y_1}$ $l = e^{\lambda Y}$

$$p_{\mathsf{SIMD}}(n,Y) = (\frac{l}{m})^{k_1-k_2}(1-\frac{1}{l})^{n-a}(\frac{1}{l})^{k_1}\frac{(n+k_1-1)!}{(k_1+a-1)!(n-a)!} \times {}_2F_1(k_2-k_1,-n+a,1-k_1-n,\frac{l-m}{l-1})$$

Constraints:

1. Evolution parameter ordering $0 < Y_1 < Y$
2. In SUSY-QCD, $C_F = C_A = N_c$ (N_c number of colors),

$I = J = A = \tilde{A}$, $k_1 = \dfrac{lI + pJ + m\tilde{A}}{A} = l + p + m$,
$k_2 = l + 2p + m$, we know $k_1 < k_2$.

Fitting Parameter Table

Table: SUSY included Multiplicity Distribution best fit parameters and χ^2/dof for CMS $\sqrt{s} = 0.9, 2.36, 7$TeV data at different pseudorapidity intervals $|\eta| < 0.5, 1.0, 1.5, 2.0, 2.4$.

$\sqrt{s}$(TeV)	η_c	m	l	k_1	k_2	a	χ^2/dof(dof)
0.9	0.5	3.25	3.25	1.61	1.61	0.102	0.106(17)
	1.0	4.84	4.84	2.07	2.07	8.88×10^{-3}	1.04(34)
	1.5	6.77	6.77	2.05	2.05	1.74×10^{-8}	0.768(51)
	2.0	8.65	8.68	2.04	2.04	2.73×10^{-3}	0.526(54)
	2.4	9.65	9.83	2.13	2.13	3.59×10^{-4}	0.700(62)
2.36	0.5	1.24	4.26	1.42	1.42	0.0746	0.338(17)
	1.0	1.10	6.29	1.90	1.90	5.47×10^{-12}	1.58(34)
	1.5	1.57	9.75	1.70	1.70	4.72×10^{-3}	0.555(44)
	2.0	2.51	12.7	1.69	1.69	5.86×10^{-3}	0.603(54)
	2.4	3.21	14.4	1.77	1.77	3.98×10^{-3}	0.624(64)
7	0.5	5.37	5.37	1.57	1.57	6.52×10^{-9}	2.05(35)
	1.0	9.58	9.58	1.54	1.72	3.44×10^{-11}	2.16(64)
	1.5	13.48	13.48	1.58	1.89	6.60×10^{-10}	2.53(89)
	2.0	17.69	18.1	1.55	1.55	3.61×10^{-13}	1.97(109)
	2.4	22.5	22.5	1.48	1.60	0	1.08(121)

Fitting Plots $\sqrt{s} = 0.9\text{TeV}$

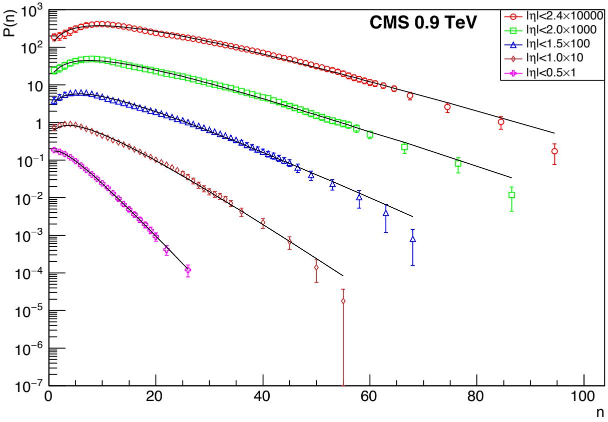

Figure: Charged particle multiplicity distribution measures by CMS at $\sqrt{s} = 0.9\text{TeV}$, fitted with SUSY included model.

Fitting Plots $\sqrt{s} = 2.36\text{TeV}$

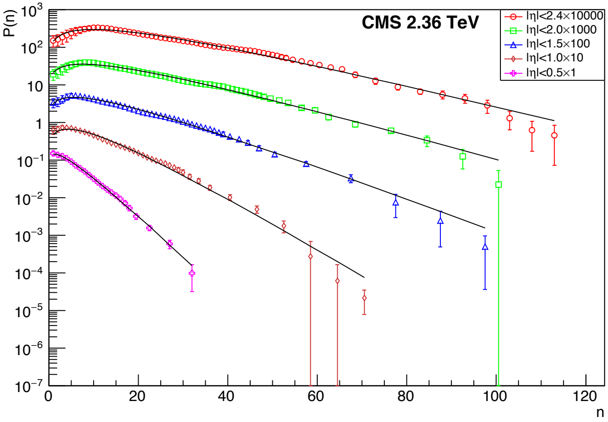

Figure: Charged particle multiplicity distribution measures by CMS at $\sqrt{s} = 2.36\text{TeV}$, fitted with SUSY included model.

Fitting Plots $\sqrt{s} = 7\text{TeV}$

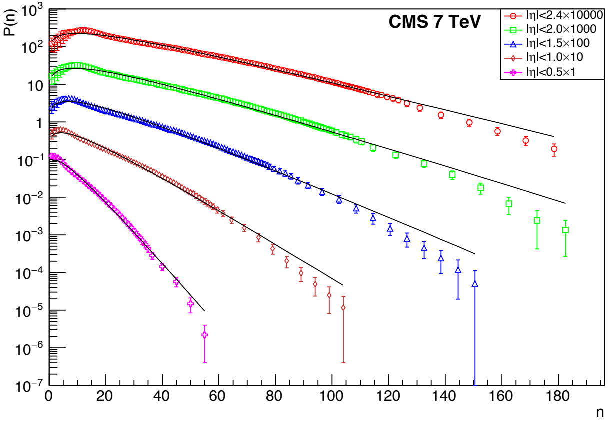

Figure: Charged particle multiplicity distribution measures by CMS at $\sqrt{s} = 7\text{TeV}$, fitted with SUSY included model.

Summary

Summary

- Supersymmetric particle branching model established, generating function and multiplicity distribution derived.
- General case of initial partons contain both Supersymmetric and ordinary partons are investigated. The SUSY included multiplicity distribution(SIMD) is derived and fitted with current data.
- Future Work: This SIMD can be used to fit with
  - Higher energy data.
  - Ultra-high energy cosmic ray data.
  - Monte Carlo Calculation data[Berezinsky,2001].

Thanks for Listening !

References

A. Giovannini (1979)
QCD JETS AS MARKOV BRANCHING PROCESSES
Nuclear Physics B 2-3, 429–448.

K. Konishi, A. Ukawa, G, Veneziano (1979)
Jet calculus: a simple algorithm for resolving QCD jets
Nuclear Physics B 1, 45–107

V. Berezinsky, M. Kachelriess (2001)
Monte Carlo simulation for jet fragmentation in SUSY QCD
Physical Review D 3, 034007

A. Giovannini, R. Ugoccioni (2008)
Monte Coherence and Incoherence in QCD Jets Dynamics (QCD Jets and Branching Processes)
String Theory and Fundamental Interactions 223–234

C.K. Chew, D. Kiang, H. Zhou (1987)
A generalized non-scaling multiplicity distribution
Physics Letters B 3-4, 411–415

Backups– Concept

Pseudorapidity η – a measure of the angle of particle relative to the beam axis.

$$\eta = -\ln[\tan(\frac{\theta}{2})]$$

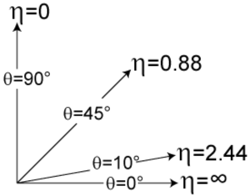

Figure: Pseudorapidity and angle relation.

Backups – Leading Logarithm Approximation

In Leading Logarithm Approximation, in the parton density function $G_h^a(x, Q^2)$

$$G_h^a(x, \ln Q^2) \approx \sum_{n=0}^{\infty} f_n(x)(\frac{\alpha_s}{\pi}\ln Q^2)^n$$

Only consider the terms $(\frac{\alpha_s}{\pi}\ln Q^2)^n$ is the LLA. Energy conservation and momentum conservation will be considered in Next to Leading Logarithm Approximation and higher order approximation.

Backups– Branching Probability

The branching probability $A = A_0^{gg}$, $\tilde{A} = A_0^{gq}$ and $B = A_0^{qg}$ is the zeroth moment of parton decay probability function $P_{ba}(x)$.

$$A_n^{ba} = \int_0^1 dx x^n P_{ba}(x); P_{ba}(x) = P(a \to b(x) + c(1-x))$$

[Giovannini,2008] propose a way to calculate branching probability, with a fixed cutoff for x, $x_{\text{min}} = \epsilon'$ and $x_{\text{max}} = 1 - \epsilon'$ (fix infrared divergence problem). For $P_{gq}(x) = C_F \frac{1+(1-x)^2}{x}$

$$\tilde{A} = A_0^{gq} = \int_{\epsilon'}^{1-\epsilon'} P_{gq}(x) dx$$

with substitution $\epsilon = (-2 \ln \epsilon')^{-1}$, we have $\tilde{A} = \frac{C_F}{\epsilon}$. Similarly, $A = \frac{C_A}{\epsilon}$, $B = \frac{N_f}{3}$. (C_A C_F color factors, N_f flavor number)

Backups – Convolution

Two non-negative random variable X and Y, independent with each other. If we define $S = X + Y$. S is the convolution of X and Y:

- For $S = n$, it comprises $(X = 0, Y = n)$, $(X = 1, Y = n - 1)$,...,$(X = n, Y = 0)$.
- The generating function of S is the multiplying of GF of X and Y.

The independence of quark and gluon jets ensure that

$$P_{\mathsf{GMD}}(x, Y) = P_Q(x, Y) \times P_G(x, Y)$$

Backups – Prove the system from $Y = Y_{\text{max}}$ to $Y = 0$

Start from scratch, the probability of having n gluons at $Y - \Delta Y$

$$p_n(Y - \Delta Y) = p_{n-1}(Y)A(n-1)\Delta Y + p_n(t)(1 - An\Delta Y)$$

The differential equation:

$$-\frac{dp_n(Y)}{dY} = -\lim_{\Delta Y \to 0} \frac{p_n(Y) - p_n(Y - \Delta Y)}{\Delta Y} = A(n-1)p_{n-1}(Y) - Anp_n$$

Then solve it, we get the generating function at $Y = 0$

$$P(x, 0) = \left[\frac{1}{1 - (1 - x^{-1})e^{AY_{\text{max}}}}\right]^a = \left[\frac{x}{(1 - e^{AY_{\text{max}}})x + e^{AY_{\text{max}}}}\right]^a$$

$P(x, 0)$ is the same as $P(x, Y_{\text{max}})$ for system from $Y = 0$ to $Y = Y_{\text{max}}$.

Backups– Hypergeometric Function Evaluation

The biggest difficulty with our fitting is the Hypergeometric Function ${}_2F_1(a, b, c, d)$ evaluation.

- We use CERN-ROOT-MINUIT to do the fitting, the math library in ROOT is GSL library (open source), gives error message when d goes near 1 (its singularity).
- In mathematica, the hypergeometric function can be calculated fast and no error message near singularity.

We learn how to call mathematica from C code under ROOT environment, thanks to CERN-ROOT discussion forum!