Electroweak Phase Transition Models and Higgs Couplings at the CEPC, From A Cosmologist's Perspective

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WORKSHOP ON CEPC PHYSICS
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Based on

… 1608.06619 (PRD) with Peisi Huang & Lian-Tao Wang.

… see also Barger, Chung, AL, Wang [1112.5460]
and Chung, AL, Wang [1209.1819].

Why are cosmologists interested in future colliders?

Early Universe Cosmology

early time

high energy

Particle Physics

Electroweak Phase Trans.
- ➔ Gravitational Waves?
- ➔ Primordial Mag Fields?
- ➔ Baryogenesis?

QCD Phase Transition
- ➔ Continuous crossover

Nucleosynthesis
- ➔ Abundance of light elements

Recombination
- ➔ Cosmic microwave bkg.

observables are cosmological relics

Higgs Precision (CEPC)

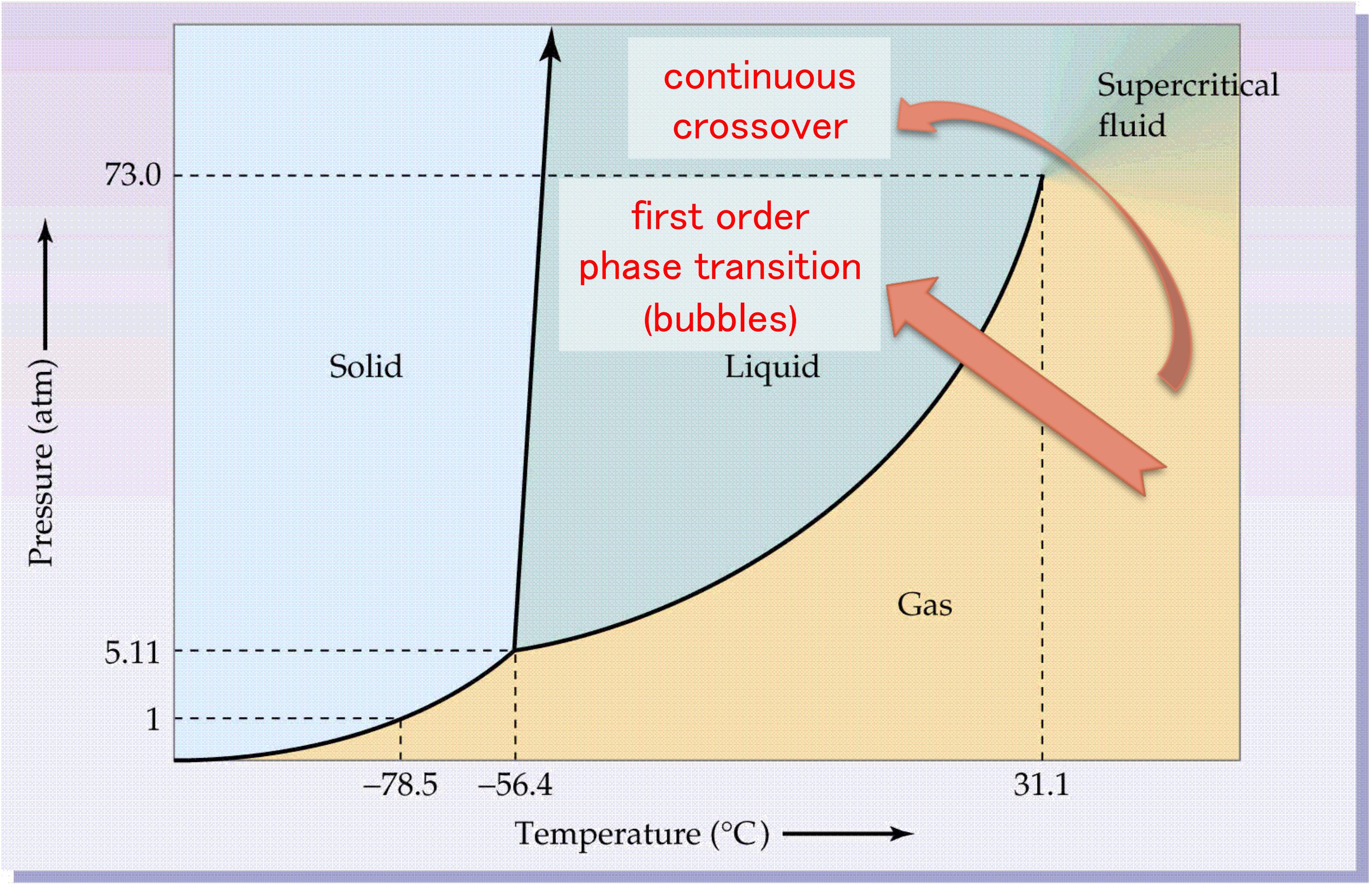
...

Deep Inelastic Scattering

Nuclear Decay, Neutrinos

Atomic Spectra

What is the Higgs phase diagram?...



Cosmological Relics of the EW Epoch

Matter / Anti-Matter Asymmetry (electroweak baryogenesis)

... SM processes called EW sphalerons violate B-number outside of the bubbles

... To avoid *washout* these processes must be suppressed inside the bubbles

$$v(T_c)/T_c \gtrsim 1.3 \qquad \text{("strongly first order")}$$

... This scenario is one of a few models of baryogenesis that's accessible to lab tests.

Stochastic Gravitational Wave Background

... When the bubbles collide some of their energy is tranferred to gravitational radiation

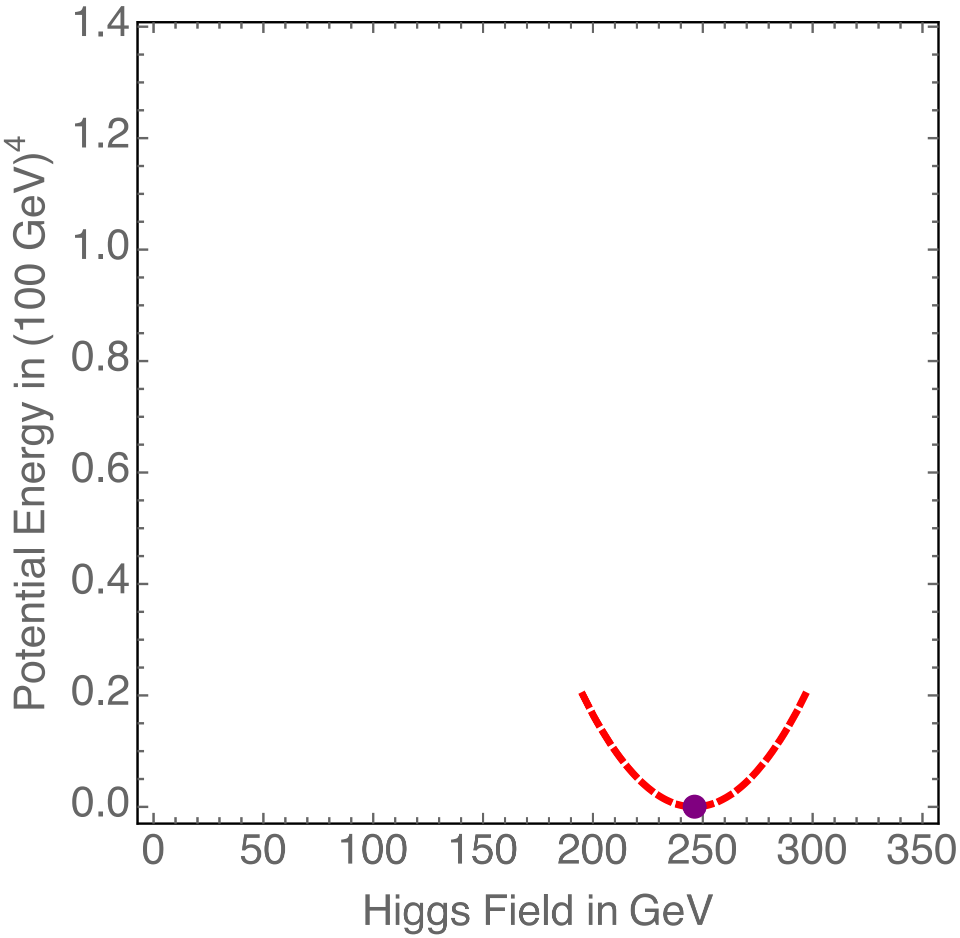
... Persists today as stochastic GW background

... Could be detected by space-based GW interferometer, like LISA

We discovered the Higgs!

We know that it's responsible for EW symmetry breaking!

Isn't that enough information to let us study the EW phase transition?

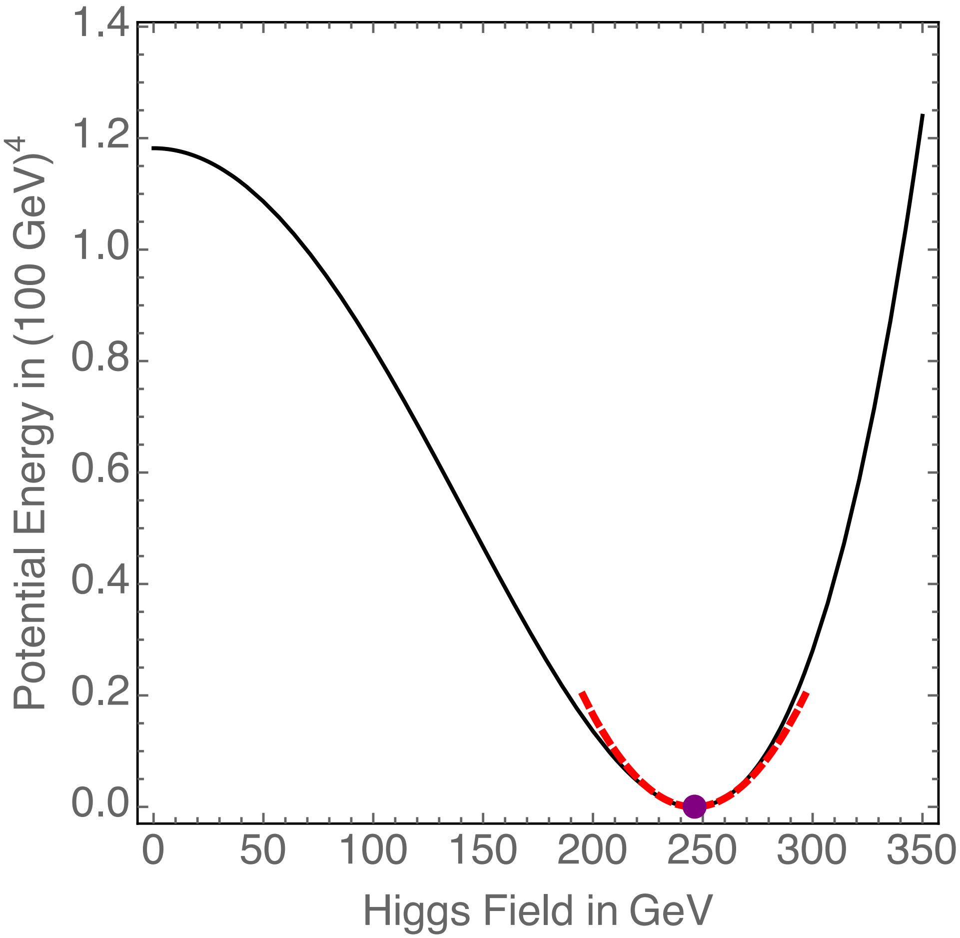


Measured Directly:

$$v \simeq 246 \text{ GeV}$$

$$M_h \simeq 125 \text{ GeV}$$

Assuming SM particle content & interactions



$$V = -\mu^2 H^\dagger H + \lambda_h (H^\dagger H)^2$$

$$\begin{cases} \mu^2 = M_h^2/2 \simeq (88\ \mathrm{GeV})^2 \\ \lambda_h = M_h^2/(2v^2) \simeq 0.13 \end{cases}$$

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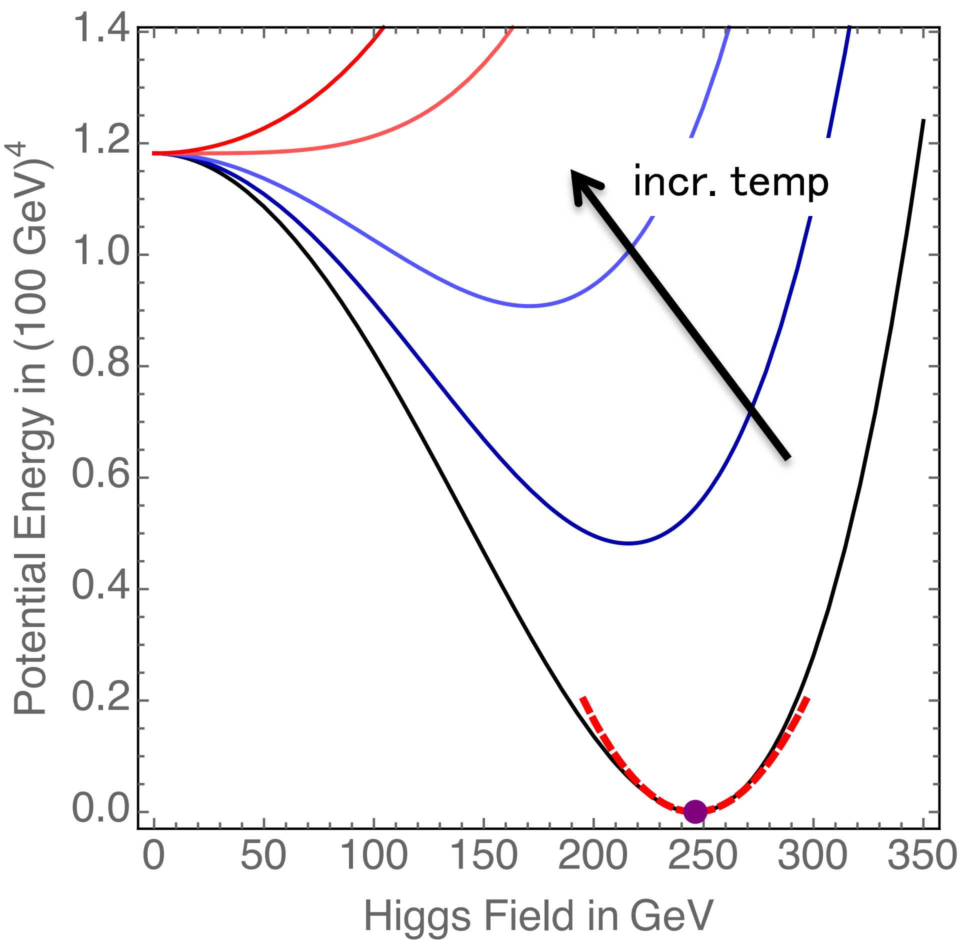

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Thermal support from Higgs interactions with W, Z, t, …

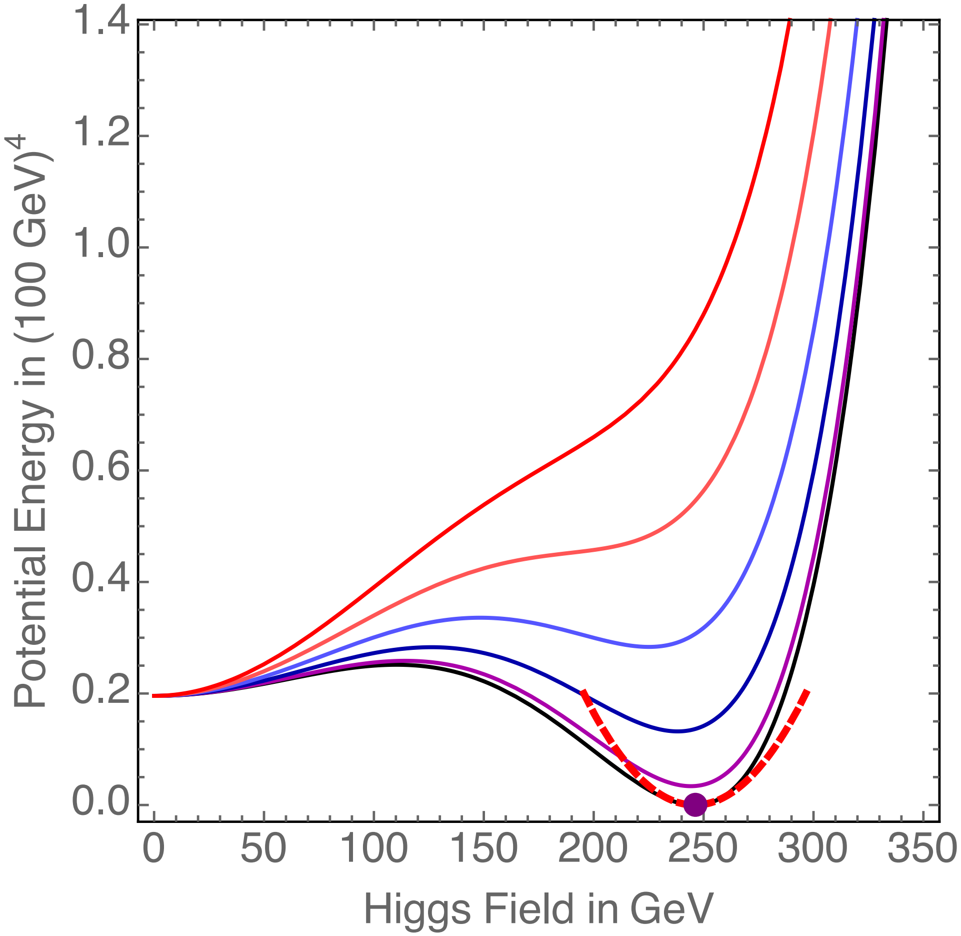
- EWPT is continuous crossover
- v(T) changes smoothly
- No energy barrier; no bubbles; no cosmological relics

Measured Directly: $v \simeq 246\ \mathrm{GeV}$

$M_h \simeq 125\ \mathrm{GeV}$

Variant #1 –SM with low cutoff

Recently studied by
P. Huang, Jokelar, Li, Wagner (2015)
F.P. Huang, Gu, Yin, Yu, Zhang (2015)
F.P. Huang, Wan, Wang, Cai, Zhang (2016)



$$V = \mu^2 H^\dagger H - \lambda_h (H^\dagger H)^2 + \Lambda^{-2}(H^\dagger H)^3$$

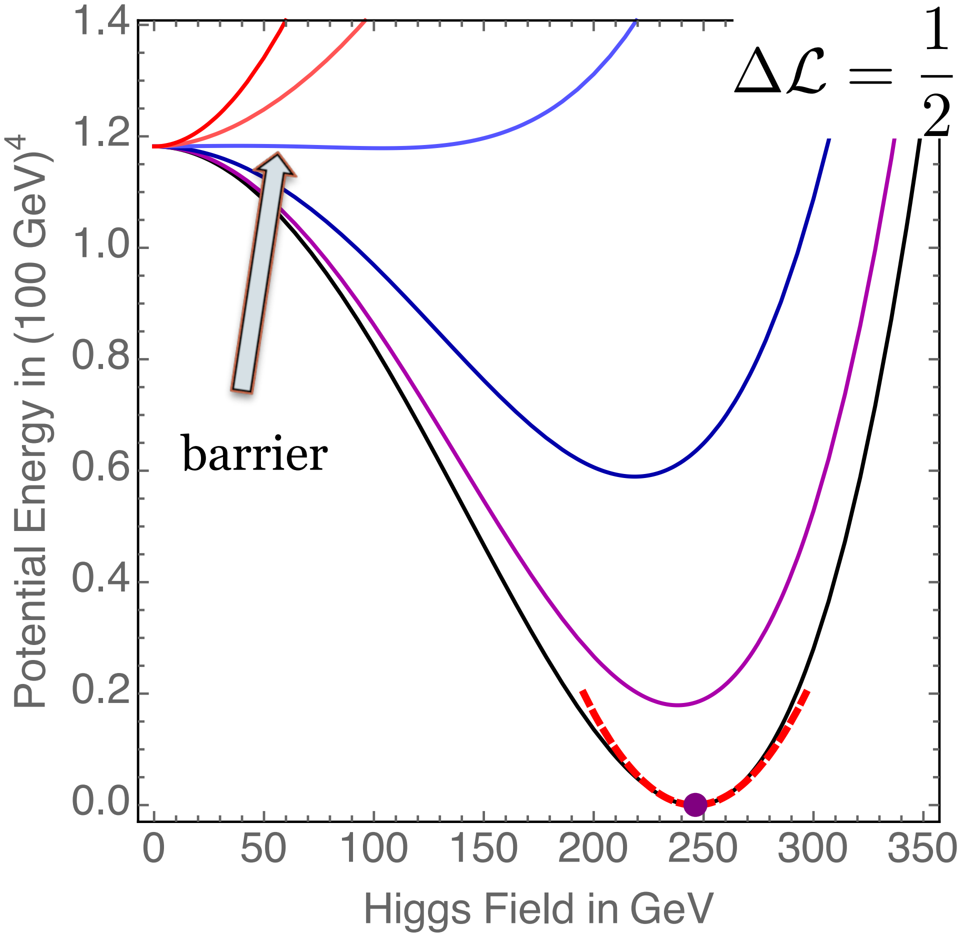
$$\begin{cases} \mu^2 \simeq (44\ \mathrm{GeV})^2 \\ \lambda_h \simeq 0.19 \\ \Lambda \simeq 530\ \mathrm{GeV} \end{cases}$$

Energy barrier may be present already at T=0.

- EWPT is first order
- Possibly interesting cosmological relics!

Measured Directly: $v \simeq 246\ \mathrm{GeV}$
$M_h \simeq 125\ \mathrm{GeV}$

Variant #2 –SM with new EW-scale matter coupled to Higgs



$$\Delta\mathcal{L} = \frac{1}{2}\left(\partial\phi_s\right)^2 - \frac{1}{2}m_s^2\phi_s^2 - \lambda_{hs}H^\dagger H\phi_s^2$$

The presence of new particles in the EW plasma can induce an energy barrier.

Heuristic understanding: these particles get their mass (in part) from the Higgs. It costs energy to bring <H> away from zero.

Measured Directly:

$$v \simeq 246 \text{ GeV}$$

$$M_h \simeq 125 \text{ GeV}$$

What can future colliders teach us about the electroweak phase transition?

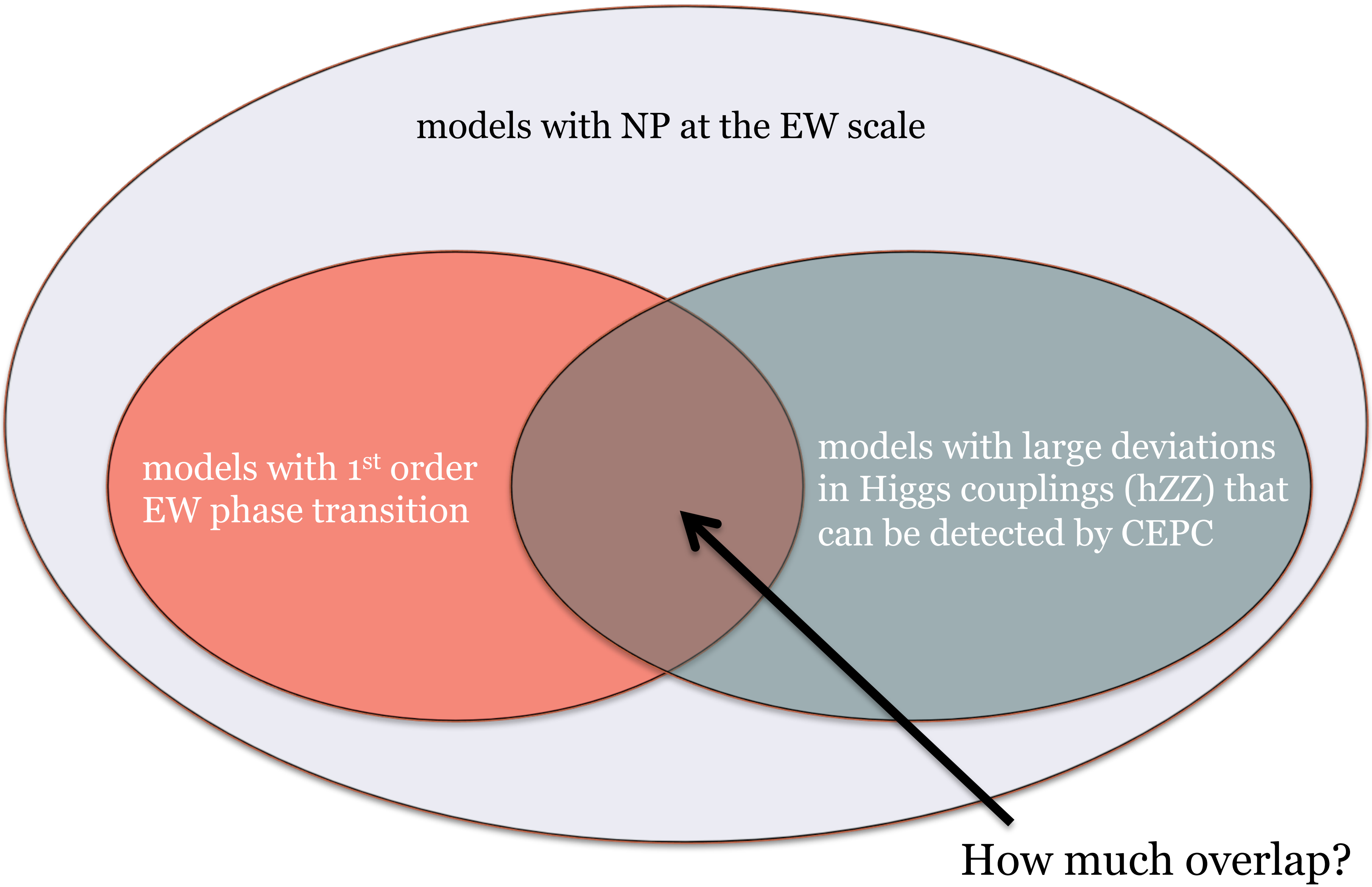
What can CEPC teach us about EWPT?

Future colliders will not recreate the conditions of the EWPT
… they reach the required energy but not the density

Instead, future colliders will measure the Higgs couplings (hhh, hZZ, h-gam-gam)

	current	**CEPC**
hZZ	27%	0.25%
$\Gamma(h \rightarrow \gamma\gamma)$	20%	4%

In models with a first order EW phase transition, there must be new physics coupled to the Higgs. It is reasonable to expect that this NP may also induce deviations in the Higgs couplings with other SM fields.


models with NP at the EW scale
models with 1st order EW phase transition
models with large deviations in Higgs couplings (hZZ) that can be detected by CEPC
How much overlap?

What Kinds of Models?

Model	References
SM + Scalar Singlet	Espinosa & Quiros, 1993; Benson, 1993; Choi & Volkas, 1993; McDonald, 1994; Vergara, 1996; Branco, Delepine, Emmanuel-Costa, & Gonzalez, 1998; Ham, Jeong, & Oh, 2004; Ahriche, 2007; Espinosa & Quiros, 2007; Profumo, Ramsey-Musolf, & Shaughnessy, 2007; Noble & Perelstein, 2007; Espinosa, Konstandin, No, & Quiros, 2008; Ashoorioon & Konstandin, 2009; Das, Fox, Kumar, & Weiner, 2009; Espinosa, Konstandin, & Riva, 2011; Chung & AL, 2011; Wainwright, Profumo, & Ramsey-Musolf, 2012; Barger, Chung, AL, & Wang, 2012; Huang, Shu, Zhang, 2012; Jiang, Bian, Huang, Shu, 2015; Huang & Li 2015
SM + Scalar Doublet	Davies, Froggatt, Jenkins, & Moorhouse, 1994; Huber, 2006; Fromme, Huber, & Seniuch, 2006; Cline, Kainulainen, & Trott, 2011; Kozhushko & Skalozub, 2011;
SM + Scalar Triplet	Patel, Ramsey-Musolf, 2012; Patel, Ramsey-Musolf, Wise, 2013; Huang, Gu, Yin, Yu, Zhang 2016
SM + Chiral Fermions	Carena, Megevand, Quiros, Wagner, 2005
MSSM	Carena, Quiros, & Wagner, 1996; Delepine, Gerard, Gonzales Felipe, & Weyers, 1996; Cline & Kainulainen, 1996; Laine & Rummukainen, 1998; Cohen, Morrissey, & Pierce,; Carena, Nardini, Quiros, & Wagner, 2012;
NMSSM / nMSSM / μνSSM	Pietroni, 1993; Davies, Froggatt, & Moorhouse, 1995; Huber & Schmidt, 2001; Ham, Oh, Kim, Yoo, & Son, 2004; Menon, Morrissey, & Wagner, 2004; Funakubo, Tao, & Toyoda, 2005; Huber, Kontandin, Prokopec, & Schmidt, 2006; Chung, AL, 2010, Huang, Kang, Shu, Wu, Yang, 2014
EFT-like Approach (H^6 operator)	Grojean, Servant, Wells, 2005; Huang, Gu, Yin, Yu, Zhang 2015; Huang, Joglekar, Li, Wagner, 2015; Huang, Wan, Wang, Cai, Zhang 2016; Huang, Gu, Yin, Yu, Zhang 2016

Can we systematize the calculation?

(similar concerns raised in:
Damgaard, Haarr, O'Connell, Tranberg 2015)

There is **no systematic formalism** for studying BSM models that give rise to a first order electroweak phase transition *and* associated collider phenomenology.

Can we use *effective field theory*?
➔ Not if there are EW-scale particles present in the plasma.
➔ Not if the particles get their mass from the Higgs (light in symmetric phase).

Can we use *phase transition model classes*? Chung, AL, Wang (2012)
➔ This framework organizes the PT-side of the calculation, but it is ignorant of the particle physics (phenomenology). E.g., from the PT perspective

SM + 1 colored scalar = SM + 3 singlet scalars with SO(3)

Models with very different collider phenomenology can have similar phase transition dynamics.

Models are typically studied on a case-by-case basis.

A Survey of Simplified Models

Model #1 – SM + chiral fermions (like MSSM gauginos)

Model #2 – SM + scalar doublet (like MSSM stops)

Model #3 – SM + real scalar singlet (like NMSSM singlet)

In the simplified / minimal models, the new degrees of freedom are responsible for *both* the 1PT and hZZ

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SM + Scalar Doublet ("stops")

In the MSSM, the stops play a critical role in making the EWPT first order. Here we considered a simplified version of the SUSY stop sector.

$$\tilde{Q} \sim (\mathbf{1}, \mathbf{2}, 1/3) \times 3 \text{ flavor}$$

$$\tilde{U} \sim (\mathbf{1}, \mathbf{1}, 4/3) \times 3 \text{ flavor}$$

four model parameters

The full Lagrangian is

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\text{SM}} &+ (D_\mu \tilde{Q})^\dagger (D^\mu \tilde{Q}) + (D_\mu \tilde{U})^* (D^\mu \tilde{U}) - \left[a_{hQU} \tilde{Q} \cdot H \tilde{U}^* + \text{h.c.} \right] \\ &- m_Q^2 \tilde{Q}^\dagger \tilde{Q} - m_U^2 \tilde{U}^* \tilde{U} - \lambda_Q (\tilde{Q}^\dagger \tilde{Q})^2 - \lambda_U (\tilde{U}^* \tilde{U})^2 \\ &- \lambda_{QU} (\tilde{Q}^\dagger \tilde{Q})(\tilde{U}^* \tilde{U}) - \lambda_{hU} (H^\dagger H)(\tilde{U}^* \tilde{U}) \\ &- \lambda_{hQ} (H^\dagger H)(\tilde{Q}^\dagger \tilde{Q}) - \lambda'_{hQ} (\tilde{Q} \cdot H)^* (\tilde{Q} \cdot H) - \lambda''_{hQ} (\tilde{Q}^\dagger H)^* (\tilde{Q}^\dagger H)\end{aligned}$$

and for simplicity we focus on

$$\langle \tilde{Q} \rangle = (0\,,\,0) \qquad \text{and} \qquad \langle \tilde{U} \rangle = 0$$

$$\lambda_Q = \lambda_U = \lambda_{QU} = \lambda_{hU} = \lambda_{hQ} = \lambda'_{hQ} = \lambda''_{hQ} \equiv \lambda$$

Spectrum

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_Q^2 + \frac{1}{2}(\lambda_{hQ} + \lambda'_{hQ})v^2 & \frac{a_{hQU}v}{\sqrt{2}} \\ \frac{a_{hQU}v}{\sqrt{2}} & m_U^2 + \frac{1}{2}\lambda_{hU}v^2 \end{pmatrix}$$

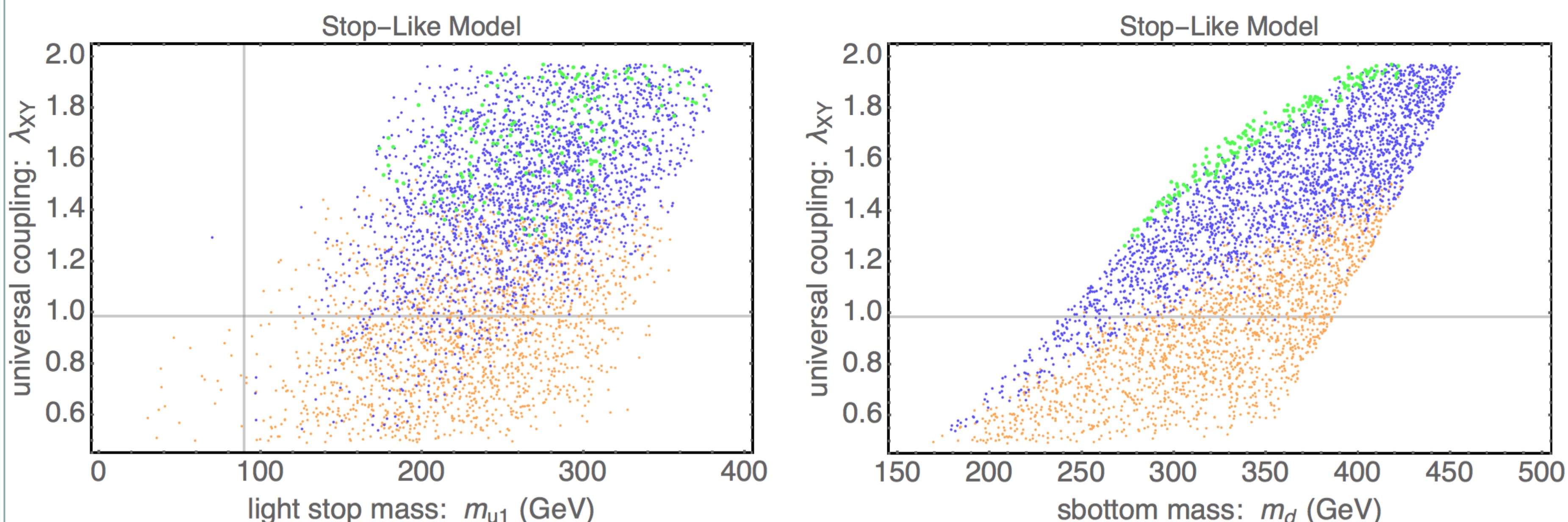
2 "stops"

$$\tan 2\theta = \frac{\sqrt{2}a_{hQU}v}{m_Q^2 - m_U^2 + \frac{1}{2}(\lambda_{hQ} + \lambda'_{hQ} - \lambda_{hU})v^2}$$

(mixing)

$$M_{\tilde{b}}^2 = m_Q^2 + \frac{1}{2}(\lambda_{hQ} + \lambda''_{hQ})v^2$$

1 "sbottom"

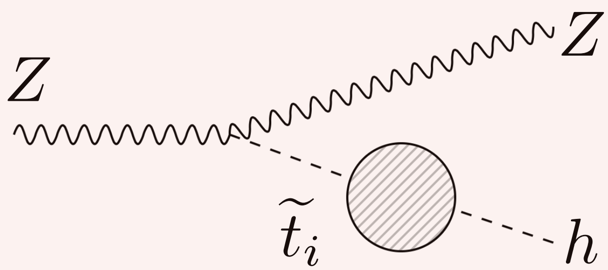


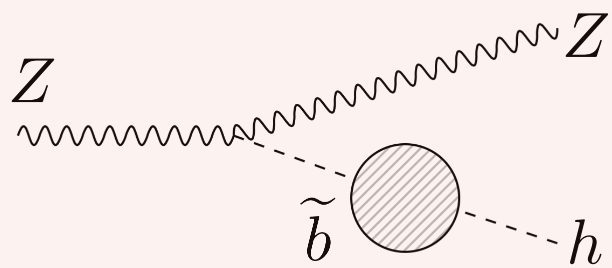
Effective hZZ coupling

(adapted from: Fan, Reece, Wang, 2014)

+vertex correction (suppressed by g/λ)

$$\delta Z_h = -n_f \sum_{i,j=1}^{2} \frac{|g_{h\tilde{t}_i\tilde{t}_j}|^2}{32\pi^2} I_B(M_h^2; M_{\tilde{t}_i}^2, M_{\tilde{t}_j}^2) - n_f \frac{|g_{h\tilde{b}\tilde{b}}|^2}{32\pi^2} I_B(M_h^2; M_{\tilde{b}}^2, M_{\tilde{b}}^2\ )$$





$$g_{h\tilde{t}_1\tilde{t}_1} = -\cos^2\theta\,(\lambda_{hQ} + \lambda'_{hQ})v - \sin^2\theta\,\lambda_{hU}v + \frac{a_{hQU}\sin 2\theta}{\sqrt{2}}$$

$$g_{h\tilde{t}_2\tilde{t}_2} = -\sin^2\theta\,(\lambda_{hQ} + \lambda'_{hQ})v - \cos^2\theta\,\lambda_{hU}v - \frac{a_{hQU}\sin 2\theta}{\sqrt{2}}$$

$$g_{h\tilde{t}_1\tilde{t}_2} = -\frac{\sin 2\theta}{2}(\lambda_{hQ} + \lambda'_{hQ})v + \frac{\sin 2\theta}{2}\lambda_{hU}v - \frac{a_{hQU}\cos 2\theta}{\sqrt{2}}$$

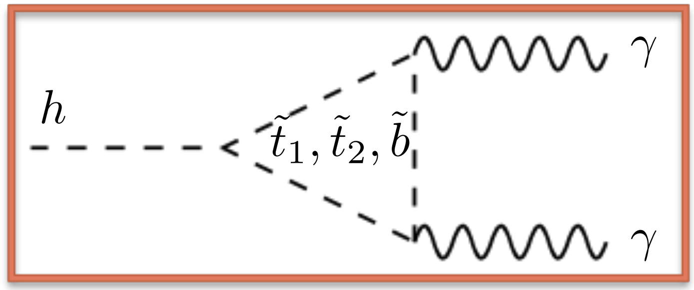
$$g_{h\tilde{b}\tilde{b}} = -(\lambda_{hQ} + \lambda''_{hQ})v$$

Higgs di-photon decay width (adapted from: Djouadi, Driesen, Hollik, Illana, 2005)

$$\Gamma_{h\to\gamma\gamma} = \frac{1}{64\pi}\frac{\alpha^2 M_h^3}{16\pi^2}\left|\bar{A}_W + \bar{A}_t + \bar{A}_{\tilde{t}} + \bar{A}_{\tilde{b}}\right|^2$$

$$\bar{A}_W = \frac{g_{hWW}}{M_W^2} F_1\big(M_h^2/4M_W^2\big)$$

$$\bar{A}_t = 2N_c Q_t^2 \frac{g_{htt}}{M_t} F_{1/2}\big(M_h^2/4M_t^2\big)$$



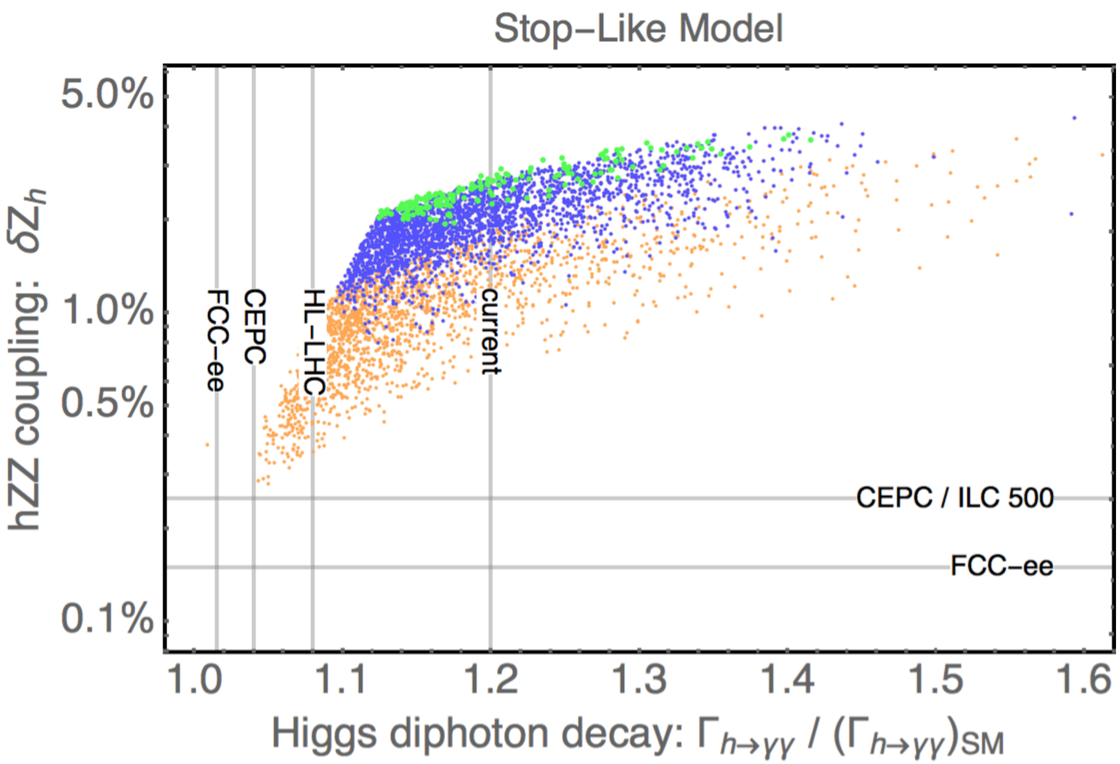
$$\bar{A}_{\tilde{t}} = \sum_{i=1}^{2} N_c Q_t^2 \frac{g_{h\tilde{t}_i\tilde{t}_i}}{M_{\tilde{t}_i}^2} F_0\big(M_h^2/4M_{\tilde{t}_i}^2\big)$$

$$\bar{A}_{\tilde{b}} = N_c Q_b^2 \frac{g_{h\tilde{b}\tilde{b}}}{M_{\tilde{b}}^2} F_0\big(M_h^2/4M_{\tilde{b}}^2\big)$$

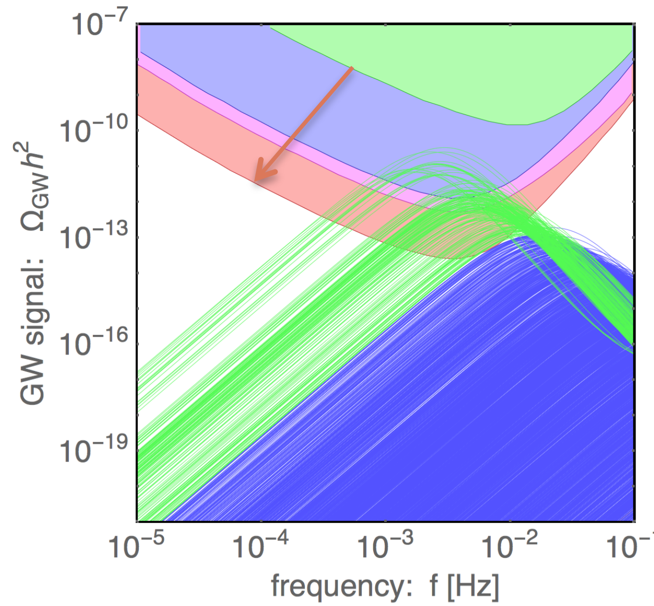
$$F_1(\tau) = \frac{2\tau^2 + 3\tau + 3(2\tau - 1)\arcsin(\tau^{1/2})^2}{\tau^2}$$

$$F_{1/2}(\tau) = -2\frac{\tau + (\tau - 1)\arcsin(\tau^{1/2})^2}{\tau^2}$$

$$F_0(\tau) = \frac{\tau - \arcsin(\tau^{1/2})^2}{\tau^2}$$



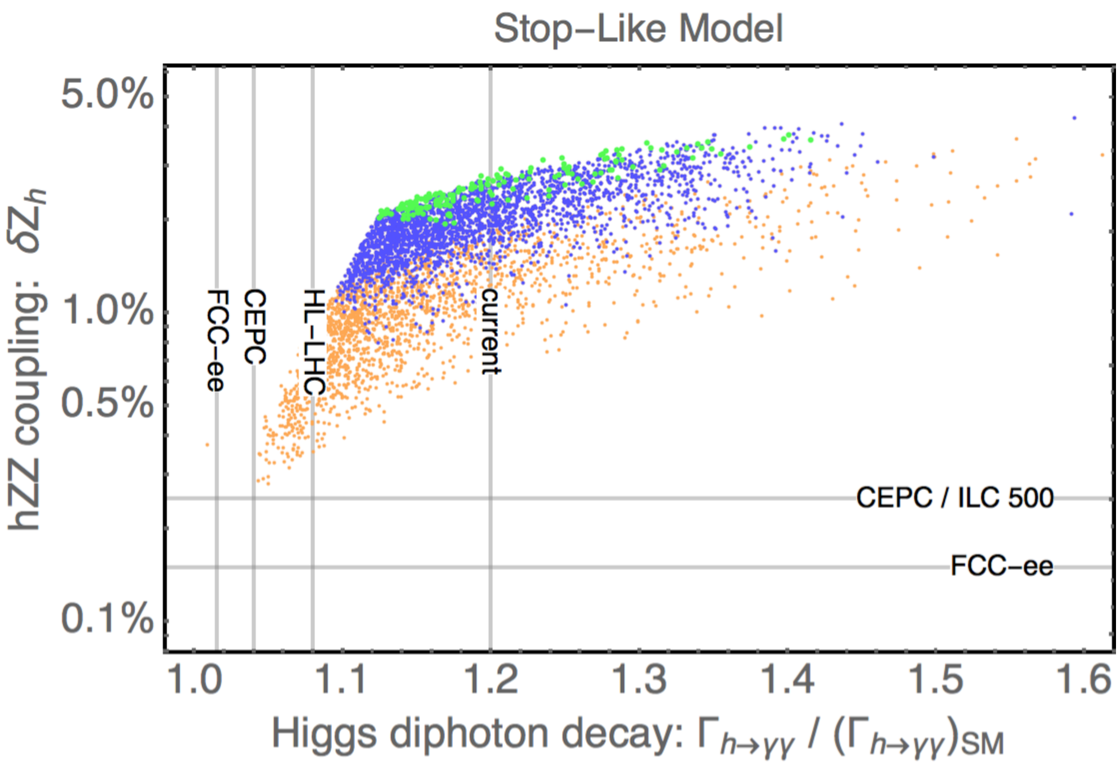
(sensitivities of four different proposed eLISA configurations)



Orange = first order phase transition, $v(T_c)/T_c > 0$

Blue = "strongly" first order phase transition, $v(T_c)/T_c > 1.3$

Green = very strongly 1PT, could detect GWs at LISA



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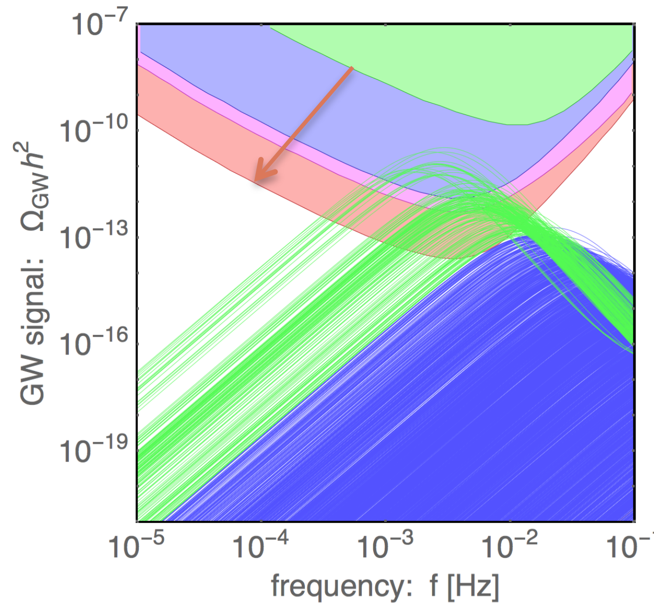

Models with a first order electroweak phase transition (orange, blue, or green) have large **deviation in hZZ** that can be probed by CEPC.

These models also have large enhancement to **Higgs diphoton decay** rate (b/c of charged particles) that can be probed by HL-LHC & CEPC.

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SM + Real Scalar Singlet

Consider

five model parameters

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial\phi_s)^2 - \frac{m_s^2}{2}\phi_s^2 - \frac{a_s}{3}\phi_s^3 - \frac{\lambda_s}{4}\phi_s^4 - \lambda_{hs}H^\dagger H\phi_s^2 - 2a_{hs}H^\dagger H\phi_s$$

real scalar singlet

Higgs portal

In the vacuum

$$\langle H \rangle = (0\,,\, v/\sqrt{2}) \qquad \text{and} \qquad \langle \phi_s \rangle = v_s$$

$$\sin 2\theta = \frac{4v(a_{hs} + \lambda_{hs}v_s)}{M_h^2 - M_s^2}$$ (mixing)

Effective hhh coupling

(adapted from: McCullough, 2014; Curtin, Meade, Yu, 2014)

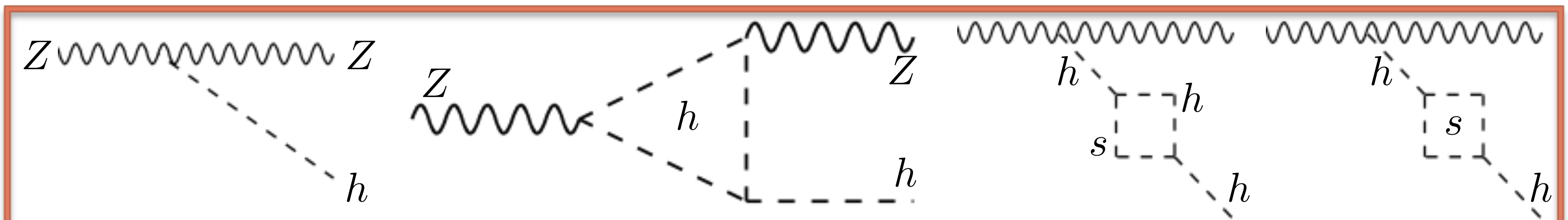
$$\lambda_3 = (6\lambda_h v)\cos^3\theta + (6a_{hs} + 6\lambda_{hs}v_s)\sin\theta\cos^2\theta + (6\lambda_{hs}v)\sin^2\theta\cos\theta + (2a_s + 6\lambda_s v_s)\sin^3\theta + 4\frac{|\lambda_{hs}|^3 v^3}{16\pi^2 M_s^2}$$

(one-loop)

Effective hZZ coupling

$$\delta Z_h \approx (1-\cos\theta) - 0.006\left(\frac{\lambda_3}{\lambda_{3,\mathrm{SM}}} - 1\right) - \frac{1}{2}\frac{|\lambda_{hs}v_s + a_{hs}|^2}{16\pi^2} I(M_h^2; M_h^2, M_s^2) - \frac{1}{2}\frac{|\lambda_{hs}|^2 v^2}{16\pi^2} I(M_h^2; M_s^2, M_s^2)$$

(leading effect is from mixing)



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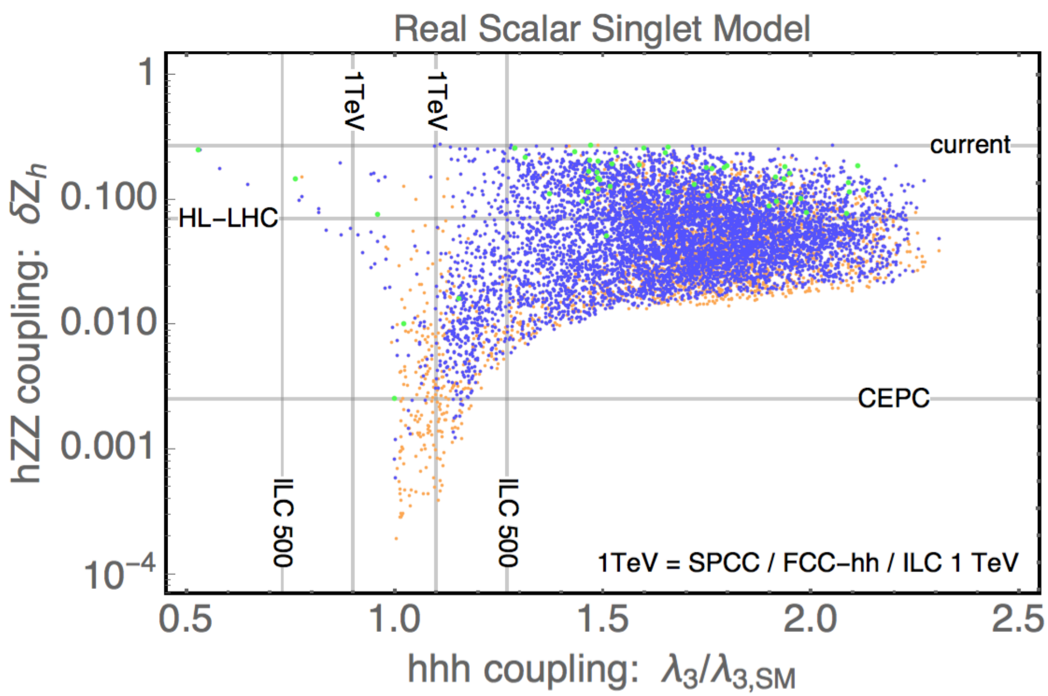
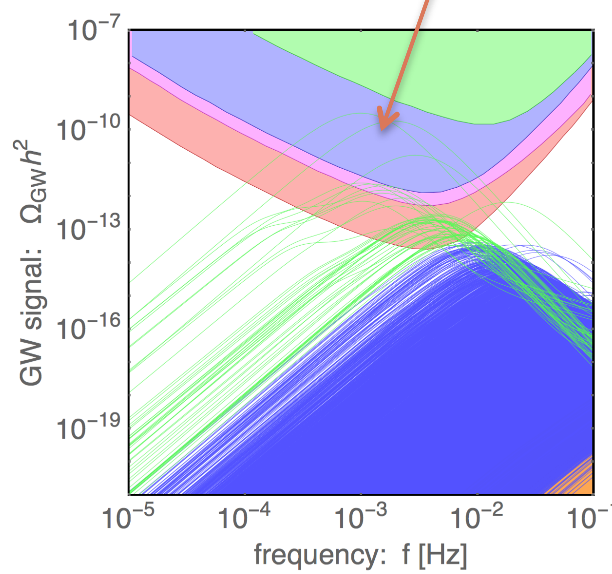
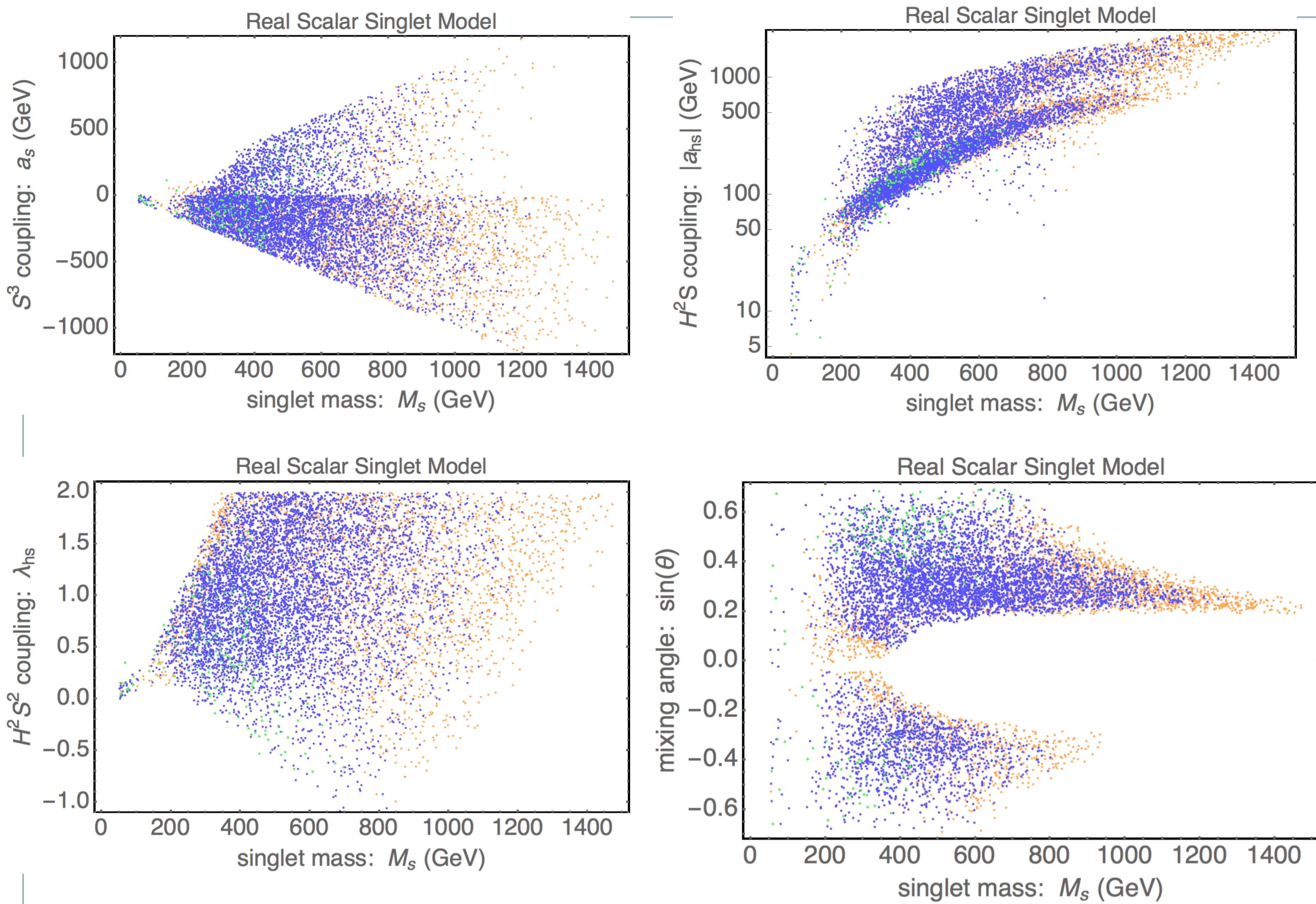
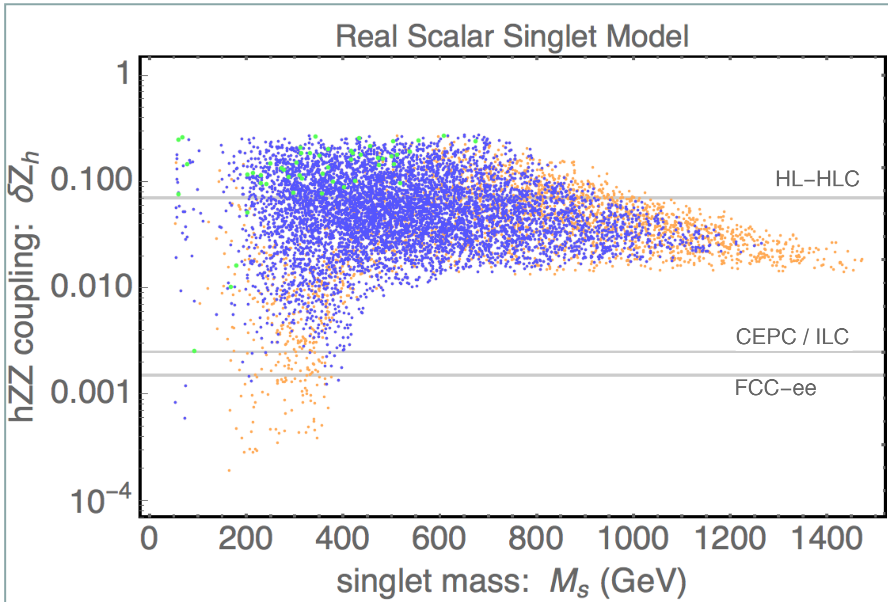


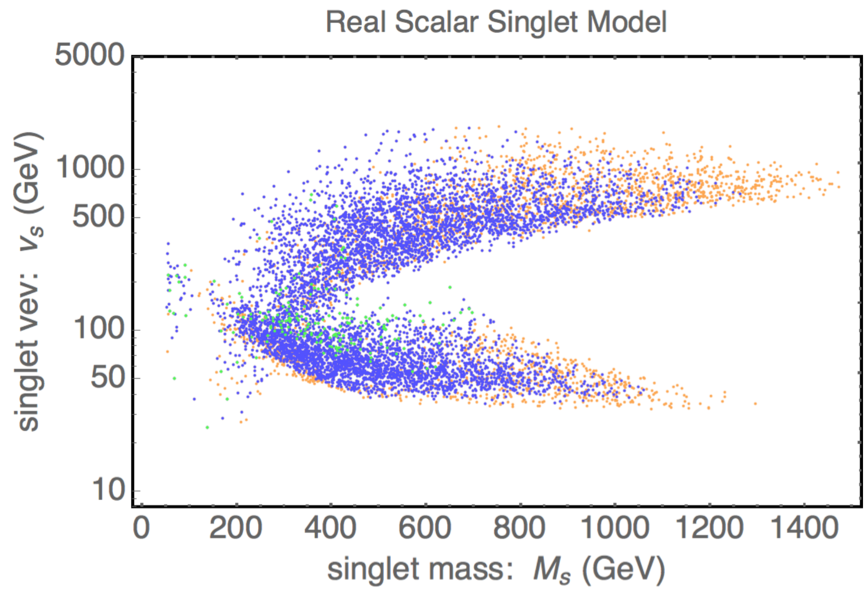
(sensitivities of four different proposed eLISA configurations)

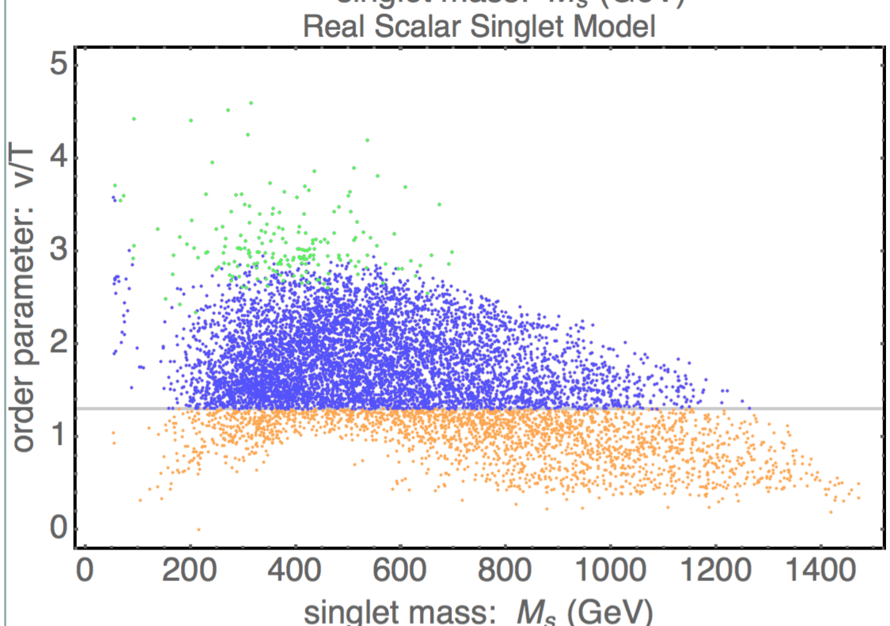


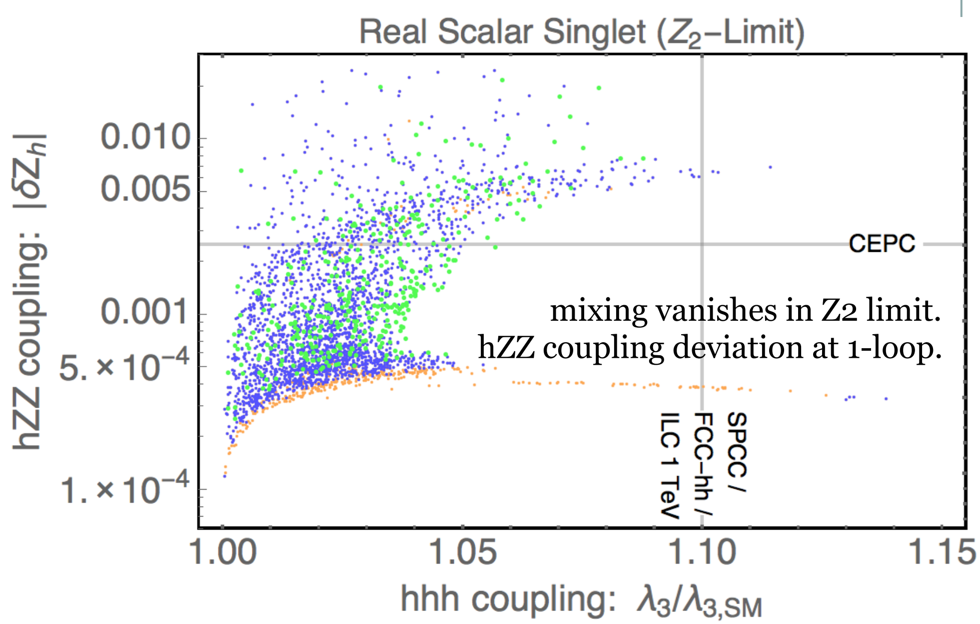
Parameter space with first order electroweak phase transition has large deviation in hZZ, which can be probed by CEPC


Real Scalar Singlet Model
S^3 coupling: a_s (GeV)
singlet mass: M_s (GeV)
Real Scalar Singlet Model
H^2S coupling: $|a_{hs}|$ (GeV)
singlet mass: M_s (GeV)
Real Scalar Singlet Model
H^2S^2 coupling: λ_{hs}
singlet mass: M_s (GeV)
Real Scalar Singlet Model
mixing angle: $\sin(\theta)$
singlet mass: M_s (GeV)


Real Scalar Singlet Model
hZZ coupling: δZ_h
singlet mass: M_s (GeV)
HL-HLC
CEPC / ILC
FCC-ee
1
0.100
0.010
0.001
10^-4
0 200 400 600 800 1000 1200 1400


Real Scalar Singlet Model
singlet vev: v_s (GeV)
singlet mass: M_s (GeV)
5000
1000
500
100
50
10
0 200 400 600 800 1000 1200 1400


Real Scalar Singlet Model
order parameter: v/T
singlet mass: M_s (GeV)
5 4 3 2 1 0
0 200 400 600 800 1000 1200 1400


Real Scalar Singlet (Z2–Limit)
hZZ coupling: |δZ_h|
hhh coupling: λ3/λ3,SM
CEPC
SPCC / FCC–hh / ILC 1 TeV
mixing vanishes in Z2 limit.
hZZ coupling deviation at 1-loop.
0.010
0.005
0.001
5.×10^-4
1.×10^-4
1.00 1.05 1.10 1.15

Comparing Cosmologists' & Particle Physicists' Approach

Cosmologist's Approach

… direct: uses GW interferometry

… with the sensitivity of LISA, only models with VERY strongly first order transitions can be probed

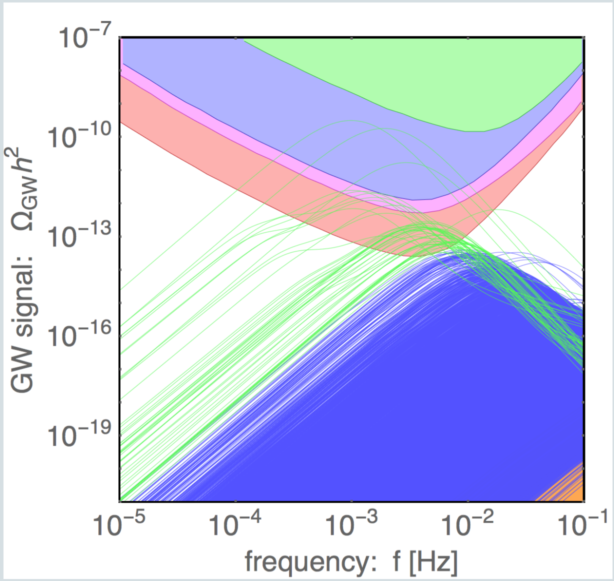
best for confirmation

Particle Physics Approach

… indirect: looks for modifications to hZZ couplings

… with the sensitivity of CEPC, most models with strong first order phase transitions can be probed

best for falsification



green = can probe GW with eLISA
green & blue = can probe hZZ with CEPC

Summary & Outlook

Cosmologists working on the EW epoch of the early universe **find ourselves on shaky ground**.

Our predictions for cosmological relics from the EW phase transition (matter / anti-matter asymmetry, primordial gravitational waves, primordial magnetic fields, …) are subject to large uncertainties.

Precision measurements of Higgs couplings indirectly probe the electroweak phase transition.

Large deviations in hZZ coupling seems to be generic in models with first order EWPT

Summary & Outlook

Cosmologists working on the EW epoch of the early universe **find ourselves on shaky ground**.

Our predictions for cosmological relics from the EW phase transition (matter / anti-matter asymmetry, primordial gravitational waves, primordial magnetic fields, ...) are subject to large uncertainties.

Precision measurements of Higgs couplings indirectly probe the electroweak phase transition.

Large deviations in hZZ coupling seems to be generic in models with first order EWPT

Keep up the hard work!

Backup

Gravitational Wave Spectrum

See Caprini et. al.
eLISA study [1512.06293]

Bubble nucleation temperature

$$\left.\frac{S_3(T)}{T}\right|_{T=T_n} \simeq 142$$

Energy liberation

$$\alpha = \left.\frac{\rho_{\text{vac},u} - \rho_{\text{vac},b}}{\rho_{\text{rad},b}}\right|_{T=T_n}$$

Phase transition duration

$$\frac{\beta}{H} \equiv -\left.\frac{dS_3}{dt}\right|_{t=t_n} \approx T\left.\frac{d(S_3/T)}{dT}\right|_{T=T_n}$$

Gravitational Wave Spectrum

Gravitational Waves are produced by three sources

(1) Bubble collisions

$$\Omega_\phi h^2 = (1.67 \times 10^{-5}) \left(\frac{\beta}{H_{\rm PT}}\right)^{-2} \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{g_{*,\rm PT}}{100}\right)^{-1/3} \left(\frac{0.11 v_w^3}{0.42 + v_w^2}\right) \frac{3.8 (f/f_\phi)^{2.8}}{1 + 2.8 (f/f_\phi)^{3.8}}$$

$$f_\phi = (1.65 \times 10^{-5}\ {\rm Hz}) \left(\frac{0.62}{1.8 - 0.1 v_w + v_w^2}\right) \left(\frac{\beta}{H_{\rm PT}}\right) \left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right) \left(\frac{g_{*,\rm PT}}{100}\right)^{1/6}$$

(2) decaying turbulence

$$\Omega_{\rm turb} h^2 = (3.35 \times 10^{-4}) \left(\frac{\beta}{H_{\rm PT}}\right)^{-1} \left(\frac{\kappa_{\rm turb} \alpha}{1+\alpha}\right)^{3/2} \left(\frac{g_*}{100}\right)^{-1/3} v_w \frac{(f/f_{\rm turb})^3}{(1 + f/f_{\rm turb})^{11/3} (1 + 8\pi f/h_*)}$$

$$f_{\rm turb} = (2.7 \times 10^{-5}\ {\rm Hz}) \frac{1}{v_w} \left(\frac{\beta}{H_{\rm PT}}\right) \left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right) \left(\frac{g_{*,\rm PT}}{100}\right)^{1/6}$$

(3) and sound waves

$$\Omega_{\rm sw} h^2 = (2.65 \times 10^{-6}) \left(\frac{\beta}{H_{\rm PT}}\right)^{-1} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-1/3} v_w \frac{7^{7/2} (f/f_{sw})^3}{[4 + 3(f/f_{sw})^2]^{7/2}}$$

$$f_{\rm sw} = (1.9 \times 10^{-5}\ {\rm Hz}) \frac{1}{v_w} \left(\frac{\beta}{H_{\rm PT}}\right) \left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right) \left(\frac{g_{*,\rm PT}}{100}\right)^{1/6}$$

Gravitational Wave Spectrum

The efficiency factors (kappa's) depend on the strength of the phase transition.

For a strongly first order transition, the pressure gradient drives the bubble wall to expand and "run away" with $v_w \rightarrow 1$.

In this regime, the amount of energy transferred to the plasma saturates, and the surplus energy causes the bubble wall to accelerate.

$$\kappa_\phi = 1 - \frac{\alpha_\infty}{\alpha} \ , \quad \kappa_v = \frac{\alpha_\infty}{\alpha} \kappa_\infty \ , \quad \kappa_{\text{therm}} = 1 - \kappa_\phi - \kappa_v$$

$$\kappa_\infty = \frac{\alpha_\infty}{0.73 + 0.083 \alpha_\infty^{1/2} + \alpha_\infty}$$

$$\alpha_\infty \simeq (4.9 \times 10^{-3}) \left(\frac{v(T_{\text{PT}})}{T_{\text{PT}}} \right)^2$$

$$\kappa_{\text{turb}} = (5\%) \times \kappa_v$$

(summarized in eLISA study: Caprini, et. al. 1512.06239)

Exceptions (nightmare scenarios)

Models with first order phase transitions *generically* have large deviations in hhh & hZZ. This is largely due to the tree-level mixing:

$$\sin 2\theta = \frac{4v(a_{hs} + \lambda_{hs} v_s)}{M_h^2 - M_s^2}$$

Without the mixing, it becomes difficult to probe the models at colliders.

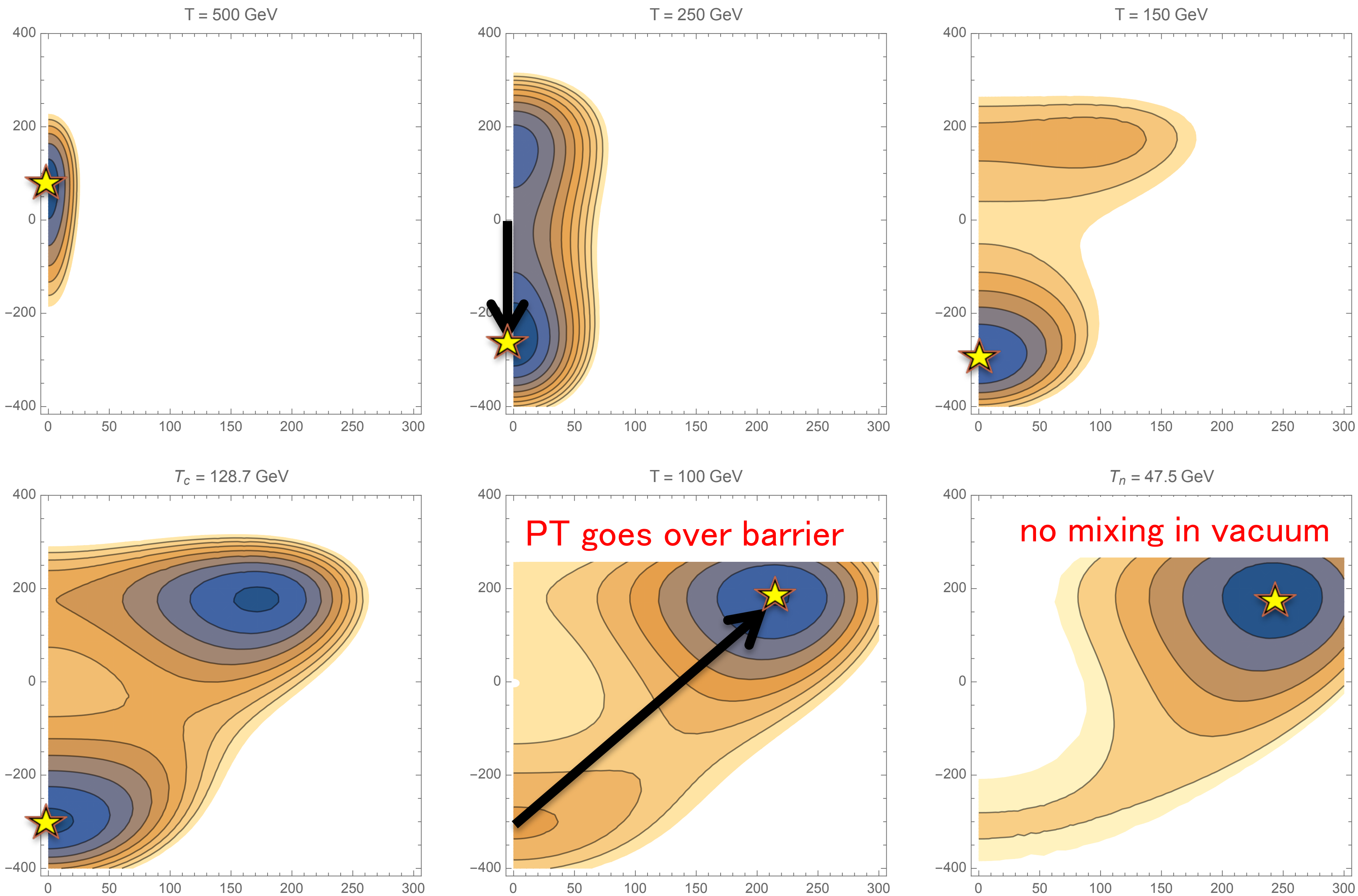
Nightmare Scenario #1 – impose Z_2 to forbid mixing (Curtin, Meade, Yu, 2014)

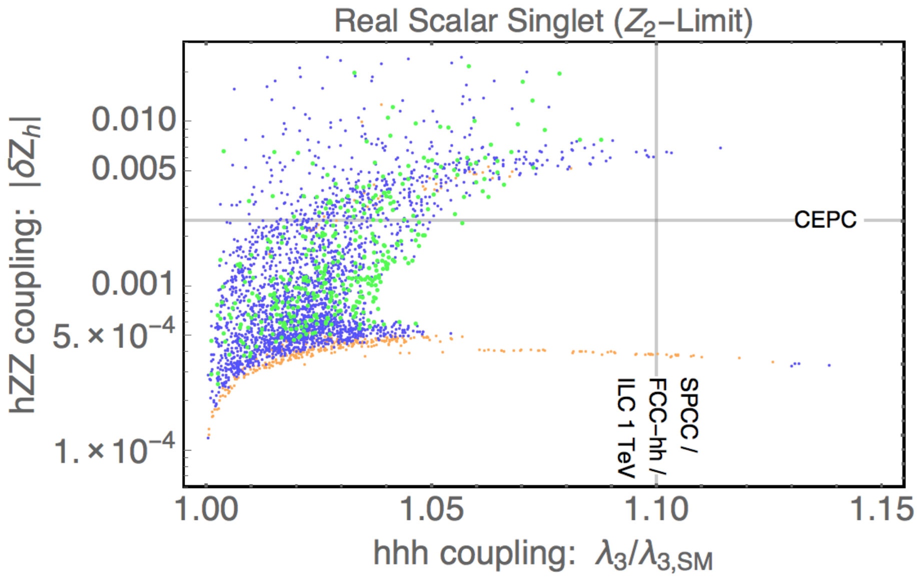
$$a_s = 0 \quad , \quad a_{hs} = 0 \quad , \quad \text{and} \quad v_s = 0$$

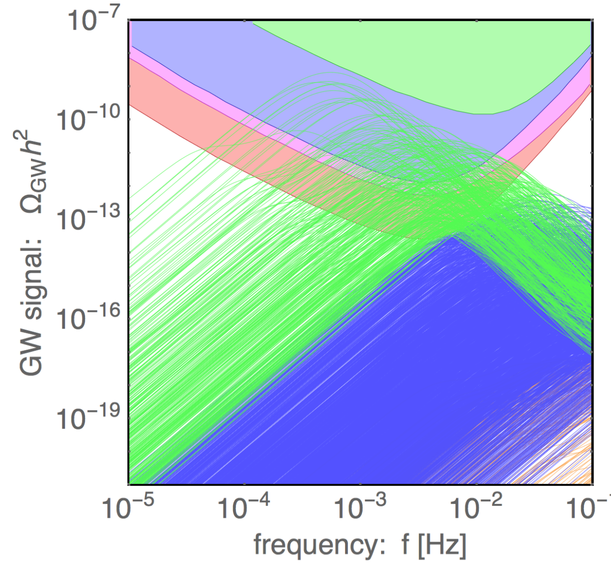
Nightmare Scenario #2 – tune the mixing to zero

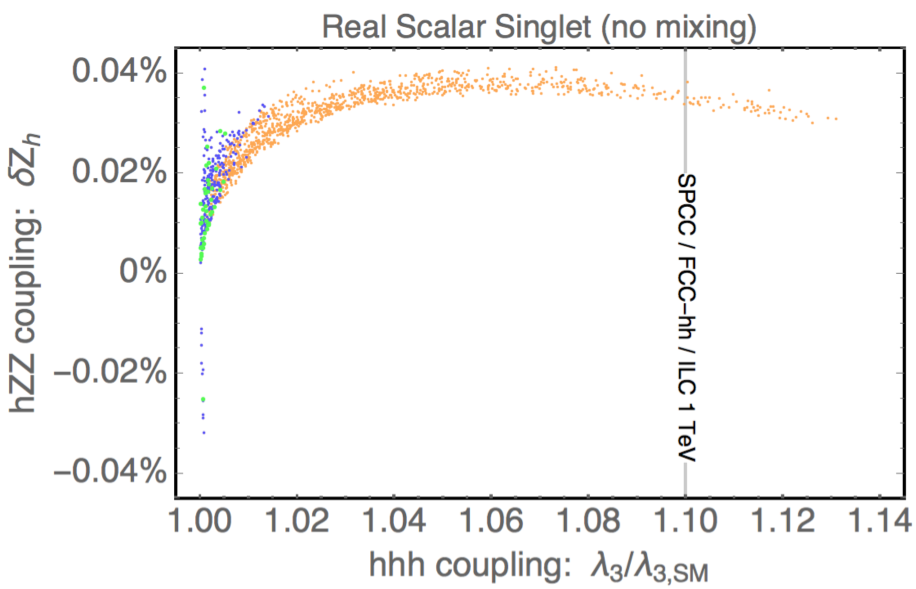
$$a_{hs} + \lambda_{hs} v_s = 0$$

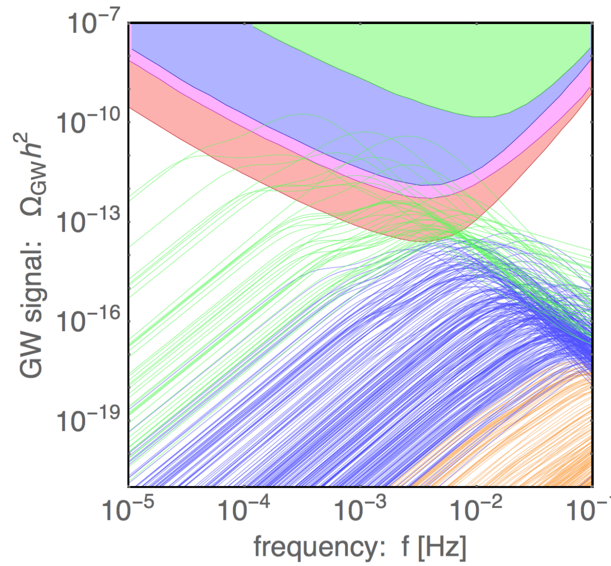
“Nightmare Scenario”




Real Scalar Singlet (Z_2–Limit)
hZZ coupling: $|\delta Z_h|$
0.010
0.005
0.001
$5.\times10^{-4}$
$1.\times10^{-4}$
CEPC
SPCC / FCC–hh / ILC 1 TeV
1.00
1.05
1.10
1.15
hhh coupling: $\lambda_3/\lambda_{3,\mathrm{SM}}$


10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
GW signal: $\Omega_{\mathrm{GW}}h^2$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
frequency: f [Hz]


Real Scalar Singlet (no mixing)
hZZ coupling: δZ_h
0.04%
0.02%
0%
−0.02%
−0.04%
SPCC / FCC–hh / ILC 1 TeV
1.00
1.02
1.04
1.06
1.08
1.10
1.12
1.14
hhh coupling: $\lambda_3/\lambda_{3,\mathrm{SM}}$


10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
GW signal: $\Omega_{\mathrm{GW}}h^2$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
frequency: f [Hz]

PT Model Classes

Chung, AL, Wang [1209.1819]