中国科学院高能物理研究所
Institute of High Energy Physics Chinese Academy of Sciences

Probing the nature of Higgs boson from particle colliders to gravitational wave detectors

Fa Peng Huang/Xinmin Zhang

based on our recent work Phys. Rev. D 93, 103515 (2016) ,
Phys. Rev. D 94, 041702(R)(2016) and work in progress

CEPC workshop 2016@IHEP, 15 Dec. 2016

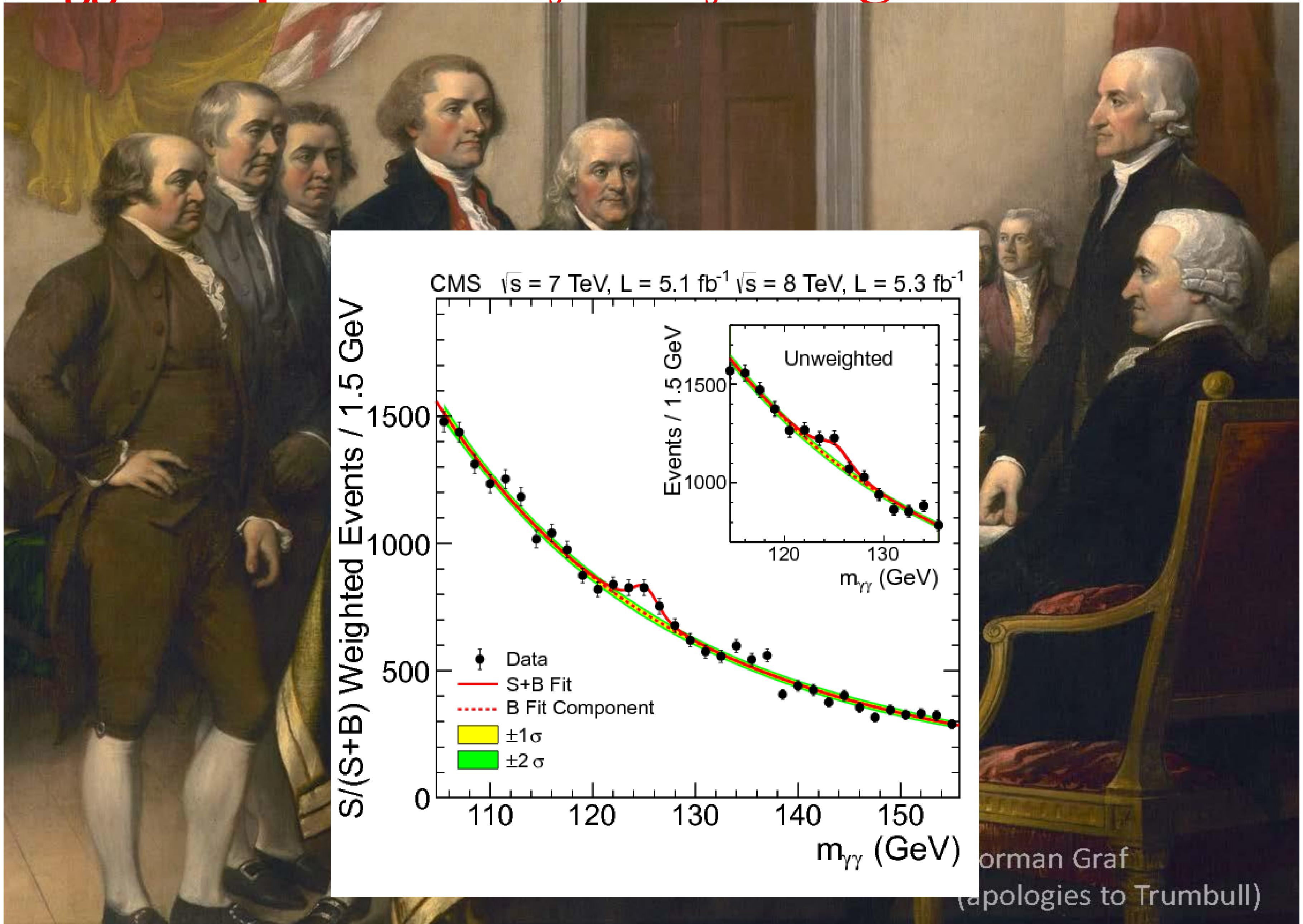
Outline

- Research background
- Electroweak baryogenesis in a nutshell
- The effective Lagrangian and concrete model
- Hints at the hadron collider
- Test at the lepton collider
- Gravitational wave from EWPT
- Summary and outlook

Higgs Independence Day:4 July 2012@LHC deliberate !

Norman Graf
(apologies to Trumbull)

Higgs Independence Day:4 July 2012@LHC deliberate !

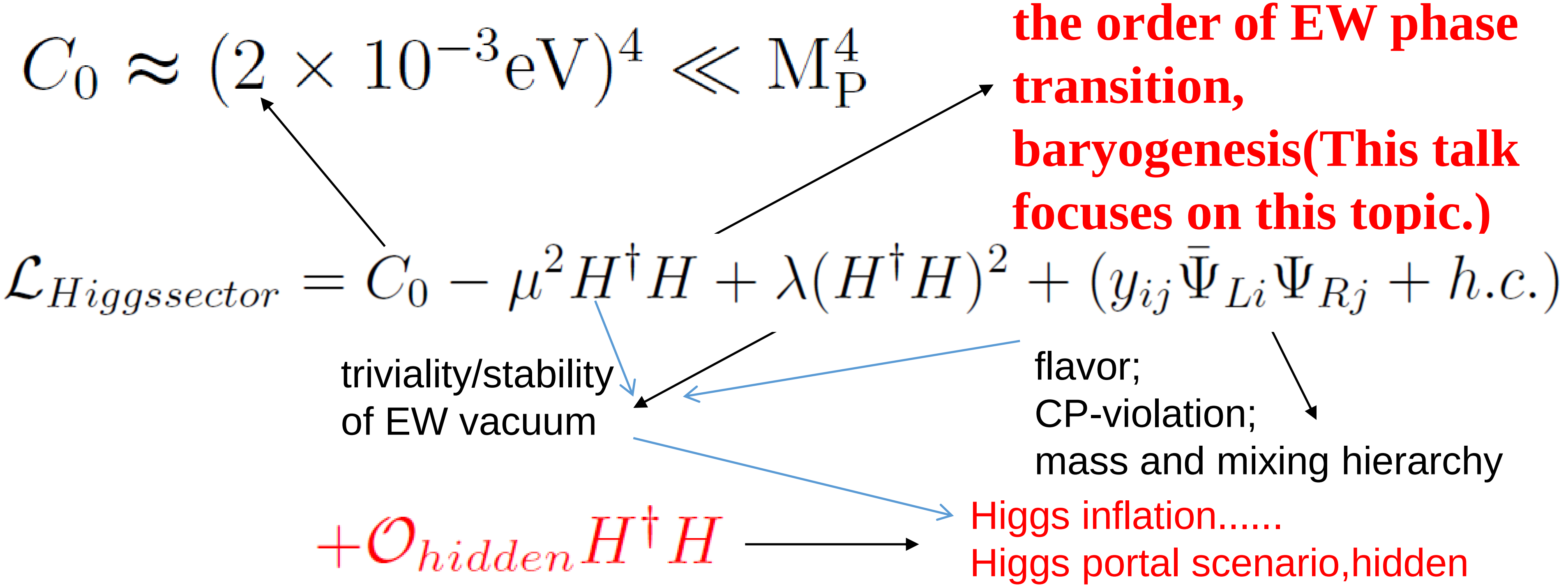


orman Graf
(apologies to Trumbull)

Physics after Higgs
Independence Day

- **The discovery of 125 GeV scalar at the LHC opens a new window to study the fundamental physics, such as neutrino mass, Higgs (portal) inflation, Higgs dark energy, baryogenesis (Higgs cosmology), and the "cosmological relaxion", Nnaturalness…**
- **However, these fundamental problems may related to the nature of Higgs potential, namely, the true shape of Higgs potential, the nature of EW SSB, and the type of the EW phase transition, the EW baryogenesis.**

The nature of the Higgs potential may related to all the fundamental problems!



Cosmological Constant problem:inflation,dark energy;
the largest hierarchy:

The 125 GeV Higgs boson–so simple, yet not so natural !

CEPC/SPPC helps to understand these problems by precise measurement of the Higgs properties.

$$C_0 \approx (2 \times 10^{-3}\mathrm{eV})^4 \ll \mathrm{M}_\mathrm{P}^4$$

hierarchy problem,

the order of EW phase transition, baryogenesis(This talk focuses on this topic.)

$$\mathcal{L}_{Higgssector} = C_0 - \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + (y_{ij}\bar{\Psi}_{Li}\Psi_{Rj} + h.c.)$$

triviality/stability of EW vacuum

flavor;
CP-violation;
mass and mixing hierarchy

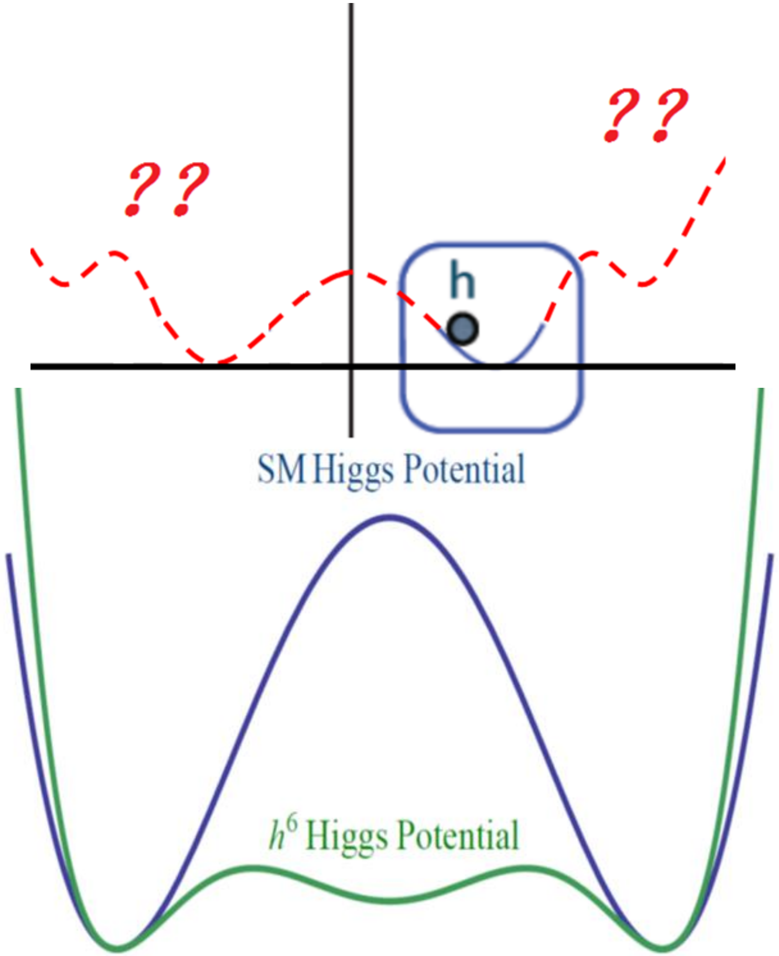
$$+\mathcal{O}_{hidden} H^\dagger H$$

Higgs inflation......
Higgs portal scenario,hidden sector of new physics,

Precise higher order calculations in SM.

Case by case study of new physics Phenomenology.

What is the nature of Higgs potential and the type of EW phase transition?



For the Higgs potential, we know nothing but the quadratic oscillation around the vev=246GeV with the mass 125 GeV from the current LHC data.

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

or $$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4} h^4 + \frac{1}{\Lambda^2} h^6$$

Pre CDR of CEPC

arXiv:1511.06495 Nima Arkani-Hamed, Tao Han, Michelangelo Mangano, Lian-Tao Wang

The nature of Higgs potential and the type of EW phase transition

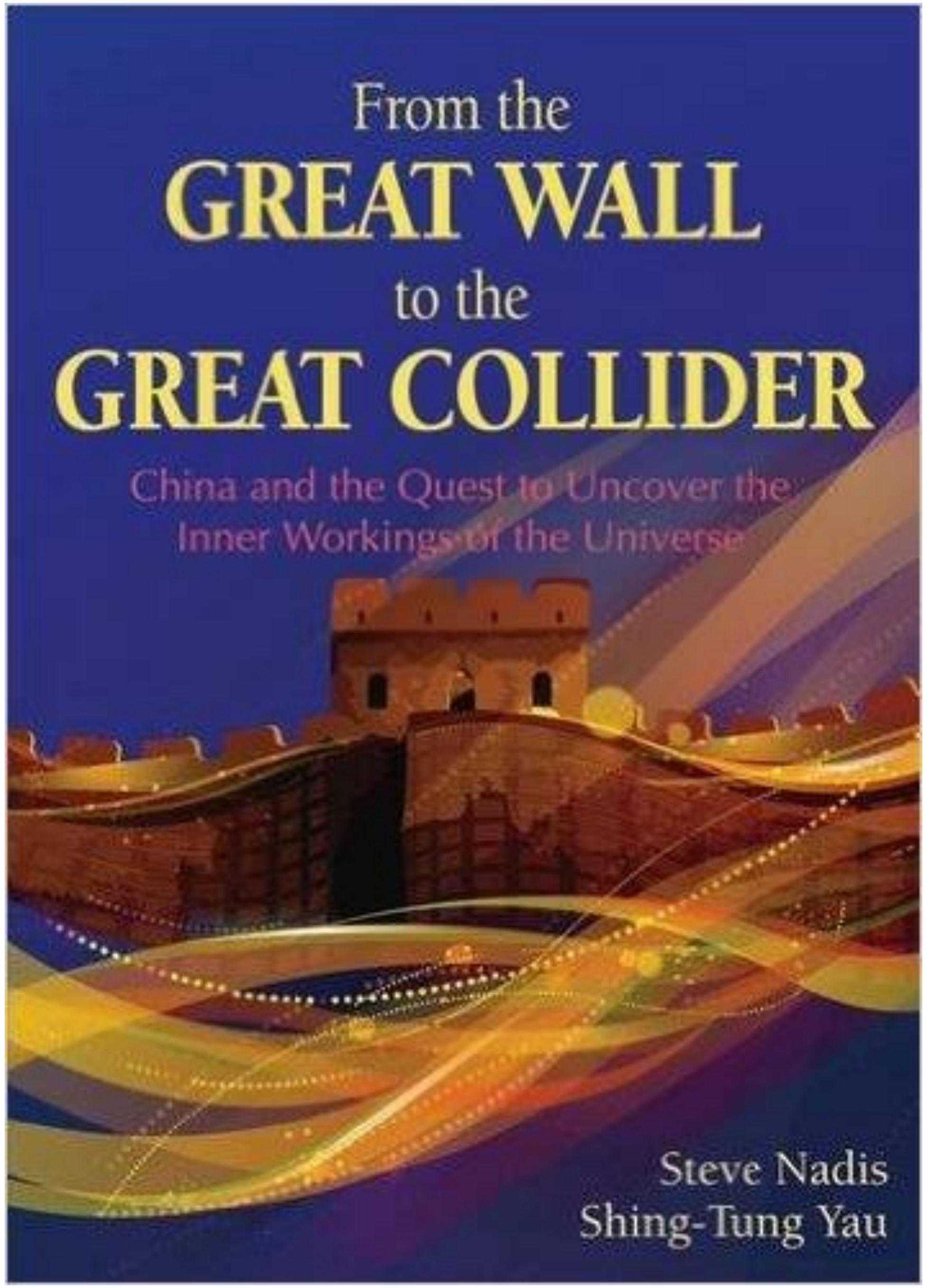
First order EW phase transition ⟺

- **The true shape of Higgs potential**
- Baryon **asymmetry of the universe (baryogenesis)**
- **Gravitational wave**
- **Help to know the evolution history of the universe at hundred GeV temperature.**

Current particle collider has no ability to unravel the true potential of the Higgs boson, we need new experiments.

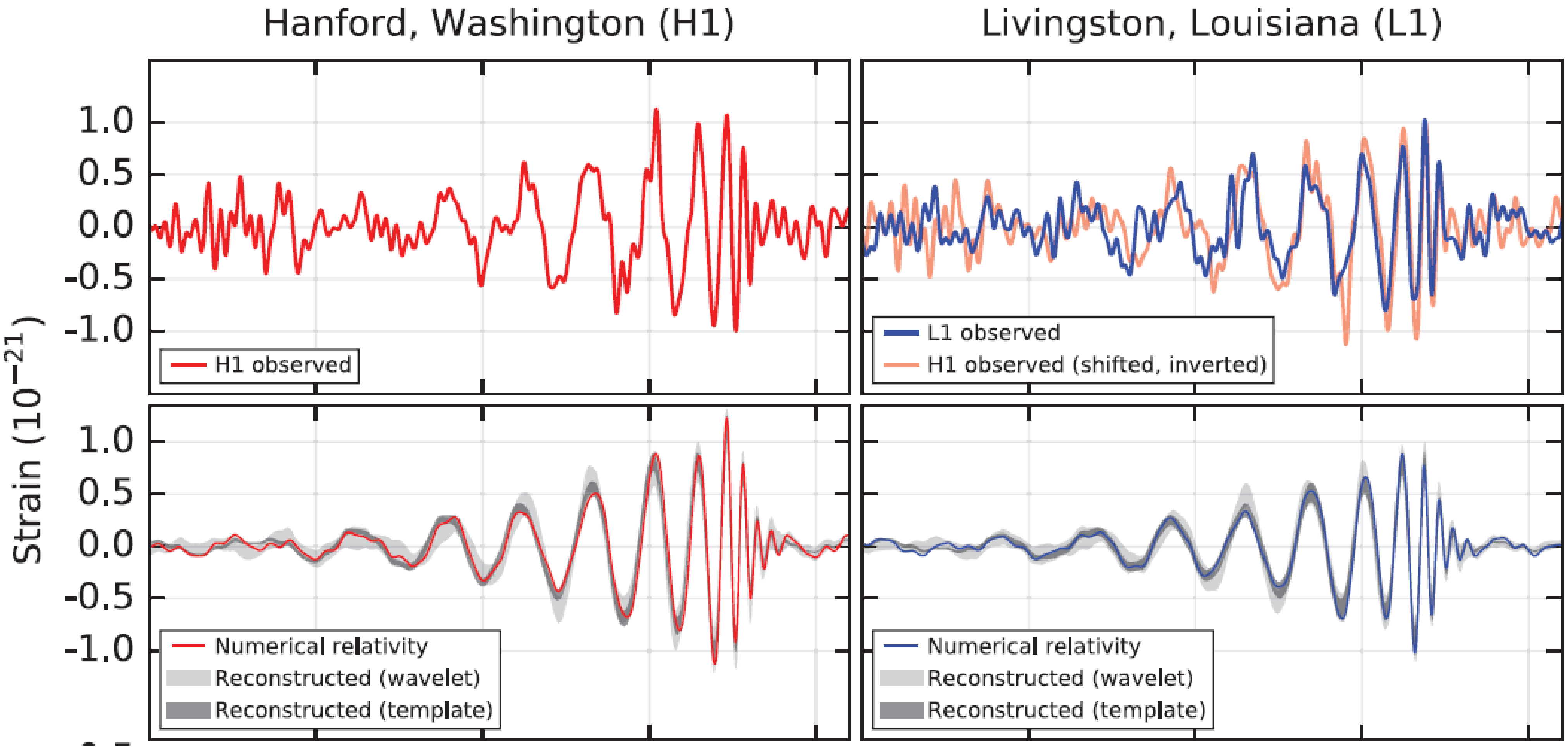
A：Directly, we can build more powerful colliders, such as the planned CEPC/SppC.

Pre CDR of CEPC



B:Indirectly, GW detectors can test Higgs potential as complementary approach.

The Dawn of the Gravitational Wave physics



Motivation from cosmology: origin of the baryon asymmetry of the universe

A long standing problem in particle cosmology is to unravel the origin of baryon asymmetry of the universe (BAU).

After the discovery of the 125 GeV Higgs, electroweak (EW) baryogenesis becomes a timely and testable scenario for explaining the BAU.

$$\eta = n_B/n_\gamma = 6.05(7) \times 10^{-10} \text{ (CMB, BBN)}$$

Baryogenesis-Sakharov conditions(1967)

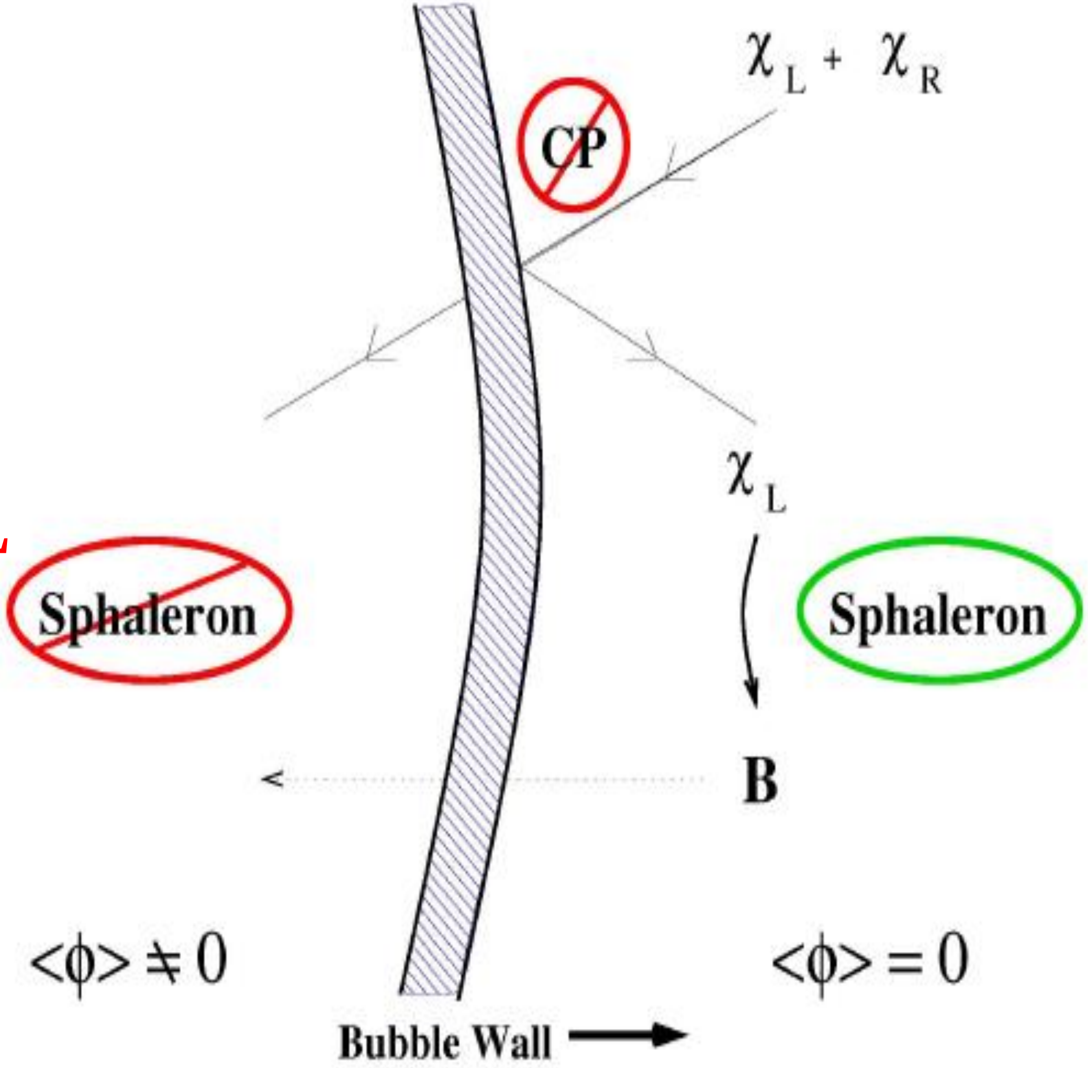
To produce the observed BAU, three necessary conditions are needed at the same cosmic epoch:

- ➢**Baryon number violation: create baryonic charge**
- ➢**C and CP violation: distinguish matter from anti-matter**
- ➢**Departure from thermal equilibrium or CPT violation: provide a time arrow**

EW baryogenesis:

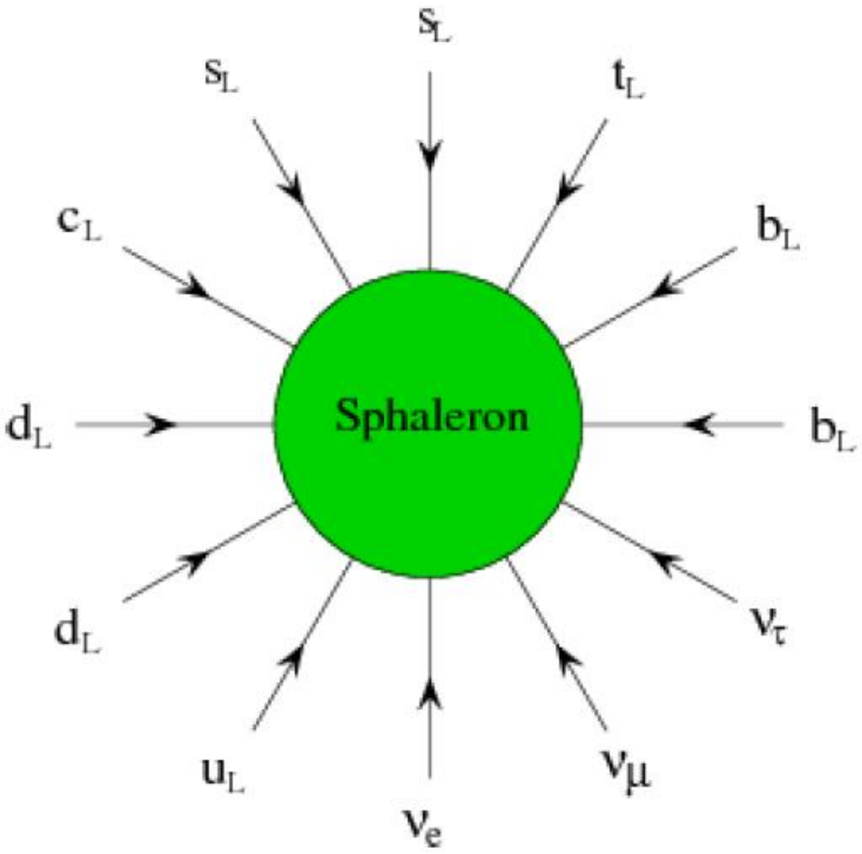
**SM technically has all the three elements for baryogenesis ,
but not enough.**

- **B violation from anomaly in B+L current.**
- **CKM matrix, but too weak.**
- **First order phase transition with expanding Higgs Bubble wall.**



From D. E. Morrissey and M. J. Ramsey-Musolf, New J. Phys. 14, 125003 (2012).

B-violation in SM——Sphaleron process



$$\partial_\mu J^\mu_{B_L+L_L} = \frac{3g^2}{32\pi^2}\epsilon_{\alpha\beta\gamma\delta}W_a^{\alpha\beta}W_a^{\gamma\delta}$$

Baryon number is violated in electroweak interactions through non-perturbative effect ,i.e.triangle anomaly in baryonic current

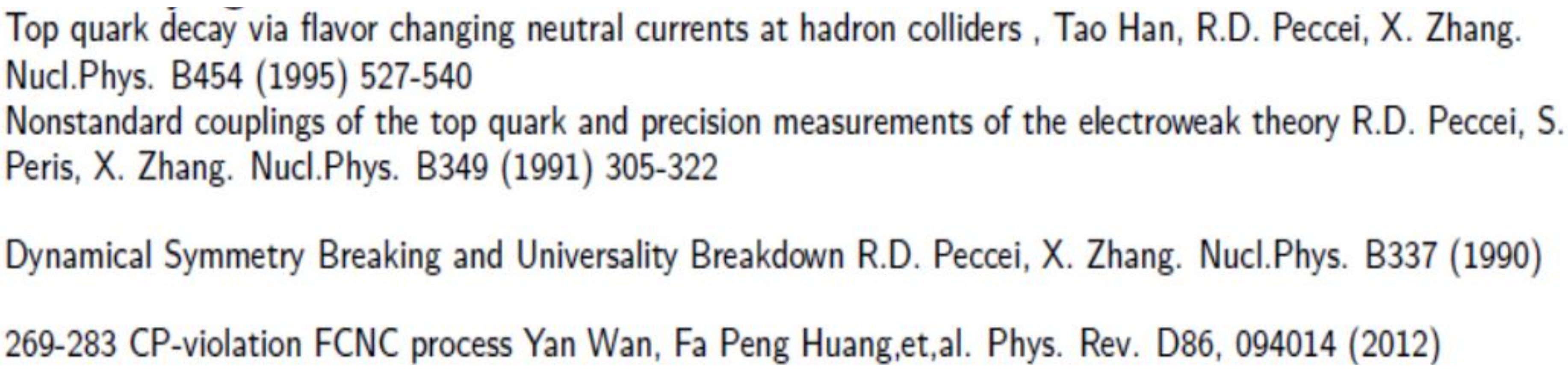
This process trades three leptons, one from each generation, for nine quarks, three within each generation, and one of each color per generation

Manton, P.R.D28(‘83)

Boris M. Kastening, R.D. Peccei, Xinmin. Zhang
 Phys.Lett. B266 (1991) 413-418

CP-violation souce

Driven by the current LHC data, it still leaves room for the anomalous top quark Yukawa coupling, which may provide the CP-violation source for EW baryogenesis.



Top quark decay via flavor changing neutral currents at hadron colliders , Tao Han, R.D. Peccei, X. Zhang.
Nucl.Phys. B454 (1995) 527-540
Nonstandard couplings of the top quark and precision measurements of the electroweak theory R.D. Peccei, S.
Peris, X. Zhang. Nucl.Phys. B349 (1991) 305-322

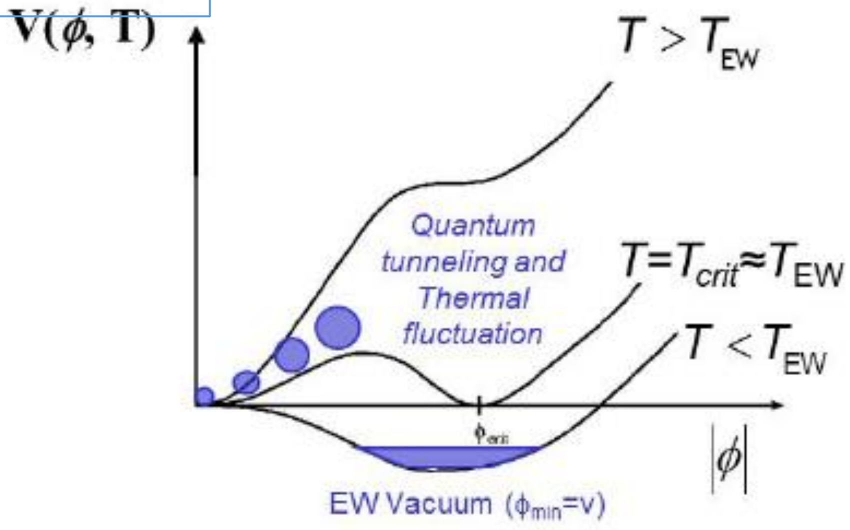
Dynamical Symmetry Breaking and Universality Breakdown R.D. Peccei, X. Zhang. Nucl.Phys. B337 (1990)

269-283 CP-violation FCNC process Yan Wan, Fa Peng Huang,et,al. Phys. Rev. D86, 094014 (2012)

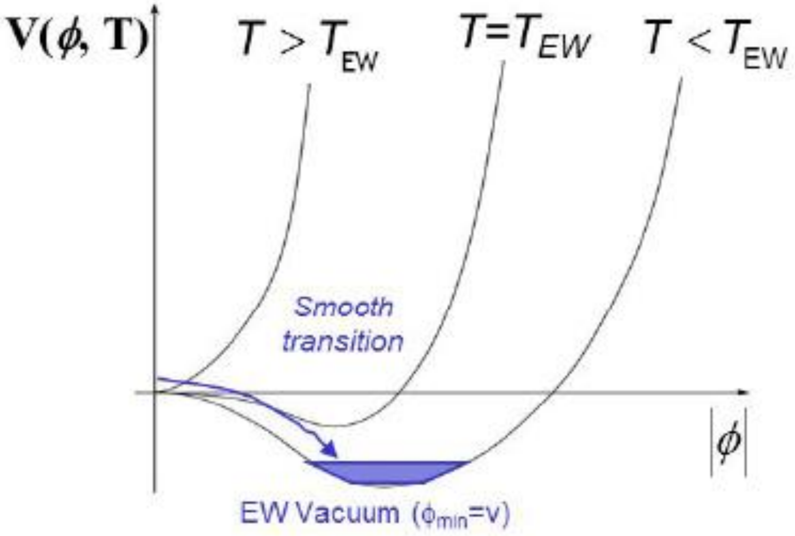
Departure from thermal equilibrium —— Strong first order phase transition

From lattice simulation

Strong First Order phase transition for $m_H < 75$ GeV



Cross over for $m_H > 75$ GeV



Extension of the Higgs sector is needed to produce strong first order phase transition for 125 GeV Higgs boson.

Operators analysis for Higgs potential and cosmological bound on Higgs mass
Xinmin Zhang **Phys.Rev. D47 (1993) 3065-3067**

Extending the Higgs sector taking the effective field theory approach

$$\delta\mathcal{L} = -x_u^{ij}\frac{\phi^\dagger\phi}{\Lambda^2}\bar{q}_{Li}\tilde{\phi}u_{Rj} + \mathrm{H.c.} - \frac{\kappa}{\Lambda^2}(\phi^\dagger\phi)^3$$

provide sizable CP violation source

provide another possible Higgs potential or EW symmetry breaking; provide strong first order phase transition

Cedric Delaunay, Christophe Grojean, James D. Wells
JHEP 0804:029,2008

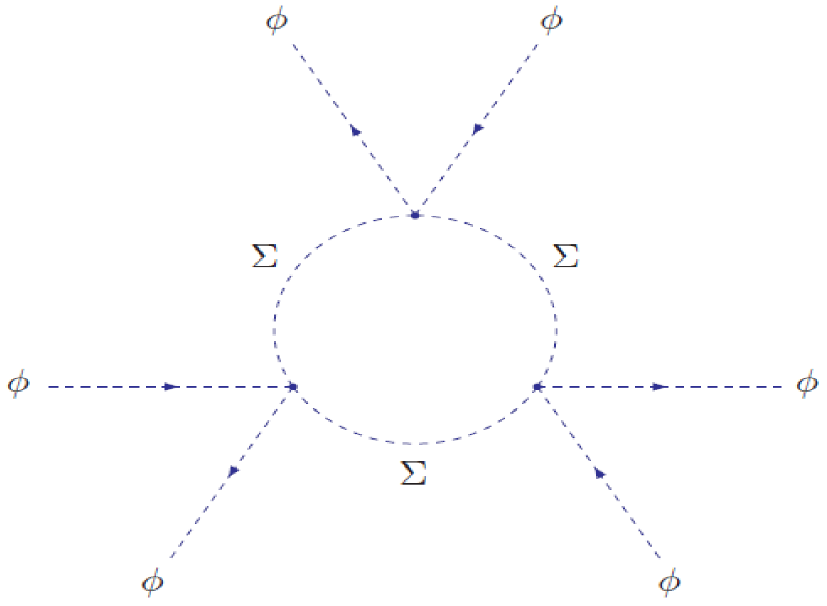
X. m. Zhang, Phys. Rev. D **47**, 3065 (1993) [hep-ph/9301277].
X. Zhang and B. L. Young, Phys. Rev. D **49**, 563 (1994) [hep-ph/9309269].
K. Whisnant, B. L. Young and X. Zhang, Phys. Rev. D **52**, 3115 (1995) [hep-ph/9410369].
X. Zhang, S. K. Lee, K. Whisnant and B. L. Young, Phys. Rev. D **50**, 7042 (1994) [hep-ph/9407259]

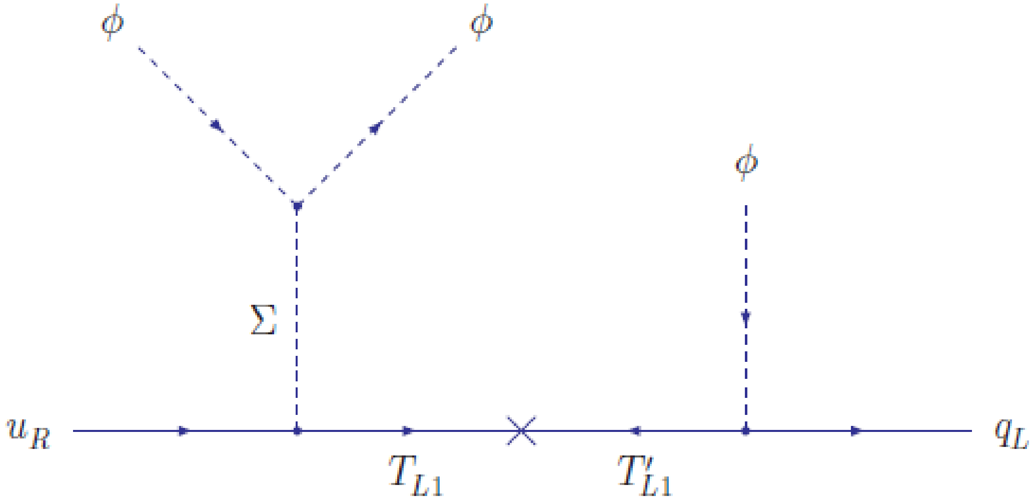
FPH, C.S. Li, Phys.Rev. D92 (2015) , 075014
FPH, P.H. Gu, P.F. Yin, Z. H. Yu, Xinmin Zhang **Phys.Rev. D93 (2016) 103515**
FPH, Y. Wan, D.G. Wang, Y.F. Cai, Xinmin Zhang **Phys. Rev. D 94, 041702(R) 2016**

Renormalizable realization of the effective Lagrangian

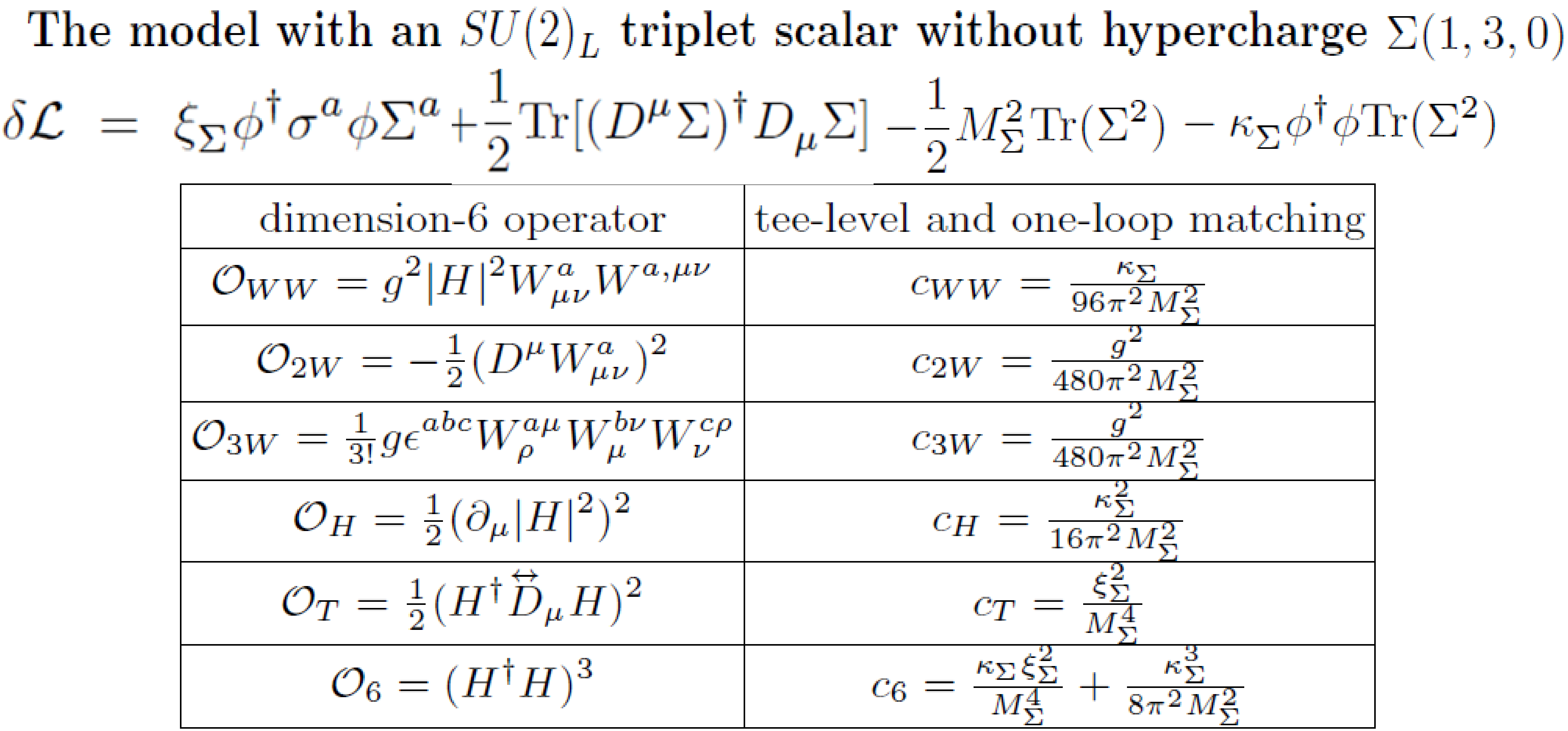
- The concerned dim-6 operators can be induced from certain renormalizable extension of the SM.
- We built simplified model with vector-like quark and triplet scalar.

model details see FPH, et. al **Phys.Rev. D93 (2016) 103515**





Renormalizable realization of the effective Lagrangian: an example to get h^6 term

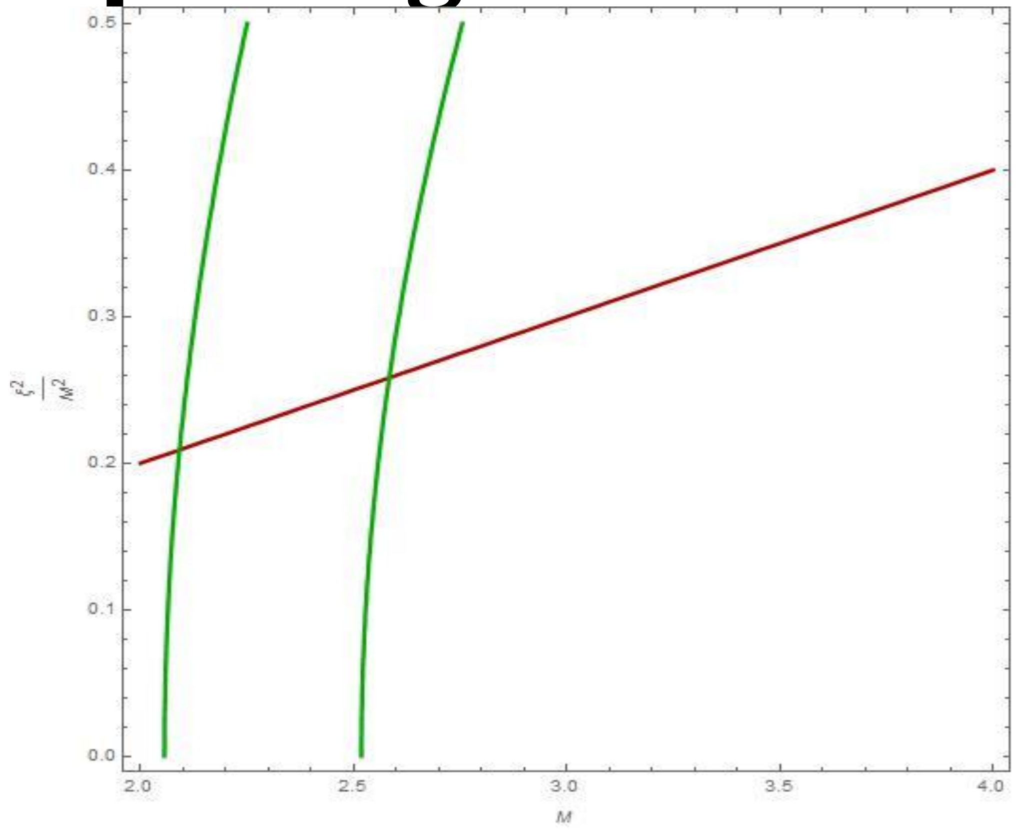


The model with an $SU(2)_L$ triplet scalar without hypercharge $\Sigma(1,3,0)$

$$\delta\mathcal{L} \;=\; \xi_\Sigma \phi^\dagger \sigma^a \phi \Sigma^a + \frac{1}{2}\mathrm{Tr}[(D^\mu\Sigma)^\dagger D_\mu \Sigma] - \frac{1}{2} M_\Sigma^2 \mathrm{Tr}(\Sigma^2) - \kappa_\Sigma \phi^\dagger \phi \mathrm{Tr}(\Sigma^2)$$

dimension-6 operator	tee-level and one-loop matching
$\mathcal{O}_{WW} = g^2 \lvert H\rvert^2 W^a_{\mu\nu} W^{a,\mu\nu}$	$c_{WW} = \frac{\kappa_\Sigma}{96\pi^2 M_\Sigma^2}$
$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$	$c_{2W} = \frac{g^2}{480\pi^2 M_\Sigma^2}$
$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon^{abc} W^{a\mu}_{\rho} W^{b\nu}_{\mu} W^{c\rho}_{\nu}$	$c_{3W} = \frac{g^2}{480\pi^2 M_\Sigma^2}$
$\mathcal{O}_H = \frac{1}{2}(\partial_\mu \lvert H\rvert^2)^2$	$c_H = \frac{\kappa_\Sigma^2}{16\pi^2 M_\Sigma^2}$
$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$	$c_T = \frac{\xi_\Sigma^2}{M_\Sigma^4}$
$\mathcal{O}_6 = (H^\dagger H)^3$	$c_6 = \frac{\kappa_\Sigma \xi_\Sigma^2}{M_\Sigma^4} + \frac{\kappa_\Sigma^3}{8\pi^2 M_\Sigma^2}$

Renormalizable realization of the effective Lagrangian:an example to get h^6 term



Considering the existed electroweak precise observables (work in progress) There exists parameter space to get the needed size of h^6 term.

Next step: compare the EFT and the UV Completed model

$T = 0.08 \pm 0.12$ $\quad T \approx -7.8 c_T$

$$\frac{\sigma_{ZH} - \sigma_{ZH}^{SM}}{\sigma_{ZH}^{SM}} \approx (0.26 c_{WW} + 0.01 c_{BB} + 0.04 c_{WB} - 0.06 c_H - 0.04 c_T)(\mathrm{TeV})^2 + 0.016 \delta_{\mathrm{h}}$$

New Higgs potential and EW phase transition

For simplicity to investigate the signals from particle colliders to GW detector, we use the effective Lagrangian form for future discussion.

$$V_{\rm tree}(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4 + \frac{\kappa}{8\Lambda^2}h^6$$

To study the EW phase transition, we need to calculate the one-loop finite temperature effective potential using the finite temperature field theory:

$$V_{\rm eff}(h,T) = V_{\rm tree}(h) + V_1^{T=0}(h) + \Delta V_1^{T\neq 0}(h,T) + V_{daisy}$$

C. Grojean, G. Servant, J. Well PRD71(2005)036001
A.Noble, M. Perelstein Phys.Rev. D78 (2008) 063518
D. Bodeker, L. Fromme,S.J. Huber, M. Seniuch,JHEP 0502 (2005) 026
D.J.H. Chung, Andrew J. Long, Lian-tao Wang Phys.Rev. D87 (2013) , 023509
Lots of discussions, sorry that I can't cover all

Strong first order phase transition leads to obvious deviation of tri-linear Higgs coupling

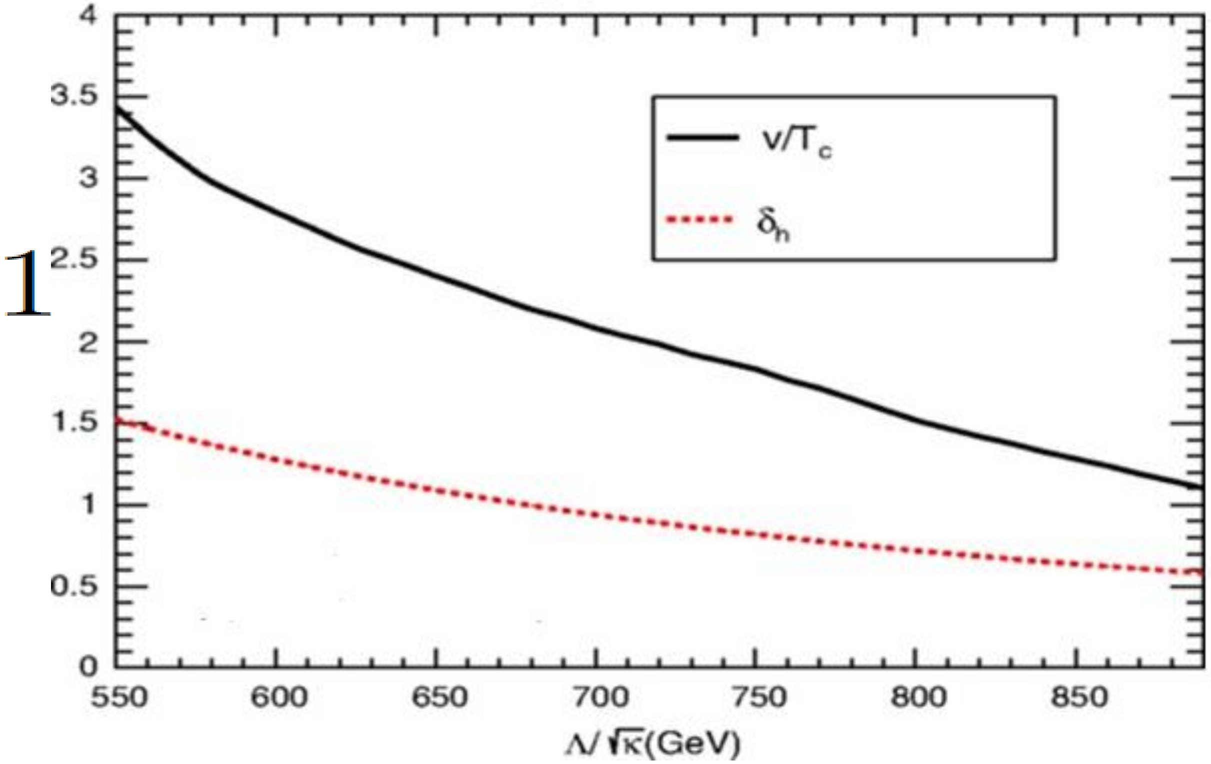
$$T_c = \frac{\sqrt{\lambda^2\Lambda^2 - 4\kappa\mu^2}}{2\sqrt{c\kappa}} > 0$$

$$\frac{v(T_c)}{T_c} = \frac{2\Lambda\sqrt{-c\lambda}}{\sqrt{\lambda^2\Lambda^2 - 4\kappa\mu^2}} \gtrsim 1$$

At one loop level, deviation of the tri-linear Higgs coupling

$$\delta_h \in (0.6, 1.5)$$

$$\mathcal{L}_{hhh} = -\frac{1}{3!}(1+\delta_h)A_h h^3$$



CP violation source

$$-x_u^{ij}\frac{\phi^\dagger\phi}{\Lambda^2}\bar{q}_{Li}\tilde{\phi}u_{Rj}+\mathrm{H.c.}$$

Provide sizable CP violation source

We only consider the top quark, and then the operator can be parameterized as

$$\mathcal{L}=-\frac{m_t}{v}h\bar{t}(1+\delta_t^+ + i\delta_t^-\gamma^5)t$$

CP violation source

The top quark acquires a complex mass inside the bubble during the phase transition:

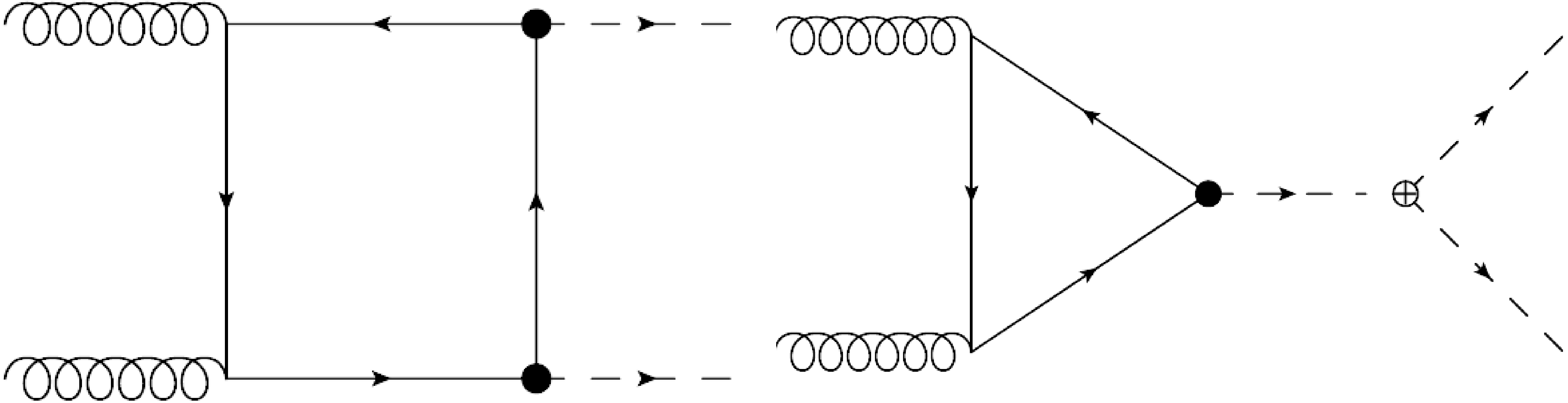
$$m_t(z) = \frac{m_t}{v}(1 + \delta_t^+ + i\delta_t^- \gamma^5)h(z)$$

$$\eta = 6.05(7) \times 10^{-10} \Longrightarrow \delta_t^- = \mathcal{O}(0.01 - 1)$$

We assume that the severe constraints from the EDM are relaxed by other new physics beyond the SM .

Hints at the large hadron collider (LHC): Higgs pair production

$$-\mathcal{L} = \frac{1}{3!}\left(\frac{3m_h^2}{v}\right)(1+\delta_h)\,h^3 \;+\; \frac{m_t}{v}h\bar{t}(1+\delta_t^+ + i\delta_t^-\gamma^5)t$$

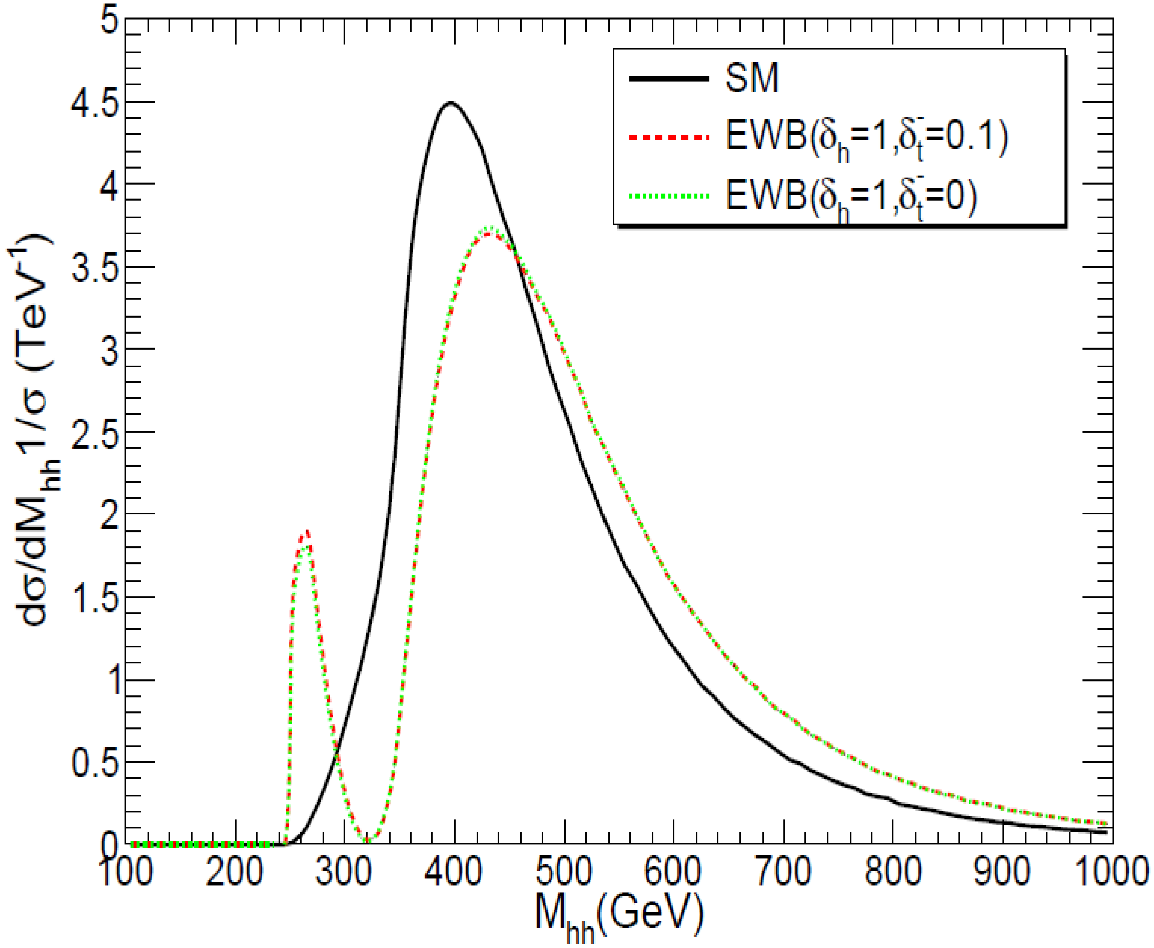


Modify the Higgs pair production at the LHC

$$
\begin{aligned}
\frac{d\hat{\sigma}(gg \to hh)}{d\hat{t}} = \frac{G_F^2 \alpha_s^2}{512(2\pi)^3} \Big\{ \Big| (1+\delta_h)(1+\delta_t^+)\mathcal{P}(\hat{s}) F_\triangle^A \\
+ (1+\delta_t^+)^2 F_\Box^{AA} + (\delta_t^-)^2 F_\Box^{BB} \Big|^2 \\
+ \Big| (1+\delta_t^+)\delta_t^- G_\Box^{AB} \Big|^2 + \Big| (1+\delta_t^+)^2 G_\Box^{AA} + (\delta_t^-)^2 G_\Box^{BB} \Big|^2 \\
+ \Big| (1+\delta_h)\delta_t^- \mathcal{P}(\hat{s}) F_\triangle^B + (1+\delta_t^+)\delta_t^- F_\Box^{AB} \Big|^2 \Big\},
\end{aligned}
$$

F and G are the 8 form factors after loop calculation, respectively.

Invariant mass distribution of Higgs pair production at the LHC induced by the dim-6 operators

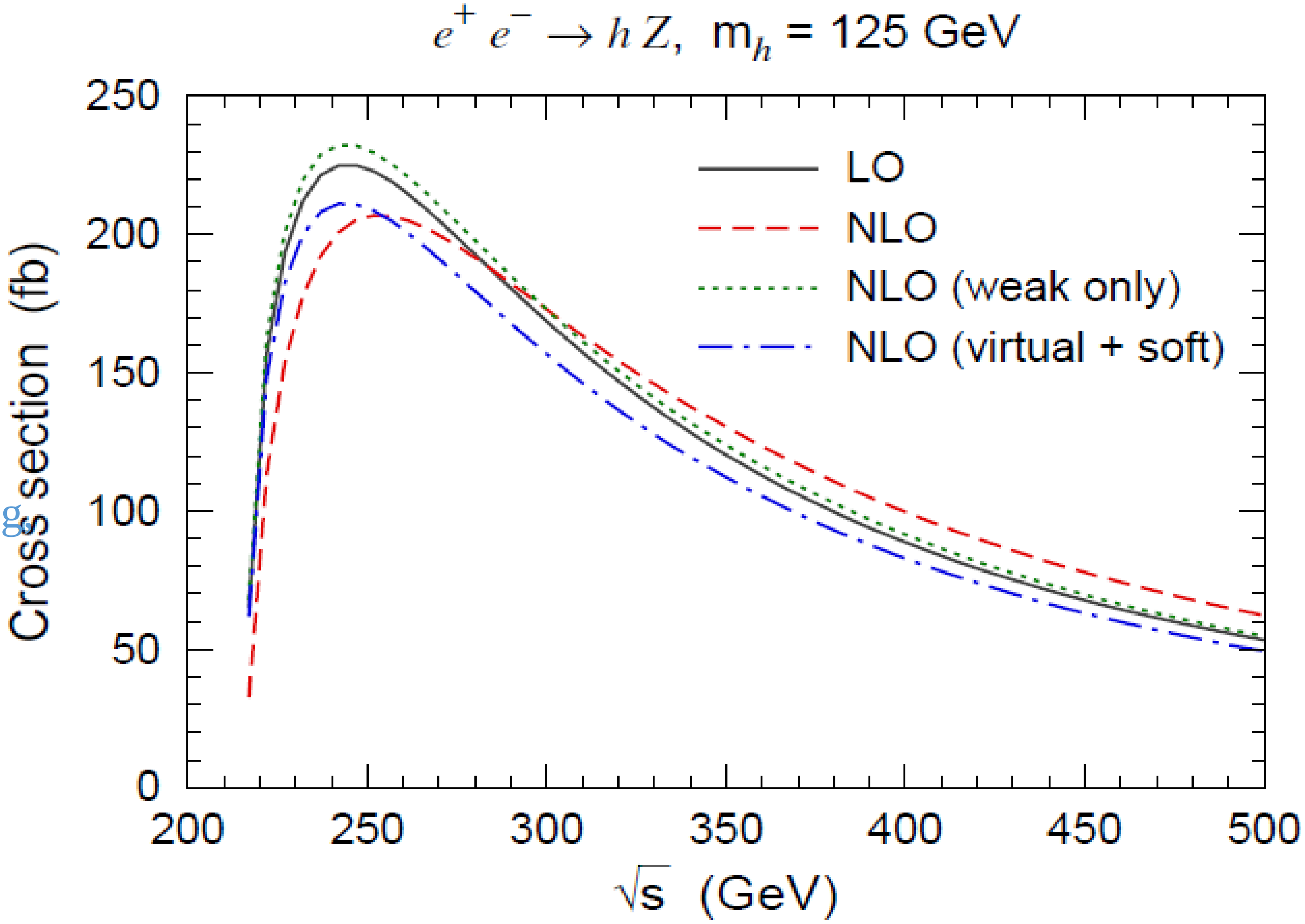


- Two peaks for the baryogenesis scenario, one peak for the SM.
- Due to the difficulties to suppress backgrounds at the LHC, it will be difficult to completely pin down these anomalous coupling at 14 TeV LHC, even with 3000 ab^{-1} integrated luminosity.
- Exploiting boosted tricks helps to increase ability to extract the anomalous couplings.
- More precise information may come from future100 TeV hadron collider, such as SppC, or future lepton collider, such as CEPC .

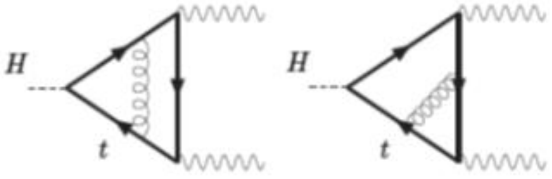
The Circular Electron Positron Collider (CEPC) can precisely test this scenario through precise measurements of the Zh production.

The NLO Zh cross-section in the SM.

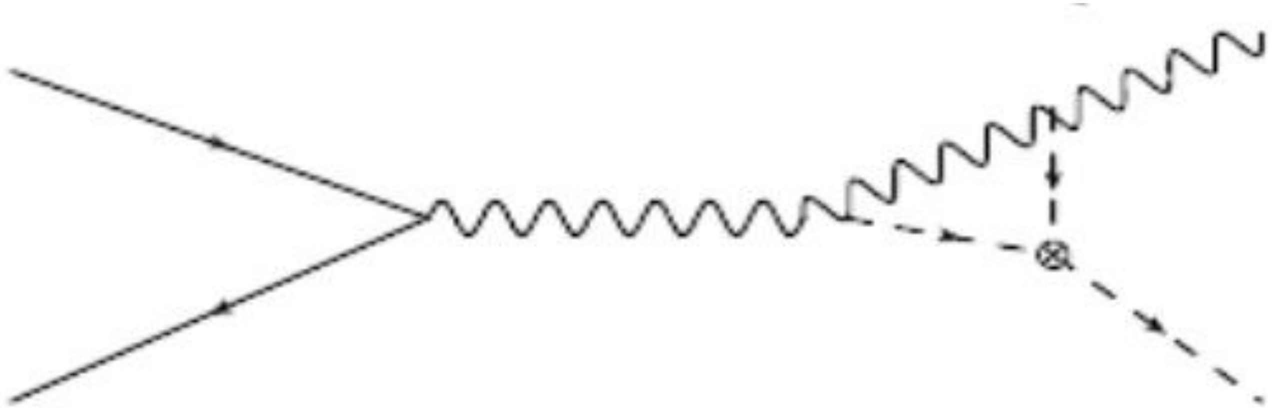
Towards NNLO Zh cross section recently by Lilin Yang et al 2016
Yu Jia et at 2016



The “simpler”: $O(\alpha\alpha_s)$

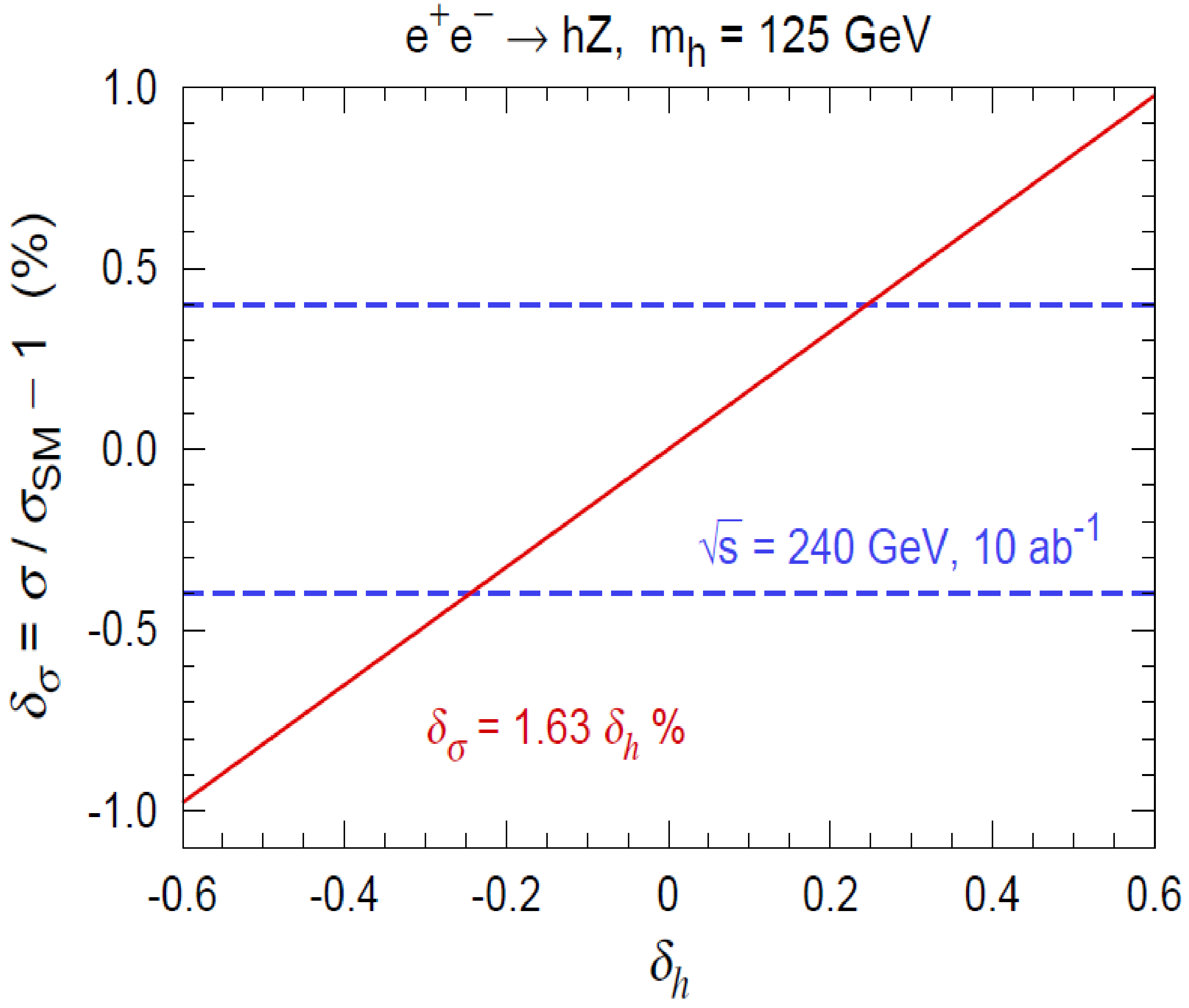


At CEPC, for the EW phase transition /baryogenesis scenario, the anomalous tri-linear Higgs coupling will contribute to the Zh production beyond the SM.



Firstly, we extract the anomalous Higgs tri-linear coupling at 240 GeV.

$$\delta_\sigma = \frac{\sigma_{hz,\delta_h\neq 0}}{\sigma_{hz,SM}} - 1$$



Pin down the tri-linear Higgs coupling

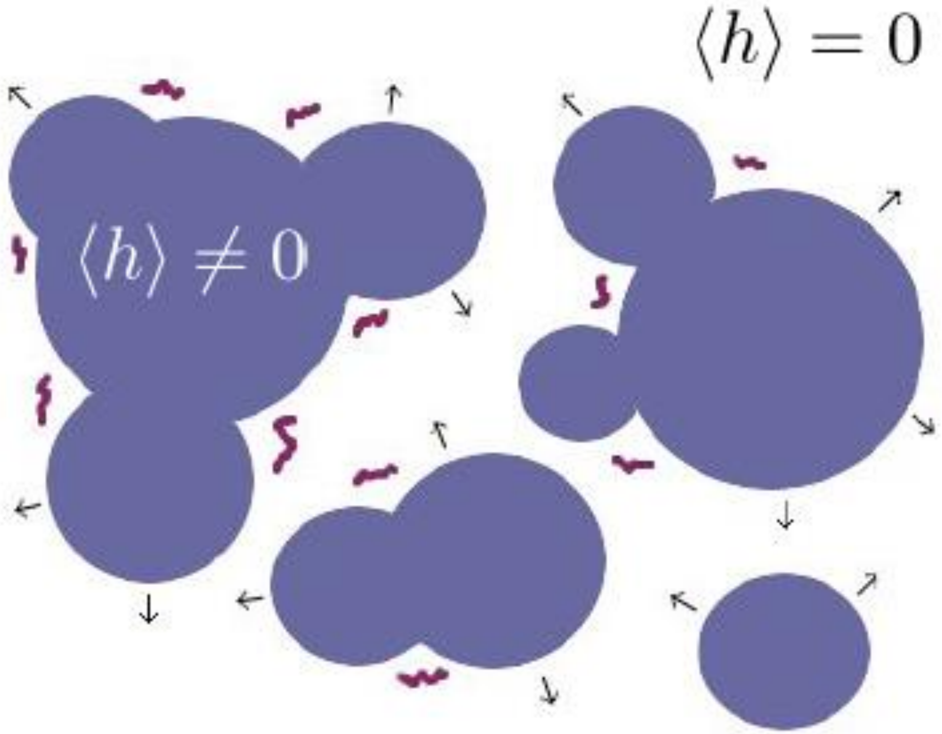
For the future CEPC with the integrated luminosity of 10 ab^{-1} at $\sqrt{s} = 240$ GeV , the precision of the σ_{zh} may be about 0.4% . Therefore, it is possible to test the δ_h ~25% at the CEPC.

Matthew McCullough arXiv:1312.3322

Since the new type of the Higgs potential (or the strong first order phase transition) leads to the modification of the tri-linear Higgs boson coupling from 0.6 to 1.5, which is well within the CEPC's precision.

Thus, the CEPC has the ability to test the shape of the Higgs potential and the type of the EW phase transition.

EW phase transition gravitational wave(GW)



E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs boson by LHC and GW by LIGO.

First order EW phase transition can drive the plasma of the early universe out of thermal equilibrium, and Higgs bubble nucleate during it, which will produce GW.

Pictures from Prof. Huber and Konstandin

Source of GW during EW phase transition

- **Bubble collision:** **well-known source**
- **Turbulence in the plasma fluid:**

  a fraction of the bubble wall energy converted into turbulence.
- **Sound wave in the plasma fluid** :

  after the collision a fraction of bubble wall energy converted into motion of the fluid (and is only later dissipated).

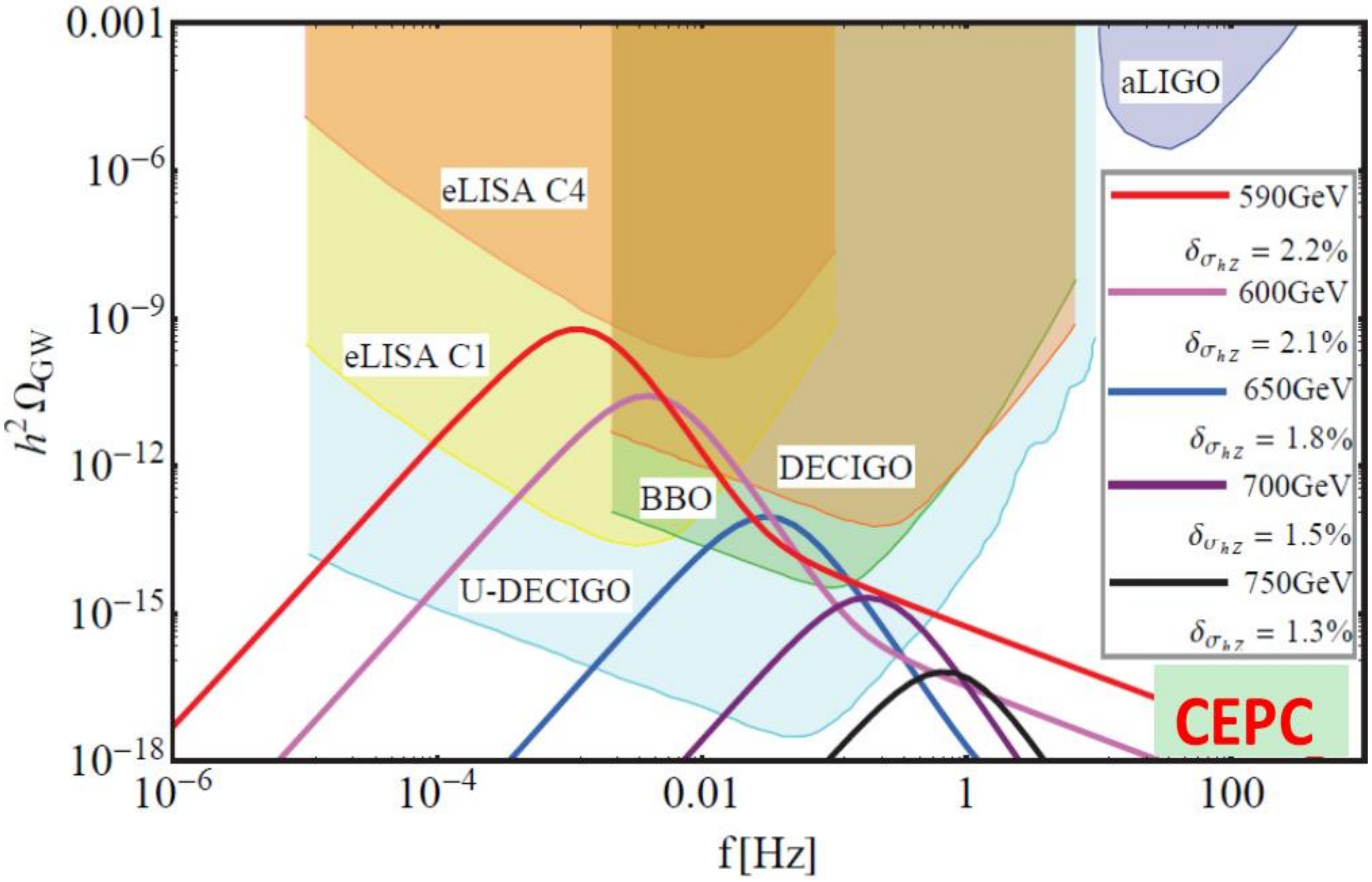
New source of GW: sound wave

M.Hindmarsh, *et al.*, PRL **112**, 041301 (2014);
G. Christophe *et al.* Phys.Rev. D75 (2007)
Caprini, Chiara *et al.* JCAP 1604 (2016)

Detectable GW signals will be produced during the EW phase transition.

Correlate particle collider and GW signals

Double test on Higgs nature from particle to wave



- **eLISA,BBO,U-DECIGO are capable of detecting the GW from EWPT**
- **The study on EWPT naturally bridges the particle physics at collider with GW survey, astrophysics and cosmology.**

We shown the combined results in this Figure:

The colored regions correspond to the expected sensitivities of GW detectors eLISA, BBO, and U-DECIGO.

The colored lines represents the GW spectrum for different cutoff scales, which also corresponds the deviation of hZ cross section at CEPC, respectively.

Firstly, we can see that the eLISA, BBO, U-DECIGO are capable of detecting the GW from EWPT.

Secondly, the study on EWPT naturally bridges the particle physics at collider with GW survey, astrophysics and cosmology. For example, the red line depicts the GW spectrum for 590 GeV, which corresponds collider signal of the deviation of hZ cross section 2.2% at the CEPC.

Last but not the least, the GW survey can provide a complementary approach to probe the nature of the EWPT alternative to particle colliders, and vice versa.

The future lepton collider and GW detector can make a double check on the nature of the Higgs boson.

Summary

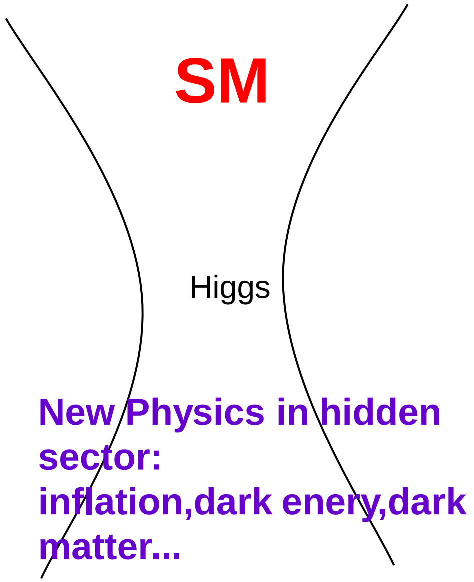
- Since the 125 GeV Higgs boson has been discovered at the LHC, it becomes a central issue to unravel the structure of the Higgs potential and the nature of the EW baryogenesis.
- The SM Higgs sector is extended using the EFT approach and a concrete renormalizable model is also built to realize the EFT . The Collider and GW signals and their correlation are investigated.
- However, there are so many possibilities, related to all possibilities of the extending the Higgs sector. Each type of extension is need to study in detail both at colliders and GW detectors. (See Andrew's talk)
- The joint analysis will contribute to deeply understand the nature of Higgs boson, which can build an deep connection between astrophysics a particle physics, and GW physics in fundamental physics.

Outlook

Theoretical phenomenology study?

A:Higgs as a portal to search for the new physics beyond the SM!

B:Unravel the nature of spontaneous symmetry breaking, the order of the EW phase transition and the true potential of the Higgs boson.



SM

Higgs

New Physics in hidden sector: inflation,dark enery,dark matter...

$$\mathcal{L} = \mathcal{O}_{hidden} H^\dagger H$$

Outlook

Experimental test?

Two complementary approaches to study the Higgs physics:

- **Particle Colliers, such as CEPC/SppC:**

to precisely measure the Higgs boson.

- **Gravitational Wave Detectors, such as eLisa, Ali:**

The LIGO observation has initiated a new era of exploring fundamental physics with GW detectors.

Thanks for your attention !

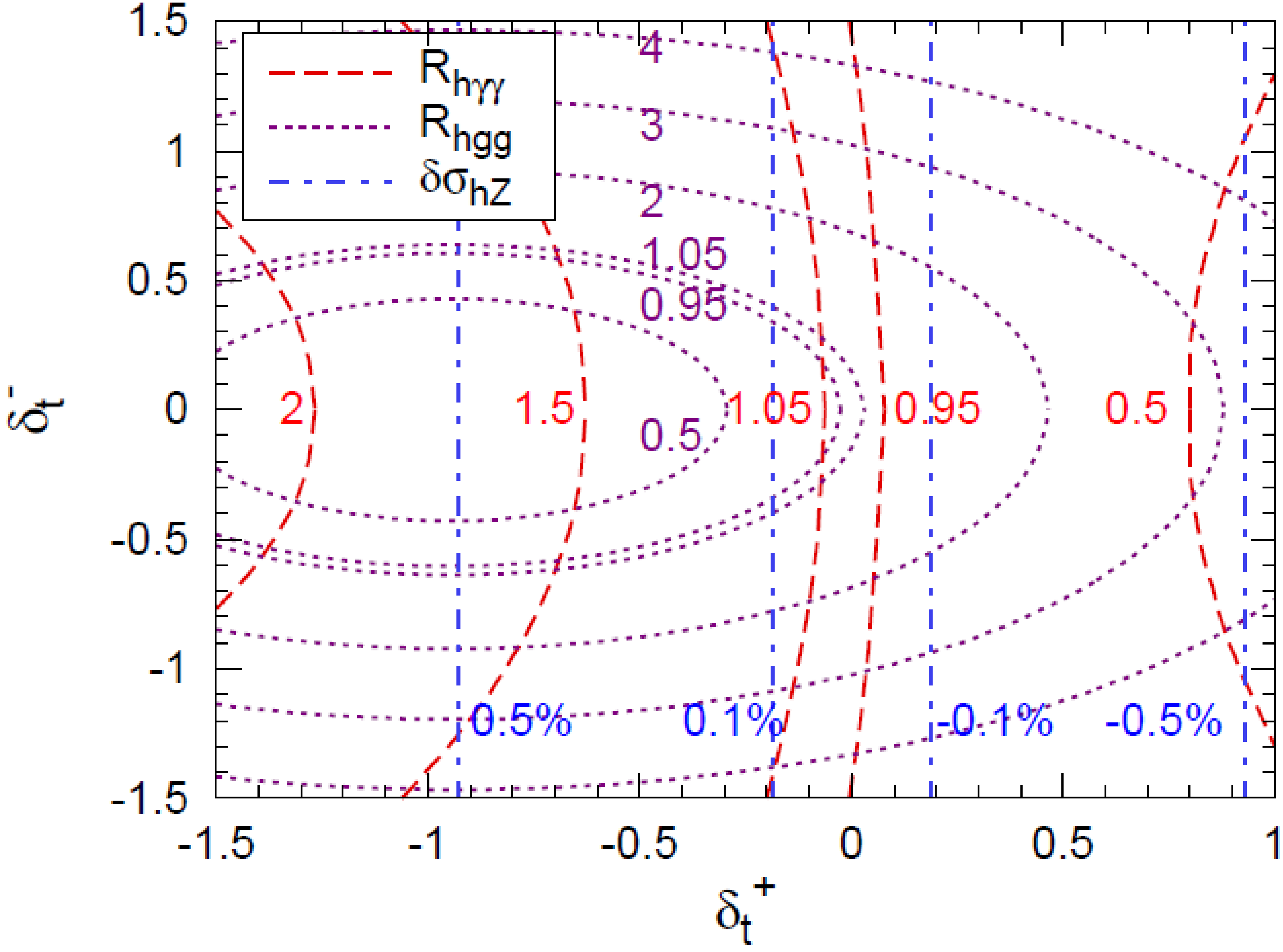
What is the truth of the Higg boson ?

Backup Slides

Challenging to test the anomalous top quark Yukawa coupling : need precise measurements of the Higgs partial decay width

$$\begin{aligned} g^2_{hgg}/g^2_{hgg,SM} &\simeq (1+\delta_t^+)^2 + 0.11\delta_t^+(1+\delta_t^+) + 2.6(\delta_t^-)^2 \\ g^2_{h\gamma\gamma}/g^2_{h\gamma\gamma,SM} &\simeq (1-0.28\delta_t^+)^2 + (0.43\delta_t^-)^2. \end{aligned}$$

$$\begin{aligned} R_{hXX} &= \frac{\sigma_h \times Br(h \to XX)}{\sigma_h^{SM} \times Br(h \to XX)^{SM}} \\ &= \frac{\sigma_h}{\sigma_h^{SM}} \frac{\Gamma_{hXX}}{\Gamma_{hXX}^{SM}} \frac{\Gamma_{tot}^{SM}}{\Gamma_{tot}}, \end{aligned}$$



$R_{h\gamma\gamma}$
R_{hgg}
$\delta\sigma_{hZ}$
4
3
2
1.05
0.95
0.5
2
1.5
1.05
0.95
0.5
0.5%
0.1%
-0.1%
-0.5%
δ_t^-
δ_t^+
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1

Dynamics of the EW phase transition

At the critical temperature Tc, the two minma are degenerate.
Phase transition and bubble nucleate at T<Tc with rate :

$$\Gamma = A T^4 e^{-S_3/T}$$

Here, the Higgs bubble energy is

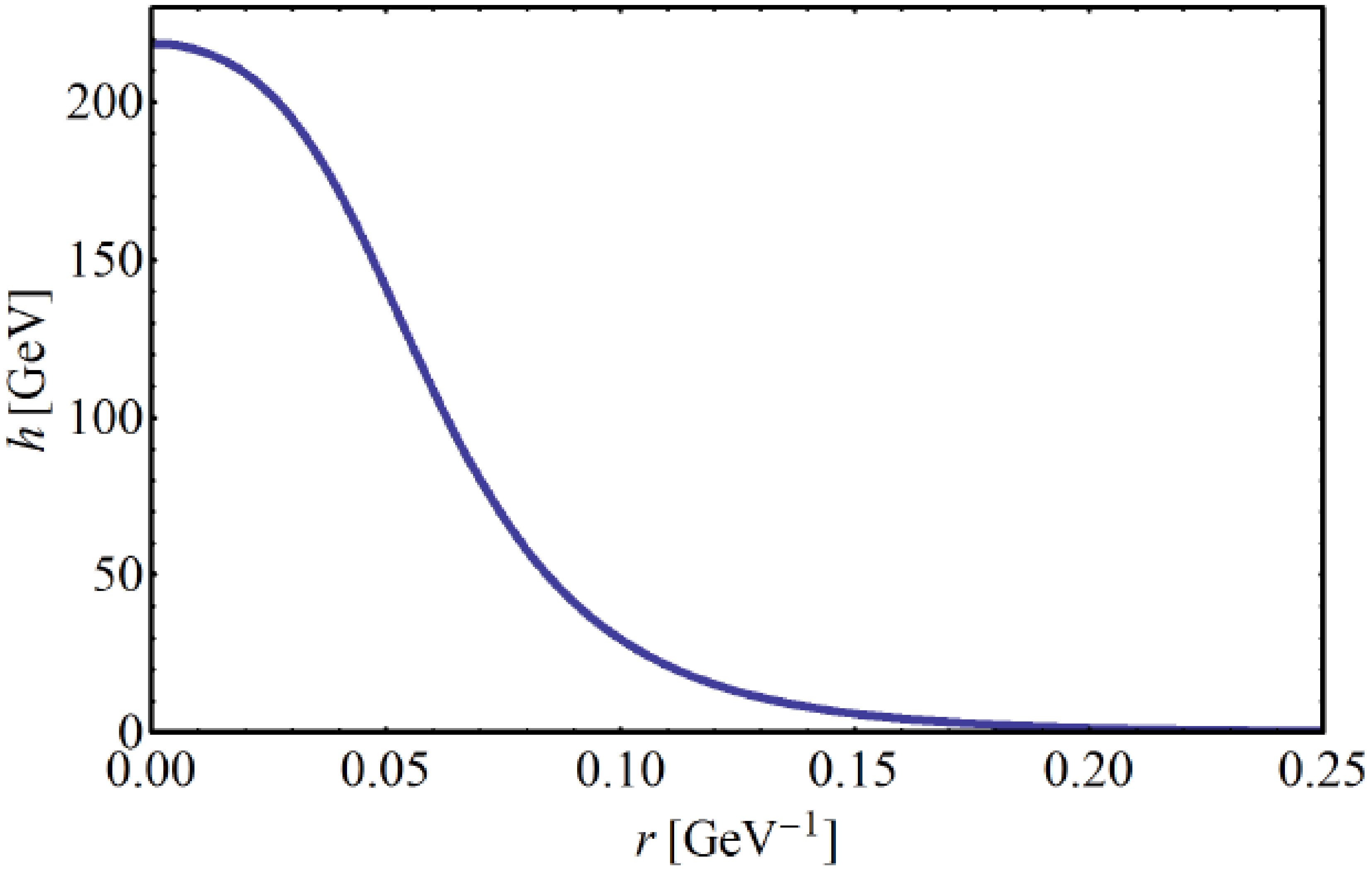
$$S_3 = \int dr 4\pi r^2 \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r}\frac{d\phi_b}{dr} = \frac{\partial V(\phi_b, T)}{\partial \phi_b}$$

$$\phi_b(r \to \infty) = 0 \text{ and } \frac{d\phi_b(r=0)}{dr} = 0$$

Profile of the Higgs field

Using the over-shoot/ Under-shoot method, we can obtain the Higgs profile. For T=51.94 GeV and Λ=600 GeV.



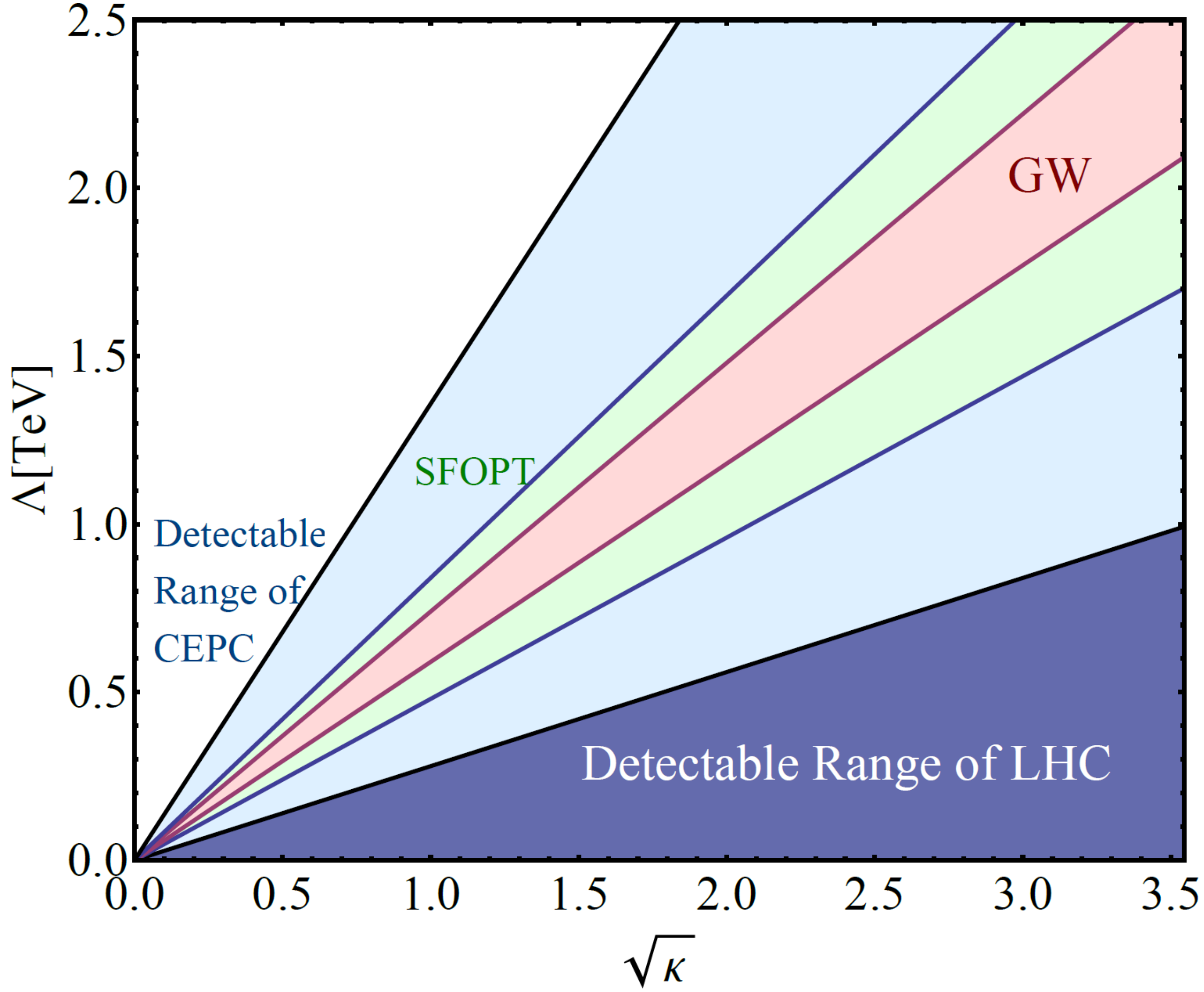
Dynamics of the EW phase transition

Two import parameters α *and* β

$$\alpha \equiv \frac{\epsilon(T)}{\rho_{\text{rad}}(T)} \quad \tilde{\beta} \equiv \frac{\beta}{H_*} = T_* \, \frac{dS}{dT}\bigg|_{T_*} = T_* \frac{d}{dT} \left(\frac{S_3}{T} \right) \bigg|_{T_*}$$

$$v_b(\alpha) = \frac{\frac{1}{\sqrt{3}} + \sqrt{\alpha^2 + \frac{2\alpha}{3}}}{1 + \alpha}$$

Complementary probing of the electroweak phase transition with relic gravitational wave



The observational abilities of different from particle colliders to GW detectors.

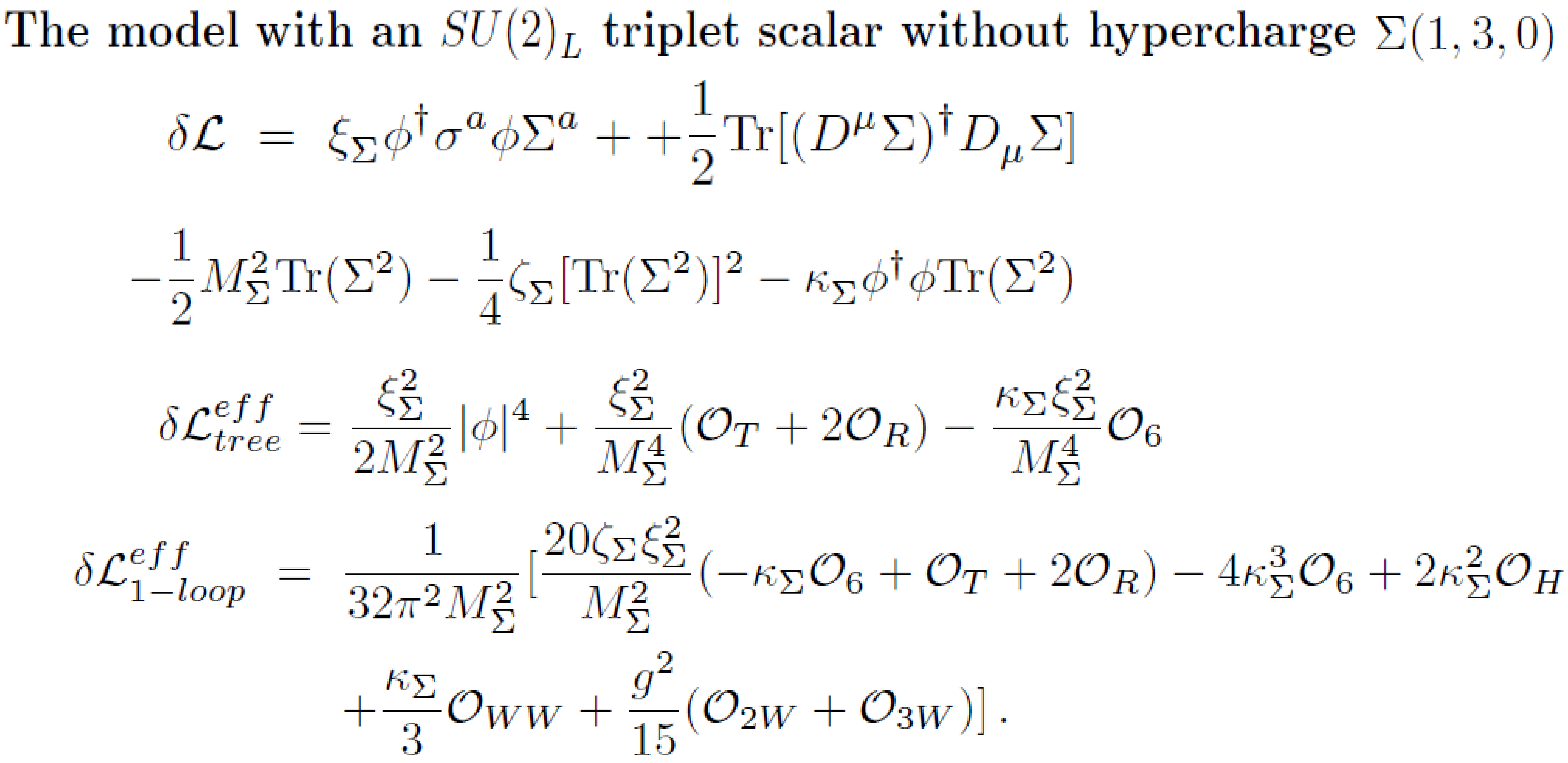
The nature of Higgs helps to understand the universe

Well understanding on Nuclear physics leads to well understanding of the BBN process in the history of universe (MeV)

electroweak physics

Well understanding on electroweak physics leads to well understanding of the electroweak transition and baryon asymmetry of the universe in the history of universe (hundred GeV)

Renormalizable realization of the effective Lagrangian: an example to get h^6 term



The model with an $SU(2)_L$ triplet scalar without hypercharge $\Sigma(1,3,0)$

$$\delta\mathcal{L} = \xi_\Sigma \phi^\dagger \sigma^a \phi \Sigma^a + + \frac{1}{2}\mathrm{Tr}[(D^\mu \Sigma)^\dagger D_\mu \Sigma]$$

$$-\frac{1}{2}M_\Sigma^2 \mathrm{Tr}(\Sigma^2) - \frac{1}{4}\zeta_\Sigma [\mathrm{Tr}(\Sigma^2)]^2 - \kappa_\Sigma \phi^\dagger \phi \mathrm{Tr}(\Sigma^2)$$

$$\delta\mathcal{L}_{tree}^{eff} = \frac{\xi_\Sigma^2}{2M_\Sigma^2}|\phi|^4 + \frac{\xi_\Sigma^2}{M_\Sigma^4}(\mathcal{O}_T + 2\mathcal{O}_R) - \frac{\kappa_\Sigma \xi_\Sigma^2}{M_\Sigma^4}\mathcal{O}_6$$

$$\delta\mathcal{L}_{1-loop}^{eff} = \frac{1}{32\pi^2 M_\Sigma^2}[\frac{20\zeta_\Sigma \xi_\Sigma^2}{M_\Sigma^2}(-\kappa_\Sigma \mathcal{O}_6 + \mathcal{O}_T + 2\mathcal{O}_R) - 4\kappa_\Sigma^3 \mathcal{O}_6 + 2\kappa_\Sigma^2 \mathcal{O}_H$$

$$+\frac{\kappa_\Sigma}{3}\mathcal{O}_{WW} + \frac{g^2}{15}(\mathcal{O}_{2W} + \mathcal{O}_{3W})]\,.$$