

Baryon-Strangeness Correlations in Au+Au Collisions at RHIC BES energies from UrQMD Model



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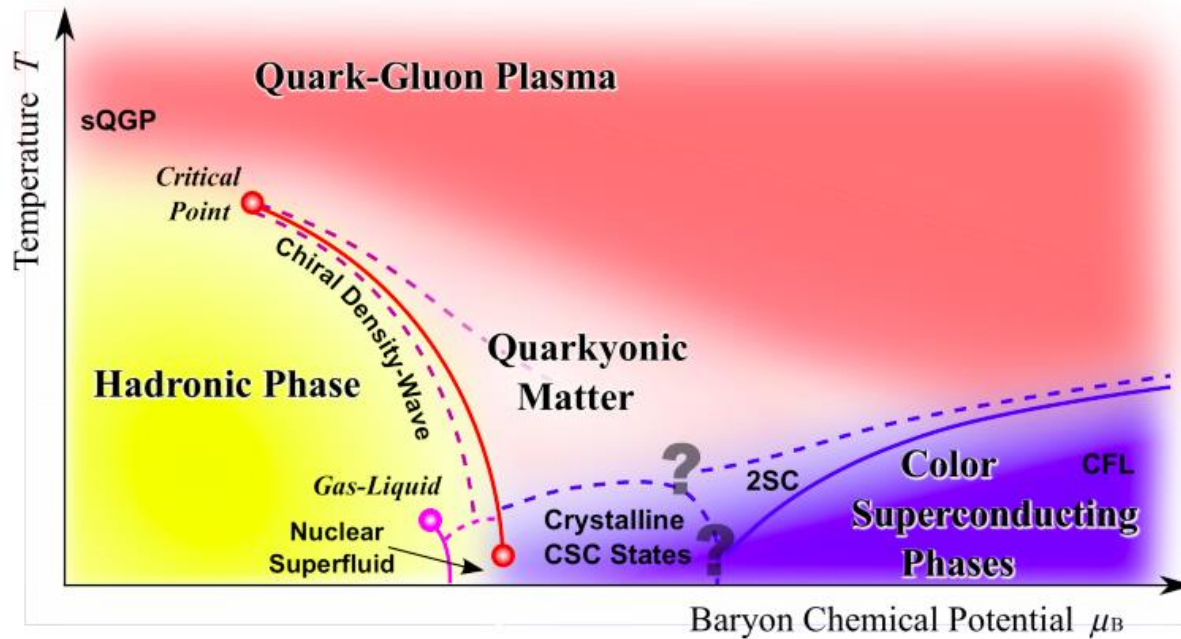
arXiv: 1610.07580

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Outline

- Motivation
- Analysis Details
- Results From UrQMD Model
- Summary



- LQCD calculations show that various order netS fluctuations and their correlations with netB and netQ are sensitive to phase transition.
- One of the STAR experimental at RHIC is to measure the B-S correlations to study the QCD phase transition.
- UrQMD model may provide a baseline for the experimental measure.



Mixed-Cumulants: Related to the Susceptibility

➤ Pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, \mu_B, \mu_Q, \mu_S)$

➤ Susceptibility: $\chi_{mn}^{XY}(T, \vec{\mu}) = \frac{\partial^{(m+n)} [P/T^4]}{\partial(\mu_X/T)^m \partial(\mu_Y/T)^n} \Big|_{\vec{\mu} = 0}$

where X, Y = net - baryon, net - charge, net - strangeness

$$\chi_{0n}^{XY} = \chi_n^Y \quad \text{and} \quad \chi_{m0}^{XY} = \chi_m^X.$$

➤ Mixed-cumulants of the conserved quantities:

$$C_{mn}^{XY} = VT^3 \chi_{mn}^{XY}(T, \vec{\mu}).$$

Observables: Mixed-Cumulant Ratios

$$\chi_{mn}^{BS} = \frac{1}{VT^3} C_{mn}^{BS}$$

$$\frac{\chi_{mn}^{BS}}{\chi_{m+n}^S} = \frac{(-1)^n}{3^m}$$

where $m + n = 2, 4$

B = the number of net - baryon

S = net - strangeness

In order to cancel
volume dependence.



Reference :

A.Bazavov, H.-T.Ding, *et al*,
Phys. Rev. Lett. 111, 082301
(2013).

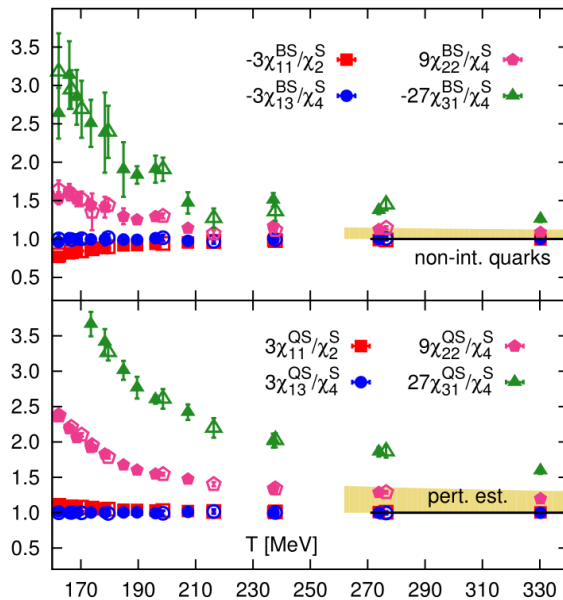
Normalized ratios:

$$-3 \frac{\chi_{11}^{BS}}{\chi_2^S} = R_{11}^{BS} = -3 \frac{C_{11}^{BS}}{C_2^S}$$

$$-3 \frac{\chi_{13}^{BS}}{\chi_4^S} = R_{13}^{BS} = -3 \frac{C_{13}^{BS}}{C_4^S}$$

$$9 \frac{\chi_{22}^{BS}}{\chi_4^S} = R_{22}^{BS} = 9 \frac{C_{22}^{BS}}{C_4^S}$$

$$-27 \frac{\chi_{31}^{BS}}{\chi_4^S} = R_{31}^{BS} = -27 \frac{C_{31}^{BS}}{C_4^S}$$



- B-S (top) and Q-S (bottom) correlations from Lattice QCD.
- In the non-interacting quark gas, all these ratios are unity .
- Higher order ratios are more sensitive.



Mixed-Cumulant Ratios Calculation

➤ The joint-cumulant of random variables, X_1, X_2, \dots, X_n ($n \geq 2$):

$$C(X_1, X_2, \dots, X_n) = \sum_{\pi} (|\pi| - 1)! (-1)^{|\pi|-1} \prod_{B \in \pi} E \left(\prod_{i \in B} X_i \right)$$

➤ The mixed-cumulant ratios of B and S are:

$$R_{11}^{BS} = -3 \frac{C_{11}^{BS}}{C_2^S} = -3 \frac{C(B, S)}{C(S, S)} = -3 \frac{\langle \delta B \delta S \rangle}{\langle (\delta S)^2 \rangle} = -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2}$$

$$R_{13}^{BS} = -3 \frac{C_{13}^{BS}}{C_4^S} = -3 \frac{C(B, S, S, S)}{C(S, S, S, S)} = -3 \frac{\langle \delta B (\delta S)^3 \rangle - 3 \langle \delta B \delta S \rangle \langle (\delta S)^2 \rangle}{\langle (\delta S)^4 \rangle - 3 \langle (\delta S)^2 \rangle^2}$$

$$R_{22}^{BS} = 9 \frac{C_{22}^{BS}}{C_4^S} = 9 \frac{C(B, B, S, S)}{C(S, S, S, S)} = 9 \frac{\langle (\delta B)^2 (\delta S)^2 \rangle - 2 \langle \delta B \delta S \rangle^2 - \langle (\delta B)^2 \rangle \langle (\delta S)^2 \rangle}{\langle (\delta S)^4 \rangle - 3 \langle (\delta S)^2 \rangle^2}$$

$$R_{31}^{BS} = -27 \frac{C_{31}^{BS}}{C_4^S} = -27 \frac{C(B, B, B, S)}{C(S, S, S, S)} = -27 \frac{\langle (\delta B)^3 \delta S \rangle - 3 \langle \delta B \delta S \rangle \langle (\delta B)^2 \rangle}{\langle (\delta S)^4 \rangle - 3 \langle (\delta S)^2 \rangle^2}$$



Error Calculation

$$f_{m,n} = \langle B^m S^n \rangle$$

$$F_{m,n} = \langle (\delta B)^m (\delta S)^n \rangle = \frac{\partial F_{m,n}}{\partial f_{i,j}} f_{i,j}$$

$$= \sum_{i=0}^m \sum_{j=0}^n C_m^i C_n^j (-1)^{m+n-i-j} f_{1,0}^{m-i} f_{0,1}^{n-j} f_{i,j}$$

Reference : Xiaofeng Luo,
Phys. Rev. C. 91, 034907 (2015).



General Error propagation formula :

$$V(\phi) = \sum_{i=1, j=1}^n \frac{\partial \phi(X_1, \dots, X_n)}{\partial X_i} \frac{\partial \phi(X_1, \dots, X_n)}{\partial X_j} \text{Cov}(X_i, X_j)$$

The covariance of multivariate moments :

$$\text{Cov}(f_{i,j}, f_{k,h}) = \frac{1}{N} (f_{i+k, j+h} - f_{i,j} f_{k,h})$$



G. Maurice and M. A. Kendall, The
Advanced Theory of Statistic, Vol.
Ind ed. (Charles Griffin & Company,
London, 1945)

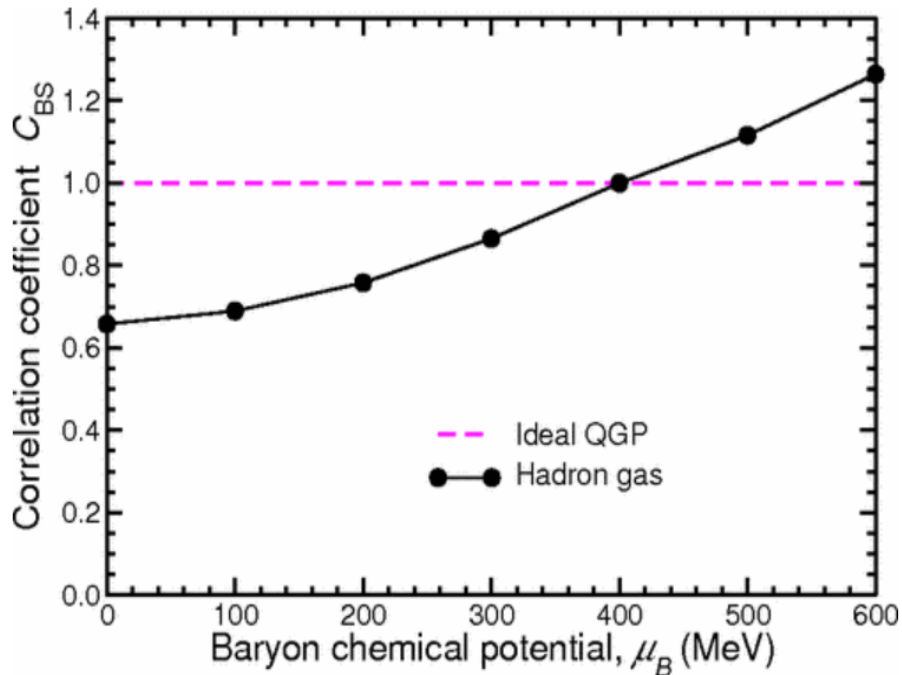
$$R_{11}^{BS} = -3 \frac{C_{11}^{BS}}{C_2^S} = -3 \frac{F_{1,1}}{F_{0,2}} = -3 \frac{f_{1,1} - f_{1,0} f_{0,1}}{f_{0,2} - f_{0,1}^2}$$

$$\frac{\partial R_{11}^{BS}}{\partial f_{i,j}} = \frac{\partial R_{11}^{BS}}{\partial F_{1,1}} \frac{\partial F_{1,1}}{\partial f_{i,j}} + \frac{\partial R_{11}^{BS}}{\partial F_{0,2}} \frac{\partial F_{0,2}}{\partial f_{i,j}} = \frac{-3}{C_{02}^{BS}} \frac{\partial F_{1,1}}{\partial f_{i,j}} + \frac{3C_{11}^{BS}}{(C_{02}^{BS})^2} \frac{\partial F_{0,2}}{\partial f_{i,j}}$$

$$\text{Error}(R_{11}^{BS}) = \sqrt{V(R_{11}^{BS})} = \sqrt{\sum_{i,k=0}^1 \sum_{j,h=0}^2 \frac{\partial R_{11}^{BS}}{\partial f_{i,j}} \frac{\partial R_{11}^{BS}}{\partial f_{k,h}} \text{Cov}(f_{i,j}, f_{k,h})}$$



Results from HRG Model



Reference:

V. Koch, A. Majumder, and J. Randrup,
Phys.Rev.Lett.95,182301(2005).

$$C_{BS} = R_{11}^{BS} = -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} \stackrel{\langle s \rangle=0}{\Rightarrow} -3 \frac{\langle BS \rangle}{\langle S^2 \rangle}$$

➤ For uncorrelated hadron gas:

$$R_{11}^{BS} = -3 \frac{\sum_k \sigma_k^2 B_k S_k}{\sum_k \sigma_k^2 S_k^2} \underset{Poisson}{\approx} -3 \frac{\sum_k \langle n_k \rangle B_k S_k}{\sum_k \langle n_k \rangle S_k^2}$$

C_{BS} increases as μ_B increasing.

➤ For non-interacting quark-gluon plasma:

$$C_{BS} = R_{11}^{BS} = \frac{\langle (u + d + s)(s) \rangle}{\langle s^2 \rangle} = 1$$



Analysis Details

$\sqrt{s_{NN}}$ (GeV)	7.7	11.5	19.6	27	39	62.4	200
Statistics(million)	72.5	105	106	81	133	38	56

- UrQMD (Ultra Relativistic Quantum Molecular Dynamics) model is a microscopic transport model.
- For centrality divided: charged particles ($0.5 < |\eta| < 1.0$)
- **For the analysis : ($|\eta| < 0.5$)**
 - The particle multiplicities : net - baryon number: $B = N_B - N_{\bar{B}}$
net - strangeness : $S = N_S - N_{\bar{S}}$
 - The weighted mean values: $\langle B \rangle$, $\langle S \rangle$, $\langle B^m S^n \rangle$
 - Observables: $C_{mn}^{BS} (C_{11}^{BS}, C_{13}^{BS}, C_{22}^{BS}, C_{31}^{BS}, C_2^S, C_4^S)$
 $R_{mn}^{BS} (R_{11}^{BS}, R_{22}^{BS}, R_{13}^{BS}, R_{31}^{BS})$



Particle Details

- For the B-S correlations: $p, n, K^+, K^0, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$ and corresponding anti-particles are included.

Particle		Component	Mass (GeV/c ²)	PID	Baryon Number	Strangeness
proton	non-strange baryons	uud	0.938	2212	1	0
neutron		udd	0.939	2112	1	0
K ⁺	strange mesons	u \bar{s}	0.493	321	0	1
K ⁰		d \bar{s}	0.497	311	0	1
Λ	strange baryons	uds	1.115	3122	1	-1
Σ^-		dds	1.197	3112	1	-1
Σ^0		uds	1.192	3212	1	-1
Σ^+		uus	1.189	3222	1	-1
Ξ^-		dss	1.321	3312	1	-2
Ξ^0		uss	1.314	3322	1	-2
Ω^-		sss	1.672	3334	1	-3



Particle Details

- In order to study the contributions of different particle species to the B-S correlations, We consider several situations.

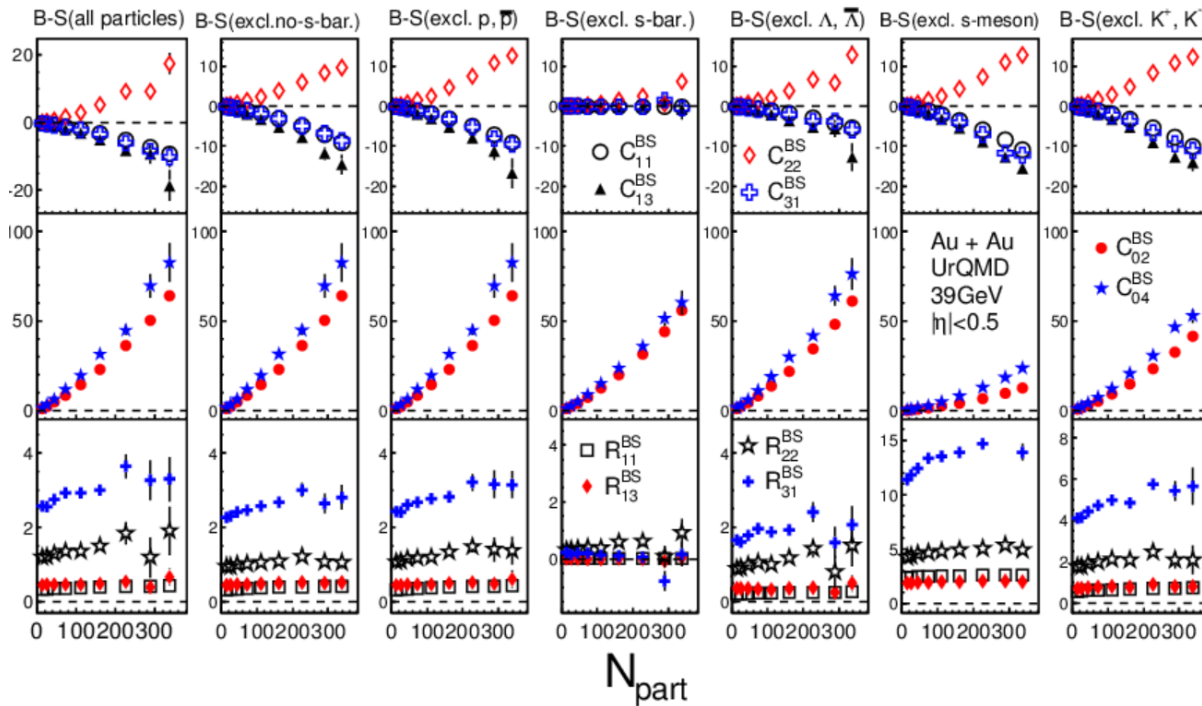
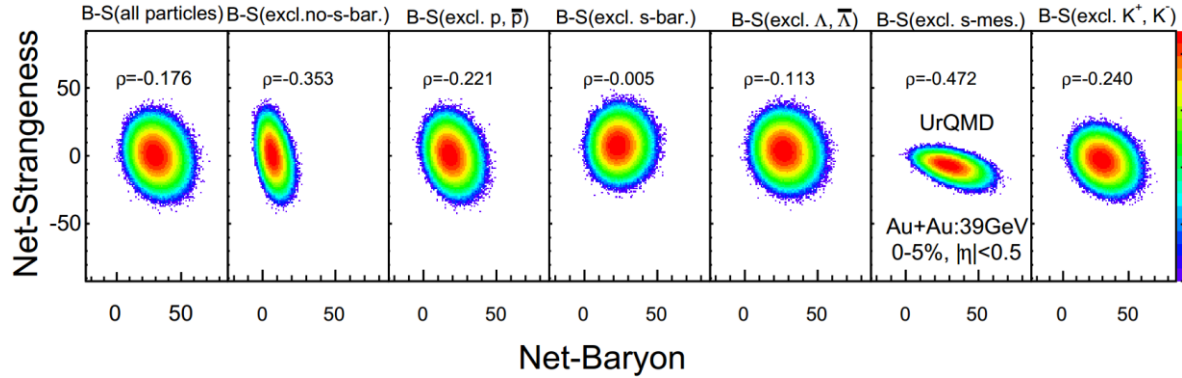
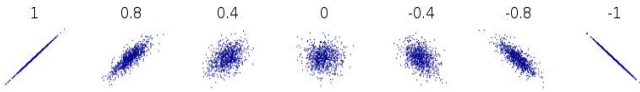
Different situations	Particles(and anti-particles) included
(i) Net- Λ vs. Net-K	Λ, K^+
(ii) Net-P vs. Net-K	p, K^+
(iii) Net-P vs. Net- Λ	p, Λ
(iv) Net-B vs. Net-S	$p, n, K^+, K^0, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$
(v) B-S(excl. s-baryon)	p, n, K^+, K^0
(vi) B-S(excl. Λ)	$p, n, K^+, K^0, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$
(vii) B-S(excl. no-s-baryon)	$K^+, K^0, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$
(viii) B-S(excl. p)	$n, K^+, K^0, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$
(ix) B-S(excl. s-meson)	$p, n, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$
(x) B-S(excl. K^+)	$p, n, K^0, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Omega^-$



Centrality Dependence(I): iv-x

Correlation Coefficient:

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle}{\sqrt{(\langle X^2 \rangle - \langle X \rangle^2)(\langle Y^2 \rangle - \langle Y \rangle^2)}}$$



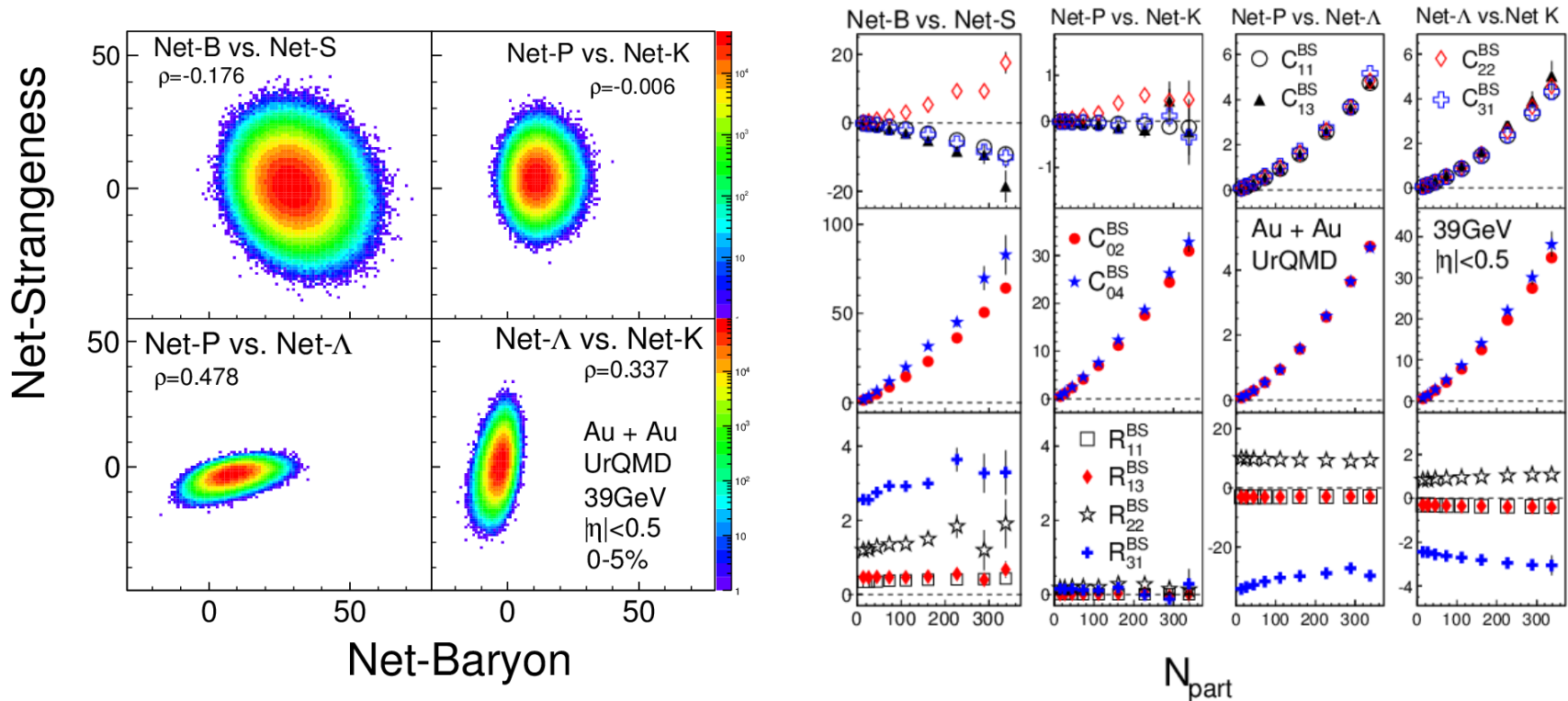
➤ Almost these cases are anti-correlated.

➤ S-baryons have contributions to B-S correlations.

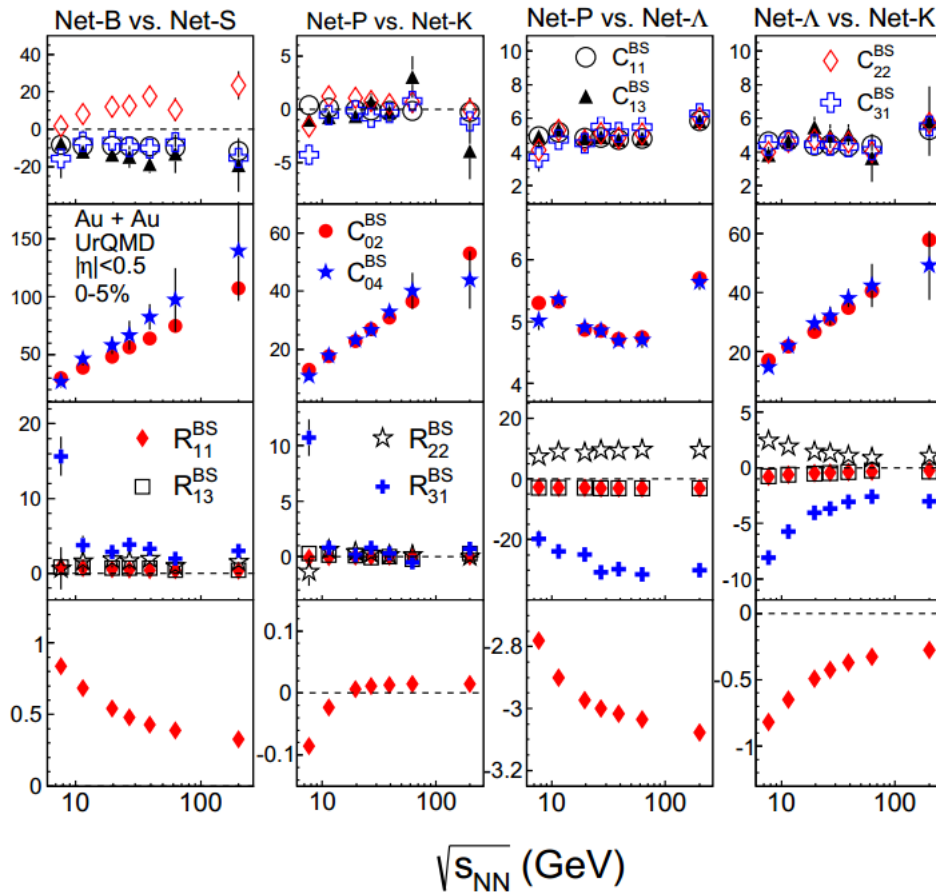
➤ S-mesons have contributions to S-fluctuations.



Centrality Dependence(II): i-iv



- Net-B and Net-S show strong anticorrelated;
- Net-P and Net-K show less correlated;
- The fluctuation of net- Λ is smaller than net-K.
- These ratios does not show any large centrality dependence.



➤ Net-B and Net-S, Net-P and Net-K :

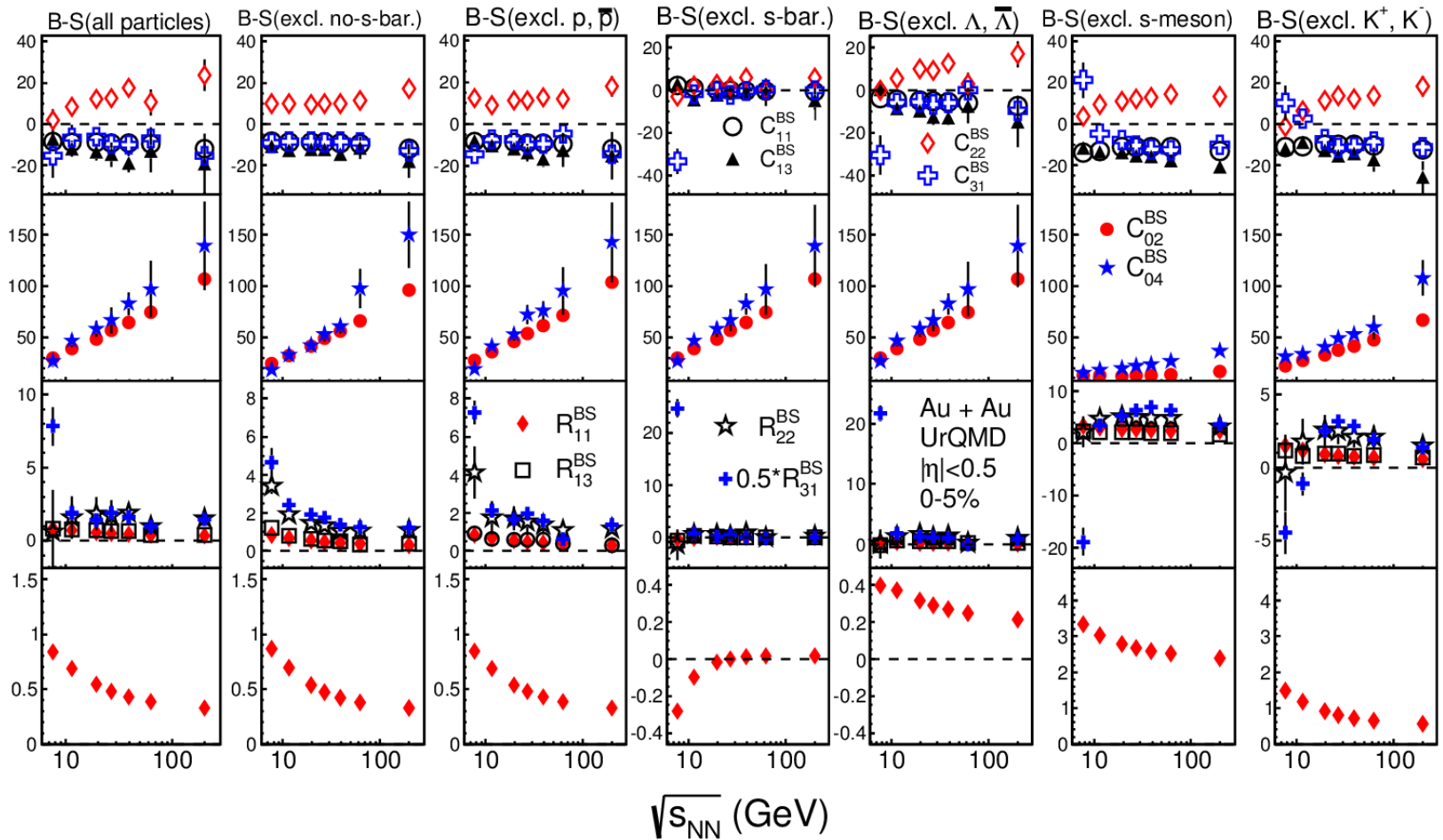
R_{31} sharply increases at low energies.

Reaction channel: $NN \rightarrow NYK$
where N is nucleon, Y is hyperon.

➤ Net-P and Net- Λ ,
 R_{11} increases with baryon
chemical increases.

➤ Net- Λ and Net-K:
 R_{11} decreases with baryon
chemical increases.

Energy Dependence (II): iv-x



- The non-strange baryons have small effects on the B-S correlations.
- Exclude strange-baryons: These ratios are close to zero at high energies.
- Exclude K^+, K^- : The ratios R_{22}, R_{31} become negative at low energies.



Summary

The centrality and energy dependence of B-S correlations.

- These ratios are comparable with the results from Lattice QCD at low temperatures.
- These ratios show weak centrality dependence.
- The higher order ratios are more sensitive to energy.

The contributions of particle species to B-S correlations.

- The strange-baryons and strange-mesons have contributions to these mixed-cumulant ratios, especially at low energies.
- Strong correlations between Net-Proton, Net- Λ and Net-K at low energies.

Thank you !