Higher order corrections to $e^+ e^- \to HZ$ and $e^+ e^- \to H+\gamma$ production at CEPC

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Part 1

$$e^{+}\, e^{-} \rightarrow \mathbf{H+Z}$$

Golden production channel for Higgs at CEPC

so-called *Higgsstrahlung* process

Motivations

Three next-generation e^+e^- colliders have been proposed to serve as Higgs factory: International Linear Collider (ILC), Future Circular Collider (FCC-ee), formerly called TLEP , and Circular Electron-Positron Collider (CEPC). All of them intend to operate at center-of-mass energy within the 240–250 GeV range. At such energy, the Higgsstrahlung process, $\mathrm{e^+e^- \to HZ}$, is the dominant Higgs production mechanism.

Motivations

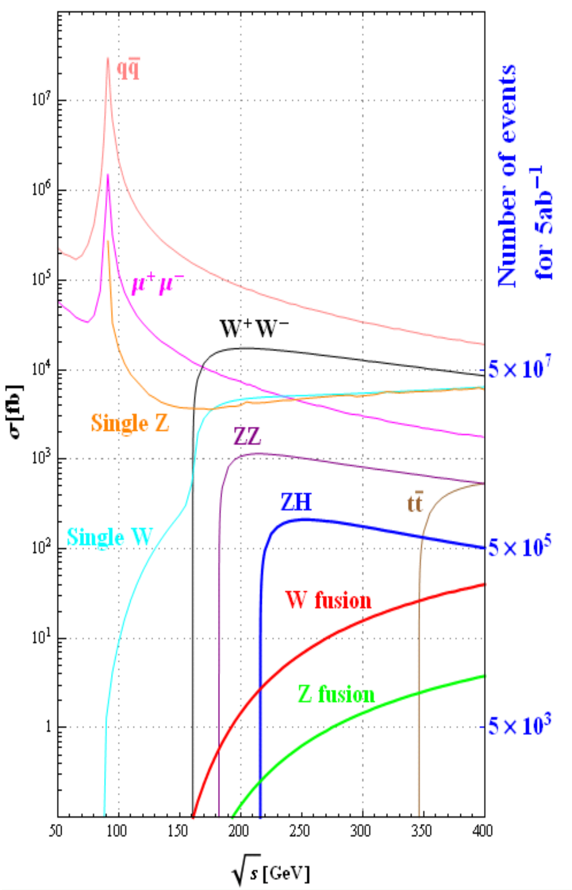
CEPC can measure production cross section for **$\sigma(ZH)$** to an exquisite precision of **0.51%**

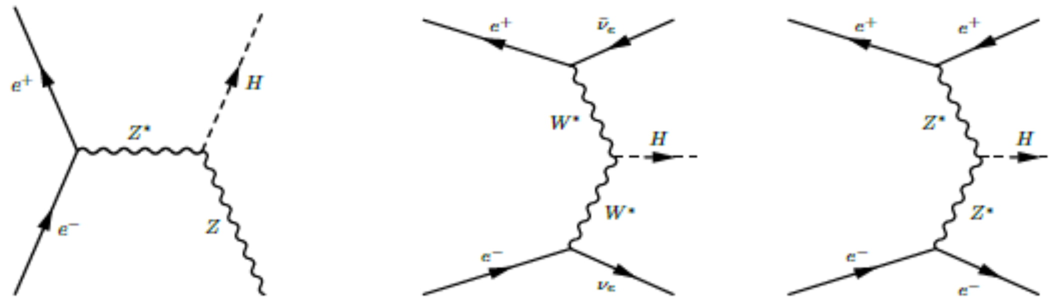
Knowing the NLO EW correction (a few percent) is not sufficient to meet experimental precision!

$O(\alpha^2)$ and **$O(\alpha\alpha_s)$** corrections need be considered!

We will investigate the latter, which should be more manageable and even more important!

Various Higgs Production mechanism





Process	Cross section	Events in 5 ab^{-1}
Higgs boson production, cross section in fb		
$e^+e^- \to ZH$	212	1.06×10^6
$e^+e^- \to \nu\bar{\nu}H$	6.72	3.36×10^4
$e^+e^- \to e^+e^-H$	0.63	3.15×10^3
Total	219	1.10×10^6

As is seen above, the Higgsstrahlung process, $e^+e^- \to HZ$ is dominant in e^+e^- colliders at energy below 400 GeV. Its contribution is much more important than that of WW and ZZ fusion mechanism.

Previous NLO work on $e^+e^- \rightarrow$ HZ

The $\mathcal{O}(\alpha)$ corrections to $e^+e^- \rightarrow$ HZ have been calculated independently by three groups:

- J. Fleischer and F. Jegerlehner,

Nucl. Phys. B 216 (1983) 469.

- B. A. Kniehl,

Z. Phys. C 55 (1992) 605.

- A. Denner, J. Kublbeck, R. Mertig and M. Bohm,

Z. Phys. C 56 (1992) 261.

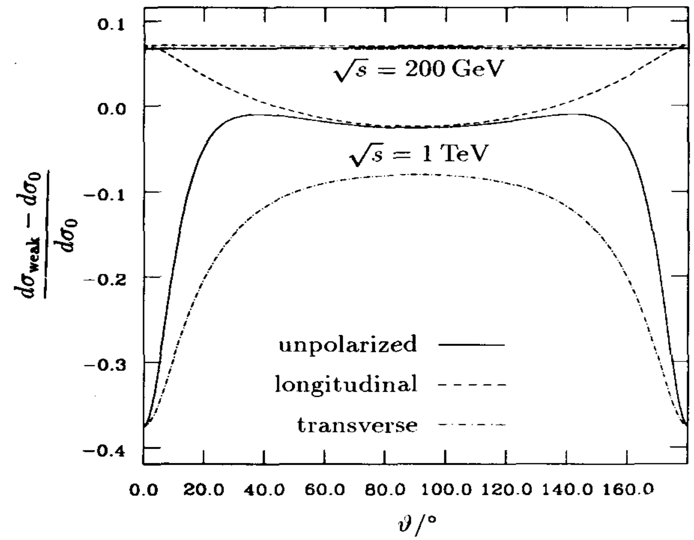


Fig. 15. The relative corrections to the differential cross section for different polarizations of the Z-boson and different CMS energies

Our recent work on $e^+e^- \to$ HZ at $\mathcal{O}(\alpha\alpha_s)$

arXiv:1609.03995 by **Sun, Feng, Jia and Sang, PRD RC 2017**

arXiv.org > hep-ph > arXiv:1609.03995

Search or Article-id

High Energy Physics - Phenomenology

Mixed electroweak-QCD corrections to $e^+e^- \to HZ$ at Higgs factories

Qing-Feng Sun, Feng Feng, Yu Jia, Wen-Long Sang

(Submitted on 13 Sep 2016)

The prospective e^+e^- Higgs factories, exemplified by ILC, FCC-ee, CEPC, plan to conduct the precision Higgs measurements at center-of-mass energy around 250 GeV. The cross sections for the dominant Higgs production channel, the Higgsstrahlung process, can be measured to a sub-percent accuracy. Apart from the well-known next-to-leading order electroweak correction, this unprecedented precision also necessitates our knowledge about the mixed electroweak-QCD corrections. In this work, we calculate the $\mathcal{O}(\alpha\alpha_s)$ correction to $e^+e^- \to HZ$ for both unpolarized and polarized Z boson. The corrections turn out to reach one percent level of the leading order cross section, thereby must be incorporated in the future confrontation with the data.

Comments: 11 pages, 4 figures, 1 table
Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex)
Cite as: arXiv:1609.03995 **[hep-ph]**
(or **arXiv:1609.03995v1 [hep-ph]** for this version)

Submission history

From: Yu Jia [view email]
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Collaborators: Feng Feng (China Univ. of Mining), Wen-Long Sang (Southwest Univ.)
Qing-Feng Sun (PhD student at USTC, will join IHEP /EPC as a postdoc fellow)

An independent calculation by another group

arXiv:1609.03955 **by** **Gong, Li, Xu, Yang, PRD 2017**

arXiv.org > hep-ph > arXiv:1609.03955

Search or Article-id

High Energy Physics - Phenomenology

Mixed QCD-EW corrections for Higgs boson production at e^+e^- colliders

Yinqiang Gong, Zhao Li, Xiaofeng Xu, Li Lin Yang

(Submitted on 13 Sep 2016)

Since the discovery of the Higgs boson at the Large Hadron Collider, a future electron-position collider has been proposed for precisely studying its properties. We investigate the production of the Higgs boson at such an e^+e^- collider and calculate for the first time the mixed QCD-electroweak corrections to the total cross sections. We find that the $\mathcal{O}(\alpha\alpha_s)$ corrections amount to a 1.2% increase of the cross section for a center-of-mass energy around 250 GeV. This is larger than the expected experimental accuracy and has to be included for extracting the properties of the Higgs boson from the measurements of the cross sections in the future.

Subjects: **High Energy Physics - Phenomenology (hep-ph)**
Cite as: arXiv:1609.03955 **[hep-ph]**
(or **arXiv:1609.03955v1** **[hep-ph]** for this version)

Submission history

From: Li Lin Yang [view email]
[v1] Tue, 13 Sep 2016 18:05:00 GMT (6kb)

Typical higher-order Feynman diagrams to the *Higgsstrahlung* process

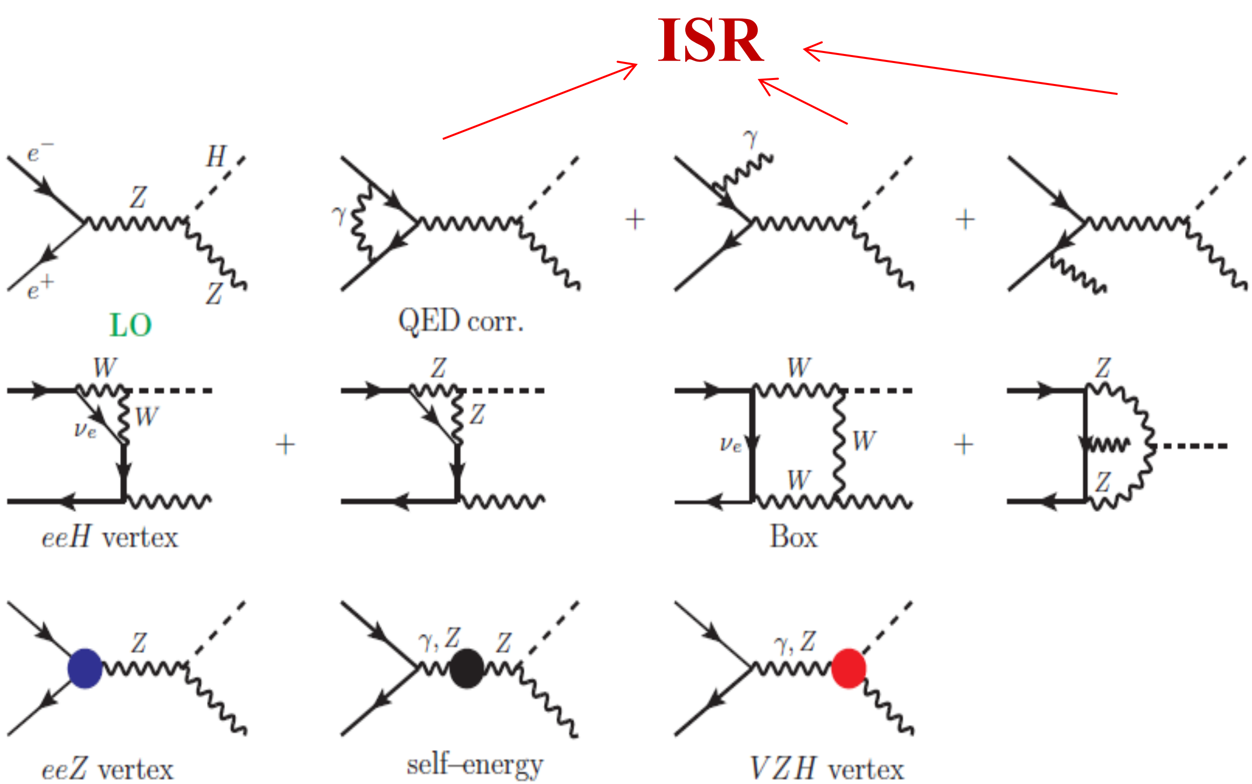


FIG. 1: LO diagram for $e^+e^- \to HZ$, together with some representative higher-order diagrams up to order-$\alpha\alpha_s$.

Typical Feynman diagrams for eeZ vertex, self-energy and ZZH vertex.

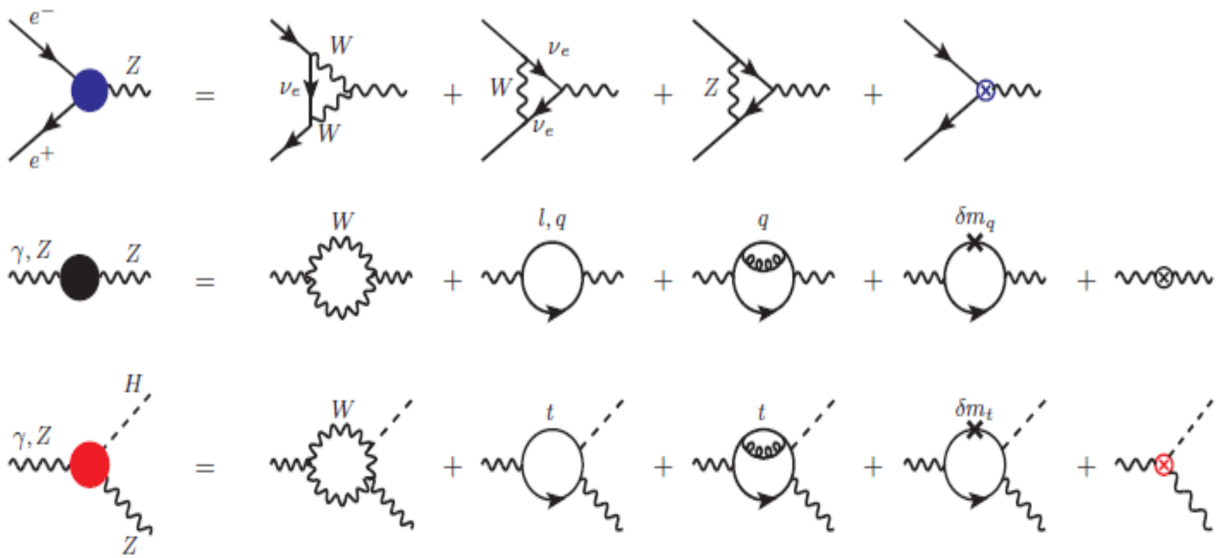


FIG. 2: Representative diagrams for the radiative corrections to the renormalized eeZ vertex, γ/Z self-energy, and VZH vertex, through order-$\alpha\alpha_s$. The cross represents the quark mass counterterm in QCD, cap denotes the electroweak counterterm in on-shell scheme.

Detailed Techniques and results

- **Techniques**

We renormalize the UV divergences with on-mass-shell scheme. Ref.:

A. Sirlin, Phys. Rev. D 22, 971 (1980).

A. Denner, Fortsch. Phys. 41, 307 (1993).

The top quark mass appears in internal top quark propagator and the $Ht\bar{t}$ Yukawa vertex, which is renormalized in on-shell scheme as:

$$\delta m_t = -m_t \Gamma(1+\epsilon)\left(\frac{4\pi\mu^2}{m_t^2}\right)^{\epsilon} \frac{C_F}{4}\frac{\alpha_s}{\pi}\frac{3-2\epsilon}{\epsilon(1-2\epsilon)}$$

We calculate the electroweak counter terms $\delta Z_e,\ \delta M_Z^2,$ $\delta M_W^2,\ \delta M_H^2,\ \delta Z_{ZZ},\ \delta Z_{\gamma Z}$ and δZ_H analytically, and adopt the so-called $\alpha(0)$ scheme where $\alpha = 1/137.035999$ and the charge renormalization constant $\delta Z_e = e\, Z_e$ in $\alpha(0)$ scheme is expressed as:

$$\delta Z_e\Big|_{\alpha(0)} = \frac{1}{2}\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) + \frac{1}{2}\mathrm{Re}\,\Pi^{\gamma\gamma(5)}(M_Z^2) + \frac{1}{2}\Pi_{\mathrm{rem}}^{\gamma\gamma}(0) - \frac{s_W}{c_W}\frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2}$$

Where $\Delta\alpha_{had}^{(5)} = 0.02771$ represents the non-perturbative hadronic contributions to the running effect of the electroweak coupling and $\Pi_{rem}^{\gamma\gamma(5)}(0)$ is the remaining possible photon vacuum polarizations from other charged SM particles.

Methodology and Comparison with existing NLO predictions

Throughout this work, we only retain the top quark mass and treat the remaining five quarks massless. We work in Feynman gauge and adopt the dimensional regularization to regularize UV divergence. The Feynman diagrams and corresponding amplitudes are generated by FeynArts [25]. The packages FeynCalc/FormLink [26, 27] are employed to carry out the trace over Dirac/color matrices, and the packages Apart [28] and FIRE [29] are utilized to perform partial fraction together with integration-by-parts (IBP) reduction. We combine FIESTA/CubPack [30, 31] to perform sector decomposition and subsequent numerical integrations for Master Integrals (MI) with quadruple precision. We also employed LoopTools [32] to perform an independent cross-check. The counterterms are computed analytically [20]. After completing the renormalization procedure, we have compared our UV-finite NLO predictions with numerous differential and integrated cross sections enumerated in [18], and found gross agreement. Reassuringly, we also compare our integrated NLO cross sections with those high-precision predictions rendered by the automatic package GRACE-loop [33] for a variety of input values of $\sqrt{s}$ and M_H, and always found better-than-per-mille agreement.

We confirm Denner et al.'s analytic NLO results

Our NLO predictions to integrated cross sections are accurate to an exquisite degree, actually we get fully analytic results.

We apply three variants of On-Shell renormalization schemes:

α(0) scheme

$$\delta Z_e\big|_{\alpha(0)} = \frac{1}{2}\Delta\alpha^{(5)}_{\rm had}(M_Z^2) + \frac{1}{2}\mathrm{Re}\,\Pi^{\gamma\gamma(5)}(M_Z^2) + \frac{1}{2}\Pi^{\gamma\gamma}_{\rm rem}(0) - \frac{s_W}{c_W}\frac{\Sigma^{\gamma Z}_T(0)}{M_Z^2},$$

$$\Delta\alpha(M_Z^2) = \Pi^{\gamma\gamma}_{f\neq t}(0) - \mathrm{Re}\,\Pi^{\gamma\gamma}_{f\neq t}(M_Z^2)$$

α(MZ) scheme

$$\delta Z_e\big|_{\alpha(M_Z^2)} = \delta Z_e\big|_{\alpha(0)} - \tfrac{1}{2}\Delta\alpha(M_Z^2)$$

G_μ scheme

$$\delta Z_e|_{G_\mu} = \delta Z_e|_{\alpha(0)} - \tfrac{1}{2}\Delta r$$

$$\alpha\left(M_Z^2\right) = \frac{\alpha(0)}{1-\Delta\alpha\left(M_Z^2\right)},$$

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi}G_\mu M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)$$

Input Parameters

Phenomenology. We will take $\sqrt{s} = 240, 250$ GeV as two benchmark energy points at Higgs factory. We adopt the following values for the input parameters: $M_H = 125.09$ GeV, $M_Z = 91.1876$ GeV, $M_W = 80.385$ GeV, $m_t = 174.2$ GeV, $m_e = 0.510998928$ MeV,

$m_\mu = 105.6583715$ MeV, $m_\tau = 1.77686$ GeV, $\alpha(0) = 1/137.035999$, $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02764 \pm 0.00013$ [24] and $G_\mu = 1.1663787 \times 10^{-5}$ GeV^2. We take $\alpha(M_Z^2) = 1/128.943$ in the $\alpha(M_Z^2)$ scheme and evaluate the QCD running coupling $\alpha_s(\mu)$ using package RunDec [42].

Our predictions in α(0) scheme (including corrections for polarized cross section)

$\sqrt{s}$ (GeV)		LO (fb)	NLO Weak (fb)		NNLO mixed EW-QCD (fb)			
		$\sigma^{(0)}$	$\sigma^{(\alpha)}$	$\sigma^{(0)}+\sigma^{(\alpha)}$	$\sigma_Z^{(\alpha\alpha_s)}$	$\sigma_\gamma^{(\alpha\alpha_s)}$	$\sigma^{(\alpha\alpha_s)}$	$\sigma^{(0)}+\sigma^{(\alpha)}+\sigma^{(\alpha\alpha_s)}$
240	Total	**223.14**	6.64	229.78	2.42	0.008	2.43	232.21
	L	88.67	3.18	91.86	0.96	0.003	0.97	92.82
	T	134.46	3.46	137.92	1.46	0.005	1.46	139.39
250	Total	**223.12**	6.08	229.20	2.42	0.009	2.42	231.63
	L	94.30	3.31	97.61	1.02	0.004	1.02	98.64
	T	128.82	2.77	131.59	1.40	0.005	1.40	132.99

TABLE I: The (un)polarized Higgsstrahlung cross sections at $\sqrt{s}=240(250)$ GeV in $\alpha(0)$ scheme. We enumerate the NLO weak corrections, together with the NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections. For the latter, we also list individual contribution given in (13).

NLO enhances the LO prediction by about **3.1%**

NNLO mixed EW-QCD correction is sizable, about **1.1%** of the LO prediction!

Our predictions for unpolarized cross section in three different schemes (including uncertainty)

$\sqrt{s}$	schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z^2)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$
250	$\alpha(0)$	223.12 ± 0.47	229.20 ± 0.77	$231.63^{+0.75+0.12}_{-0.75-0.21}$
	$\alpha(M_Z^2)$	252.01 ± 0.60	$227.67^{+0.82}_{-0.81}$	$230.58^{+0.80+0.14}_{-0.79-0.25}$
	G_μ	239.62 ± 0.06	231.82 ± 0.07	$232.65^{+0.07+0.04}_{-0.07-0.07}$

TABLE II: The unpolarized Higgsstrahlung cross sections at $\sqrt{s} = 240(250)$ GeV in three different schemes. To estimate the errors caused by the input parameters, we take $M_W = 80.385 \pm 0.015\,\text{GeV}$, $m_t = 174.2 \pm 1.4\,\text{GeV}$ and $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02764 \pm 0.00013$. We also change the strong coupling constant from $\alpha_s(M_Z)$ to $\alpha_s(\sqrt{s})$ with its centeral value taken as $\alpha_s = \alpha_s(\sqrt{s}/2)$. The remaining parameters are taken the same as in Table I.

We also redo the calculation retaining finite bottom quark mass, effect too small to include

Observe strong scheme dependence!

The NNLO predictions now range from 230 To 233 fb!

Need go to 2-loop EW correction to reduce Scheme dependence!

Bad news for prescribed 0.51% precision at CEPC?

Angular distribution of the (polarized) Z boson in the *Higgsstrahlung* process

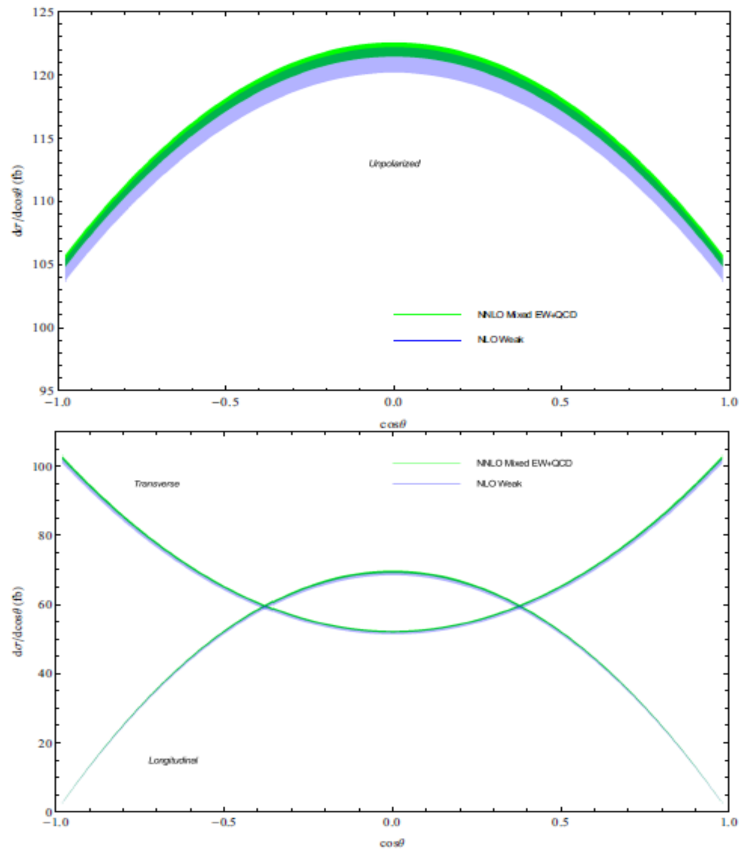


FIG. 3: Differential unpolarized/polarized cross sections for Higgsstrahlung at $\sqrt{s} = 240$ GeV for the NLO $\mathcal{O}(\alpha)$ and NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections. The green band indicates the uncertainties from the input parameters as adopted in Table II and three different schemes.

ISR effect

- ISR effect

$$\sigma = \int \mathrm{d}z\, \hat{\sigma}_{e^+e^-\to ZH}(z\, s)\, \mathcal{L}_{e^+e^-}(z) \qquad \mathcal{L}_{e^+e^-} = \phi_S[2\alpha, z]$$

Structure function approach

- LL resummation
- ad hoc exponentiation to reproduce higher order several leading logarithmic photonic corrections (a^2 L^2, a^2 L)

$$\begin{aligned}
\phi_S[\alpha, z] \;=\; & e^{\frac{\alpha}{2\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)} \frac{e^{\frac{1}{2}\beta_{\mu_F}\left(\frac{3}{4}-\gamma_E\right)}}{\Gamma\left(1+\frac{1}{2}\beta_{\mu_F}\right)} \frac{\beta_{\mu_F}}{2} (1-z)^{-1+\frac{\beta_{\mu_F}}{2}} \\
& \times \left(\frac{1}{2}(1+z^2) + \frac{\alpha}{4\pi}\beta_{\mu_F}\left(\frac{3}{32}-\frac{\pi^2}{8}+\frac{3}{2}\zeta[3]\right)\right. \\
& + \frac{\beta_{\mu_F}}{8}\left(\frac{-1}{2}(1+3z^2)\log(z) - (1-z)^2\right) \\
& + \frac{\alpha}{16\pi}\left(-(1+3z^2)\log^2(z) + 4(1+z^2)\left(\mathrm{Li}_2(1-z)+\log(z)\log(1-z)\right)\right. \\
& \qquad \left. + 2(1-z)(3-2z) + 2(3+2z+z^2)\log(z)\right) \\
& \left. + \frac{\beta^2_{\mu_F}}{32}\left(\frac{1}{2}(3z^2-4z+1)\log(z) + \frac{1}{12}(1+7z^2)\log^2(z) + (1-z^2)\mathrm{Li}_2(1-z) + (1-z)^2\right)\right)
\end{aligned}$$

0203120v1 $\beta_{\mu_F} = \frac{2\alpha}{\pi}\left(\log\frac{\mu_F^2}{m_l^2} - 1\right)$

ISR effect is well-understood (structure function approach)

- ISR effect

$$\sigma = \int \mathrm{d}z\, \hat{\sigma}_{e^+e^-\to ZH}(z\, s)\, \mathcal{L}_{e^+e^-}(z) \qquad \mathcal{L}_{e^+e^-} = \phi_S[2\alpha, z]$$

$$\begin{aligned}
\phi_S[\alpha, z] \;=\; & e^{\frac{\alpha}{2\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)} \frac{e^{\frac{1}{2}\beta_{\mu_F}\left(\frac{3}{4}-\gamma_E\right)}}{\Gamma\left(1+\frac{1}{2}\beta_{\mu_F}\right)} \frac{\beta_{\mu_F}}{2} (1-z)^{-1+\frac{\beta_{\mu_F}}{2}} \\
\times & \left(\frac{1}{2}(1+z^2) + \frac{\alpha}{4\pi}\beta_{\mu_F}\left(\frac{3}{32} - \frac{\pi^2}{8} + \frac{3}{2}\zeta[3]\right)\right. \\
& + \frac{\beta_{\mu_F}}{8}\left(\frac{-1}{2}(1+3z^2)\log(z) - (1-z)^2\right) \\
& + \frac{\alpha}{16\pi}\left(-(1+3z^2)\log^2(z) + 4(1+z^2)\left(\mathrm{Li}_2(1-z) + \log(z)\log(1-z)\right)\right. \\
& \qquad \left. + 2(1-z)(3-2z) + 2(3+2z+z^2)\log(z)\right) \\
& \left. + \frac{\beta_{\mu_F}^2}{32}\left(\frac{1}{2}(3z^2-4z+1)\log(z) + \frac{1}{12}(1+7z^2)\log^2(z) + (1-z^2)\mathrm{Li}_2(1-z) + (1-z)^2\right)\right)
\end{aligned}$$

Structure function approach

- Go beyond that, we add more NNLO logarithmic terms from both photons and lepton pairs extracted from Nucl. Phys. B 297, 429 (1988)

hep-ph/0203120v1

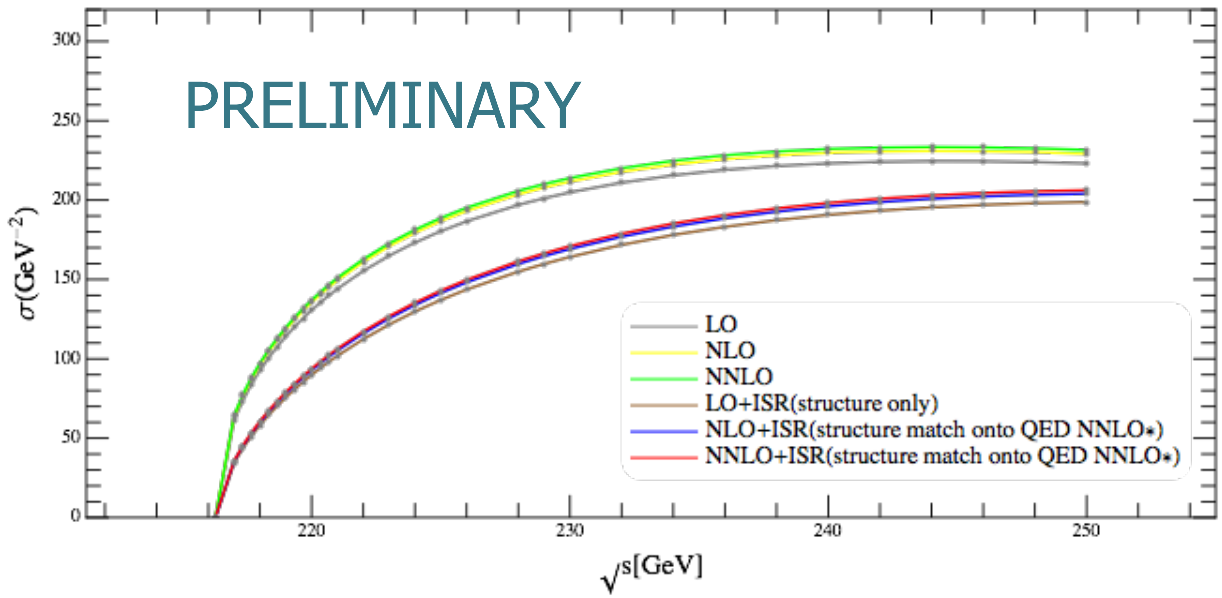
$$\beta_{\mu_F} = \frac{2\alpha}{\pi}\left(\log\frac{\mu_F^2}{m_l^2} - 1\right)$$

ZH cross section incorporating ISR effect

Very preliminary

Significant ISR effects

α 0 scheme



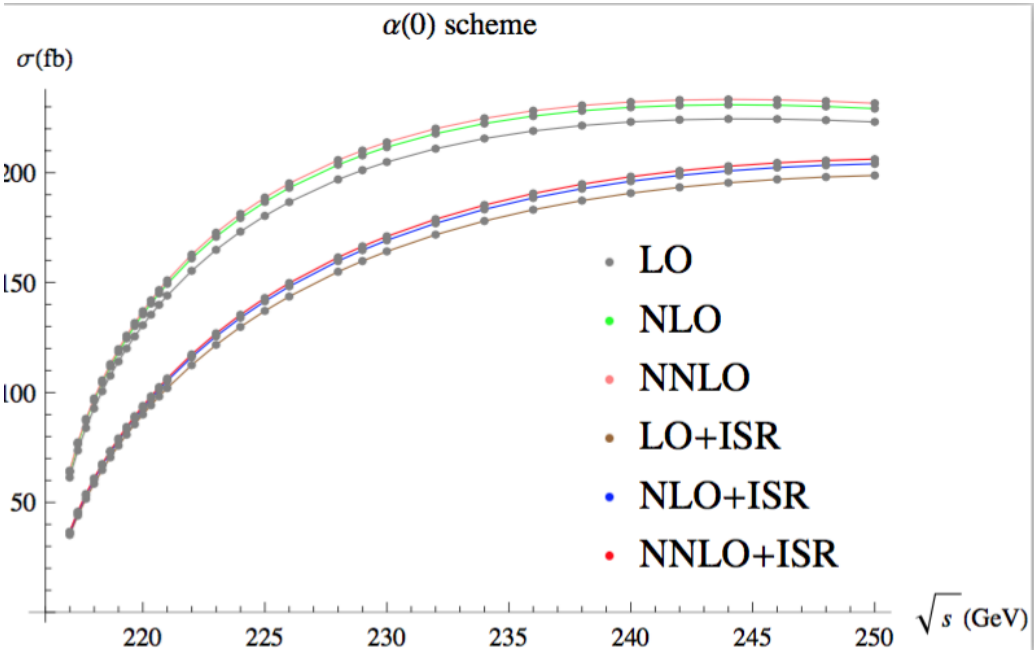
F. Feng, Y. Jia, X.-H. Liu, W.-L. Sang, Q.-F. Sang, in preparation

We present the state-of-the-art ZH cross section at CEPC

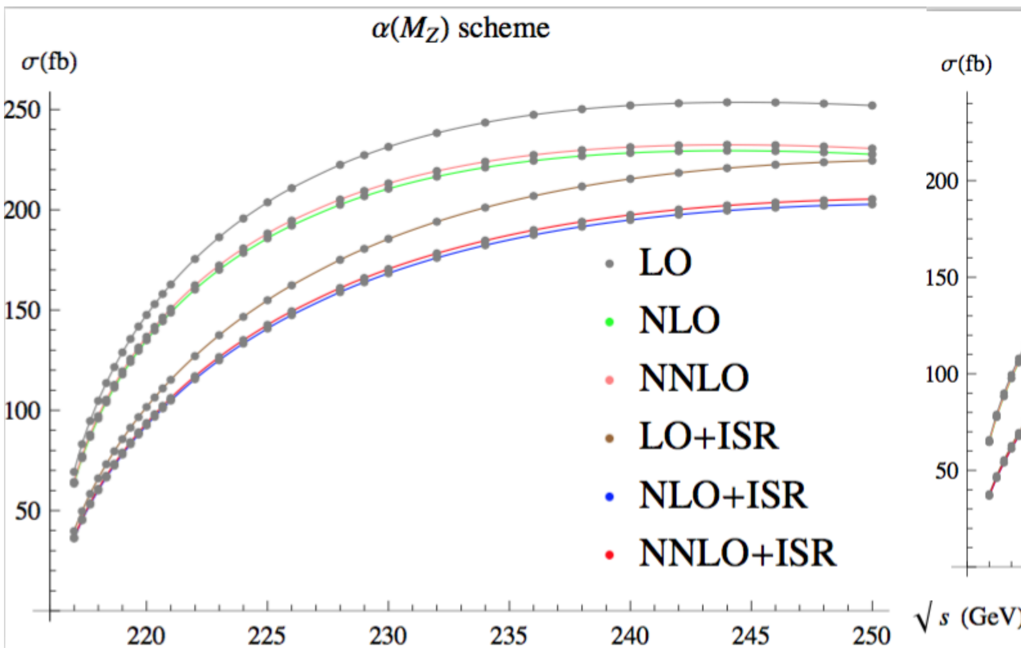
But the scheme-dependence is strong; >0.5% experimental precision

$\sqrt{s}$	schemes	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	$\sigma^{\mathrm{ISR}}_{\mathrm{LO}}$ (fb)	$\sigma^{\mathrm{ISR}}_{\mathrm{NLO}}$ (fb)	$\sigma^{\mathrm{ISR}}_{\mathrm{NNLO}}$ (fb)
240	$\alpha(0)$	223.14	229.78	232.21	190.72	196.14	198.22
	$\alpha(M_Z^2)$	252.03	228.36	231.28	215.41	194.95	197.44
	G_μ	239.64	232.46	233.29	204.82	198.44	199.15
250	$\alpha(0)$	223.12	229.20	231.63	198.77	204.06	206.22
	$\alpha(M_Z^2)$	252.01	227.67	230.58	224.51	202.72	205.32
	G_μ	239.62	231.82	232.65	213.47	206.40	207.14

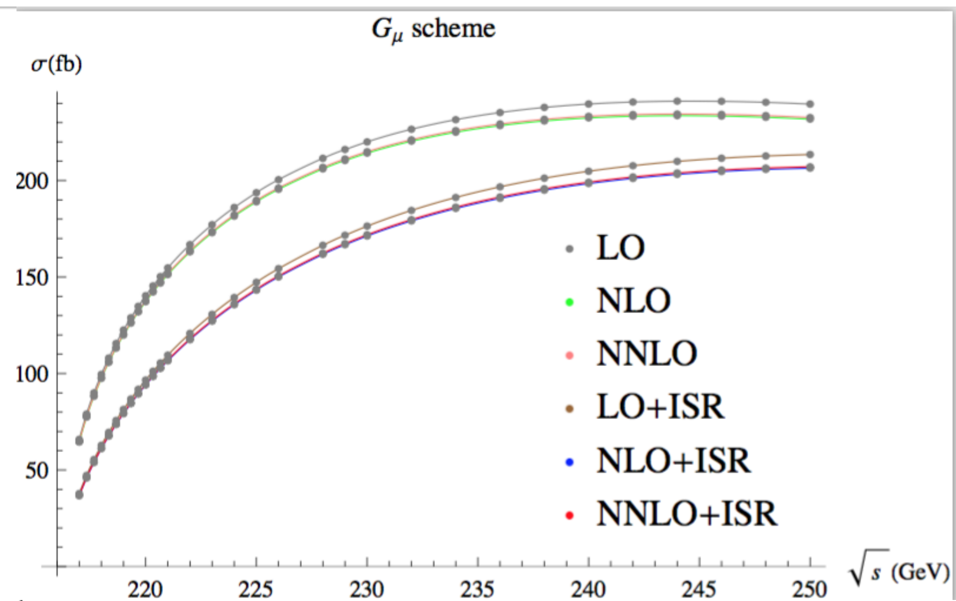
Very preliminary; will include the uncertainty in input parameters later



α(0) scheme
σ(fb)
200
150
100
50
LO
NLO
NNLO
LO+ISR
NLO+ISR
NNLO+ISR
220
225
230
235
240
245
250
√s (GeV)



α(M_Z) scheme
σ(fb)
250
200
150
100
50
LO
NLO
NNLO
LO+ISR
NLO+ISR
NNLO+ISR
220
225
230
235
240
245
250
√s (GeV)



G_μ scheme
σ(fb)
200
150
100
50
LO
NLO
NNLO
LO+ISR
NLO+ISR
NNLO+ISR
220
225
230
235
240
245
250
√s (GeV)

Part 2

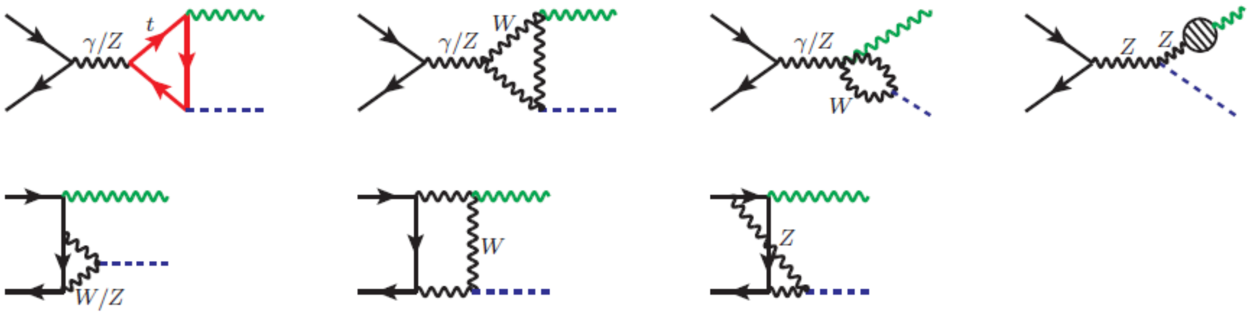
$$e^+ e^- \rightarrow H + \gamma$$

A very **rare** Higgs production channel at CEPC

several orders-of-magnitude smaller than HZ production

Loop-induced process, a sensitive channel to seek the footprint of new physics

arXiv:1706.03572 by Sang, Chen, Feng, Jia and Sun, published in PLB



LO results are known since Abbasabadi et al. (95); Gounaris et al. (95); Djouadi et al. (96)

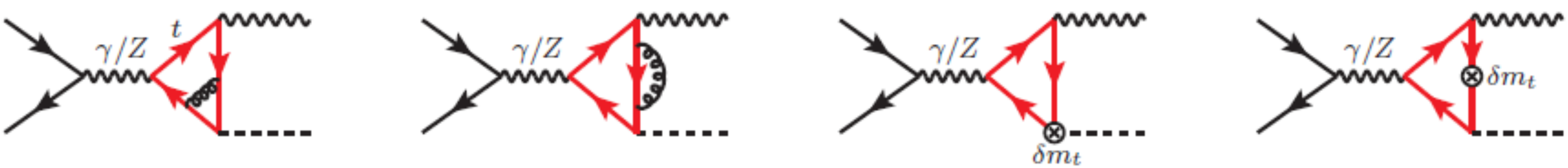


Figure 2: Typical Feynman diagrams for the NLO QCD corrections to $e^+e^- \to H\gamma$. The cap signifies the insertion of the top quark mass counterterm δm_t, as given in (7).

For the first time, we have computed the NLO QCD correction

Angular distribution of Higgs

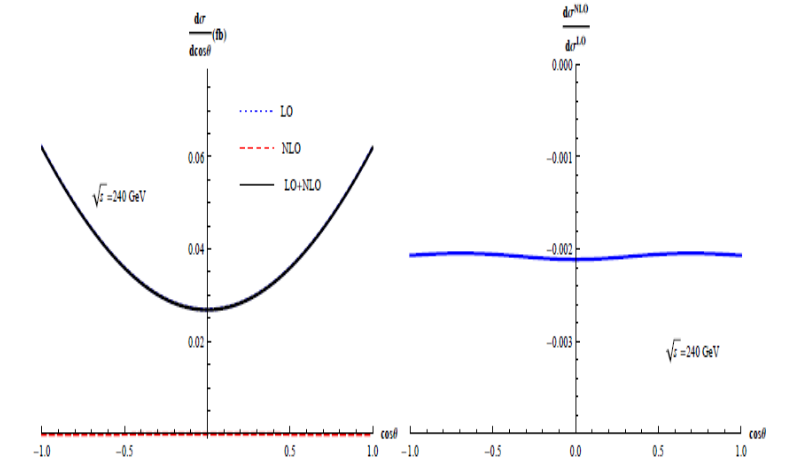


Figure 3: Angular distributions of the Higgs boson in the $e^+e^- \to H\gamma$ process at $\sqrt{s}$ = 240 GeV. The right panel embodies the relative magnitude of the NLO QCD corrections.

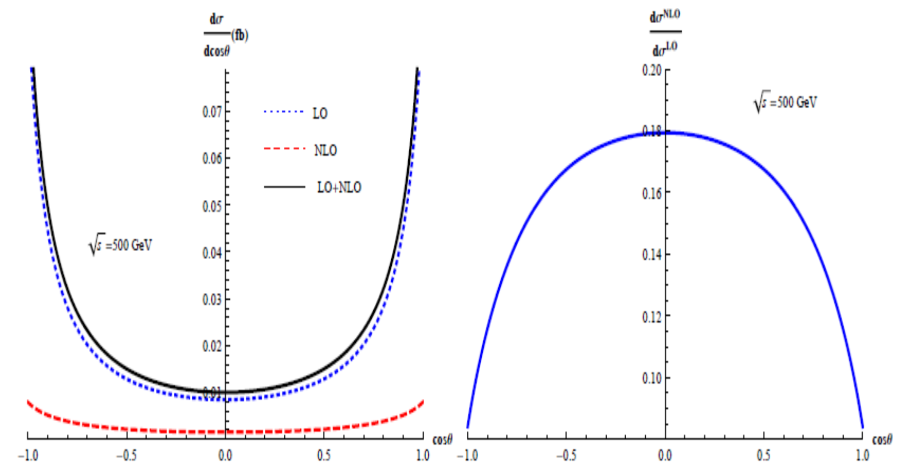


Figure 4: Angular distributions of the Higgs boson in the $e^+e^- \to H\gamma$ process at $\sqrt{s}$ = 500 GeV. The right panel signifies the relative magnitude of the NLO QCD corrections.

CEPC

ILC

The NLO QCD correction is negligible at CEPC energy

Integrated cross section versus CM energy (LO)

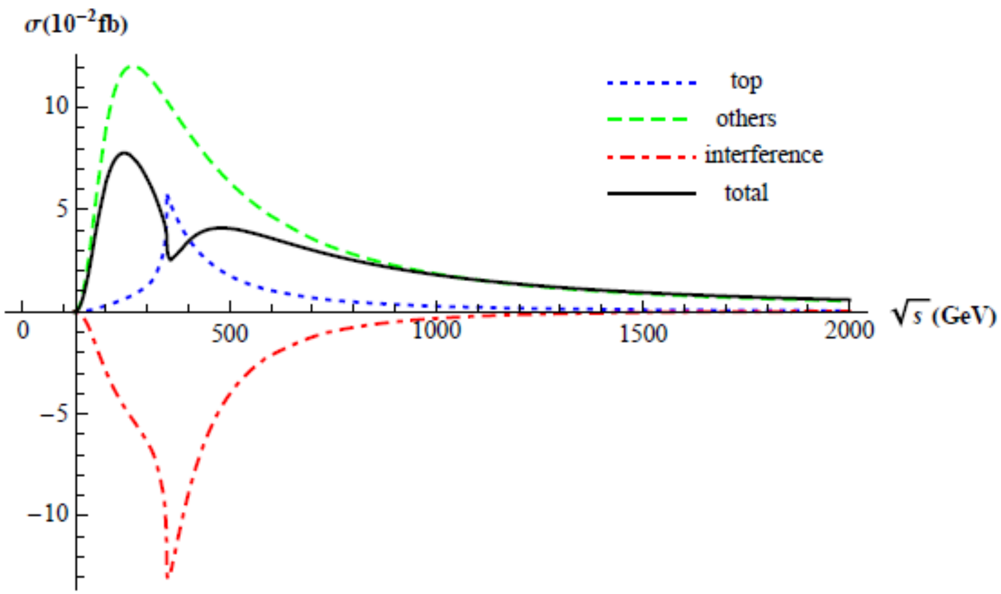


Figure 5: The LO cross section as a function of $\sqrt{s}$ (the solid line). To trace the origin of the nontrivial line shape, we deliberately isolate the contributions from two classes of diagrams. The dotted, dashed and dot-dashed lines represent the contribution from diagrams involving the top quark loop, that from all other diagrams involving weak gauge bosons in the loop, and their interference, respectively.

At asymptotically high energy, sigma_LO ~ 1/s

Integrated cross section versus CM energy (NLO)

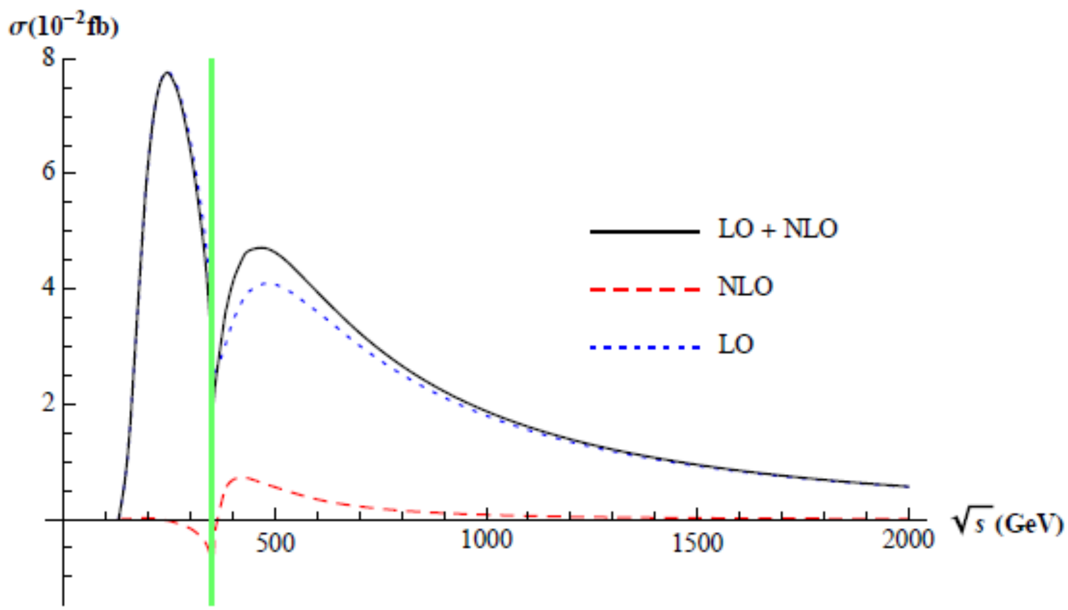


Figure 6: The total cross section as a function of $\sqrt{s}$, both at LO and NLO in α_s. The vertical band with $\sqrt{s} = 2m_t \pm 5$ GeV signifies the threshold region, inside which the perturbative expansion is expected to break down and our fixed-order predictions become invalid.

At asymptotically high energy, sigma_NLO ~ 1/s^2

Caveat: fixed order breaks down near ttbar threshold, need resummation of Coulomb gluon to all orders

Summary and Outlook

1. Mixed EW-QCD correction for the *Higgsstrahlung* process appears to be significant, about 1% of LO cross sections

2. Strong α-scheme dependence observed, also sizable uncertainty arising from input parameters. So far, cannot meet precision of CEPC measurement.

   What we can do to improve?

   Compute NNLO EW correction??

   Technically feasible?? Computational Resources?

Summary and Outlook (cont')

3. **ISR** effect is very important, and must be included to match the realistic experimental measurements.

4. Finite $\mathbf{Z^0}$ width effect? How good is the *narrow width approximation* ? Tries mu mu H, q qbar H final states?

5. For $e^+ e^- \rightarrow$ **H+γ** (Harbor of new physics) **,** the QCD correction at CEPC seems completely negligible at CEPC energy. But, maximum X section of 0.08 fb occurs at CEPC energy.

Thanks for your attention!