The Higgs couplings and self-coupling in the EFT framework

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DESY & IHEP

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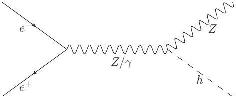
[arXiv:1704.02333] G. Durieux, C. Grojean, JG, K. Wang
and current work, S. Di Vita, G. Durieux, C. Grojean, JG, Z. Liu, G. Panico, M. Riembau, T. Vantalon

Introduction

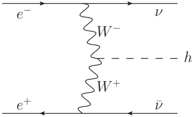
- Higgs and nothing else? What next?
- An e^+e^- collider is an obvious direction to go.
- Higgs factory ($e^+e^- \to hZ$ at 240-250 GeV, $e^+e^- \to \nu\bar{\nu}h$ at higher energies), and many more other measurements.
- The scale of new physics Λ is large $\Rightarrow$ effective field theory (EFT) is a good description at low energy.
- A global analysis of the Higgs coupling constraints, in the EFT framework. See also *e.g.*,
  - [arXiv:1510.04561, 1701.04804] Ellis *et al.*,
  - [arXiv:1708.08912, 1708.09079] Peskin *et al.* (See Sunghoon's talk)
- **Robust constraints** on the **triple Higgs coupling** at both circular and linear colliders. (current work, to appear soon)

Higgs measurements

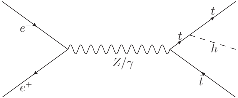
- $e^+e^- \to hZ$, cross section maximized at around 250 GeV.



- $e^+e^- \to \nu\bar{\nu}h$, cross section increases with energy.

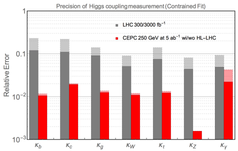


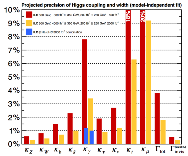
- $e^+e^- \to t\bar{t}h$, can be measured with $\sqrt{s} \gtrsim 500$ GeV.



- $e^+e^- \to Zhh$ and $e^+e^- \to \nu\bar{\nu}hh$ (triple Higgs coupling).

κ framework vs. EFT





From the CEPC preCDR and "Physics Case for the ILC" ([arXiv:1506.05992])

- Conventionally, the constraints on Higgs couplings are obtained from global fits in the so-called "κ" framework.

$$g_h^{\rm SM} \to \kappa\, g_h^{\rm SM} \,.$$

- Anomalous couplings such as $hZ^{\mu\nu}Z_{\mu\nu}$ or $hZ_\mu\partial_\nu Z^{\mu\nu}$ are assumed to be zero.
- EFT framework
  - Assuming $v \ll \Lambda$, leading contribution from BSM physics are well-parameterized by D6 operators.
  - Gauge invariance is built in the parameterization.
- Lots of parameters! (Is it practical to perform a global fit?)

The "12-parameter" framework in EFT

- Assume the new physics
  - is CP-even,
  - does not generate dipole interaction of fermions,
  - only modifies the diagonal entries of the Yukawa matrix,
  - has no corrections to Z-pole observables and W mass (more justified if the machine will run at Z-pole).
- Additional measurements
  - Triple gauge couplings from $e^+e^- \to WW$. (The LEP constraints will be improved at future colliders.)
  - Angular observables in $e^+e^- \to hZ$. (see *e.g.* [arXiv:1512.06877] N. Craig, JG, Z. Liu, K. Wang)
  - $h \to Z\gamma$ is also important.
- Only 12 combinations of operators are relevant for the measurements considered (with the inclusion of the Yukawa couplings of t, c, b, τ, μ).
- All 12 EFT parameters can be constrained reasonably well in the global fit!

EFT basis

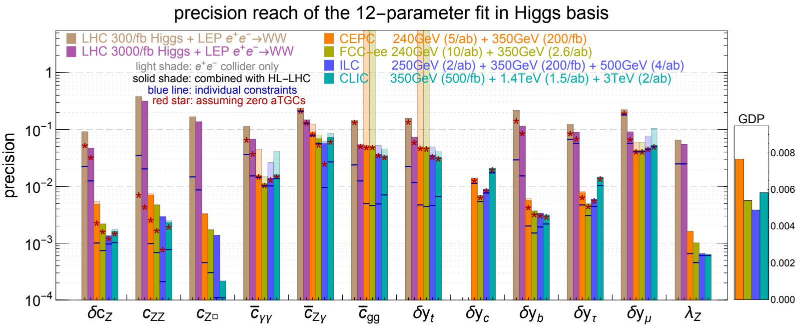
- We work in the Higgs basis (LHCHXSWG-INT-2015-001, A. Falkowski) with the following 12 parameters,

$$\delta c_Z\,,\quad c_{ZZ}\,,\quad c_{Z\Box}\,,\quad c_{\gamma\gamma}\,,\quad c_{Z\gamma}\,,\quad c_{gg}\,,\quad \delta y_t\,,\quad \delta y_c\,,\quad \delta y_b\,,\quad \delta y_\tau\,,\quad \delta y_\mu\,,\quad \lambda_Z\,.$$

- The Higgs basis is defined in the broken electroweak phase.
  - $\delta c_Z \leftrightarrow hZ^\mu Z_\mu$, $c_{ZZ} \leftrightarrow hZ^{\mu\nu} Z_{\mu\nu}$, $c_{Z\Box} \leftrightarrow hZ_\mu \partial_\nu Z^{\mu\nu}$.
- Couplings of h to W are written in terms of couplings of h to Z and γ.
- 3 aTGC parameters ($\delta g_{1,Z}$, $\delta\kappa_\gamma$, λ_Z), 2 written in terms of Higgs parameters.
- It can be easily mapped to the following basis with D6 operators.

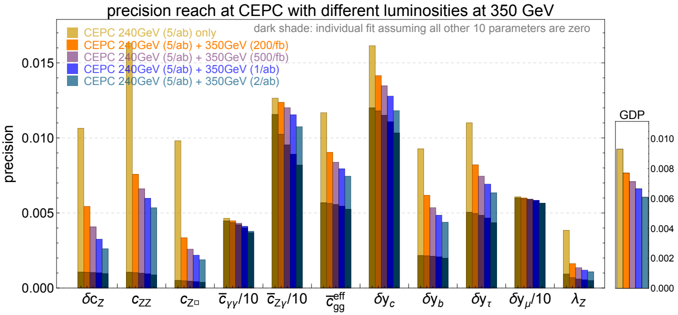
$\mathcal{O}_H = \frac{1}{2}(\partial_\mu \|H^2\|)^2$	$\mathcal{O}_{GG} = g_s^2 \|H\|^2 G^A_{\mu\nu} G^{A,\mu\nu}$
$\mathcal{O}_{WW} = g^2 \|H\|^2 W^a_{\mu\nu} W^{a,\mu\nu}$	$\mathcal{O}_{y_u} = y_u \|H\|^2 \bar{Q}_L \tilde{H} u_R$
$\mathcal{O}_{BB} = g'^2 \|H\|^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{y_d} = y_d \|H\|^2 \bar{Q}_L H d_R$
$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$	$\mathcal{O}_{y_e} = y_e \|H\|^2 \bar{L}_L H e_R$
$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W^{a\,\nu}_\mu W^b_{\nu\rho} W^{c\,\rho\mu}$

Results of the “12-parameter” fit



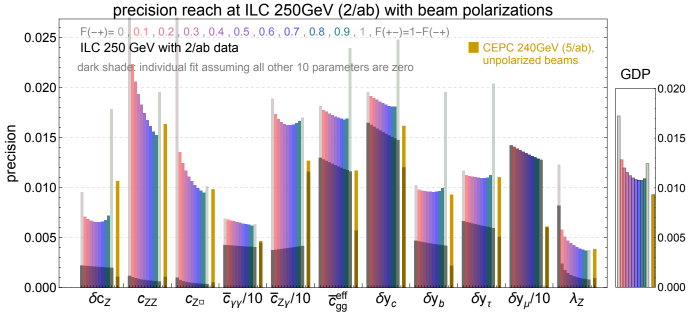
- Assuming the following run plans (no official plan for CEPC 350 GeV run)
  - CEPC 240 GeV(5/ab) + 350 GeV(200/fb)
  - FCC-ee 240 GeV(10/ab) + 350 GeV(2.6/ab)
  - ILC 250 GeV(2/ab) + 350 GeV(200/fb) + 500 GeV(4/ab)
  - CLIC 350 GeV(500/fb) + 1.4 TeV(1.5/ab) + 3 TeV(2/ab)

Impact of a 350 GeV run



- Advantages of the runs at higher energies
  - Much better measurement of the WW fusion process ($e^+e^- \to \nu\bar{\nu}h$).
  - Probing $e^+e^- \to hZ$ at different energies.
  - Improving constraints on aTGCs ($e^+e^- \to WW$).
- Very helpful in resolving the degeneracies among parameters!

Impact of beam polarization



- Beam polarization helps discriminate different parameters.
  - Two polarization configurations are considered, $P(e^-, e^+) = (-0.8, +0.3)$ and $(+0.8, -0.3)$.
  - $F(-+)$ in the range of 0.6-0.8 gives an optimal overall results.
- Runs with different polarizations probe different combinations of EFT parameters in Higgs production.

Triple Higgs coupling in the EFT framework

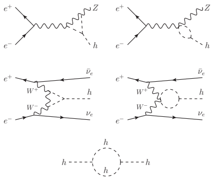
current work, S. Di Vita, G. Durieux, C. Grojean, JG, Z. Liu, G. Panico, M. Riembau, T. Vantalon

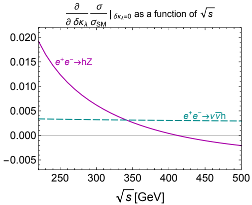
- Triple Higgs coupling

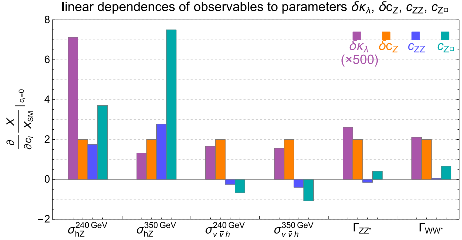
$$\kappa_\lambda \equiv \frac{\lambda_3}{\lambda_3^{\rm SM}}, \quad \delta\kappa_\lambda \equiv \kappa_\lambda - 1 \; = c_6 - \frac{3}{2} c_H, \quad \text{with } \mathcal{L} \supset -\frac{c_6 \lambda}{v^2} (H^\dagger H)^3$$

- HL-LHC: $\sim \mathcal{O}(1)$ determination. ($\kappa_\lambda \in [-0.8, \, 7.7]$ at 95% CL from Atlas projection for the $b\bar{b}\gamma\gamma$ channel, ATL-PHYS-PUB-2017-001)
- Linear colliders: direct measurements with $e^+e^- \to Zhh$, $e^+e^- \to \nu\bar{\nu}hh$.
- Circular colliders: probe indirectly via the loop contribution in $e^+e^- \to hZ$. ([arXiv:1312.3322] M. McCullough)
  - TLEP (FCC-ee) 240 GeV: $|\delta\kappa_\lambda| \lesssim 28\%$ assuming all other Higgs couplings are SM-like.
  - **What if other Higgs couplings are not SM-like?**
- **A global fit with 12+1 parameters!**

Triple Higgs coupling at circular colliders (240 & 350 GeV)

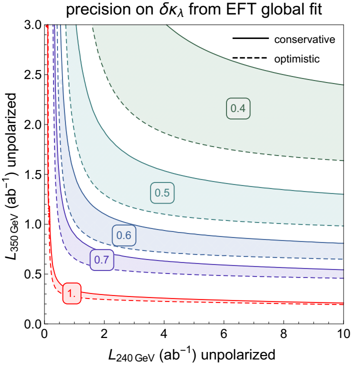






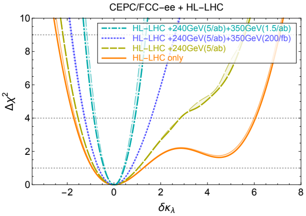
- One loop corrections to all Higgs couplings (production and decay).
- 240 GeV: hZ near threshold (more sensitive to $\delta\kappa_\lambda$)
- at 350 GeV:
  - WW fusion
  - hZ at a different energy
- $h \to WW^*/ZZ^*$ also have some discriminating power (but turned out to be not enough).

Triple Higgs coupling at circular colliders



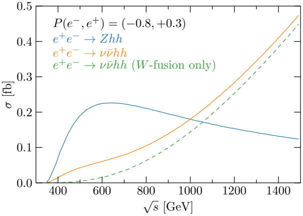
- Runs at both 240 GeV and 350 GeV are needed to obtain good constraints on $\delta\kappa_\lambda$!
- Bounds are further improved if combined with HL-LHC measurements.

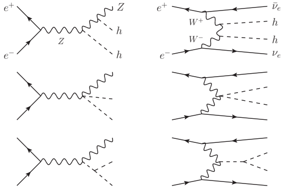
	CEPC alone		CEPC + HL-LHC	
	non-zero aTGCs	zero aTGCs	non-zero aTGCs	zero aTGCs
HL-LHC alone			$[-0.92, +1.26]$	$[-0.90, +1.24]$
240 GeV (5 ab^{-1})	$[-4.55, +4.72]$	$[-2.93, +3.01]$	$[-0.81, +1.04]$	$[-0.82, +1.03]$
+350 GeV (200 fb^{-1})	$[-1.08, +1.09]$	$[-1.04, +1.04]$	$[-0.66, +0.76]$	$[-0.66, +0.74]$
+350 GeV (1.5 ab^{-1})	$[-0.50, +0.49]$	$[-0.43, +0.43]$	$[-0.43, +0.44]$	$[-0.39, +0.40]$



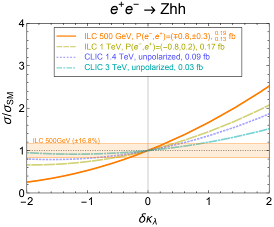
HL-LHC bounds from [arXiv:1704.01953] Di Vita, Grojean, Panico, Riembau, Vantalon

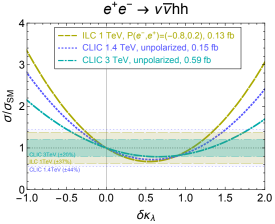
Double-Higgs measurements ($e^+e^- \to Zhh$ & $e^+e^- \to \nu\bar{\nu}hh$)

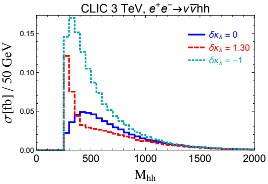




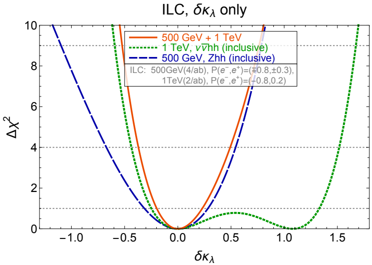
- Destructive interference in $e^+e^- \to \nu\bar{\nu}hh$! The square term is important.
- *hh* invariant mass distribution helps discriminate the "2nd solution."

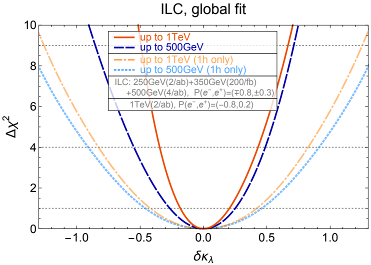






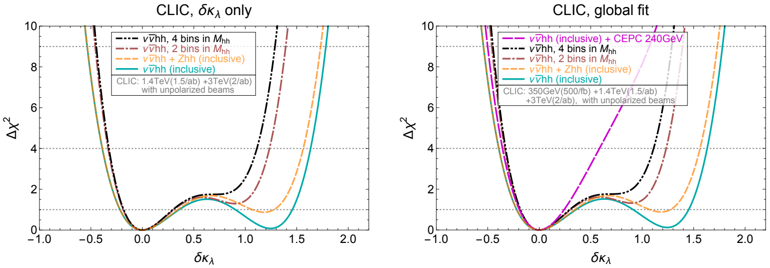
χ^2 vs. $\delta\kappa_\lambda$, ILC





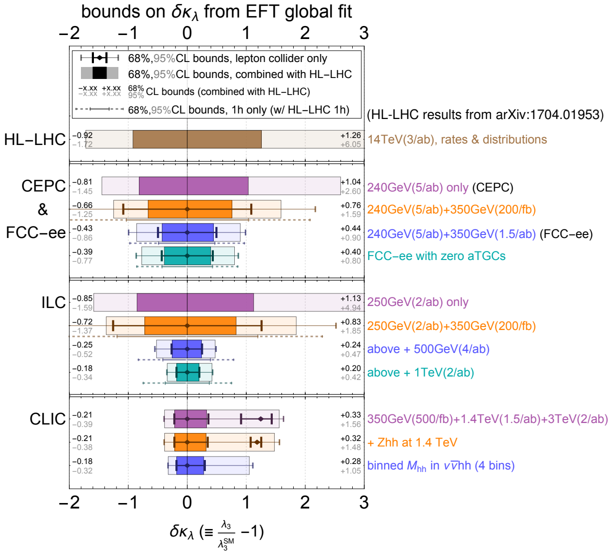
- Inputs:
  - 500 GeV (4 ab^{-1}): $\sigma(Zhh)$ measured to 16.8% [C. F. Dürig, PhD thesis, Hamburg U. (2016)]
  - 1 TeV (2 ab^{-1}): $\sigma(\nu\bar{\nu}hh)$ measured to 2.7σ significance $\Rightarrow\sim$ 37% [talk by Dürig at ALCW15]
- Complementarity between the 500 GeV run and the 1 TeV run.
- Single Higgs measurements provide non-negligible improvment.
  - up to 500 GeV: $[-0.31, +0.28] \rightarrow [-0.26, +0.25]$,
  - up to 1 TeV: $[-0.20, +0.23] \rightarrow [-0.18, +0.20]$,

χ^2 vs. $\delta\kappa_\lambda$, CLIC



- Input:
  - $\sigma(\nu\bar{\nu}hh)$ measured to 44% at 1.4 TeV and 20% at 3 TeV (Higgs Physics at the CLIC Electron-Positron Linear Collider [arXiv:1608.07538], Assuming unpolarized beam.)
  - $\sigma(Zhh)$ measured to $\sim$ 50% at 1.4 TeV (our own naive estimation).
- The measurement of *Zhh* or the M_{hh} distribution of $\nu\bar{\nu}hh$ can help resolve the “2nd solution.”
- The bounds on $\delta\kappa_\lambda$ can be further improved by having a *hZ* threshold run (*e.g.*, by combining with CEPC 240 GeV or ILC 250 GeV).

A summary of the (future) bounds on $\delta\kappa_\lambda$

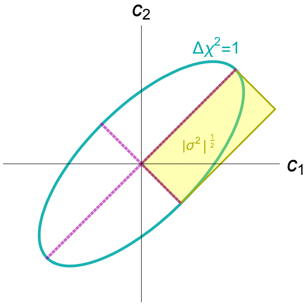


Conclusion

- Lepton colliders are great for Higgs precision measurements!
- It makes sense to go beyond the "κ" frame and study Higgs physics in the EFT framework.
- We can obtain strong and robust constraints on the coefficients of the relevant dimension-6 operators!
  - If discrepancy is observed, the EFT global fit can help identify the underlying new physics!
- We can obtain robust constraints on the triple Higgs coupling!

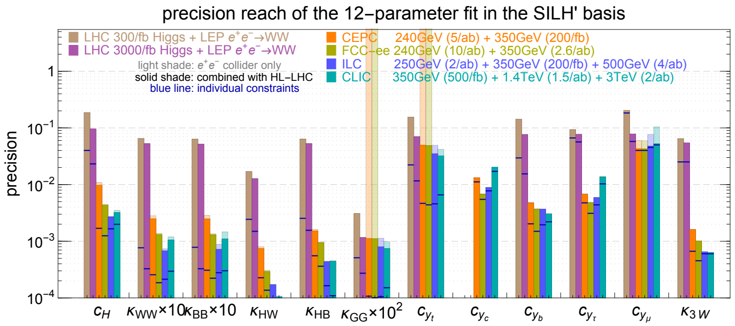
backup slides

GDP



- Global Determinant Parameter ($\mathrm{GDP} \equiv \sqrt[2n]{\det \sigma^2}$).
- Ratios of GDPs are basis-independent.
- Smaller GDP $\rightarrow$ better precision!

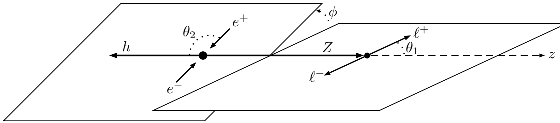
If you don't like the Higgs basis...



- Results in the SILH'(-like) basis ($\mathcal{O}_{W,B} \to \mathcal{O}_{WW,WB}$)

$$\mathcal{L}_{\rm D6} = \frac{c_H}{v^2}\mathcal{O}_H + \frac{\kappa_{WW}}{m_W^2}\mathcal{O}_{WW} + \frac{\kappa_{BB}}{m_W^2}\mathcal{O}_{BB} + \frac{\kappa_{HW}}{m_W^2}\mathcal{O}_{HW} + \frac{\kappa_{HB}}{m_W^2}\mathcal{O}_{HB} + \frac{\kappa_{GG}}{m_W^2}\mathcal{O}_{GG} + \frac{\kappa_{3W}}{m_W^2}\mathcal{O}_{3W} + \sum_{f=t,c,b,\tau,\mu}\frac{c_{y_f}}{v^2}\mathcal{O}_{y_f}\,.$$

angular observables in $e^+e^- \to hZ$

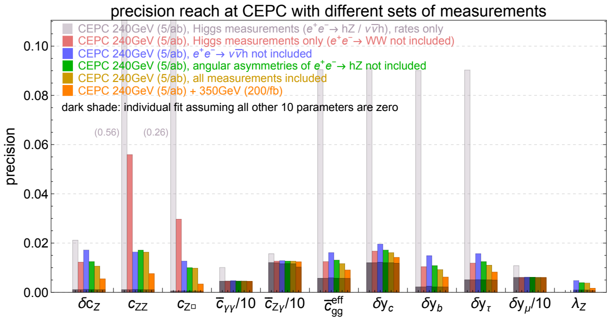


- Angular distributions in $e^+e^- \to hZ$ can provide information in addition to the rate measurement alone.
- Previous studies
  - [arXiv:1406.1361] M. Beneke, D. Boito, Y.-M. Wang
  - [arXiv:1512.06877] N. Craig, JG, Z. Liu, K. Wang
- 6 independent asymmetry observables from 3 angles

$$\mathcal{A}_{\theta_1}\ ,\ \ \mathcal{A}_{\phi}^{(1)}\ ,\ \ \mathcal{A}_{\phi}^{(2)}\ ,\ \ \mathcal{A}_{\phi}^{(3)}\ ,\ \ \mathcal{A}_{\phi}^{(4)}\ ,\ \ \mathcal{A}_{c\theta_1,c\theta_2}\ .$$

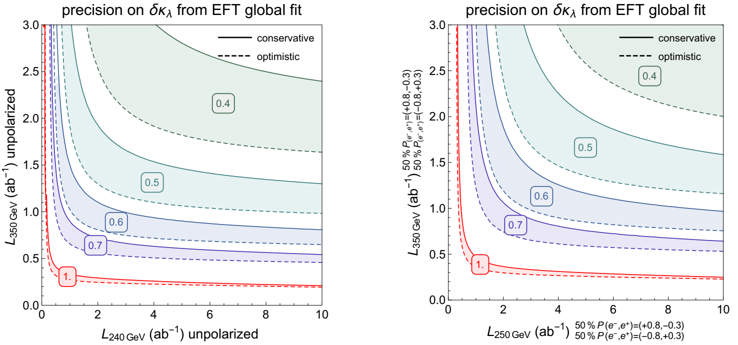
- Focusing on leptonic decays of Z (good resolution, small background, statistical uncertainty dominates).

The importance of combining all measurements



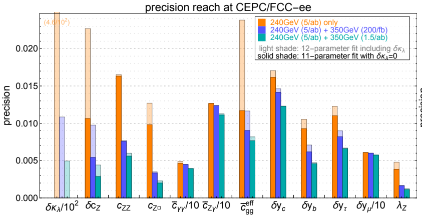
- The results are much worse if we only include the rates of Higgs measurements alone!
- There is some overlap in the information from different measurements.
- Measurements at different energies can be very helpful.

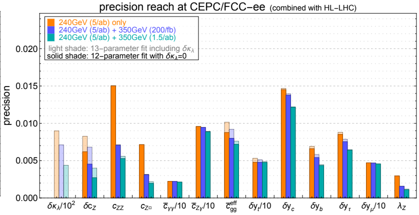
The precision reach of $\delta\kappa_\lambda$ at the low-energy ILC



- $e^+e^- \to hZ$ is more sensitive to $\delta\kappa_\lambda$ near the threshold (240 GeV *vs.* 250 GeV).
- Polarization doesn't help too much here...

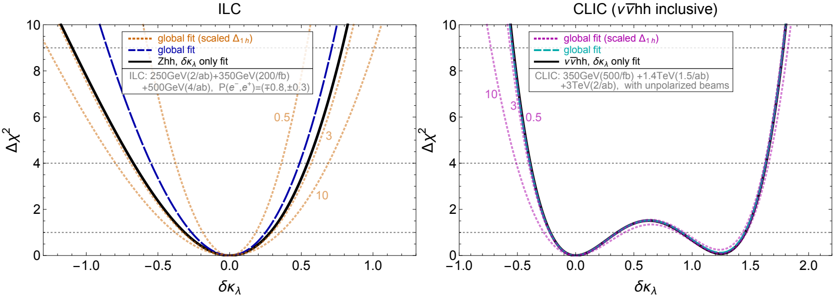
Impact of $\delta\kappa_\lambda$ on the other parameters





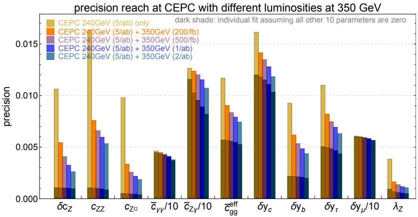
- Adding one more parameter could worsen the bounds on others.
- The effect is under control if the degeneracies are well-resolved.
- The HL-LHC bounds on $\delta\kappa_\lambda$ can also help.

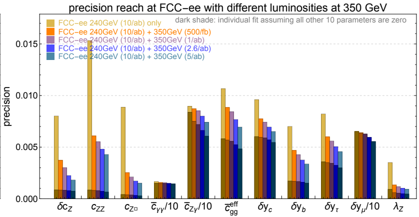
Impact of the single Higgs measurements

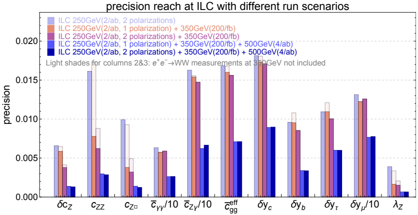


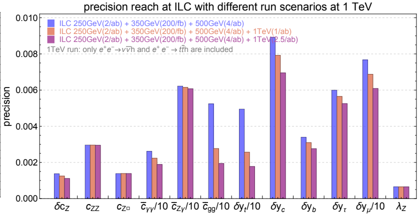
- What if the single Higgs measurements are much better or much worse?
- Much better: can further improve the bounds on $\delta\kappa_\lambda$ from double-Higgs measurements.
- Much worse: can significantly worsen the bounds on $\delta\kappa_\lambda$ from double-Higgs measurements.

Impact of the Higher energy runs

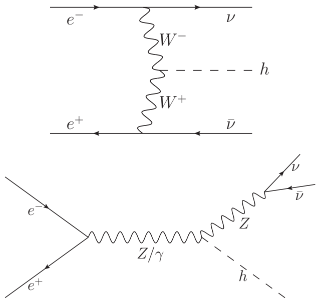


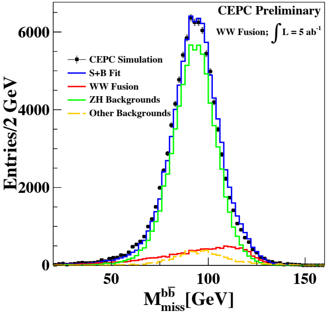






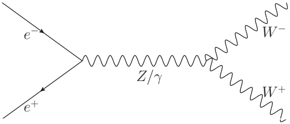
$e^+e^- \to \nu\bar{\nu}h$

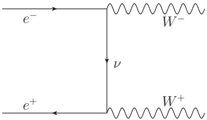




- It is hard to separate the *WW* fusion process from $e^+e^- \to hZ, Z \to \nu\bar{\nu}$ at 240 GeV.
- It is not consistent to focus on one process and treat the other one as SM-like!
- For CEPC/FCC-ee 240 GeV, we analyze the combined $e^+e^- \to \nu\bar{\nu}h$ process, assuming new physics can contribute to both processes.

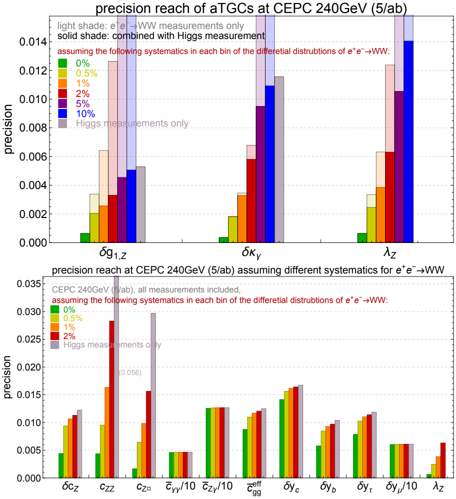
$e^+e^- \to WW$





- $e^+e^- \to WW$ offers a great way to probe the anomalous triple gauge couplings (aTGCs, parameterized by $\delta g_{1,Z}$, $\delta\kappa_\gamma$, λ_Z).
- $\delta g_{1,Z}$ and $\delta\kappa_\gamma$ are related to Higgs observables.
- CEPC with $5\,\mathrm{ab}^{-1}$ data at 240 GeV can produce $\sim 9 \times 10^7$ $e^+e^- \to WW$ events.
- With such large statistics, the aTGCs can be very well constrained ([1507.02238] Bian, Shu, Zhang), but with two potential issues:
    - Systematic uncertainties can be important!
    - If $e^+e^- \to WW$ is measured more precisely than the Z-pole measurements, is it still ok to assume the fermion gauge couplings are SM-like?

The interplay between Higgs and TGC



- $\delta g_{1,Z}$, $\delta\kappa_\gamma \leftrightarrow$ c_{ZZ} , $c_{Z\Box}$, $c_{\gamma\gamma}$, $c_{Z\gamma}$
- We try different assumptions on the systematic uncertainties (in each bin with the differential distribution divided into 20 bins).
- Detailed study of $e^+e^- \to WW$ required to estimate the systematic uncertainties!

Asymmetry observables

$$\mathcal{A}_{\theta_1} = \frac{1}{\sigma} \int_{-1}^{1} d\cos\theta_1 \,\mathrm{sgn}(\cos(2\theta_1))\, \frac{d\sigma}{d\cos\theta_1} \,,$$
$$\mathcal{A}_{\phi}^{(1)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \,\mathrm{sgn}(\sin\phi)\, \frac{d\sigma}{d\phi} \,,$$
$$\mathcal{A}_{\phi}^{(2)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \,\mathrm{sgn}(\sin(2\phi))\, \frac{d\sigma}{d\phi} \,,$$
$$\mathcal{A}_{\phi}^{(3)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \,\mathrm{sgn}(\cos\phi)\, \frac{d\sigma}{d\phi} \,,$$
$$\mathcal{A}_{\phi}^{(4)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \,\mathrm{sgn}(\cos(2\phi))\, \frac{d\sigma}{d\phi} \,, \tag{1}$$
$$\mathcal{A}_{c\theta_1,c\theta_2} = \frac{1}{\sigma} \int_{-1}^{1} d\cos\theta_1 \,\mathrm{sgn}(\cos\theta_1) \int_{-1}^{1} d\cos\theta_2 \,\mathrm{sgn}(\cos\theta_2) \frac{d^2\sigma}{d\cos\theta_1\, d\cos\theta_2} \,, \tag{2}$$

The "12-parameter" framework in the Higgs basis

- The relevant terms in the EFT Lagrangian are

$$\mathcal{L} \supset \mathcal{L}_{hVV} + \mathcal{L}_{hff} + \mathcal{L}_{\rm tgc} \,, \tag{3}$$

- the Higgs couplings with a pair of gauge bosons

$$\begin{aligned}\mathcal{L}_{hVV} = \frac{h}{v}\bigg[&(1+\delta c_W)\frac{g^2v^2}{2}W^+_\mu W^-_\mu + (1+\delta c_Z)\frac{(g^2+g'^2)v^2}{4}Z_\mu Z_\mu \\ &+ c_{WW}\frac{g^2}{2}W^+_{\mu\nu}W^-_{\mu\nu} + c_{W\Box}\, g^2(W^-_\mu \partial_\nu W^+_{\mu\nu} + \text{h.c.}) \\ &+ c_{gg}\frac{g_s^2}{4}G^a_{\mu\nu}G^2_{\mu\nu} + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A_{\mu\nu} + c_{Z\gamma}\frac{e\sqrt{g^2+g'^2}}{2}Z_{\mu\nu}A_{\mu\nu} \\ &+ c_{ZZ}\frac{g^2+g'^2}{4}Z_{\mu\nu}Z_{\mu\nu} + c_{Z\Box}\, g^2 Z_\mu\partial_\nu Z_{\mu\nu} + c_{\gamma\Box}\, gg' Z_\mu\partial_\nu A_{\mu\nu}\bigg]\,. \end{aligned} \tag{4}$$

The "12-parameter" framework in the Higgs basis

- Not all the couplings are independent, for instance one could write the following couplings as

$$
\begin{aligned}
\delta c_W &= \delta c_Z + 4\delta m \,, \\
c_{WW} &= c_{ZZ} + 2 s^2_{\theta_W} c_{Z\gamma} + s^4_{\theta_W} c_{\gamma\gamma} \,, \\
c_{W\Box} &= \frac{1}{g^2 - g'^2} \left[g^2 c_{Z\Box} + g'^2 c_{ZZ} - e^2 s^2_{\theta_W} c_{\gamma\gamma} - (g^2 - g'^2) s^2_{\theta_W} c_{Z\gamma} \right] \,, \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2} \left[2g^2 c_{Z\Box} + (g^2 + g'^2) c_{ZZ} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{Z\gamma} \right] \,, \qquad (5)
\end{aligned}
$$

- we only consider the diagonal elements in the Yukawa matrices relevant for the measurements considered,

$$
\mathcal{L}_{hff} = -\frac{h}{v} \sum_{f=t,c,b,\tau,\mu} m_f (1 + \delta y_f) \bar{f}_R f_L + \text{h.c.} \,. \qquad (6)
$$

TGC

$$
\begin{aligned}
\mathcal{L}_{\rm tgc} = \; & igs_{\theta_W} A^\mu (W^{-\nu} W^+_{\mu\nu} - W^{+\nu} W^-_{\mu\nu}) \\
& + ig(1+\delta g_1^Z) c_{\theta_W} Z^\mu (W^{-\nu} W^+_{\mu\nu} - W^{+\nu} W^-_{\mu\nu}) \\
& + ig \left[(1+\delta\kappa_Z) c_{\theta_W} Z^{\mu\nu} + (1+\delta\kappa_\gamma) s_{\theta_W} A^{\mu\nu} \right] W^-_\mu W^+_\nu \\
& + \frac{ig}{m_W^2} (\lambda_Z c_{\theta_W} Z^{\mu\nu} + \lambda_\gamma s_{\theta_W} A^{\mu\nu}) W^{-\rho}_v W^+_{\rho\mu} \,, \qquad (7)
\end{aligned}
$$

- $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ for $V = W^\pm, Z, A,$. Imposing Gauge invariance one obtains $\delta\kappa_Z = \delta g_{1,Z} - t^2_{\theta_W} \delta\kappa_\gamma$ and $\lambda_Z = \lambda_\gamma$.
- 3 aTGCs parameters $\delta g_{1,Z}$, $\delta\kappa_\gamma$ and λ_Z, 2 of them related to Higgs observables by

$$
\delta g_{1,Z} = \frac{1}{2(g^2 - g'^2)} \left[-g^2(g^2+g'^2) c_{Z\Box} - g'^2(g^2+g'^2) c_{ZZ} + e^2 g'^2 c_{\gamma\gamma} + g'^2(g^2-g'^2) c_{Z\gamma} \right]
$$

$$
\delta\kappa_\gamma = -\frac{g^2}{2} \left(c_{\gamma\gamma} \frac{e^2}{g^2+g'^2} + c_{Z\gamma} \frac{g^2-g'^2}{g^2+g'^2} - c_{ZZ} \right) . \qquad (8)
$$

CEPC/FCC-ee Higgs rate measurements

	CEPC				FCC-ee			
	[240 GeV, 5 ab^{-1}]		[350 GeV, 200 fb^{-1}]		[240 GeV, 10 ab^{-1}]		[350 GeV, 2.6 ab^{-1}]	
production	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$
σ	0.50%	-	2.4%	-	0.40%	-	0.67%	-
	$\sigma \times$ BR				$\sigma \times$ BR			
$h \to b\bar{b}$	0.21%*	0.39%$^{\diamond}$	2.0%	2.6%	0.20%	0.28%$^{\diamond}$	0.54%	0.71%
$h \to c\bar{c}$	2.5%	-	15%	26%	1.2%	-	4.1%	7.1%
$h \to gg$	1.2%	-	11%	17%	1.4%	-	3.1%	4.7%
$h \to \tau\tau$	1.0%	-	5.3%	37%	0.7%	-	1.5%	10%
$h \to WW^*$	1.0%	-	10%	9.8%	0.9%	-	2.8%	2.7%
$h \to ZZ^*$	4.3%	-	33%	33%	3.1%	-	9.2%	9.3%
$h \to \gamma\gamma$	9.0%	-	51%	77%	3.0%	-	14%	21%
$h \to \mu\mu$	12%	-	115%	275%	13%	-	32%	76%
$h \to Z\gamma$	25%	-	144%	-	18%	-	40%	-

Table: For $e^+e^- \to \nu\bar{\nu}h$, the precisions marked with a diamond $^{\diamond}$ are normalized to the cross section of the inclusive channel which includes both the WW fusion and $e^+e^- \to hZ, Z \to \nu\bar{\nu}$, while the unmarked ones include WW fusion only.

ILC Higgs rate measurements

ILC

	[250 GeV, 2 ab^{-1}]		[350 GeV, 200 fb^{-1}]		[500 GeV, 4 ab^{-1}]			[1 TeV, 1 ab^{-1}]		[1 TeV, 2.5 ab^{-1}]	
production	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	$t\bar{t}h$	$\nu\bar{\nu}h$	$t\bar{t}h$	$\nu\bar{\nu}h$	$t\bar{t}h$
σ	0.71%	-	2.1%	-	1.1%	-	-	-	-	-	-
	$\sigma \times \mathrm{BR}$										
$h \to b\bar{b}$	0.42%	3.7%	1.7%	1.7%	0.64%	0.25%	9.9%	0.5%	6.0%	0.3%	3.8%
$h \to c\bar{c}$	2.9%	-	13%	17%	4.6%	2.2%	-	3.1%	-	2.0%	-
$h \to gg$	2.5%	-	9.4%	11%	3.9%	1.4%	-	2.3%	-	1.4%	-
$h \to \tau\tau$	1.1%	-	4.5%	24%	1.9%	3.2%	-	1.6%	-	1.0%	-
$h \to WW^*$	2.3%	-	8.7%	6.4%	3.3%	0.85%	-	3.1%	-	2.0%	-
$h \to ZZ^*$	6.7%	-	28%	22%	8.8%	2.9%	-	4.1%	-	2.6%	-
$h \to \gamma\gamma$	12%	-	44%	50%	12%	6.7%	-	8.5%	-	5.4%	-
$h \to \mu\mu$	25%	-	98%	180%	31%	25%	-	31%	-	20%	-
$h \to Z\gamma$	34%	-	145%	-	49%	-	-	-	-	-	-

CLIC Higgs rate measurements

CLIC

	[350 GeV, 500 fb^{-1}]		[1.4 TeV, 1.5 ab^{-1}]		[3 TeV, 2 ab^{-1}]
production	Zh	$\nu\bar{\nu}h$	$\nu\bar{\nu}h$	$t\bar{t}h$	$\nu\bar{\nu}h$
σ	1.6%	-	-	-	-
	$\sigma \times \mathrm{BR}$				
$h \to b\bar{b}$	0.84%	1.9%	0.4%	8.4%	0.3%
$h \to c\bar{c}$	10.3%	14.3%	6.1%	-	6.9%
$h \to gg$	4.5%	5.7%	5.0%	-	4.3%
$h \to \tau\tau$	6.2%	-	4.2%	-	4.4%
$h \to WW^*$	5.1%	-	1.0%	-	0.7%
$h \to ZZ^*$	-	-	5.6%	-	3.9%
$h \to \gamma\gamma$	-	-	15%	-	10%
$h \to \mu\mu$	-	-	38%	-	25%
$h \to Z\gamma$	-	-	42%	-	30%

Table: We also include the estimations for $\sigma(hZ) \times \mathrm{BR}(h \to b\bar{b})$ at high energies in [arXiv:1701.04804] (Ellis et al.), which are 3.3% (6.8%) at 1.4 TeV (3 TeV). For simplicity, the measurements of ZZ fusion ($e^+e^- \to e^+e^-h$) are not included in our analysis.