PWA on the $e^+e^- \to \pi^+\pi^- h_c$ at $\sqrt{s} = 4.42$ GeV

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Outline

1. Introduction
2. Event selection
3. Amplitude and fit method
4. Study Zc(4020) as 1^+
5. Study Zc(4020) as other J^P states
6. Statistical significance to assign $J^P=1^+$
7. Uncertainties
8. Cross section
9. Summary and discussion

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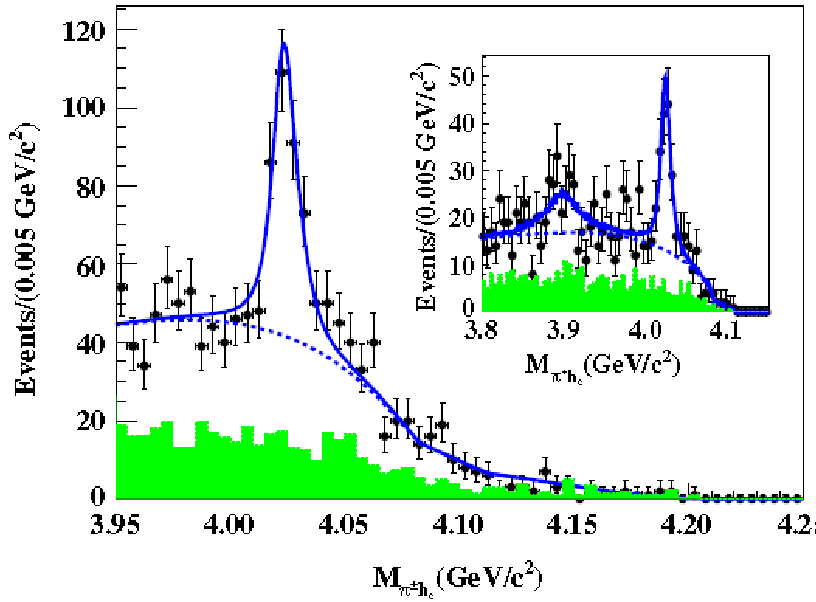
1. Introduction

- Zc(4020) was observed for the first time at BESIII using cocktail data sets taken from 3.9~4.42GeV.
- Evidence of Zc(3900) was searched in the final state $\pi^+\pi^- h_c$.
- Recent observation of charmonium and bottomnium like state motivate many theoretical investigation on the nature , structure and decay mechanism.
- To interpret them, such as tetraquark scenario, hadronic molecular, meson loop and so on.
- Using more XYZ data accumulated at BESIII, the cross section for $\pi^+\pi^- h_c$ was measured.
- We perform the PWA based on 1073.56/pb taken at 4.42 GeV.



BESIII, PRL111, 242001 (2013)

2. Event selection

- Data sets: L=1073.56/pb taken at 4.42 GeV
- $h_c \to \gamma\eta_c$ and $\eta_c \to X_i$

X_i : 16 exclusive modes , see arXiv: 1610.07044v1

$$p\bar{p},\ 2(\pi^+\pi^-),\ 2(K^+K^-),\ \pi^+\pi^-K^+K^-,$$
$$\pi^+\pi^-p\bar{p},\ 3(\pi^+\pi^-),\ 2(\pi^+\pi^-)K^+K^-,\ K^0_SK^\pm\pi^\mp,$$
$$K^0_SK^\pm\pi\mp\pi^+\pi^-,\ K^+K^-\pi^0,\ p\bar{p}\pi^0,\ K^+K^-\eta,$$
$$\pi^+\pi^-\eta,\ 2(\pi^+\pi^-)\eta,\ \pi^+\pi^-\pi^0\pi^0 \text{ and } 2(\pi^+\pi^-)\pi^0.$$

Candidates: 914
Background: 323

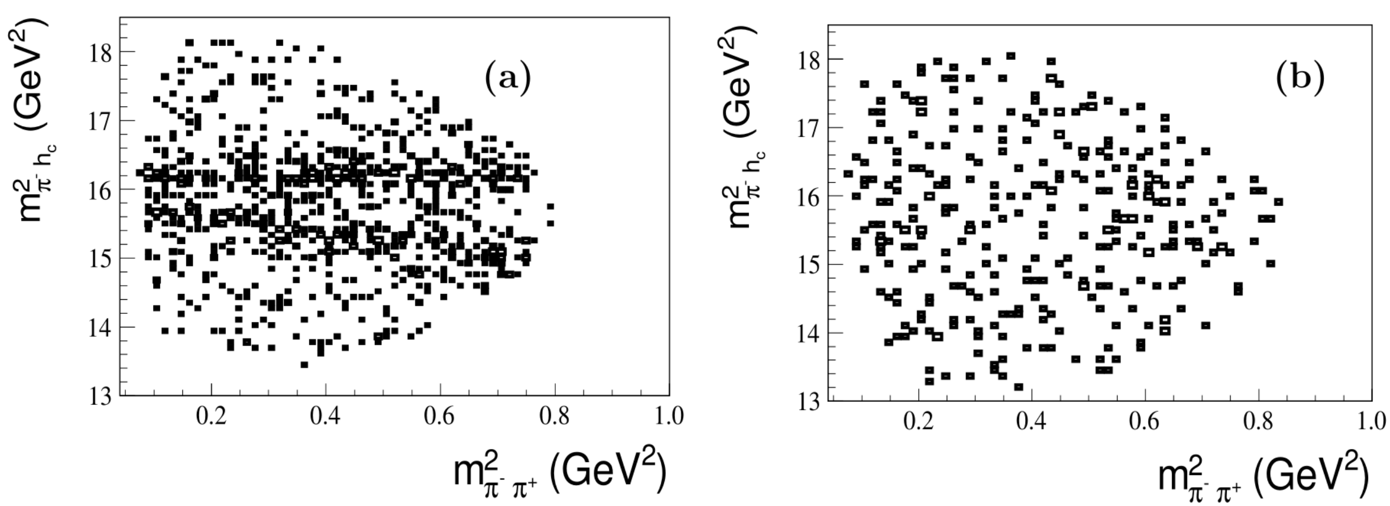
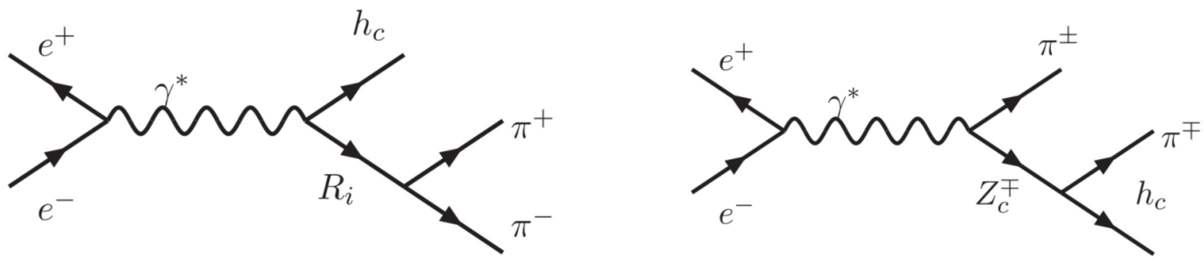


Fig. 1: Dalitz plots for the data (a) and backgrounds (b).

3. Amplitude and fit method

The quasi-two body decays for process $e^+e^- \to \gamma^* \to \pi^+\pi^- h_c$



(a) (b)

(a) $e^+e^- \to \gamma^*(\lambda_0) \to R_j(\lambda_1)h_c(\lambda_2), R_j(\lambda_1) \to \pi^+\pi^-$

$$A_1(\lambda_0,\lambda_2) = \sum_{\lambda_1,\lambda_2,j} F^{\gamma^*}_{\lambda_1,\lambda_2}(r_1) D^{1*}_{\lambda_0,\lambda_1-\lambda_2}(\theta_0,\phi_0) BW_j(m_{\pi^+\pi^-}) F^{R_j}_{0,0}(r_2) D^{J_1*}_{\lambda_1,0}(\theta_1,\phi_1),$$

(b) $\gamma^*(\lambda_0) \to Z_c^\pm(\lambda_1)\pi^\mp, Z_c^\pm(\lambda_1) \to \pi^\pm h_c(\lambda'_2)$

$$A_2(\lambda_0,\lambda_2) = \boxed{\sum_{\lambda'_2} d^{J_1}_{\lambda'_2,\lambda_2}(\theta_2)} \sum_{\lambda_1,j} F^{\gamma^*}_{\lambda_1,0}(r_1) D^{1*}_{\lambda_0,\lambda_1}(\theta_0,\phi_0) BW_j(m_{h_c\pi}) F^{Z_c}_{\lambda'_2,0}(r_2) D^{J_1*}_{\lambda_1,\lambda'_2}(\theta_1,\phi_1),$$

Helicity amplitude are expanded according to the L-S coupling scheme:

$$F^J_{\lambda\nu} = \sum_{ls} \left(\frac{2l+1}{2J+1}\right)^{1/2} \langle l0S\delta|J\delta\rangle\langle s\lambda\sigma-\nu|S\delta\rangle g_{lS} r^l B_l(r),$$

If Zc(4020) is assigned as 1^+, one has

$$F^{\gamma^*}_{1,0}(r) = F^{\gamma^*}_{-1,0}(r) = \frac{1}{\sqrt{3}} g_{01} B_0(r) + \frac{r^2}{\sqrt{6}} g_{21} B_2(r),$$

$$F^{\gamma^*}_{0,0}(r) = \frac{1}{\sqrt{3}} g_{01} B_0(r) - \frac{2r^2}{\sqrt{6}} g_{21} B_2(r),$$

Angular distribution for e+e- $\rightarrow Z_c(4020)\pi$ take the form

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \alpha\cos^2\theta_0, \text{ with } \alpha = \frac{|F^{\gamma^*}_{1,0}|^2 - |F^{\gamma^*}_{0,0}|^2}{|F^{\gamma^*}_{1,0}|^2 + |F^{\gamma^*}_{0,0}|^2},$$

The total amplitude is expressed by:

$$A(\lambda_0, \lambda_2) = \sum_{i=1}^{3} g_i A_i(\lambda_0, \lambda_2),$$

The differential cross-section is given by:

$$d\sigma = \left(\frac{3}{8\pi^2}\right) \sum_{\lambda_0, \lambda_2} A(\lambda_0, \lambda_2) A^*(\lambda_0, \lambda_2) d\Phi,$$

Breit-Wigner

We use a relativistic Breit-Wigner function in the analysis,

$$BW(m) = \frac{1}{m^2 - m_0^2 - im\Gamma_X(m)},$$

σ resonance: $\Gamma_X(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}\Gamma,$ E791 type

The $f_0(980)$ is parametrized by the Flatté-formula:

$$BW(s) = \frac{1}{M^2 - s - i(g_1\rho_{\pi\pi}(s) + g_2\rho_{K\bar{K}}(s))},$$

Flatté-like formula:

$Z_c(3900)$

$$BW(s) = \frac{1}{M^2 - s - i(g_1'\rho_{\pi J/\psi}(s) + g_2'\rho_{D^*D}(s))},$$

- **Fit method**

Likelihood function:

$$\mathcal{L} = \prod_{i=1}^{N} P(x_i), \quad P(x_i) = \frac{(d\sigma/d\Phi)_i}{\sigma_{MC}},$$

$$\sigma_{MC} = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left(\frac{d\sigma}{d\Phi}\right)_i.$$

Use MINUIT package to minimize the object function

$$S = -\ln \mathcal{L}$$

Background is subtracted from lon-likelihood function

$$\ln \mathcal{L} = \ln \mathcal{L}_{\mathrm{data}} - \ln \mathcal{L}_{\mathrm{bg}}.$$

Signal yields are calculated by

$$N_i = R_i * (N_{\mathrm{obs}} - N_{\mathrm{bg}}), \text{ with } R_i = \frac{\sigma_i}{\sigma_{\mathrm{tot}}},$$

Statistical uncertainties are calculated with covariant matrix

$$\delta N_i^2 = \sum_{m=1}^{N_{\mathrm{pars}}} \sum_{n=1}^{N_{\mathrm{pars}}} \left(\frac{\partial N_i}{\partial X_m} \frac{\partial N_i}{\partial X_n} \right)_{\mathbf{X}=\mu} V_{mn}(\mathbf{X}),$$

4. Study Zc(4020) as 1^+

- The $Zc(4020)^{\pm}$ are assumed as isospin partner
- The $f_0(980)$ is described with Flatte formula with $g_1 = 0.138 \pm 0.010 \text{ GeV}^2$ and $g_2/g_1 = 4.45 \pm 0.25.$
- The Zc(3900) is parameterized with Flatte-like formula, with

$$M = 3901.5 \pm 2.7 \pm 38.2 \text{ MeV}, \quad g_1' = 0.075 \pm 0.006 \pm 0.025 \text{ GeV}^2,$$

$$g_2'/g_1' = 27.1 \pm 2.0 \pm 1.8$$

- Determine baseline solution, Zc(4020) mass and width fixed to observed value

Resonance	$\Delta(-2\ln L)$	Δndf	significance
$h_c(\pi^+\pi^-)_{\text{S-wave}}$	41.4	4	5.6σ
$Z_c(3900)^{\pm}\pi^{\mp}$	52.2	4	6.4σ
$Z_c(4020)^{\pm}\pi^{\mp}$	269.6	4	15.9σ
$h_cf_2(1270)$	26.8	8	3.4σ

$(\pi^+\pi^-)_{\text{S-wave}}$ is parameterized with σ and $f_0(980)$ resonance.

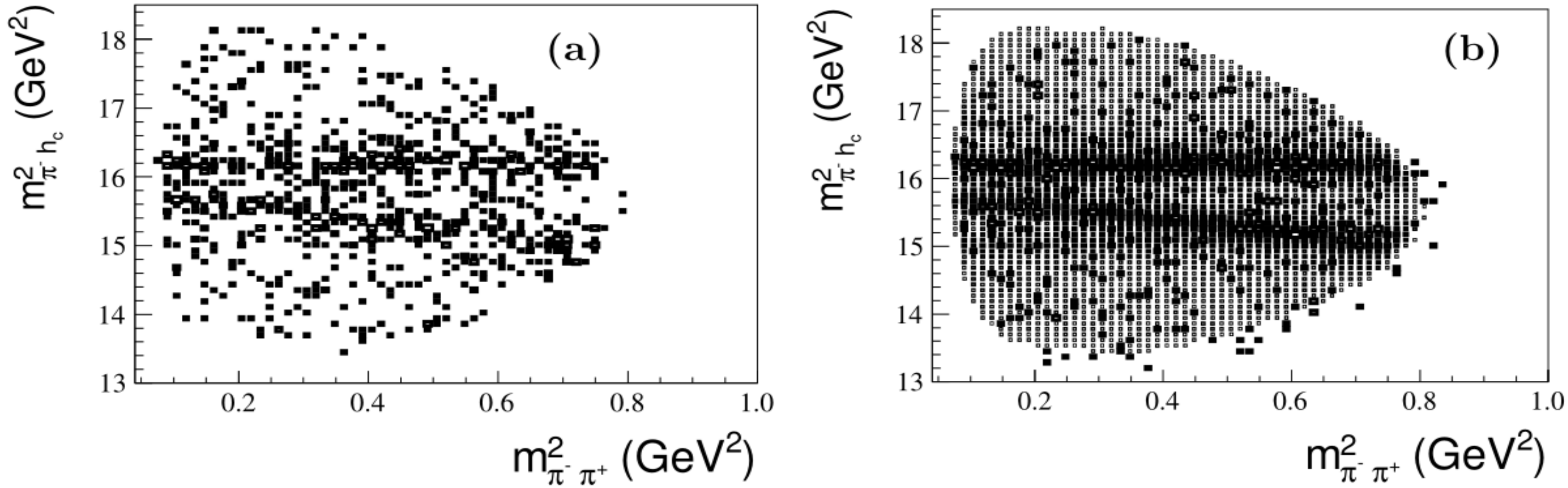


Fig. 3: (a) Dalitz plot for the data, (b) Dalitz plot for the fitted results (including background events).

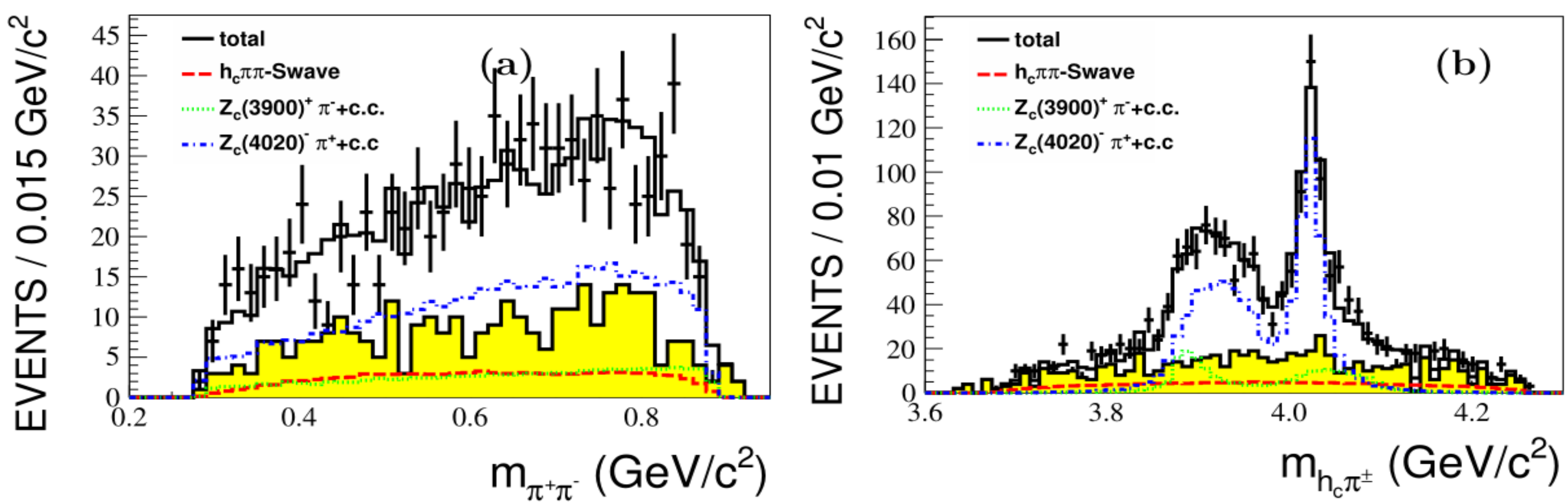


Fig. 4: Fit results with $(\pi^+\pi^-)_{\text{S-wave}}$ resonances, $Z_c(4020)^\pm$ and $Z_c(3900)^\pm$. The quantum number of Z_c is assigned as $J^P = 1^+$. (a): $m_{\pi^+\pi^-}$ distribution, (b): $m_{\pi^\pm h_c}$ distribution for data. The points with error bars are data, and black histogram is the total

The signal yields for each mode, here the errors are only statistical

$\sqrt{s}$	$(\pi^+\pi^-)_{\text{S-wave}}$	$Z_c(3900)^\pm$	$Z_c(4020)^\pm$
4.42 GeV	100.0 ± 84.0	102.1 ± 71.1	443.9 ± 153.3

- Helicity amplitude ratio for $e^+e^- \rightarrow Zc(4020)^+\pi^- + c.c.$

$$|F^{\gamma^*}_{1,0}|^2/|F^{\gamma^*}_{0,0}|^2 = 0.59 \pm 0.19,$$

Angular distribution parameter is calculated to be

$$\alpha = \frac{|F^{\gamma^*}_{1,0}|^2 - |F^{\gamma^*}_{0,0}|^2}{|F^{\gamma^*}_{1,0}|^2 + |F^{\gamma^*}_{0,0}|^2}$$

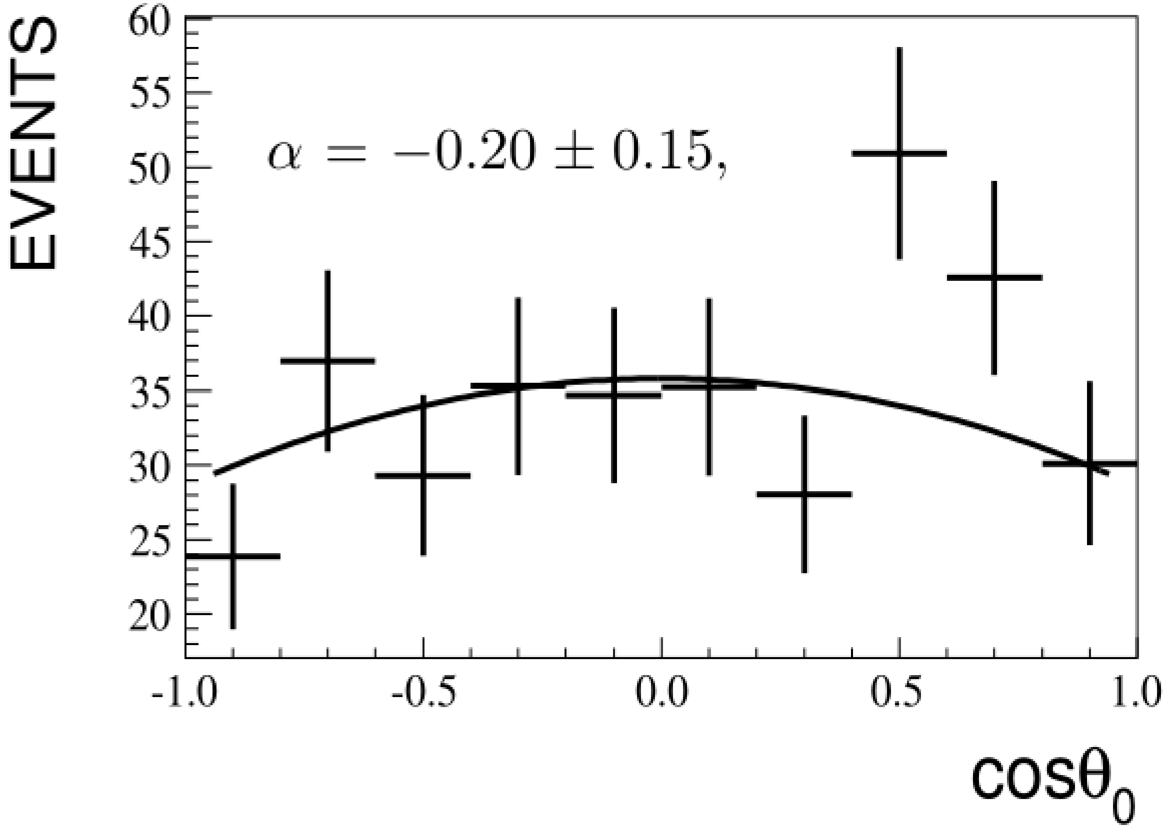
$$= -0.22 \pm 0.15$$

- A cross check on the angular distribution parameter.

$Z_c(4020)$ mass region with $4.0 < m_{\pi^\pm h_c} < 4.05$ GeV.

Detection efficiency correction is done based on the scatter plot of $\cos\theta_0$ versus $m_{\pi^\pm h}$

Background events are subtracted within the Zc(4020) mass region



5. Study Zc(4020) as other J^P states

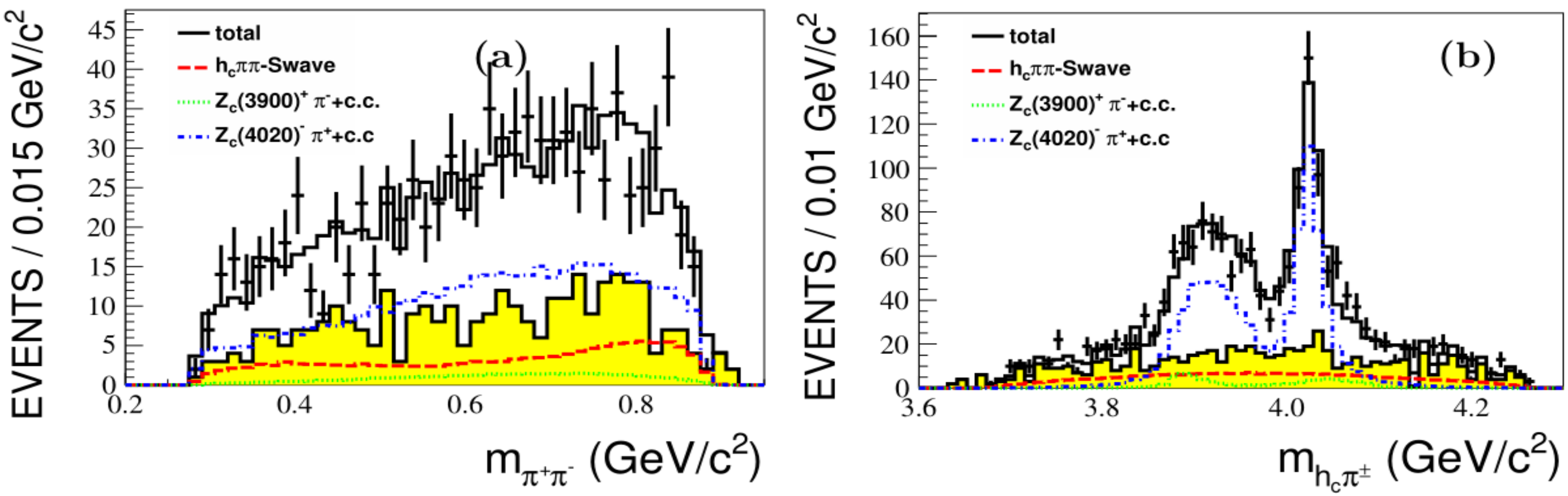
- The $J^P = 0^{\pm}$ states are not allowed to conserve the spin parity
- We only consider $J^P = 1^-$ and $2^{\pm}$.
- The mass and width of Zc(4020) are all fixed to the observed values in all cases

Comparison of minus log-likelihood

$Z_c : J^P$	$-\ln L$
1^-	-290.6
1^+	-299.6
2^-	-279.7
2^+	-265.3

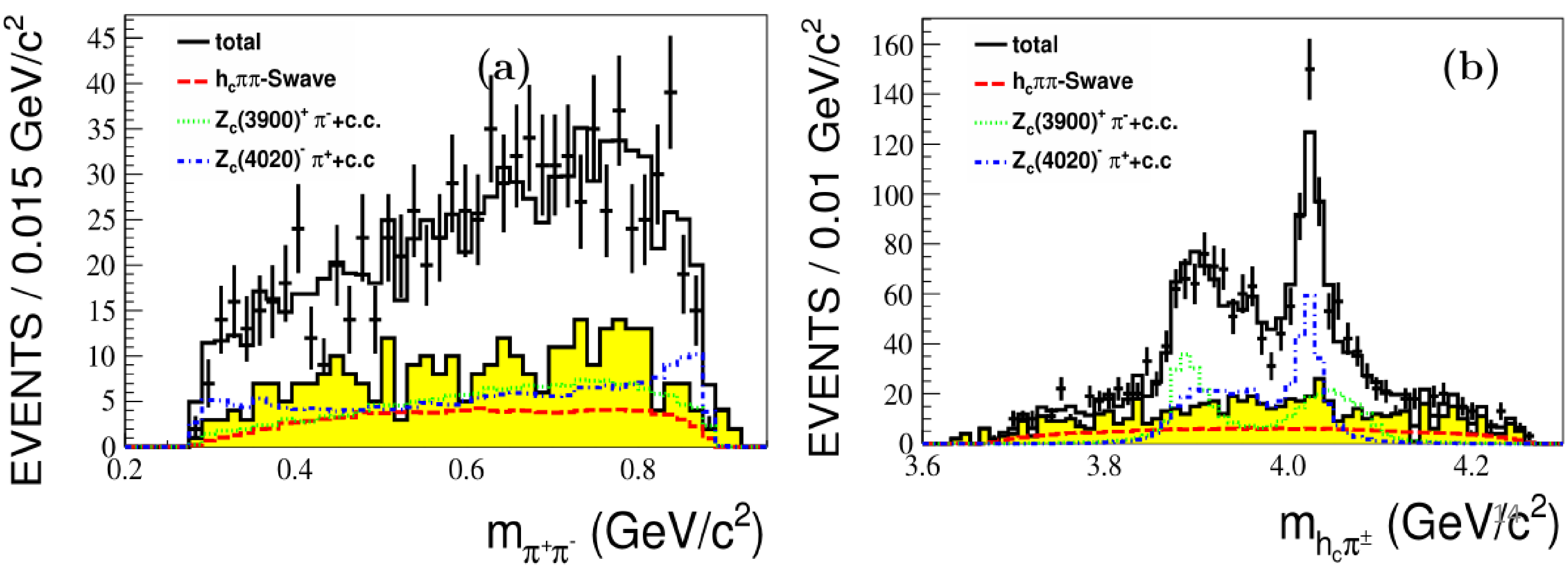
- The data favors for $J^P = 1^+$ assignment to Zc(4020)

$$J^P = 1^-.$$



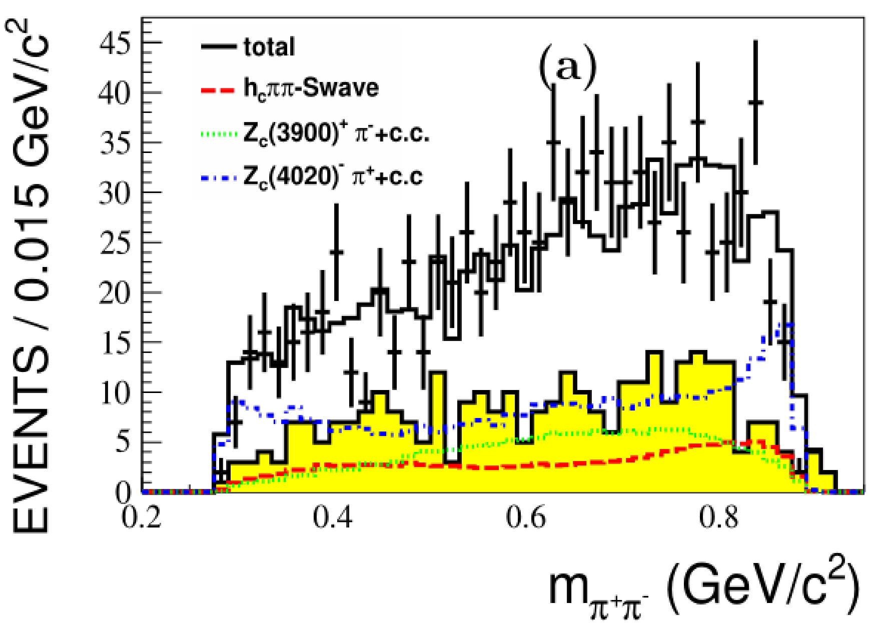
(a)
total
$h_c\pi\pi$-Swave
$Z_c(3900)^+ \pi^-$+c.c.
$Z_c(4020)^- \pi^+$+c.c
EVENTS / 0.015 GeV/c^2
$m_{\pi^+\pi^-}$ (GeV/c^2)
(b)
total
$h_c\pi\pi$-Swave
$Z_c(3900)^+ \pi^-$+c.c.
$Z_c(4020)^- \pi^+$+c.c
EVENTS / 0.01 GeV/c^2
$m_{h_c\pi^\pm}$ (GeV/c^2)

$$J^p = 2^-.$$

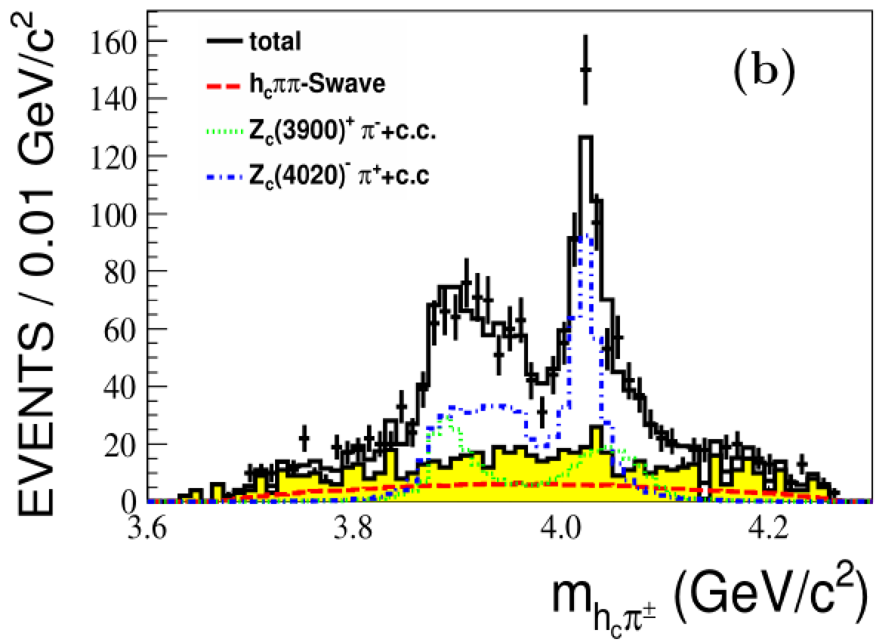


(a)
total
$h_c\pi\pi$-Swave
$Z_c(3900)^+ \pi^-$+c.c.
$Z_c(4020)^- \pi^+$+c.c
EVENTS / 0.015 GeV/c^2
$m_{\pi^+\pi^-}$ (GeV/c^2)
(b)
total
$h_c\pi\pi$-Swave
$Z_c(3900)^+ \pi^-$+c.c.
$Z_c(4020)^- \pi^+$+c.c
EVENTS / 0.01 GeV/c^2
$m_{h_c\pi^\pm}$ (GeV/c^2)

$J^p = 2^+$



EVENTS / 0.015 GeV/c²
total
$h_c\pi\pi$-Swave
$Z_c(3900)^+ \pi^-$+c.c.
$Z_c(4020)^- \pi^+$+c.c
(a)
0.2
0.4
0.6
0.8
0
5
10
15
20
25
30
35
40
45
$m_{\pi^+\pi^-}$ (GeV/c²)



EVENTS / 0.01 GeV/c²
total
$h_c\pi\pi$-Swave
$Z_c(3900)^+ \pi^-$+c.c.
$Z_c(4020)^- \pi^+$+c.c
(b)
3.6
3.8
4.0
4.2
0
20
40
60
80
100
120
140
160
$m_{h_c\pi^\pm}$ (GeV/c²)

6.Statistical significance for the Zc(4020) as 1^+ state

Null hypothesis H_0 : **data described with** $(\sigma_0, Zc(J^P))$

Alternative hypothesis H_1: data described with $(\sigma_0, Zc(1^+), \text{other } Zc(J^P))$

$$t \equiv -2\ln\lambda = 2[\ln L_{\max}(H_1) - \ln L_{\max}(H_0)],$$

See Ref.

Ilya Narsky, Nucl. Instr. Meth., A **450**, 444 (2000); Zhu Yong-Sheng, High Energy Physics and Nuclear Physics, **30**, 331 (2006).

$$p(t_{\text{obs}}) = \int_{t_{\text{obs}}}^{\infty} \chi^2(t;r)dt.$$

$$\int_{-S}^{S} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 - p(t_{\text{obs}}) = \int_0^{t_{\text{obs}}} \chi^2(t;r)dt.$$

Significance to distinguish the quantum number 1^+ over other quantum numbers.

Hypothesis	$\Delta(-2\ln L)$	$\Delta(ndf)$	significance
1^+ over 1^-	31.6	4	4.7σ
1^+ over 2^-	48.2	4	6.1σ
1^+ over 2^+	55.4	4	6.7σ

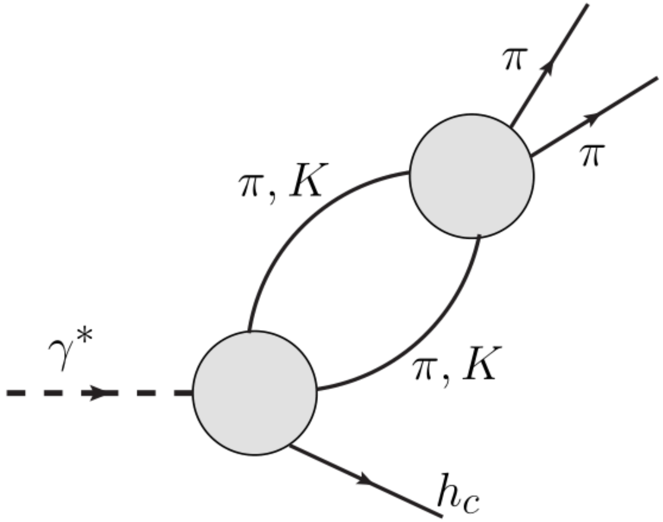
7. Uncertainties

- Uncertainty from event selection

Source	Uncertainty (%)
Luminosity	1.0
η_c mass	0.7
ISR factor	0.6
$\mathcal{B}(h_c \to \gamma\eta_c)$	11.8
$\sum_i \epsilon_i \cdot \mathcal{B}(\eta_c \to B_i)$	9.1
Total	15.0

- **Uncertainties from $\pi\pi$-S wave amplitude**

$(\pi\pi)_{\text{S-Wave}}$ is described with $\pi\pi$ – rescattering amplitude



$$S_{\pi^+\pi^-} = \frac{1+z(s)}{D(s)} = S^0_{\pi^+\pi^-} + cS^1_{\pi^+\pi^-} + ...,$$

$$D(s) = m_0^2 - s - \left[\sum_c Re\Pi_c(s) - Re\Pi_c(m_0^2)\right] - i\sum_c Im\Pi_c,$$

$$Re\Pi_c(s) = \frac{1}{\pi}\int_{s_{th}}^{\inf} \frac{Im\Pi_c(s')ds'}{(s'-s)}, \text{ with } Im\Pi_c(s) = g_c^2\rho_c(s),$$

$\pi^+\pi^-$ rescattering amplitude in the process $e^+e^- \to \gamma^* \to \pi^+\pi^- h_c$.

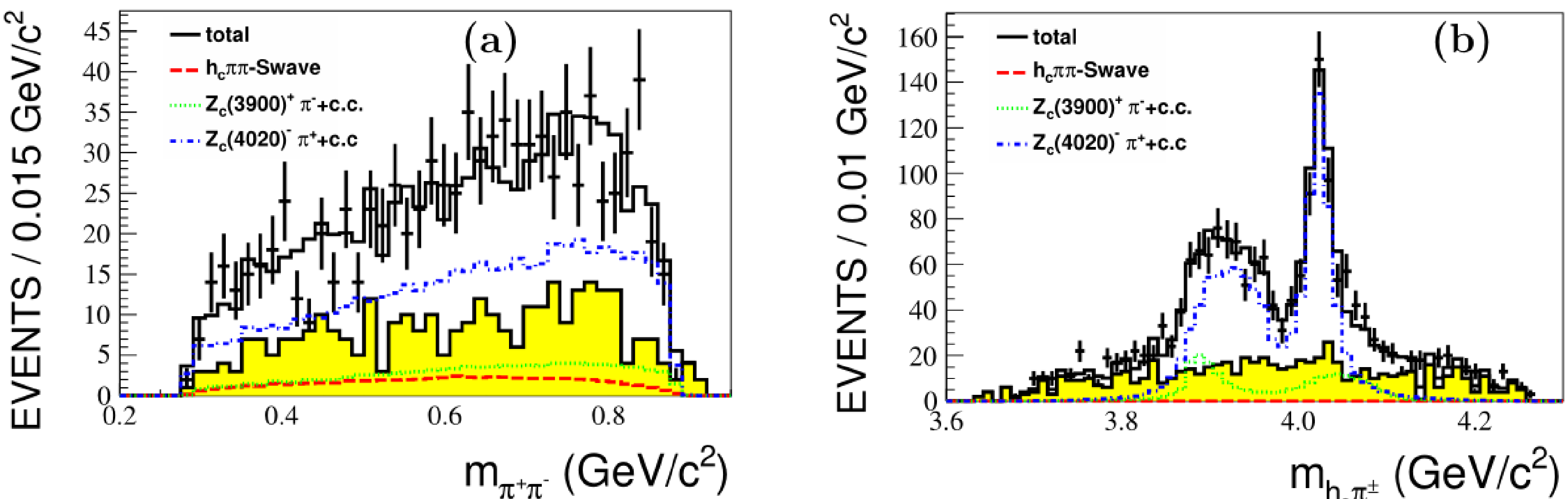


Fig. 9: Fit results with the $\pi\pi$-S wave and $Z_c(4020)^{\pm}$ components, (a) $m_{\pi^+\pi^-}$ and (b) $m_{h_c\pi^{\pm}}$.

· Uncertainties from the backgrounds

The number of background events estimated with the h_c sidebands is 323 ± 18. The uncertainties due to its statistical fluctuation are estimated with 305 background events

· Uncertainty from the Barrier radius

For meson decays, the radius of the centrifugal barrier is often taken in the range $r \in (0.25, 0.76)$ fm, the nominal values are obtained with $r_0 = 0.6$fm, and uncertainties are checked at both ends, and estimated with $r = 0.76$ fm for conservative consideration.

· Uncertainty from insignificant resonances

Uncertainty is taken as the difference in the signal yields estimated with

$h_c(\pi^+\pi^-)_{\text{S-wave}}$, $h_c f_2(1270)$, $Z_c(3900)^\pm\pi^\mp$ and $Z_c(4020)^\pm\pi^\mp$, Summary of uncertainties for the signal yields of the $\pi\pi$-S wave, $Z_c(3900)$ and $Z_c(4020)$ components (%).

Sources	$(\pi^+\pi^-)_{\text{S-wave}}$	$Z_c(3900)$	$Z_c(4020)$
Event selection	15.0	15.0	15.0
$\pi\pi$ amplitude	26.1	8.4	17.6
Backgrounds	40.8	5.4	20.2
Barrier radius	7.9	35.9	12.5
Insignificant resonance	56.8	3.0	20.8
Total	76.6	40.2	39.1

8. Cross section

$$\sigma^B(e^+e^- \to h_c(\pi^+\pi^-)_{\text{S-wave}} \to \pi^+\pi^- h_c) = 7.7 \pm 6.5 \pm 5.9 \text{ pb},$$

$$\sigma^B(e^+e^- \to \pi^+ Z_c(3900)^- + c.c. \to \pi^+\pi^- h_c) = 7.6 \pm 5.3 \pm 3.0 \text{ pb},$$

$$\sigma^B(e^+e^- \to \pi^+ Z_c(4020)^- + c.c. \to \pi^+\pi^- h_c) = 31.7 \pm 10.9 \pm 12.4 \text{ pb},$$

9. Summary and discussion

We have performed a PWA on the process $e^+e^- \to \pi^+\pi^- h_c$ at $\sqrt{s} = 4.42$ GeV. To explain the peaks observed at around $m_{\pi h_c} = 4.02$ GeV, the $Z_c(4020)$ is introduced, together with $Z_c(3900)$ and $\pi^+\pi^-$ S-wave components with significance larger than 5.0σ. The fit results show that:

- The spin parity of the $Z_c(4020)^\pm$ is determined to be $J^P = 1^+$ over other quantum numbers with statistical significance larger than 4.5σ.

- The angular distribution for $e^+e^- \to Z_c(4020)^+\pi^- + c.c.$ is measured to be

$$\frac{dN}{d\cos\theta_0} \propto 1 + \alpha\cos\theta_0 \text{ with } \alpha = -0.22 \pm 0.15.$$

- The Born cross sections are determined to be

$$\begin{aligned}
\sigma^B(e^+e^- \to h_c(\pi^+\pi^-)_{\text{S-wave}} \to \pi^+\pi^- h_c) &= 7.7 \pm 6.5 \pm 5.9 \text{ pb}, \\
\sigma^B(e^+e^- \to \pi^+ Z_c(3900)^- + c.c. \to \pi^+\pi^- h_c) &= 7.6 \pm 5.3 \pm 3.0 \text{ pb}, \\
\sigma^B(e^+e^- \to \pi^+ Z_c(4020)^- + c.c. \to \pi^+\pi^- h_c) &= 31.7 \pm 10.9 \pm 12.4 \text{ pb},
\end{aligned}$$

Sum of these cross sections is very close to that measured $\sigma(e^+e^- \to \pi^+\pi^- h_c) = 52.1 \pm 2.7 \pm 5.9 \pm 6.1)$ in [27].

Thanks for your intension!