Using K-matrix to analyze the open-charm cross sections

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Outline

As we discussed before:
Breit-Wigner is not correct in multi-channel case.
But to get the experimental results, we need the parameterization to do ISR correction or get efficiency.

The question is:

How to parameterize the line-shape correctly?

Introduction

- All the XYZ states are observed in exclusive channels.
- Open charm should be the most favored decay channels. Haven't observed for $Y(4260)$, $Y(4360)$ ...
- $\pi\pi J/\psi$ in $Y(4260)$ decays is dominant by $f_0 J/\psi$, so first study the final states with $s\bar{s}$.

1. At BESIII, the cross sections for exclusive channels will be measured as much as possible,
2. After that, how to do parameterization?
3. Coupled channel effect should be huge around thresholds and should be considered,
4. Breit-Wigner is not a correct way to fit the cross sections,
5. K-matrix can deal with the coupled channel effect.

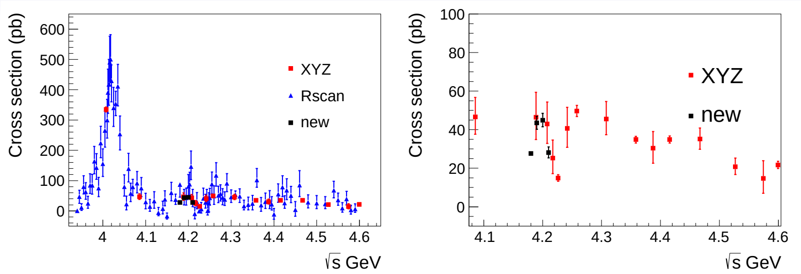
Open charm channels

Data sets: XYZ + R-scan data.

More precisely than other experiment's results.

- $e^+e^- \to D_sD_s$
- $e^+e^- \to D_sD_s^*$
- $e^+e^- \to D_s^*D_s^*$
- $e^+e^- \to D_s^*D_{s0}(2317), D_s^*D_{s1}(2460), D_sD_{s1}(2460)$ (thresholds around 4.43 GeV, not included yet)
- other channels will be included.

Born cross section of $e^+e^- \to D_sD_s$



A narrow $\psi(4040)$ from R-scan data.

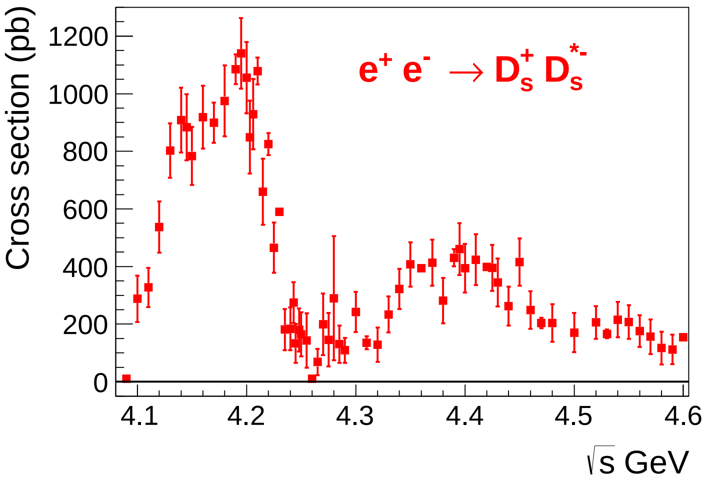
Dip at 4.23 GeV, more clear from XYZ-data.

Maybe due to the interference between $Y(4260)/Y(4220)$ and other charmonium states.

And there seems contribution from $\psi(4415)$.

Systematic uncertainty is very large when cross section changed dramatically due to paramterization.

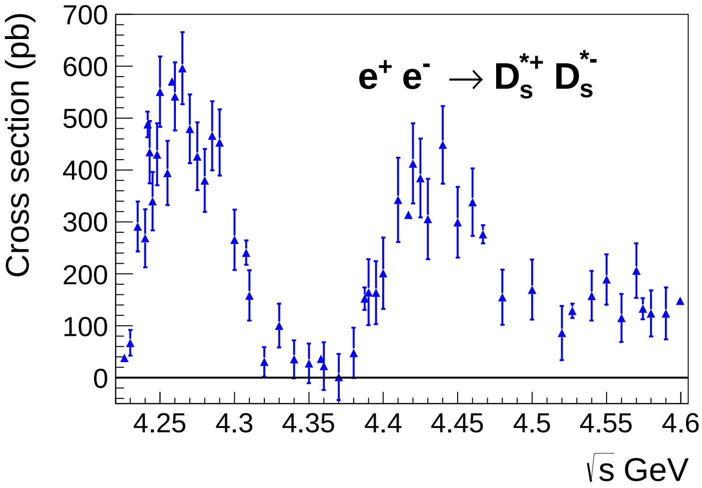
Born cross section of $e^+e^- \to D_sD_s^*$



The cross sections around 4.18 GeV is higher than others'.

There seems contribution from $\psi(4415)$.

Born cross section of $e^+e^- \to D_s^* D_s^*$



Seems related with $Y(4260)$. And contribution from $\psi(4415)$.

Simultaneous fit

- Cover $D_s D_s$, $D_s D_s^*$ and $D_s^* D_s^*$
- Use Flatte for parameterization.

$$F(s)_i = \frac{\sqrt{12\pi \cdot \Gamma_{ee}\Gamma_i}}{s - m^2 - i \cdot m \cdot (\sum_{i=1}^{3} g_i \cdot q_i)}, \quad (1)$$

here the $i = 1, 2, 3$ represent $D_s D_s$, $D_s D_s^*$ and $D_s^* D_s^*$,
q_i is momentum of D_s or D_s^*, and $\Gamma_i = q_i \cdot g_i$.

- 5 Flatte formulas for each mode.
- The phase is different for different mode and different Flatte.

$$\sigma(s)_i = \sum_{j=0}^{5} e^{i \cdot \phi_{ij}} \cdot F(s)_j \quad (2)$$

Likelihood

- Assuming the cross section and error follow Gaussian distribution.
- Symmetric error for each point of $D_sD_s^*$ and $D_s^*D_s^*$.

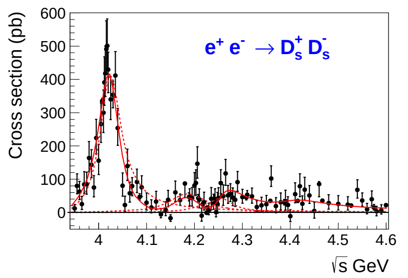
$$G(x, \sigma, t)_i = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-t)^2}{2\sigma^2}} \quad (3)$$

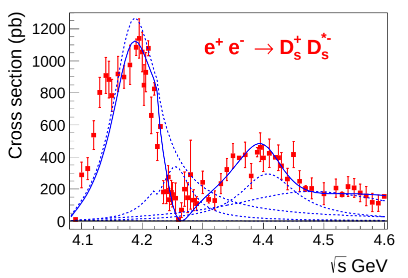
- Asymmetric error (σ_{lo} and σ_{hi}) for each point of D_sD_s.

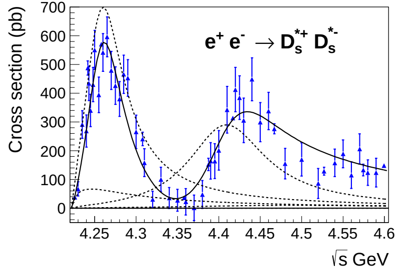
$$t < x : G(x, \sigma_{lo}, t)_i \quad and \quad t > x : G(x, \sigma_{hi}, t)_i \quad (4)$$

- x and σ are cross section and its error.
- Minuit to find the minimum of $-2 \cdot log(L)$, here $L = L_{D_sD_s} * L_{D_sD_s^*} * L_{D_s^*D_s^*}$

Fit results







Five states for three channels. $\psi(4040), \psi(4160), Y(4260), \psi(4415)$ and another one $X(4500)$. Parameters are floated.

Fit results

Mass (MeV)	Γ_{ee} (eV)	$g_{D_s D_s}$	$g_{D_s D_s^*}$	$g_{D_s^* D_s^*}$	ϕ
4030.1 ± 0.6	2.4 ± 0.3	125 ± 4	21 ± 2	0 ± 0	6.2 ± 0.01
4200.9 ± 0.8	13.6 ± 1.3	5.0 ± 0.3	172 ± 4	60 ± 4	6.3 ± 0.02
4263.0 ± 0.5	8.0 ± 0.8	2.8 ± 0.2	30 ± 1	142 ± 3	5.4 ± 0.04
4411.8 ± 1.3	8.9 ± 0.9	1.0 ± 0.1	68 ± 2	89 ± 4	0.07 ± 0.0
4500.0 ± 0.1	9.0 ± 3.4	0 ± 0	344 ± 20	22 ± 6	0

K-matrix

S. U. Chung, Ana. Physik 4, 404 (1995).

For a two-body scattering: $ab \to cd$

Cross section:

$$\sigma_{fi} = (\frac{4\pi}{q_i^2})(2J+1) \mid T_{ji}^2(s) \mid^2 \tag{5}$$

The scatter amplitude that initial state $\mid i>$ will be found in final state $< f \mid$

$$: S_{fi} =< f \mid S \mid i > \tag{6}$$

Unitary for S: $SS^\dagger = S^\dagger S = I$

Can be written as: $S = I + 2iT$,

So: $(T^{-1} + iI)^\dagger = T^{-1} + iI$

define K-matrix:

$$K^{-1} = T^{-1} + iI \tag{7}$$

Since S is unitary, K is Hermitian, $K^\dagger = K$,

$$T = \frac{K}{1 - iK}. \tag{8}$$

$$Re(T) = \frac{K}{I + K^2}, \tag{9}$$

$$Im(T) = \frac{K^2}{I + K^2} \tag{10}$$

The amplitude is the imaginary part.
K_{ij} describe the coupling effect between i and j channels.

Since in strong interaction:
$< f \mid S \mid i >=< i \mid S \mid f >$,
so K-matrix is symmetric.

one way to define K-matrix

The Lorentz invariant:

$$\hat{K}_{ij} = \sum_{\alpha} \frac{g_{\alpha i}(s) g_{\alpha j}(s)}{(m_\alpha^2 - s)(\sqrt{\rho_i \rho_j})} \quad (11)$$

$$\hat{T} = \hat{K} + i\hat{K}\rho\hat{T} \quad (12)$$

phase space matrix: $\rho = \begin{pmatrix} \rho_0 & 0 \\ 0 & \rho_1 \end{pmatrix}$

s-wave two-body: $\rho_1 = 2q_1/m$

partial decay width: $\Gamma_{\alpha i} = \frac{g_{\alpha i}^2(s)}{m_\alpha}$,

i, j means different channels, α represent poles.

Resonance

One decay channel (S-wave) for a resonance:

$$K = \frac{m_0 \Gamma(s)}{m_0^2 - s} \tag{13}$$

$$\hat{T} = [\frac{m_0 \Gamma_0}{m_0^2 - s - i m_0 \Gamma(m)}](\frac{\rho}{\rho_0}) \tag{14}$$

A normal relativistic Breit-Wigner.

Resonance

Two decay channel (S-wave) for a resonance:

$\hat{K_{11}} = \frac{g_1^2}{m_0^2 - s}$,

$\hat{K_{22}} = \frac{g_2^2}{m_0^2 - s}$,

$\hat{K_{12}} = \hat{K_{21}} = \frac{g_1 g_2}{m_0^2 - s}$,

$$\hat{T} = \frac{1}{1 - \rho_1\rho_2 D - i(\rho_1\hat{K_{11}} + \rho_2\hat{K_{22}})} \begin{pmatrix} \hat{K_{11}} - i\rho_2 D & \hat{K_{12}} \\ \hat{K_{21}} & \hat{K_{22}} - i\rho_1 D \end{pmatrix} \quad (15)$$

$D = \hat{K_{11}}\hat{K_{22}} - \hat{K_{12}}^2$,

$$\hat{T} = \frac{1}{m_0^2 - s - i(\rho_1 g_1^2 + \rho_2 g_2^2)} \begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix} \quad (16)$$

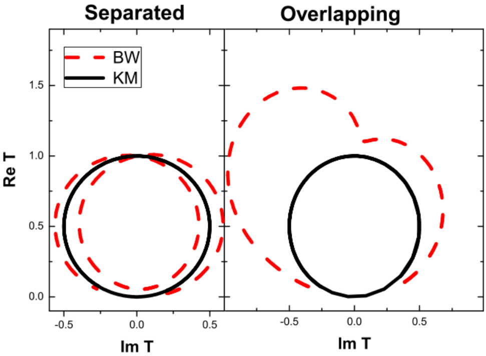
branching fraction $\mathcal{B}_i = \frac{g_i}{\sqrt{m_0\Gamma_0}}$

The Flatte formula for $f_0(980) \to \pi^+\pi^-/K^+K^0$

Comparison between BW and K-matrix

A. Wiranata *et.al* arXiv:1307.4681.

Argand diagrams



Two resonances are separated well (left), and have large overlapping (right).

BW can not preserve unitary, but K-matrix can.

BW: $| BW_1 + BW_2 |^2$, K-matrix: $K_{ij} = \frac{g_i g_j}{m_1^2 - s} + \frac{g_i g_j}{m_2^2 - s}$

Problem in K-matrix

Below threshold,

$$p = \frac{\sqrt{(s-(m1+m2)^2)(s-(m1-m2)^2)}}{2\sqrt{s}} \qquad (17)$$

is imaginary number.
Phase space factor p^{2L+1}

$L = 0, ip$, $L = 1, -ip^3$,

change the sign to real part in denominator $m_0^2 - s - i \cdot p^{2L+1}$.
change the pole position.
Thanks Guangyi Tang and A. Nefediev for explains

How to deal with the P-wave in K-matrix?

One approximation

T. V. Uglov *et. al.* arXiv: 1611.07582.
Assume:

$$K_{ij} = \sum_{\alpha} G_{i\alpha}(s) \frac{1}{m_\alpha^2 - s} G_{j\alpha}(s) \tag{18}$$

and

$$G_{i\alpha}^2(s) = g_{i\alpha}^2 \frac{p_i^{2L+1}}{\sqrt{s}} \theta(s - s_i) \tag{19}$$

$\theta(s - s_i)$ is a step function. So no problem below threshold, but not exactly correct.

Any better solution?

Another way

Dispersion relation:

$f(z) = \frac{1}{\pi} \int_{z_R}^{\infty} dz' \frac{Im(f(z'))}{z'-z}$

When $\mid z \mid \to \infty$, $\mid \frac{f(z)}{z} \mid \to 0$,

first order substraction:

$\frac{f(z)-f(z_0)}{z-z_0}$:

$f(z) =$

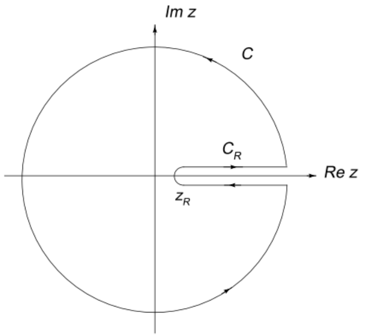
$f(z_0) + \frac{z-z_0}{\pi} \int_{z_R}^{\infty} dz' \frac{Im(f(z'))}{(z'-z_0)(z'-z)}$

second order substraction:

$\frac{f'(z)-f'(z_0)}{z-z_0}$ ...

In this case:

$f(z = s) = Im(Phsp(s))$



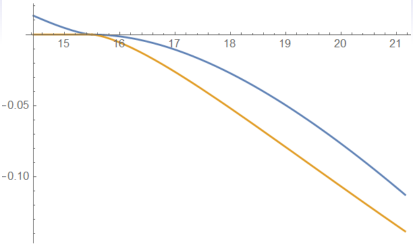
S-wave phase space p^1:

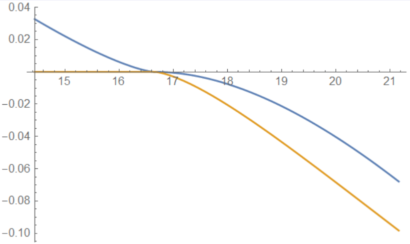
first order substraction.

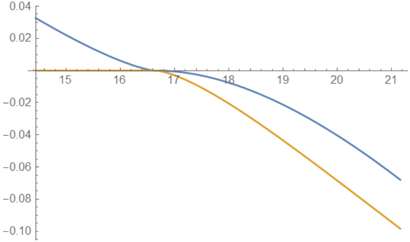
P-wave phase space p^3:

$f(z) = f(z_0) + (z - z_0)f'(z_0) + \frac{(z-z_0)^2}{\pi} \int_{z_R}^{\infty} dz' \frac{f(z')}{(z'-z_0)^2(z'-z)}$

Phase space







P-wave phase space for $D_s^+ D_s^-$, $D_s^{*+} D_s^-$ and $D_s^{*+} D_s^{*-}$.

An extra linear part: $c \cdot s$.

Take the threshold as substraction point.

Real part (blue): mass renormalization.

Imaginary part (brown): width.

Denominator in normal BW: $(m^2 - s) + Im(Phsp(s))$

A naive fit

Use

$$K_{ij} = \sum_{\alpha} G_{i\alpha}(s) \frac{1}{m_{\alpha}^2 - s} G_{j\alpha}(s) \quad (20)$$

and

$$G_{i\alpha}^2(s) = g_{i\alpha}^2 \rho_i \quad (21)$$

Then the cross section:

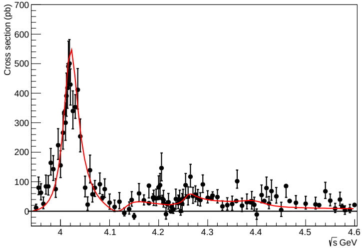
$$\sigma_i(s) = \frac{4\pi\alpha}{s}(\rho_i) \mid \sum_{\alpha,\beta} g_{e\alpha} P_{\alpha\beta}(s) g_{i\beta} \mid^2 \quad (22)$$

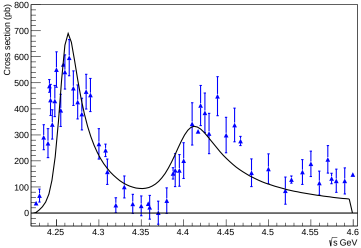
$P^{-1} = (m_{\alpha}^2 - s)\delta_{\alpha\beta} - i \sum_m G_{m\alpha} G_{m\beta}$

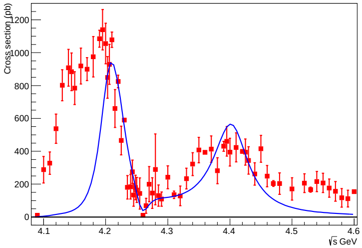
Electronic width: $\Gamma_{e\alpha} = \frac{\alpha g_{e\alpha}^2}{3 m_{\alpha}^3}$,

partial decay width: $\Gamma_{i\alpha} = \frac{g_{i\alpha}^2 \rho_i}{M_{\alpha}^2}$

Very preliminary fit result







Five resonances.

The $e^+e^- \to D_s^* D_s$ and $D_s^* D_s^*$ can be fitted.

This method works.

More channels

CPU time: only 13.4s for three channels, so we can include more channels.
Assume no difference between isospin channels, 9 open-charm in total:

- Three strange-charmed meson $D_s D_s$ ($D_s^{(*)} D_s^{(*)}$) channels,
- Three DD ($D^{0(*)} D^{0(*)} / D^{+(*)} D^{-(*)}$) channels,
- Three πDD channels, separate S and P wave,

Add hidden-charm channels, about 10:
$\pi^+\pi^-(\eta, \eta')J/\psi$, $\pi^+\pi^- h_c$, $\gamma(\omega, \phi)\chi_{cJ}$, $\gamma(\rho)\eta_c$ ... Above 4.0 GeV,

20 channels in total, parameters:
210 coupling constants, 20 phase space,
several poles: masses, electronic widths.
$\sim$300 parameters in total, operable.
Precise measurements should be provided.

Summary and questions

- How to explain the cross sections of open-charm channels ($D_sD_s, D_s^*D_s, D_s^*D_s^*$)?
- Breit-Wigner and Flatte functions are not suitable above open-charm threshold.
- *K*-matrix seems better, still has problems.
- The *p*-wave phase space can be calculated and fitted.

With the precise measurements, maybe we can find all the charmonium(-like) states and all the branching fractions.

Thanks very much for your comments and suggestions.