Amplitude analysis for $e^+e^- \rightarrow \pi^+\pi^- J/\psi$@4420

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OUT LINE

- Amplitude construction
- Finding nominal fit.
- Goodness of the fit
- Systematic uncertainty
- summary

Helicity amplitude

For decay process $Y \to Z_c + \pi^+, Z_c \to J/\psi + \pi^-, J/\psi \to l^+ l^-$, the helicity amplitude is:

$$A_{Z_c}(\lambda_Y, \lambda_{Z_c}, \lambda_{l^+}, \lambda_{l^-}) = F^{J_Y}_{\lambda_{Z_c},0} D^{J_Y}_{\lambda_Y,\lambda_{Z_c}}(\theta_{Z_c}, \phi_{Z_c}) \cdot BW(Z_c) \cdot F^{J_{Z_c}}_{\lambda_{J/\psi},0} D^{J_{Z_c}}_{\lambda_{Z_c},\lambda_{J/\psi}}(\theta_{J/\psi}, \phi_{J/\psi}) \cdot F^{J_{J/\psi}}_{\lambda_{l^+},\lambda_{l^-}} D^{J_{J/\psi}}_{\lambda_{J/\psi},\lambda_{l^+}-\lambda_{l^-}}(\theta_{l^+}, \phi_{l^+})$$

For decay process $Y \to f_0(f_2) + J/\psi, f_0(f_2) \to \pi^+ + \pi^-, J/\psi \to l^+ l^-$, the helicity amplitude is:

$$A_f(\lambda_Y, \lambda_f, \lambda_{l^+}, \lambda_{l^-}) = F^{J_Y}_{\lambda_f,\lambda_{J/\psi}} D^{J_Y}_{\lambda_Y,\lambda_f-\lambda_{J/\psi}}(\theta_f, \phi_f) \cdot BW(f) \cdot F^{J_f}_{0,0} D^{J_f}_{\lambda_f,0}(\theta_{\pi^+}, \phi_{\pi^+}) \cdot F^{J_{J/\psi}}_{\lambda_{l^+},\lambda_{l^-}} D^{J_{J/\psi}}_{\lambda_{J/\psi},\lambda_{l^+}-\lambda_{l^-}}(\theta_{l^+}, \phi_{l^+})$$

The cross section:

$$\frac{d\sigma}{d\phi} = M = \sum_{\lambda_Y,\Delta\lambda_l} |\sum_{\lambda_{Zc},\lambda_f} (A_f + e^{i\Delta\lambda_l\alpha_l(Z_c^+)} A_{Z_c^+} + e^{i\Delta\lambda_l\alpha_l(Z_c^-)} A_{Z_c^-})|^2$$

Parameterization of intermediate states

- Zc are parameterized with constant width Breit-Wigner function $\frac{1}{s - M^2 + iM\Gamma}$

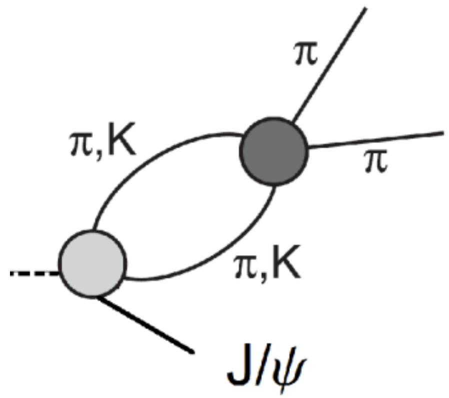
- For ππ S-wave, we utilize the amplitude from

ππ→ππ and KK→ππ scattering experiment.

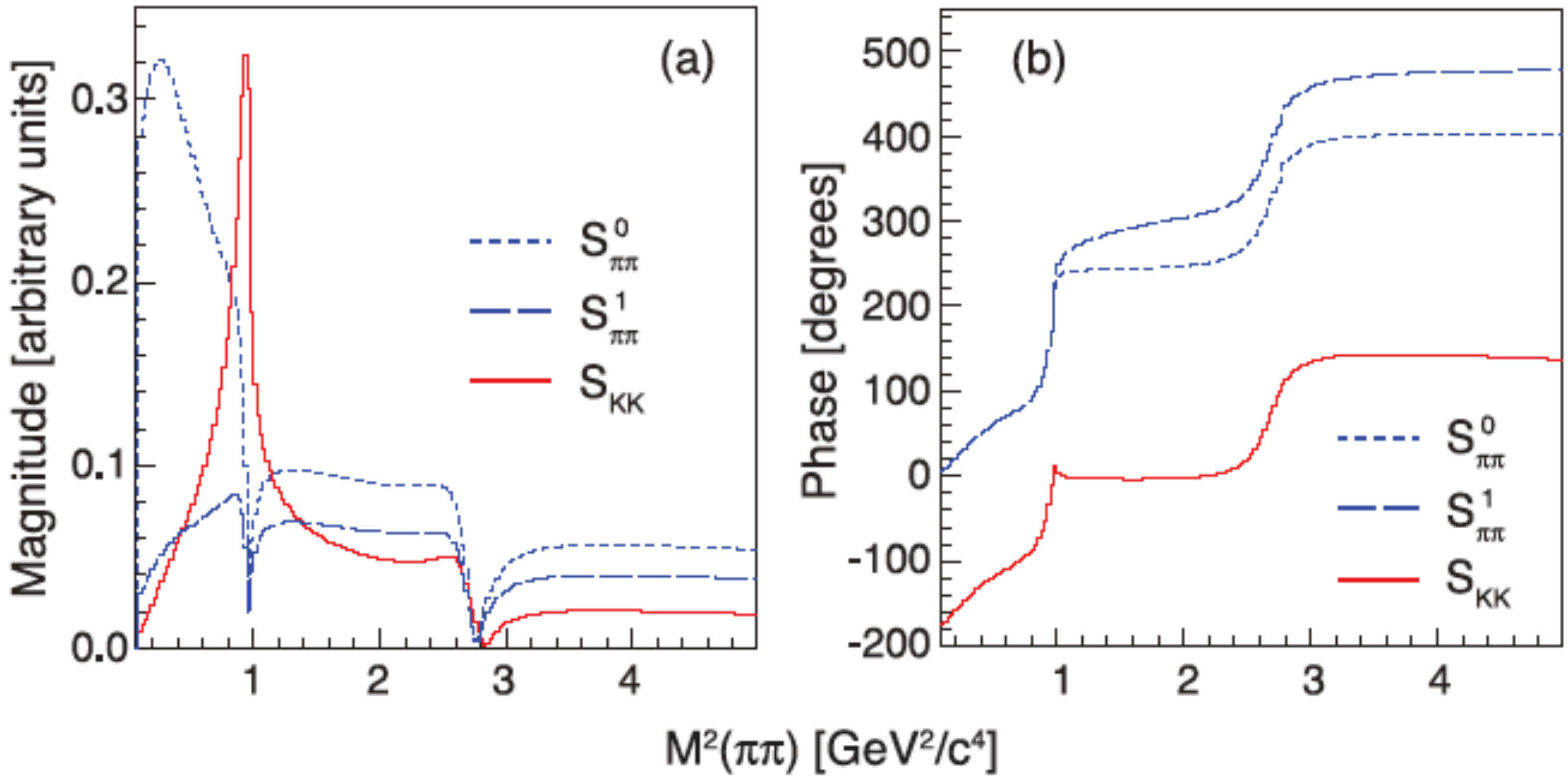
The scattering amplitude is rewritten in N(s)/D(s) form, the numerator is replaced by a polynomial expansion

$$S_{\pi\pi}(s) = \frac{1 + z(s)}{D(s)} = S^0_{\pi\pi}(s) + cS^1_{\pi\pi}(s),$$

$$z(s) = \frac{\sqrt{s + s_0} - \sqrt{4m_K^2 - s}}{\sqrt{s + s_0} + \sqrt{4m_K^2 - s}}.$$



ππ S-wave parameterization



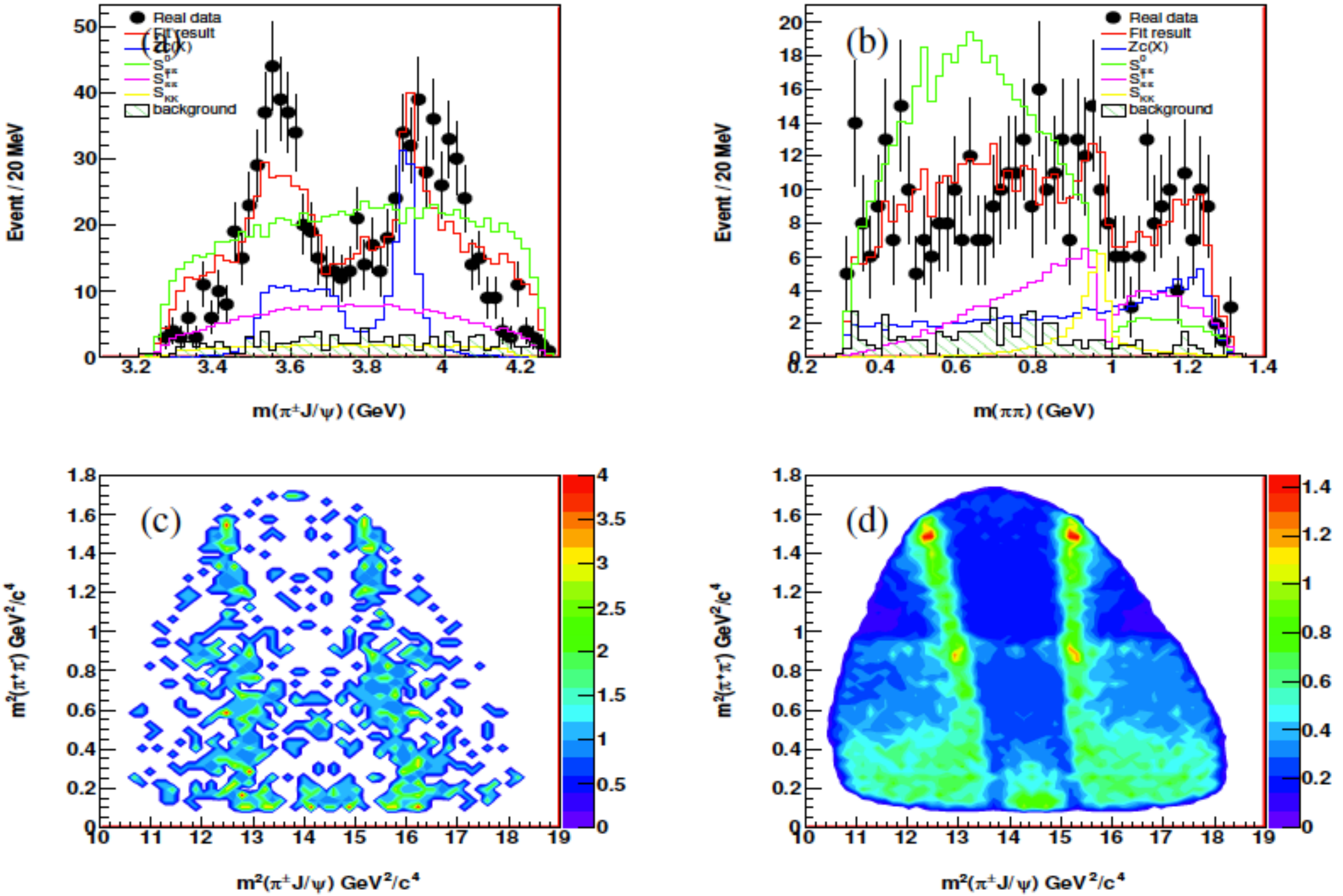
Same method as
CLEO: Phys. Rev. D 84, 112009 (2011).
BESIII: BAM 168

Finding nominal fit

Fit 0: Zc(3900) fixed mass and width	Single Zc(3900) like 4260 data can't well fit 4420 data	-5198
Fit 1: one Zc(xx)(J^p)	Zc(3550) give better fit than Zc(39xx), The structure contain significant 1^+ component	-5218
Fit 2a: one Zc(xx)(1^+) + Zc(xx)(J^p)	The best result is gotten when both Zc are 1^+, one of them has mass and width very close to Zc(3900)	-5286
Fit 2: one Zc(3900)(1^+) + Zc(xx)(J^p)	Nominal fit, mass and width of Zc(3900) are fixed Another Zc has mass aroud 4020, but much wider	-5285
Fit 3: one Zc(3900)(1^+) + Zc(4020)(1^+) + Zc(xx)(J^p)	The J^p of Zc(4020) favors 1^+, additional states are not neede	

Fit 0: Fit result with one $Z_c(3900)$

-2LnL=-5198



Fit 1: Fit result with one Zc(x)

J^P of $Z_c(35xx)$	-2lnL	$\Delta N_{dof.}$	Mass of Z_c MeV	Width MeV
0^-	-5143.32	4	3539.2 ± 9.1	91.2 ± 19.4
1^+	-5218.74	10	3546.9 ± 6.1	93.7 ± 14.9
1^-	-5159.78	4	3550.6 ± 6.5	72.8 ± 11.8
2^+	-5159.45	4	3541.8 ± 6.0	65.7 ± 11.0
2^-	-5208.12	10	3546.3 ± 5.7	106.4 ± 13.9

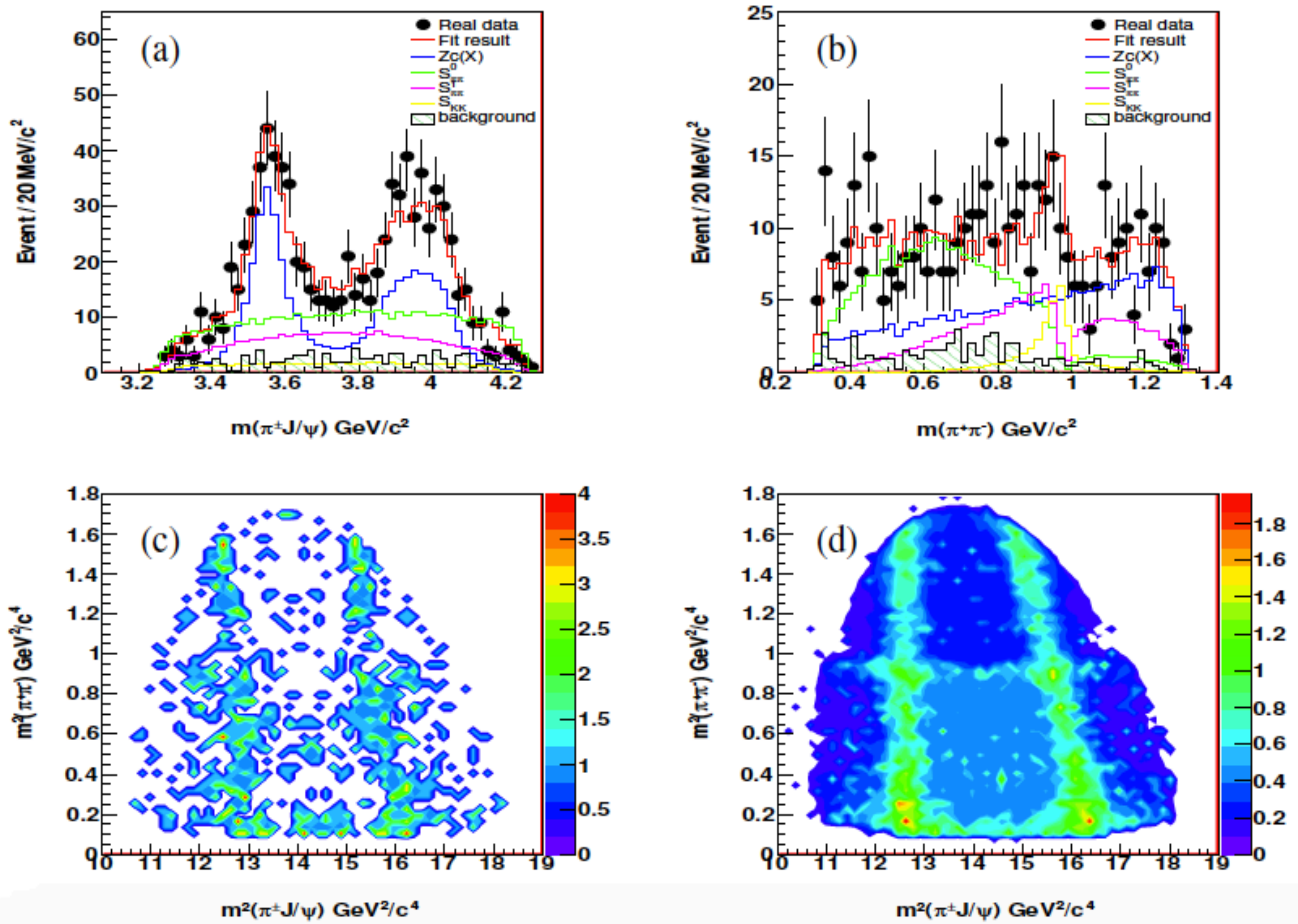
H0

Table 1: The fit result with different J^P assumption of $Z_c(35xx)$

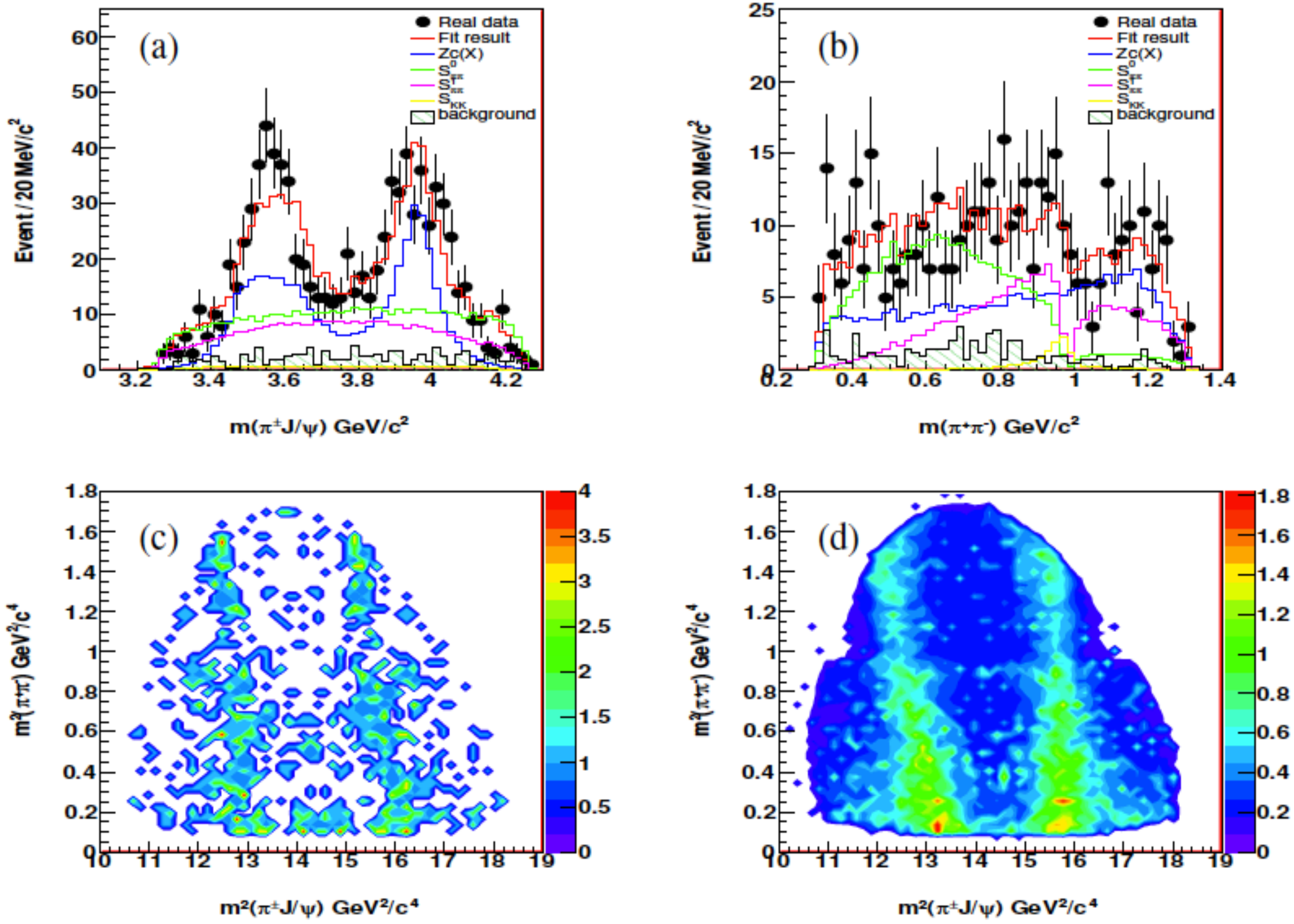
J^P of $Z_c(40xx)$	-2lnL	$\Delta N_{dof.}$	Mass of Z_c MeV	Width MeV
0^-	-5154.62	4	3917.4 ± 5.2	55.3 ± 9.3
1^+	-5202.58	10	3959.5 ± 8.9	116.0 ± 13.4
1^-	-5135.88	4	3948.5 ± 15.6	94.2 ± 24.3
2^+	-5130.10	4	3959.2 ± 13.0	112.4 ± 17.6
2^-	-5182.70	10	3949.4 ± 8.8	113.3 ± 17.1

Table 2: The fit result with different J^P assumption of $Z_c(40xx)$

Fit 1: Fit result with one Zc(35xx)



Fit 1: Fit result with one Zc(40xx)



Fit 2a: fit with two Zc

J^P of 2^{nd} Z_c	-2lnL	J^P of Z_c	$\Delta N_{dof.}$	Mass of Z_c MeV	Width MeV
0^-	-5258	0^- 1^+	4 10	3903.9 ± 9.4 4004.1 ± 12.4	41.6 ± 8.4 110.2 ± 14.9
1^+	-5286	1^+ 1^+	10 10	3893.8 ± 5.0 4018.4 ± 13.4	41.2 ± 8.7 106.7 ± 19.0
1^-	-5253	1^- 1^+	4 10	3591.0 ± 14.2 3535.1 ± 8.1	52.3 ± 11.8 109.8 ± 29.2
2^+	-5236	2^+ 1^+	4 10	4014.3 ± 13.2 3914.0 ± 5.4	114.5 ± 28.1 65.8 ± 10.0
2^-	-5264	2^- 1^+	10 10	3892.6 ± 6.4 3994.8 ± 11.5	30.6 ± 8.5 124.8 ± 16.2

H1

Table 4: The fit result with different J^P assumption of $Z_c(J^P)$+ $Z_c(1^+)$.

Compare with page 8, Likelihood ratio test shows that the 1+ component is significant.

J^p of Zc of H0	δL(H1-H0)	N_{dof}	significance
0-	52	10	8.6σ
1+	34	10	6.5σ
1-	47	10	8.0σ
2+	38.5	10	7.0σ
2-	28	10	5.6σ

Fit 2: nominal fit result with two Zc

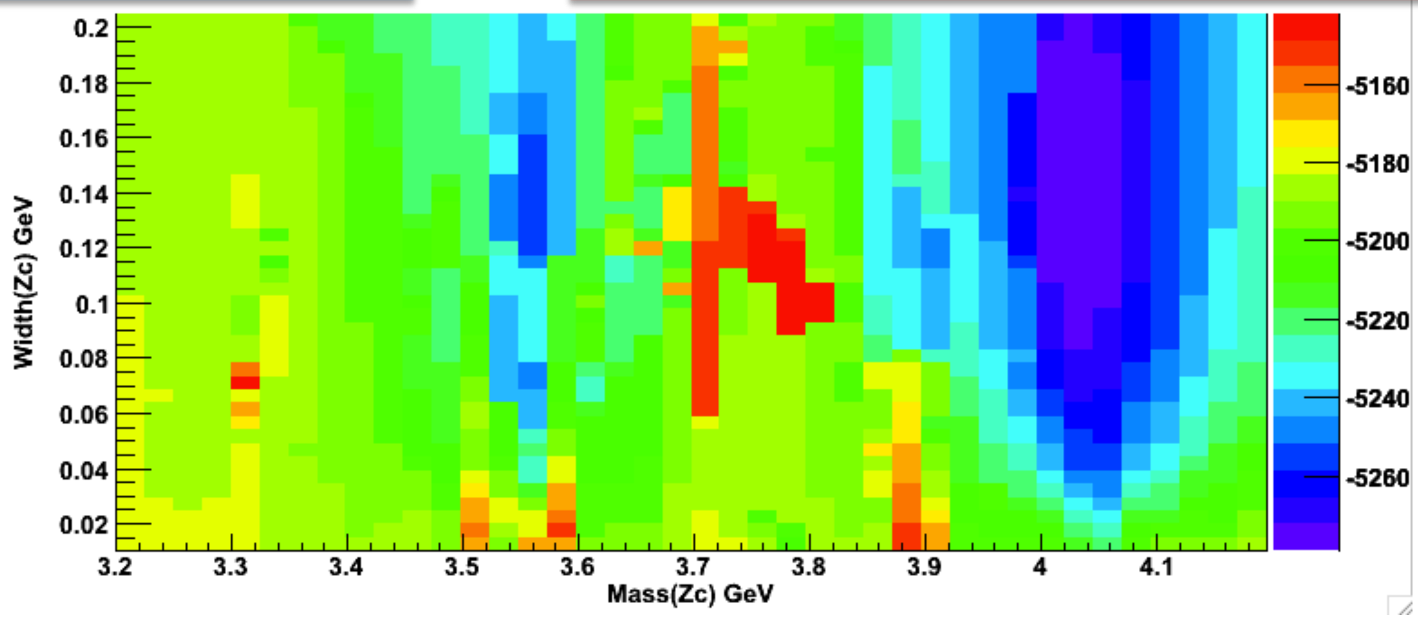
J^P of Z_c	-2lnL	$\Delta N_{dof.}$	Mass of Z_c MeV	Width MeV
0^-	-5217.67	4	4005.7 ± 18.0	130.2 ± 44.5
1^+	-5285.50	10	4027.0 ± 11.5	105.6 ± 20.2
1^-	-5237.11	4	4028.7 ± 6.4	39.0 ± 14.9
2^+	-5228.86	4	4000.0 ± 15.1	121.9 ± 28.4
2^-	-5244.44	10	3995.7 ± 10.6	108.8 ± 16.3

H0

Table 5: The fit result with different J^P assumption of $Z_c(40xx)$+ $Z_c(3900)(1^+)$.

Scan plot of mass and width of Second Zc

The mass and width of Zc(3900) are fixed,
The mass and width of second Zc are free in the fit



The completeness of fit 2

- Completeness: The components in fit 2 are all significant, no other states are needed.
- The fit with Zc(3900)(1^+)+Zc(40xx)(1^+)+Zc(40xx)(J^P), the mass and width of all states beside Zc(3900) are free in the fit.

H1

J^P of Z_c H_0	Likelihood of H_1	$\delta L(H_1 - H_0)$	N_{dof} 1^+	significance of 1^+	$\Delta L(H_1 - fit2)$	N_{dof} J^P	significance of J^P
0^-	-5306.09	44	10	7.7σ	10.3	4	3.5σ
1^+	-5312.84	8	10	–	13.7	10	3.1σ
1^-	-5293.79	28	10	5.6σ	4.1	4	1.7σ
2^+	-5294.74	33	10	6.3σ	4.6	4	1.9σ
2^-	-5320.71	38	10	7.0σ	17.6	10	3.8σ

The significance of 1+ over other J^p assumption for Zc(40xx)

The significance of extra Zc states with different J^p assumption

The completeness of fit 2

- The significance of $\pi^+\pi^-$ D-Wave ($f_2(1270)$)is 2.7σ
- Fit 2 compare with fit 1 →The significance of Zc(3900) is 6.4σ
- Fit 2 compare with fit 0→The significance of Zc(4020) is 7.6σ

The logic

Fit 0: Zc(3900) fixed mass and width	Single Zc(3900) like 4260 data can't well fit 4420 data	-5198
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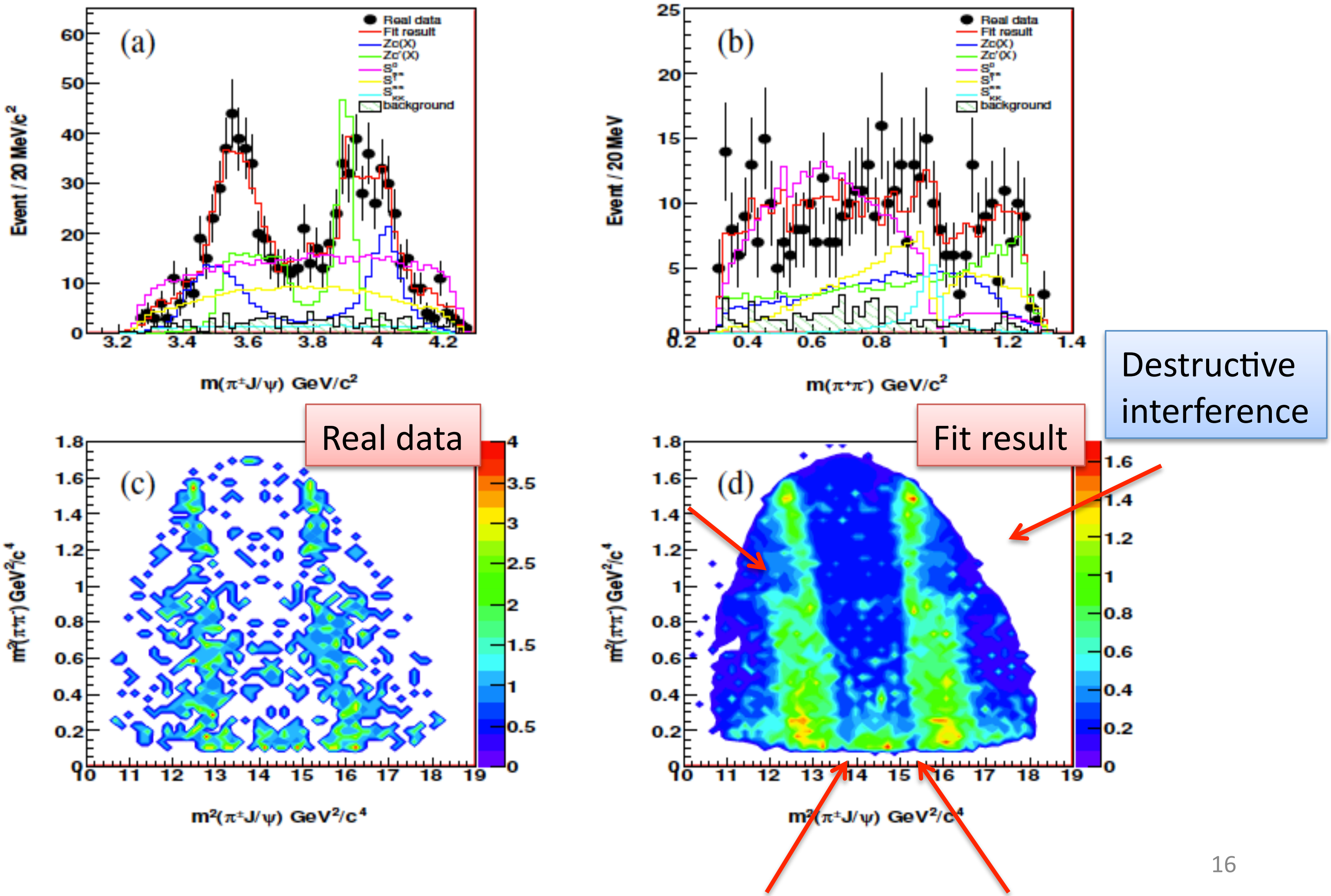
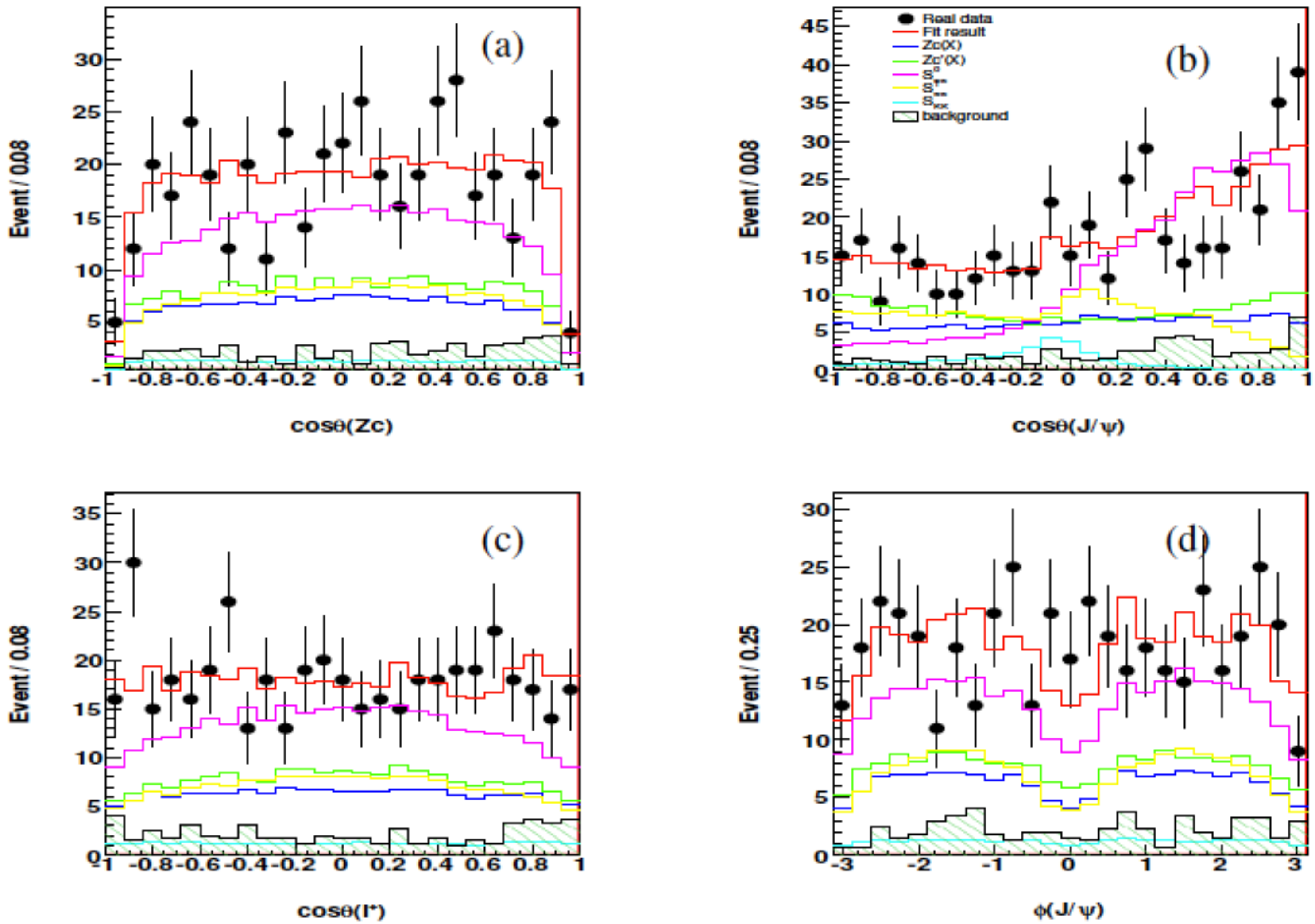
Fit 3: one Zc(3900)(1^+) + Zc(4020)(1^+) + Zc(xx)(J^p)	The J^p of Zc(4020) favors 1^+, additional states are not neede	

The projection of fit 2



The projection of fit 2



Fit fraction

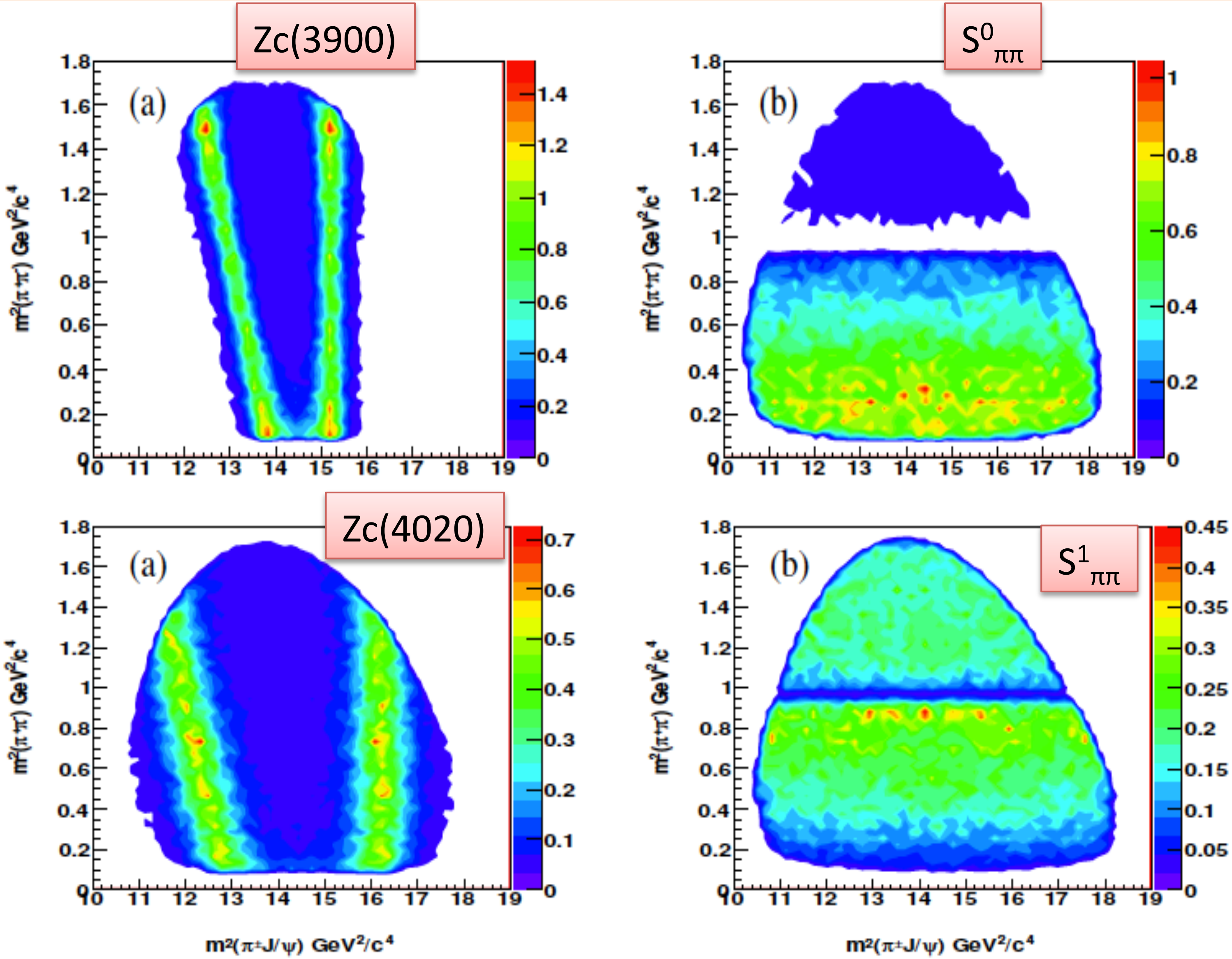
- The fit fraction is calculated by the MC integral of the cross section of intermediate process and the total cross section.

$$F_i = \sum_{j=1}^{N_{mc}} \left(\frac{d\sigma}{d\phi}\right)_j^i / \sum_{j=1}^{N_{mc}} \left(\frac{d\sigma}{d\phi}\right)_j$$

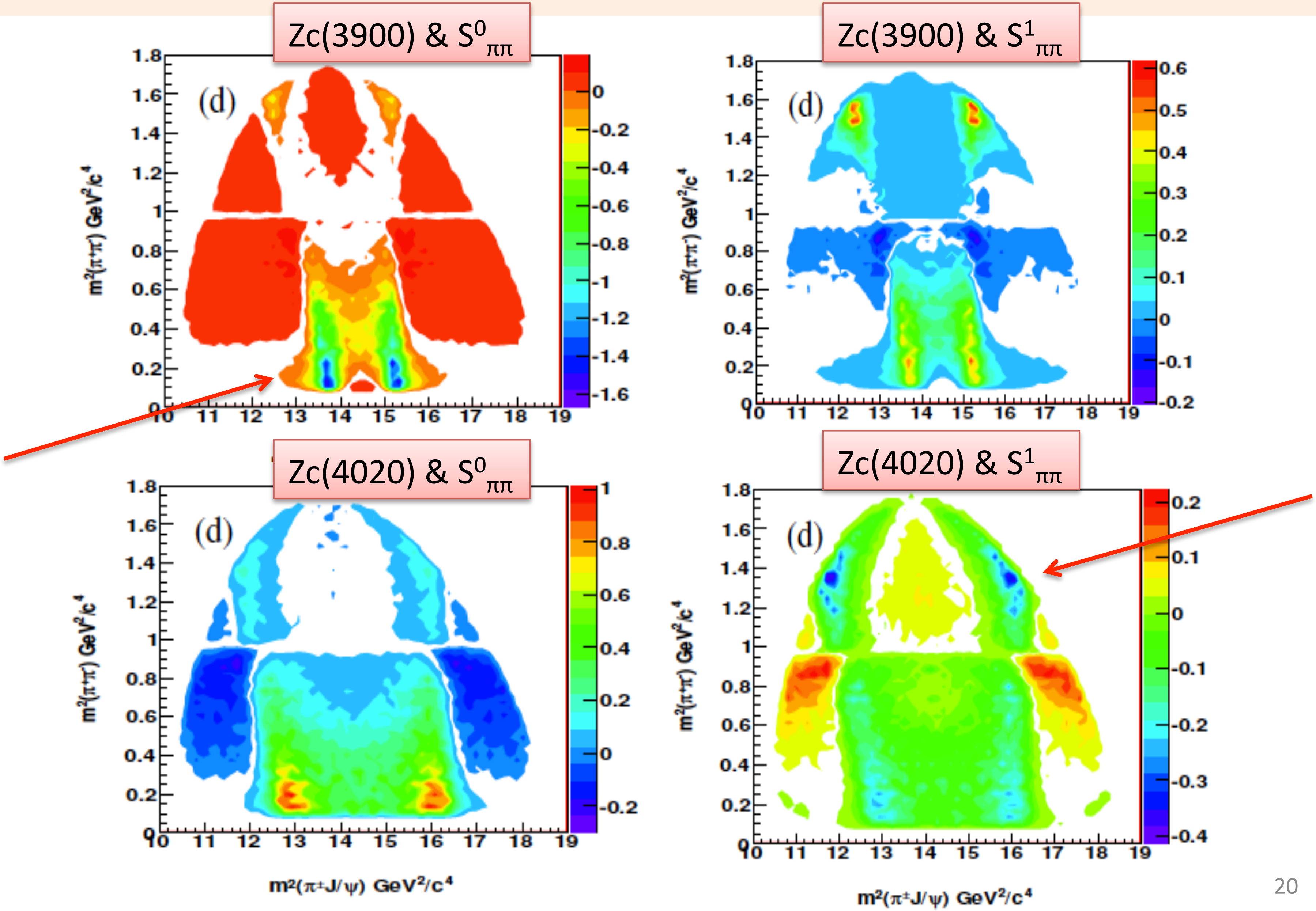
The interference table of fit 2

	$Z_c(3900)$	$Z_c(4020)$	$S^0_{\pi\pi}$	$S^1_{\pi\pi}$	S_{KK}
$Z_c(3900)$	0.43				
$Z_c(4020)$	-0.28	0.324			
$S^0_{\pi\pi}$	-0.17	0.26	0.923		
$S^1_{\pi\pi}$	0.06	-0.1	-1.0	0.42	
S_{KK}	-0.01	-0.002	0.067	0.019	0.076

Dalitz plot of Four dominant states



Dalitz plot of Four dominant states



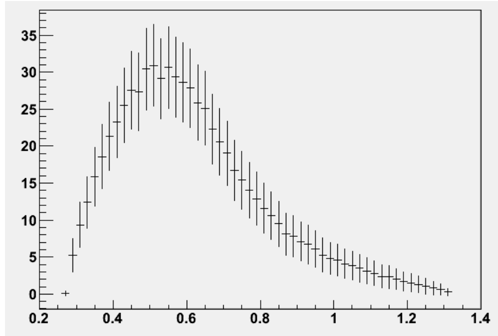
Another $m(\pi^+\pi^-)$ parameterization

- **Three pipi S-wave components are used. σ_1, $f_0(980)$, σ_2**
- **The σ_1 : The bump at the lower threshold of $m(\pi\pi)$**

$$f = \frac{G_\sigma}{M^2 - s - iM\Gamma_{tot}(s)},$$

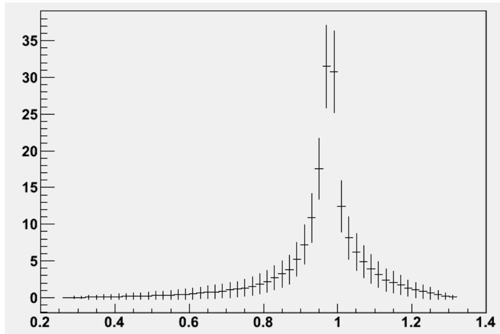
$$\Gamma_{tot}(s) = g_1 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(M^2)} + g_2 \frac{\rho_{4\pi}(s)}{\rho_{4\pi}(M^2)},$$

$$g_1 = f(s) \frac{s - m_\pi^2/2}{M^2 - m_\pi^2/2} \exp[-(s - M^2)/a].$$



- **$f_0(980)$** is parameterized with flatte formula: fixed to BESII's measurement Phys.Lett. B607 (2005) 243-253

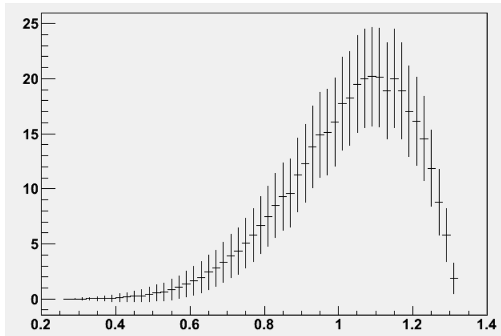
$$f = \frac{1}{M^2 - s - i(g_1\rho_{\pi\pi} + g_2\rho_{K\bar{K}})}.$$



- **σ_2 : The bump at higher threshold of $m(\pi\pi)$**

$$\frac{d\sigma}{dm_{\pi\pi}} \propto |\vec{q}| \sqrt{(q^2 - 4m_\pi^2)} \times q^4$$

q is the four momentum of the dipion system
|q| is the magnitude of space part of q



Fit result with alternative $m(\pi^+\pi^-)$ parameterization

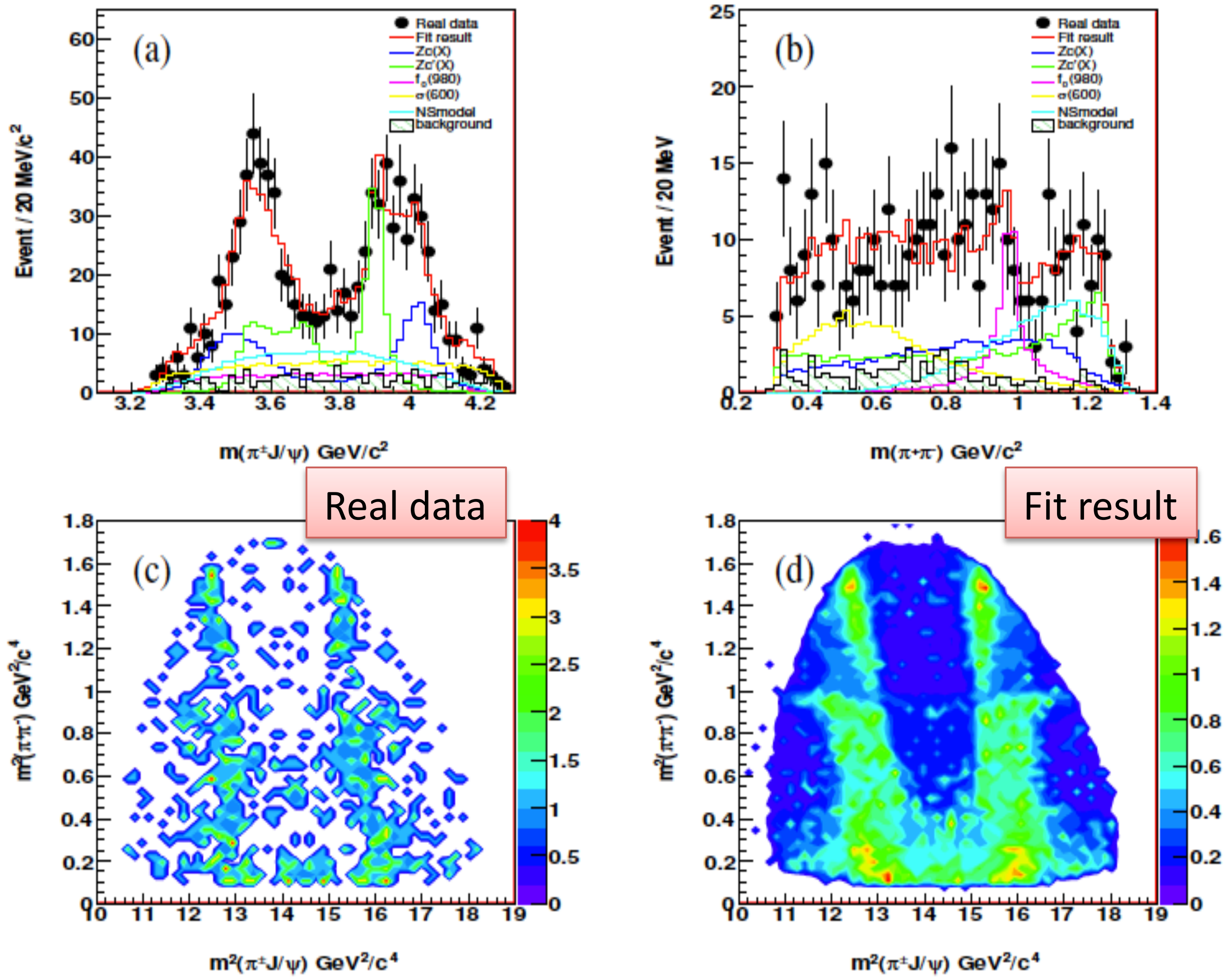
Fit result with different J^p assumption

J^P of $Z_c(40xx)$	-2lnL	$\Delta N_{dof.}$	Mass of Z_c MeV	Width MeV
0^-	-5231.66	4	3974.1 ± 14.2	96.6 ± 21.0
1^+	-5281.17	10	4025.1 ± 17.7	108.4 ± 22.7
1^-	-5240.66	4	3983.0 ± 7.2	53.0 ± 9.9
2^+	-5234.63	4	3977.2 ± 9.5	67.1 ± 18.0
2^-	-5252.47	10	4004.9 ± 12.1	97.7 ± 17.3

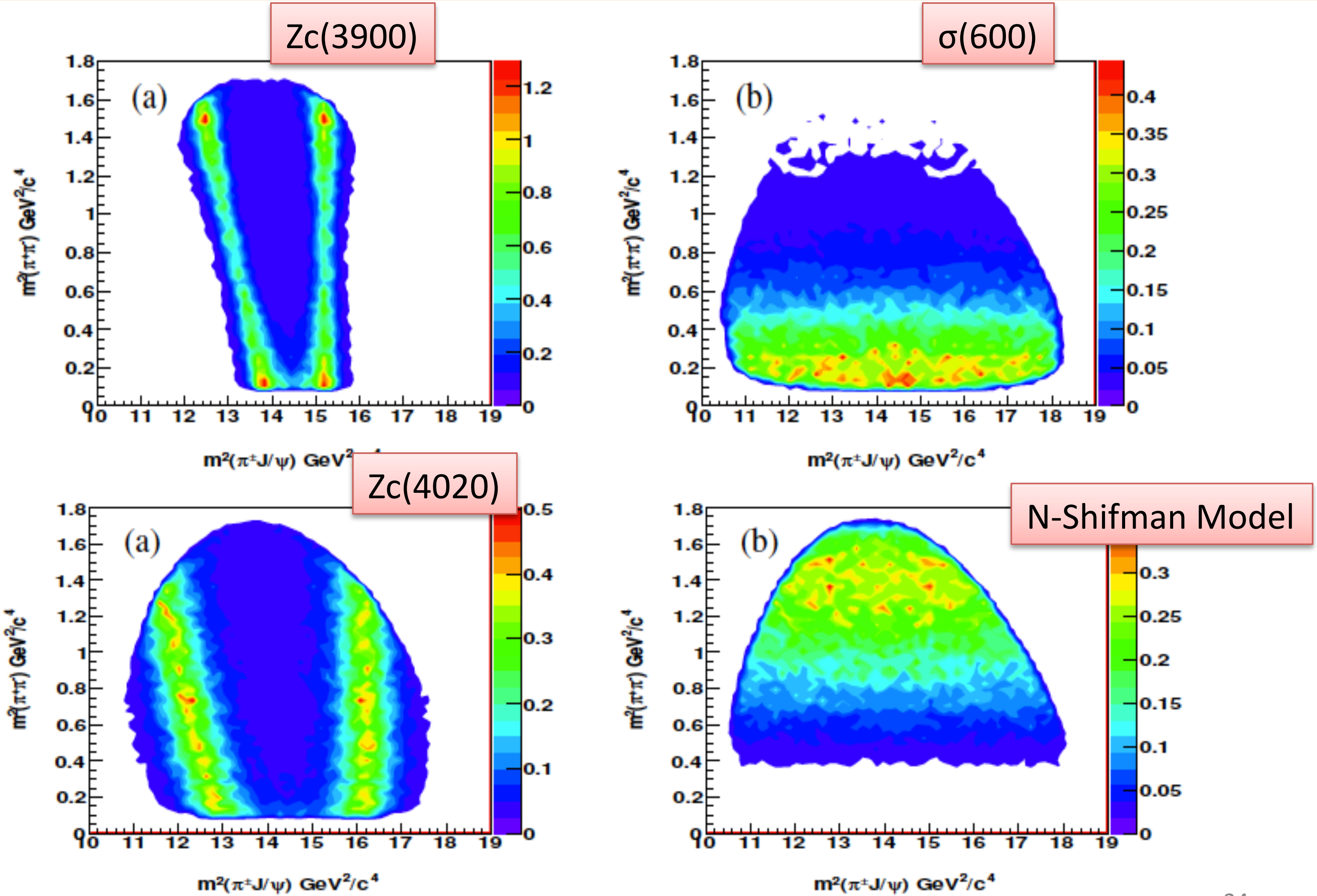
The interference table of fit

	$Z_c(3900)$	$Z_c(4020)$	$\sigma(600)$	NS model	$f_0(980)$
$Z_c(3900)$	0.324				
$Z_c(4020)$	-0.133	0.248			
$\sigma(600)$	-0.094	0.111	0.339		
NS model	0.066	-0.037	-0.124	0.252	
$f_0(980)$	-0.052	0.047	0.112	-0.221	0.157

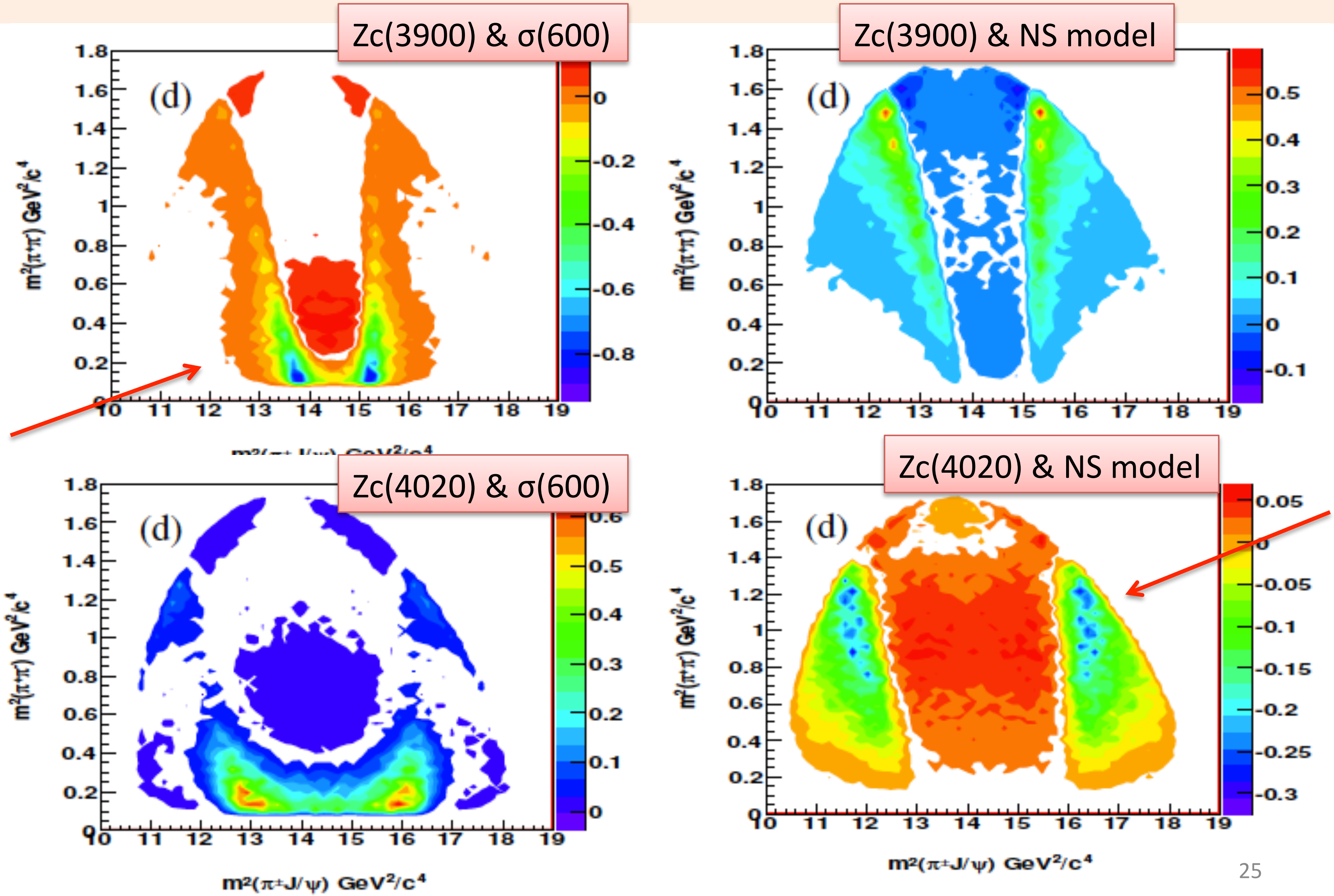
Fit result projection with alternative $m(\pi^+\pi^-)$ parameterization



Dalitz plot of Four dominant states



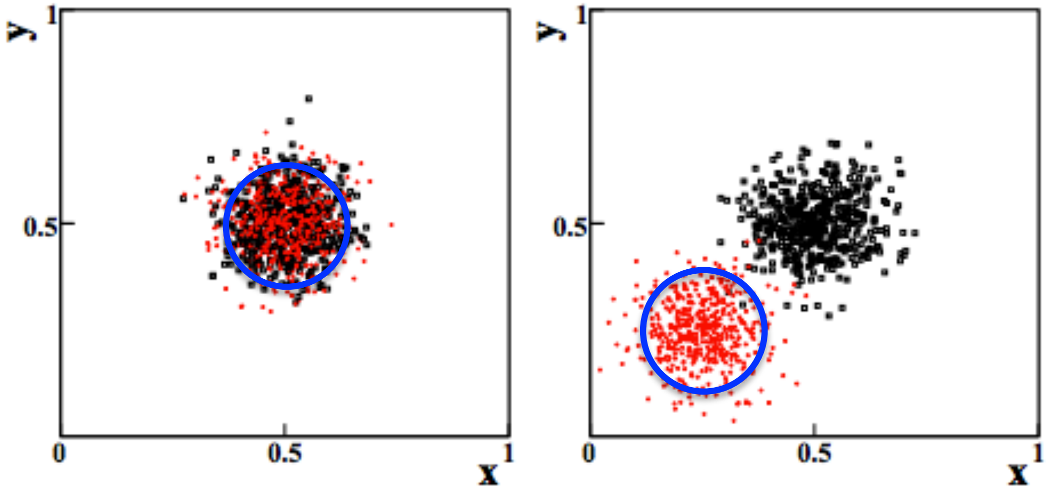
Dalitz plot of Four dominant states



Goodness of the fit

- We use the Mixed-Sample Method used in BAM278.

M. Williams, Journal of Instrumentation **5**, P09004 (2010).



$$T = \frac{1}{n_k(n_a + n_b)} \sum_{i=1}^{n_a+n_b} \sum_{k=1}^{n_k} I(i,k),$$

I(I,k)=1 if the neighbor are from same sample
I(I,k)=0 if the neighbor are from different sample

Goodness of the fit

- The expected mean value of T if two samples have same distribution

$$\mu_T = \frac{n_a(n_a - 1) + n_b(n_b - 1)}{n(n-1)},$$

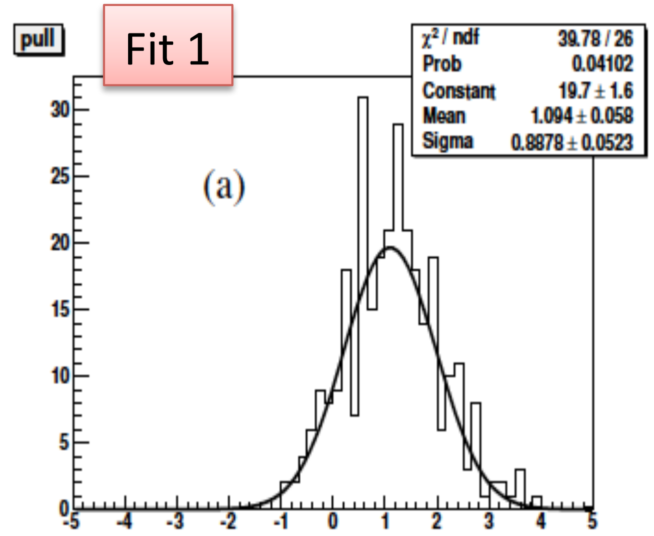
- The expected variance of T if two samples have same distribution

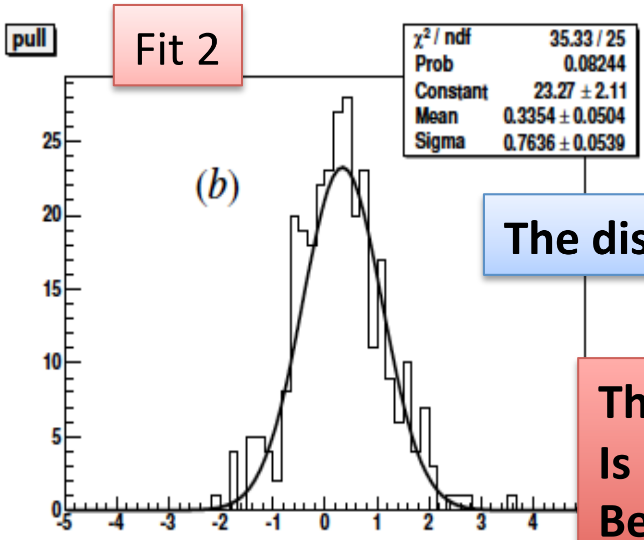
$$\lim_{n,n_k,D\to\infty} \sigma_T^2 = \frac{1}{nn_k}(\frac{n_a n_b}{n^2} + 4\frac{n_a^2 n_b^2}{n^4}).$$

- We generate MC sample according to fitted result. Then separate them into 300 samples. Then compare them with real data.
- The pull distribution of T should be a standard distribution if

these samples have same distribution.

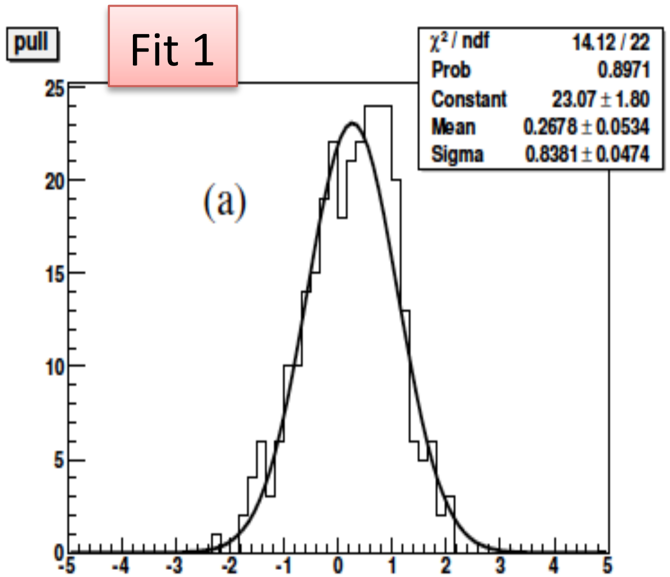
The pull distribution of T

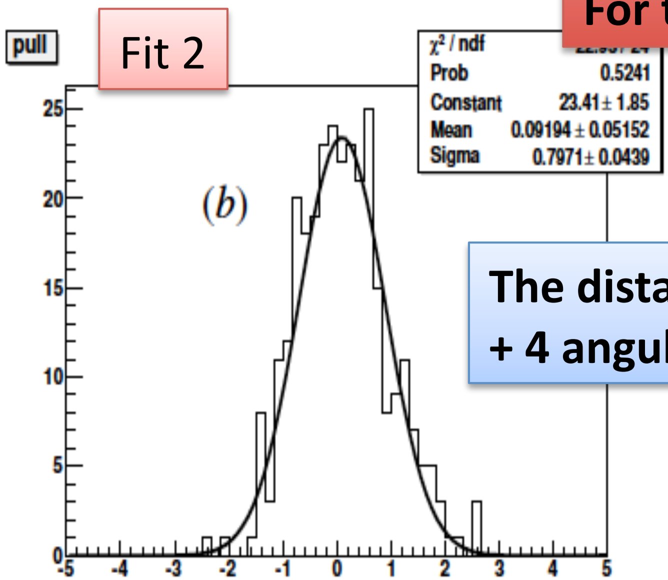




The distance is defined on dalitz plot

The mean of the pull of fit2
Is closer to zero. So Fit 2 is
Better than fit 1, especially
For the dalitz plot fit quality



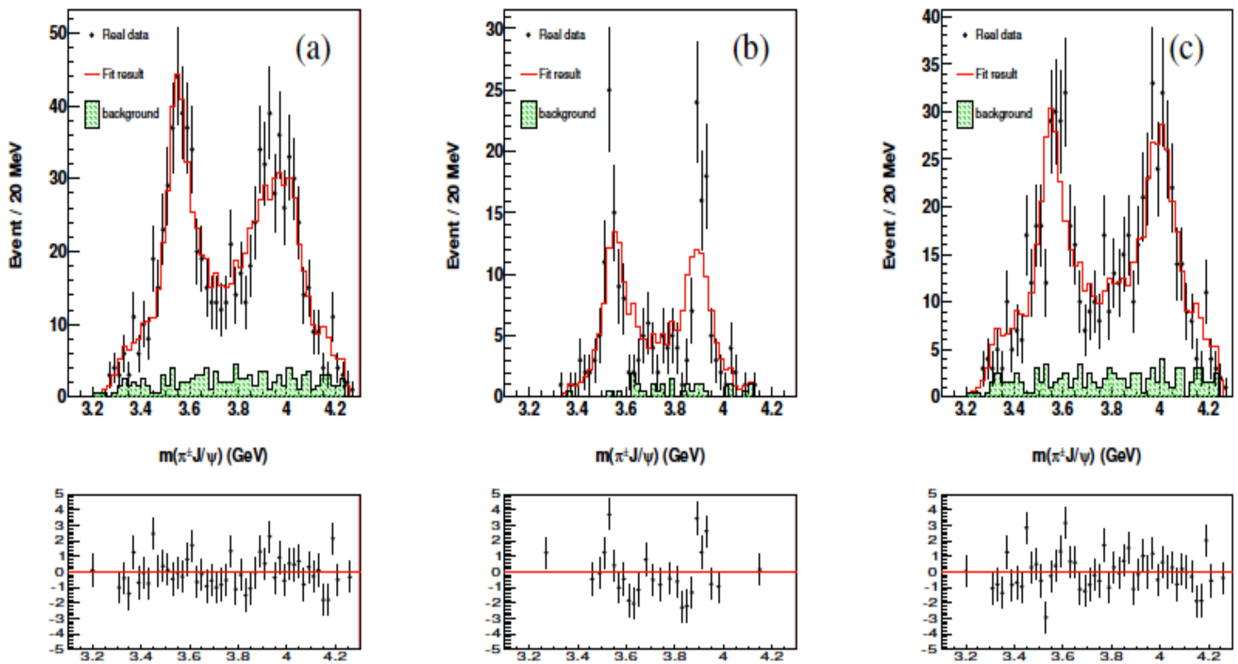


The distance is defined on dalitz plot
+ 4 angular distribution

The χ^2 check at different m(ππ) range

process	total	high ($M(\pi^+\pi^-) > 1.0GeV$)	low ($M(\pi^+\pi^-) < 1.0GeV$)
Fit 1	44.2, 44, 1.005	63.3, 24, 2.64	65.1, 47, 1.39
Fit 2	39.4, 47, 0.84	36.3, 23, 1.58	53.8, 47, 1.14

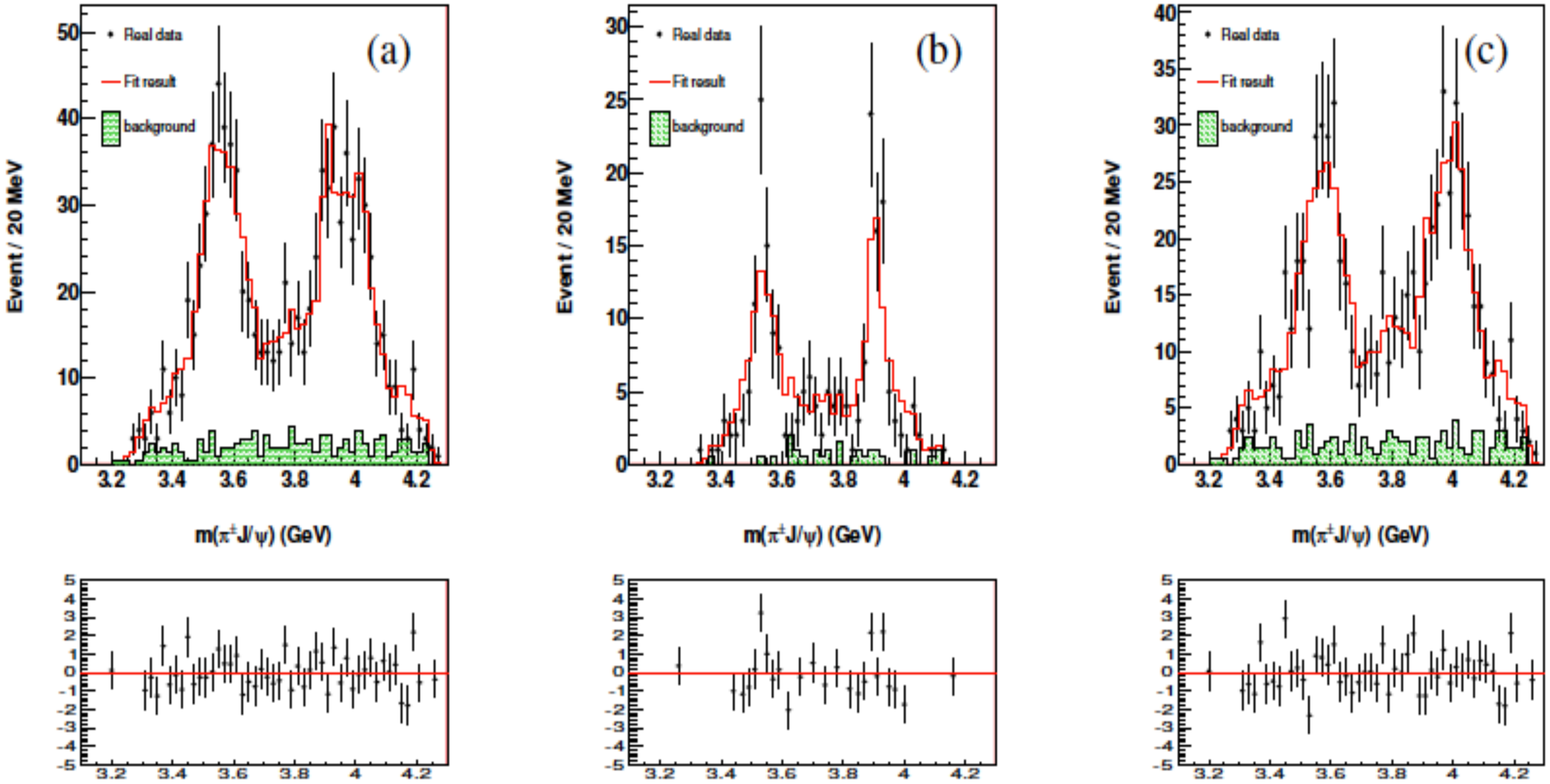
The three numbers in each box are χ^2, Ndof. and χ^2/Ndof



Fit 1

Figure 11: Fit 1: The projection result and χ^2 for each bin at different $M(\pi^+\pi^-)$ range. (a) total, (b) $M(\pi^+\pi^-) > 1.0GeV$, (c) $M(\pi^+\pi^-) < 1.0GeV$.

The χ^2 check at different m($\pi\pi$) range



Fit 2

Figure 12: Fit 2: The projection result and χ^2 for each bin at different $M(\pi^+\pi^-)$ range. (a) total, (b) $M(\pi^+\pi^-) > 1.0GeV$, (c) $M(\pi^+\pi^-) < 1.0GeV$.

Systematic uncertainty

- $\pi^+\pi^-$ S-Wave parameterization. Instead of using the scattering amplitude. We use three Breit-Wigner like structure. The difference is taken as systematic uncertainty.

- The influence of $\pi^+\pi^-$ D-Wave. The significance of $f_2(1270)$ is 2.7σ, the result difference when $f_2(1270)$ included is taken as systematic uncertainty

Systematic Uncertainty

- The J/ψ signal range and sideband range is varied, the difference is taken as systematic uncertainty.
- The barrier factor. The radius of centrifugal barrier is varied between 0.5fm and 1.5fm.
- Mass and width of Zc(3900) are varied with their uncertainty.

Systematic Uncertainty

- The resolution and fitting effect (I/O check): We use the same 300 MC samples to perform the fit. Then compare the input/output value. The mean difference is taken as uncertainty.

$$pull = \frac{x_{out} - x_{input}}{\sigma}$$

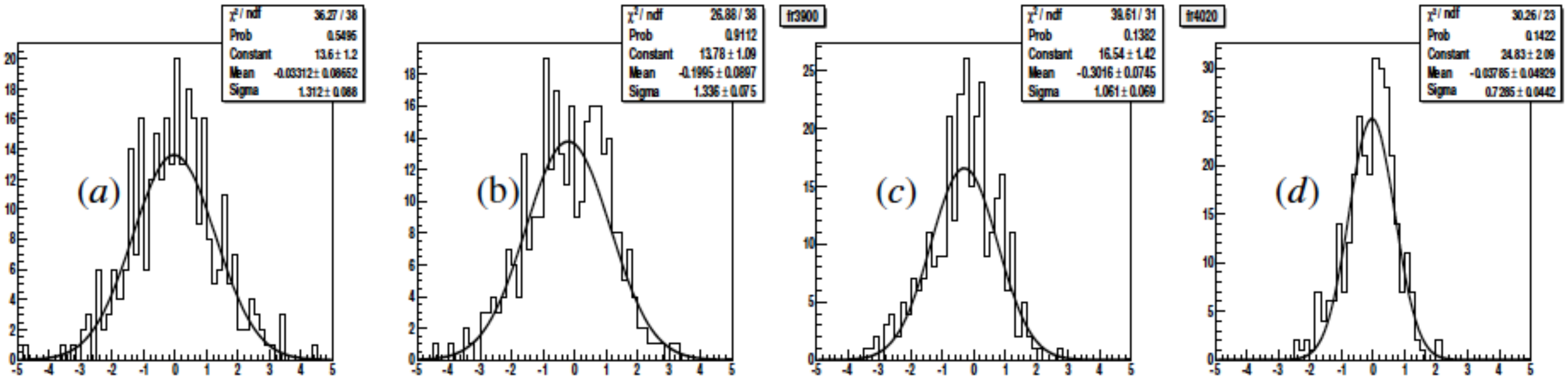


Figure 17: The pull distribution for 4 measured value. Fit 2: (a) mass of $Z_c(40xx)$, (b) width of $Z_c(40xx)$, (c) fraction of $Z_c(3900)$, (d) fraction of $Z_c(40xx)$.

Summary of the systematic uncertainty

	$M(Z_c(40xx)$	$\Gamma(Z_c(40xx))$	Fr.$(Z_c(3900))$	Fr.$(Z_c(40xx))$
$\pi^+\pi^-$ S-Wave parameterization	${}^{+0}_{-0.047}$	${}^{+2.7}_{-0}$	${}^{+0}_{-24.8}$	${}^{+0}_{-23.5}$
$\pi^+\pi^-$ D-Wave	${}^{+0}_{-0.21}$	${}^{+2.18}_{-0}$	${}^{+0}_{-3.94}$	${}^{+3.70}_{-0}$
background	${}^{+0}_{-1.24}$	${}^{+43.8}_{-0}$	${}^{+21.8}_{-0}$	${}^{+111.4}_{-0}$
Barrier factor	${}^{+0.1}_{-0}$	${}^{+1.58}_{-1.25}$	${}^{+0}_{-0.4}$	${}^{+2.87}_{-1.76}$
$Z_c(3900)$ parameter	${}^{+0.23}_{-0.53}$	${}^{+61.5}_{-4.6}$	${}^{+61.3}_{-36.7}$	${}^{+88.9}_{-13.6}$
IO	${}^{+0}_{-0}$	${}^{+0}_{-2.64}$	${}^{+0}_{-2.72}$	${}^{+0}_{-0.34}$
Total	${}^{+0.25}_{-1.37}$	${}^{+75.6}_{-5.5}$	${}^{+65.1}_{-44.6}$	${}^{+142.6}_{-27.2}$

Last result

- Multiplying the fraction by the cross section of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$, arXiv:1611.01317v2

  we get the cross section of intermediate process.

	Result
Mass of $Z_c(40xx)$	$4027.0 \pm 11.5^{+10.1}_{-55.1}$ MeV
Width of $Z_c(40xx)$	$105.6 \pm 20.2^{+79.8}_{-5.8}$ MeV
Fraction of $Z_c(3900)$	$43.1 \pm 7.0^{+28.1}_{-19.2}\%$
$\sigma(e^+e^- \to \pi^\pm Z_c(3900))$	$5.22 \pm 0.85^{+3.42}_{-2.36}$ pb
Fraction of $Z_c(40xx)$	$32.4 \pm 11.8^{+46.2}_{-8.8}\%$
$\sigma(e^+e^- \to \pi^\pm Z_c(40xx))$	$3.92 \pm 1.43^{+5.60}_{-1.11}$ pb

Summary

- The nominal fit contain a $Z_c(3900)$ and a $Z_c(4020)$. The J^p of $Z_c(4020)$ is 1^+, the width

  is much wider than that observed in $\pi^+\pi^- h_c$.

  The detailed value is shown in previous table.

- We show the detailed interference plot between different states.
- The goodness of fit are performed
- I/O check are performed