Heavy quarkonia and heavy quark diffusion coefficients at nonzero temperature from lattice QCD

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in collaboration with

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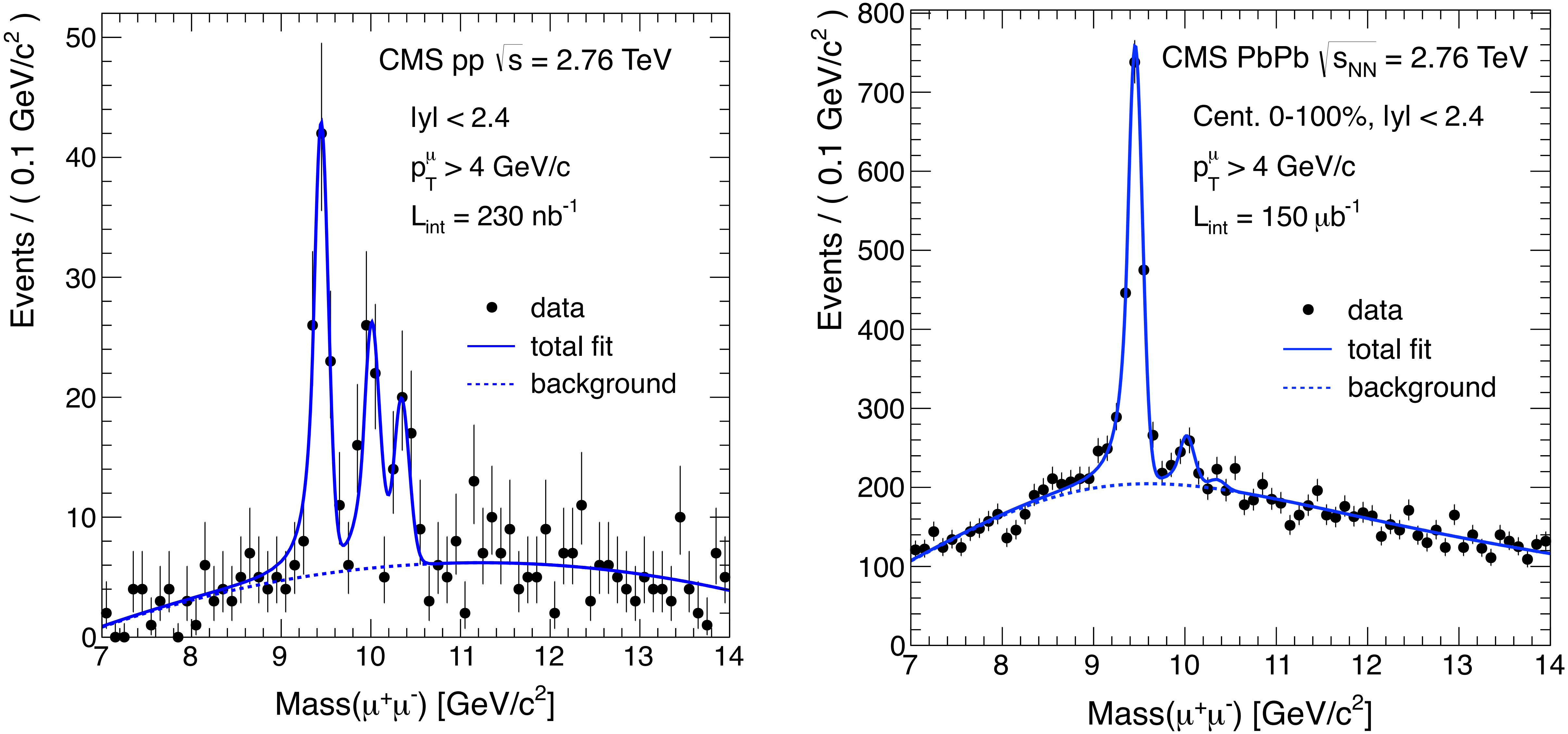
Outline

- Motivation
- Screening mass of charmonia and bottomonia
  - Temperature dependence
  - Dispersion relation
- Spectral functions of charmonia
  - Dissociation temperature
  - Heavy quark diffusion coefficient
- Summary & Conclusion

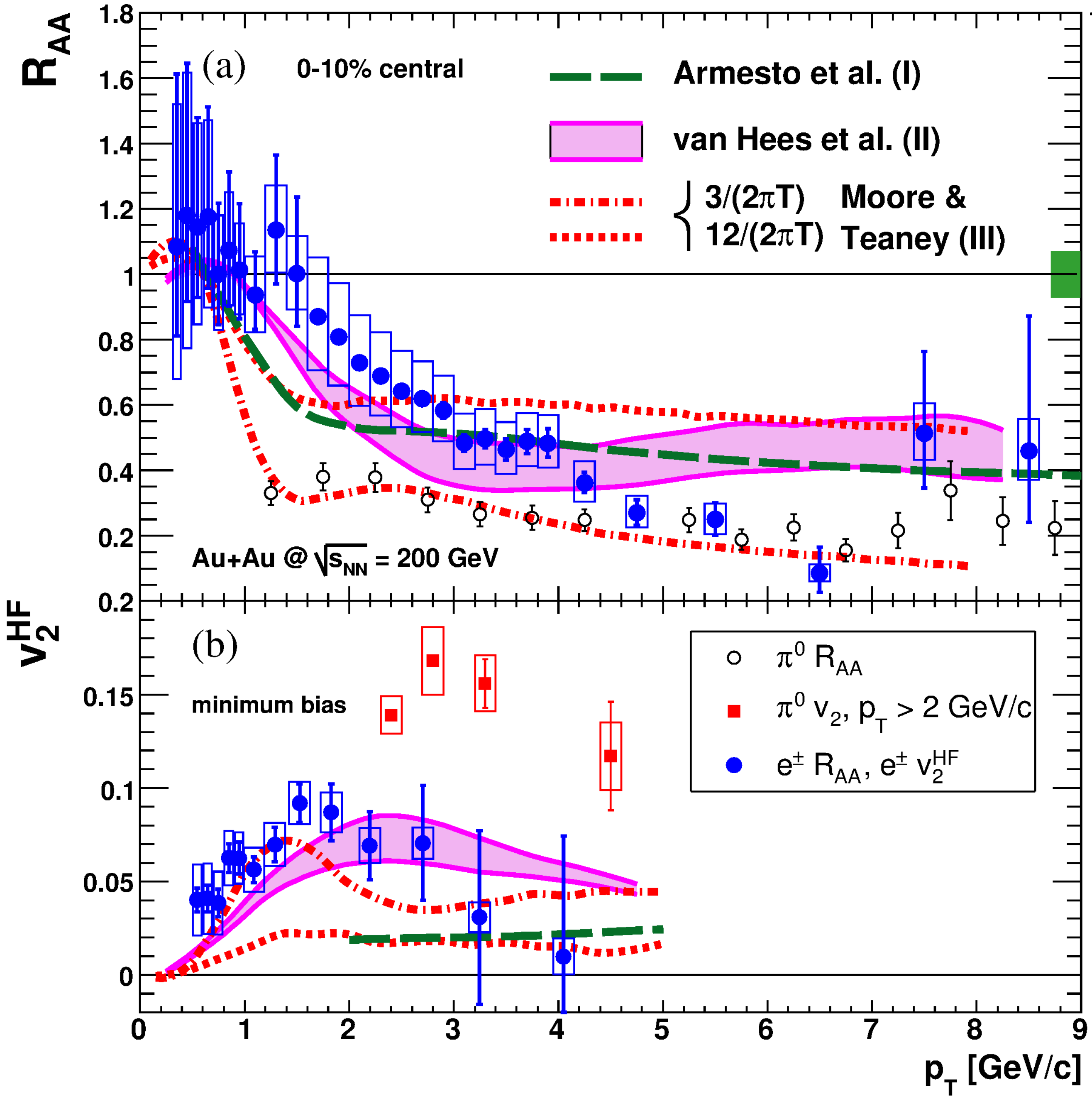
Motivation



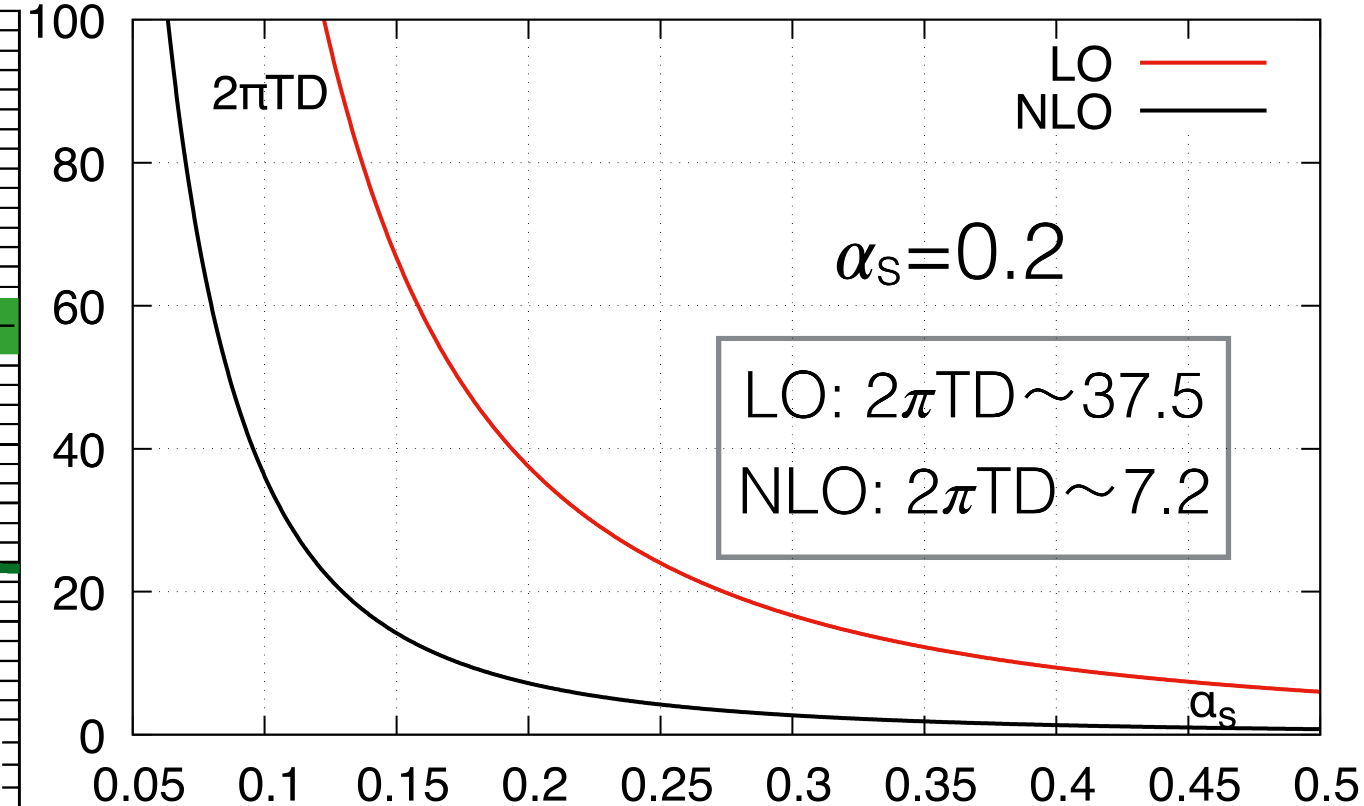
[S. Chatrchyan *et al.*, PRL 109 (2012) 222301]

- What are the dissociation temperatures of heavy quarkonia?
- Do they suffer from thermal modifications when moving in medium?

Motivation



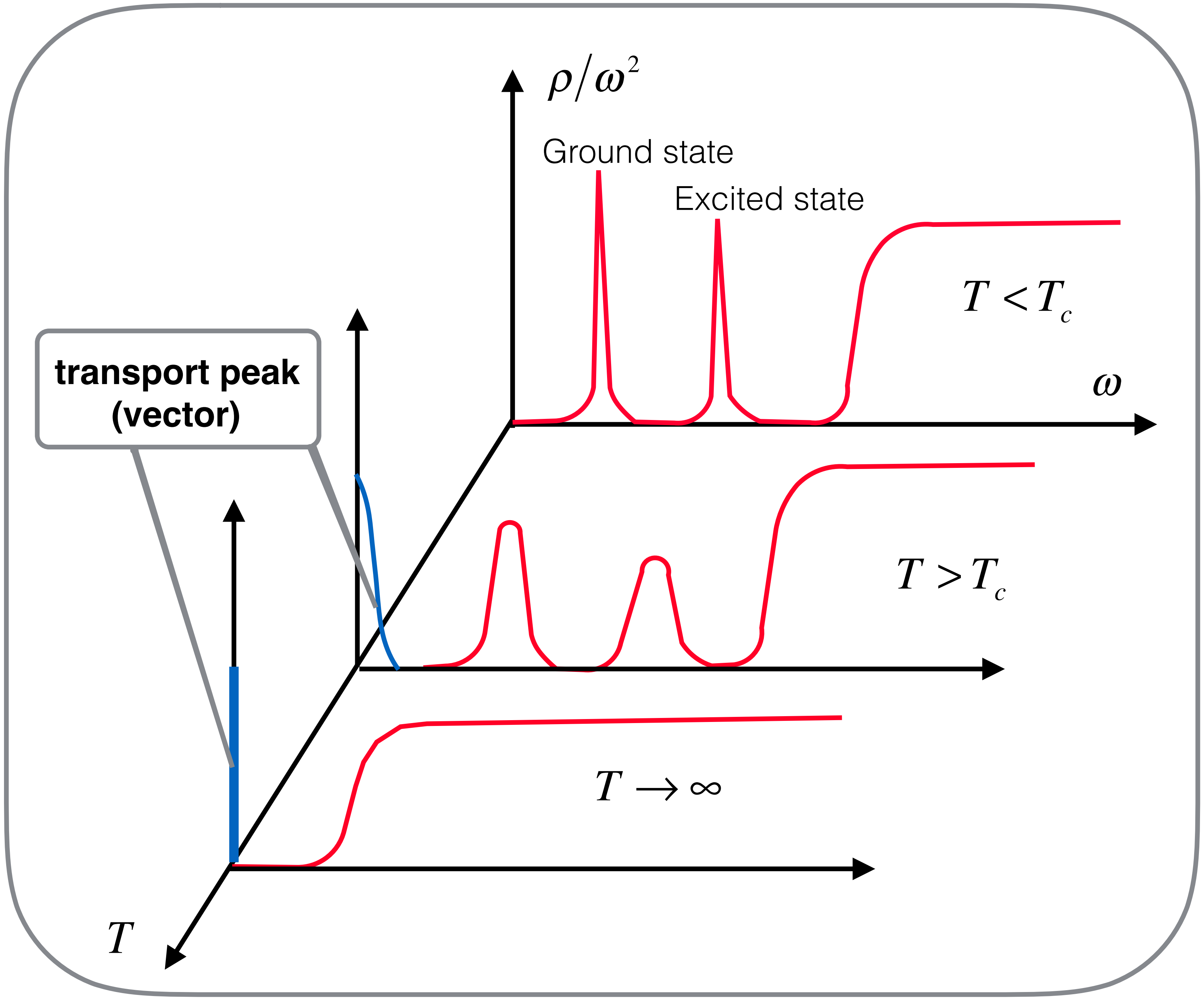
[PHENIX, PRC84 (2011) 044905 & PRL98 (2007) 172301]



Moore & Teaney, PRC71,064904
Caron-Huot & Moore, PRL100,052301

- Calculate heavy quark diffusion coefficient from first principle using lattice QCD.

Hadron spectral function



- Deformation from SPF: dissociation *T*

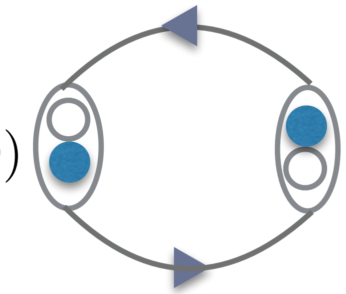
- Heavy quark diffusion coefficient:

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^{V}(\omega, \mathbf{0})}{\omega}$$

Spatial correlation function & Screening mass

- Spatial correlation function: sum over space (x,y,τ)

$$G_H(z,\boldsymbol{p}_\perp,\omega_n) = \sum_{x,y,\tau} \exp(-i\,\tilde{\boldsymbol{p}}\cdot\tilde{\boldsymbol{x}})\langle J_H(0,\boldsymbol{0}) J_H^\dagger(\tau,\boldsymbol{x})\rangle$$



$H=(\gamma^5,\gamma^\mu,\mathbb{1},\gamma^5\gamma^\mu)$

for pseudo-scalar(P), vector(V), scalar(S), axial-vector(A) channel

- Relation to the spectral function

$$G_H(z,\boldsymbol{p}_\perp,\omega_n) = \int_{-\infty}^{\infty} \frac{\mathrm{d}p_z}{2\pi} \exp(ip_z z) \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_H(\omega,\boldsymbol{p},T) \frac{\omega}{\omega^2+\omega_n^2}$$

- Long distance behavior: exponential decay in *z*

$$G_H(z,\boldsymbol{p}_\perp,\omega_n) \sim \exp(-zE_{scr}) \qquad E_{scr}{}^2 = \vec{p}^{\,2} + M^2 + \Pi(\vec{p},T)$$

at p=0: E_{scr}=screening mass
at p=0,T=0: E_{scr}=pole mass

Absorb thermal effects into *M(T)* and *A(T)*

$$E_{scr}{}^2 = A(T)\vec{p}^{\,2} + M^2(T)$$

- Non-interacting limit

$$E_{free} = 2\sqrt{(\pi T)^2 + m_q^2}$$

Temporal correlation function & Spectral function

- Relation between temporal correlation function and SPF

$$G_H(\tau,\boldsymbol{p}) = \sum_{x,y,z} \exp(-i\,\boldsymbol{p}\cdot\boldsymbol{x})\langle J_H(0,\boldsymbol{0})J_H^{\dagger}(\tau,\boldsymbol{x})\rangle = \int_0^{\infty}\frac{\mathrm{d}\omega}{2\pi}\rho_H(\omega,\boldsymbol{p},T)\frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

ill-posed

- Inversion methods to solve ill-posed problems:

Stochastic Analytic Inference
stochastic approach
based on Bayesian theorem

mean field limit → **Maximum Entropy Method**
based on Bayesian theorem
the most probable solution

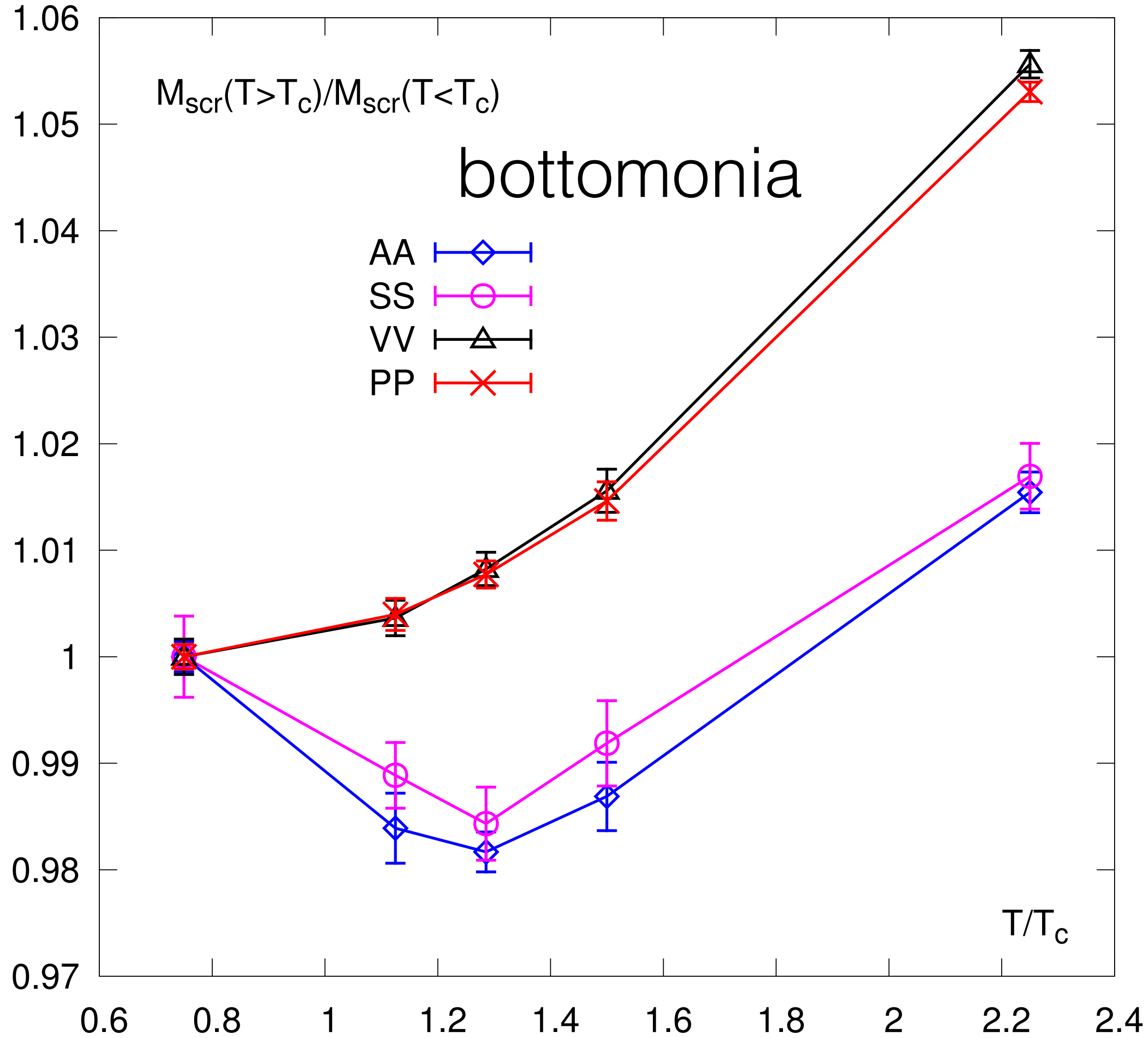
default model=*const.* → **Stochastic Optimization Method**
stochastic approach
does not need default model

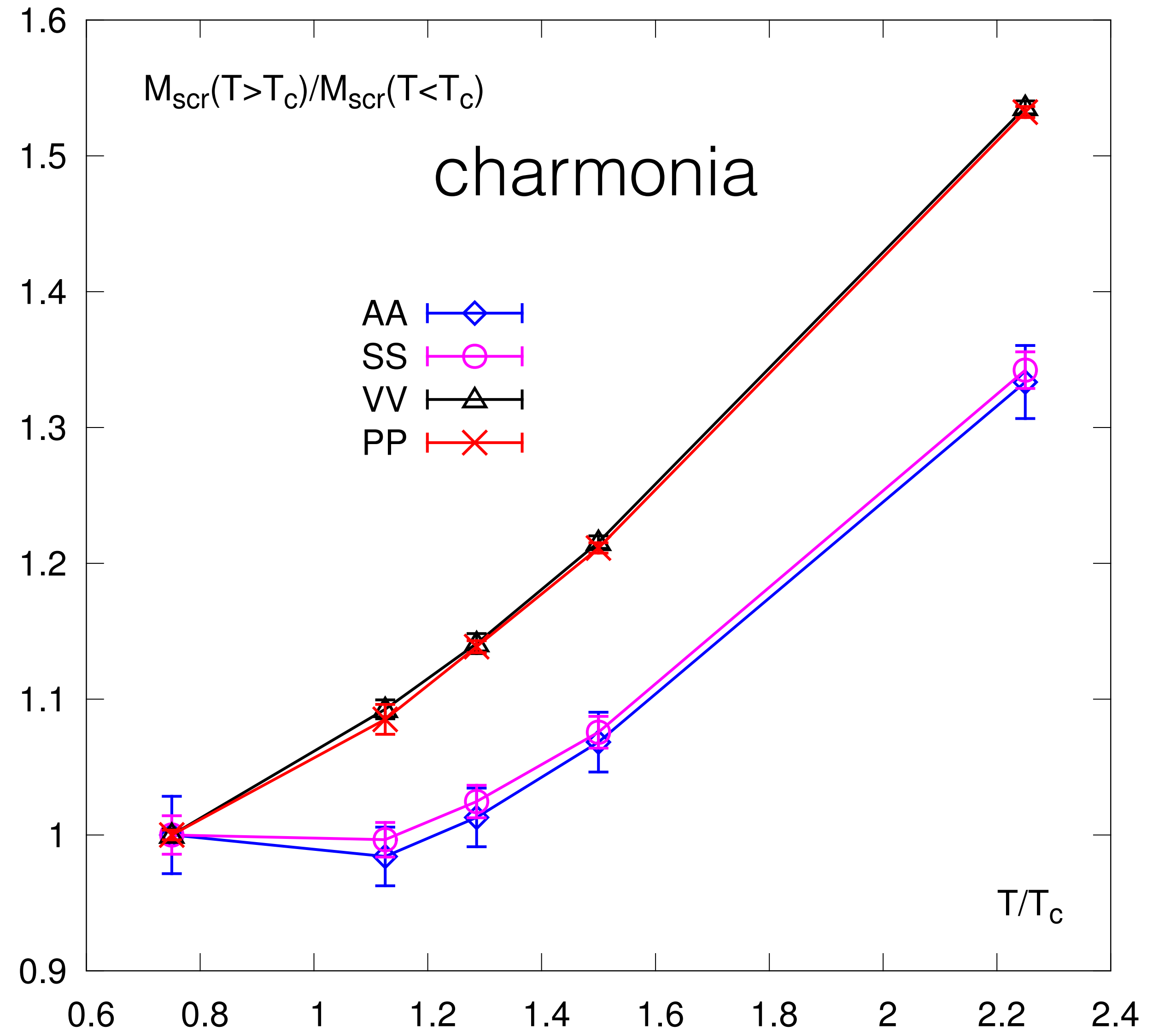
Simulation details

- Quenched QCD on isotropic lattices
- Large quenched lattices close to continuum ($aM_Q << 1$)
- Clover-improved Wilson fermions
- Quark masses tuned to reproduce nearly physical J/ψ mass and Υ mass
- $0 \le |\boldsymbol{p}| \le 3.17$GeV
  - non-zero momenta (1 source, δG/Ḡ ~1.5% at the middle point in the vector channel)
  - zero momentum (~5 sources, δG/Ḡ ~2 times smaller)

β	a^{-1}	κ	N_σ	N_τ	T/T_c	#conf
7.793	22.8GeV	$0.13221(c\bar{c})$ $0.12798(b\bar{b})$	192	96	0.75	218
				64	1.10	248
				56	1.20	190
				48	1.50	210
				32	2.25	235

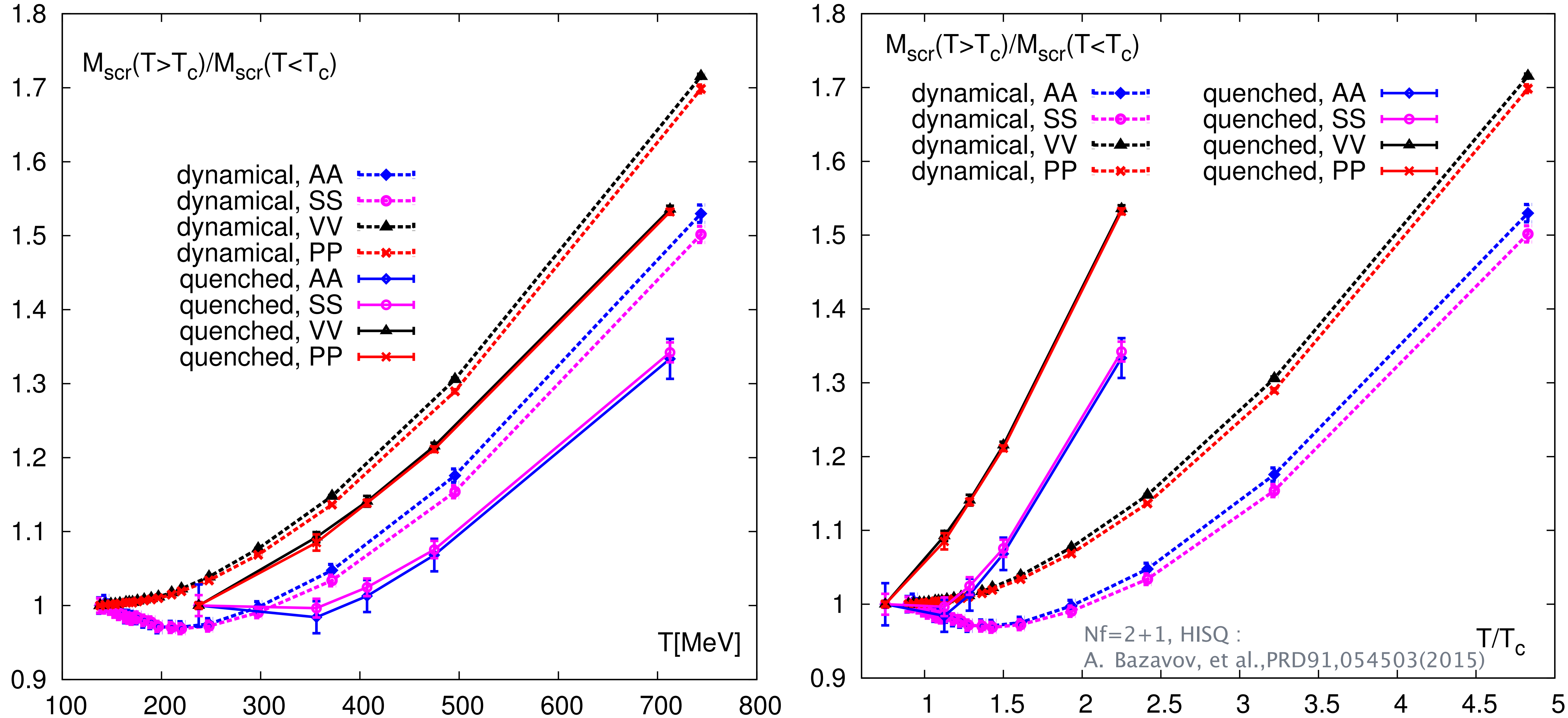
Screening masses of heavy quarkonia





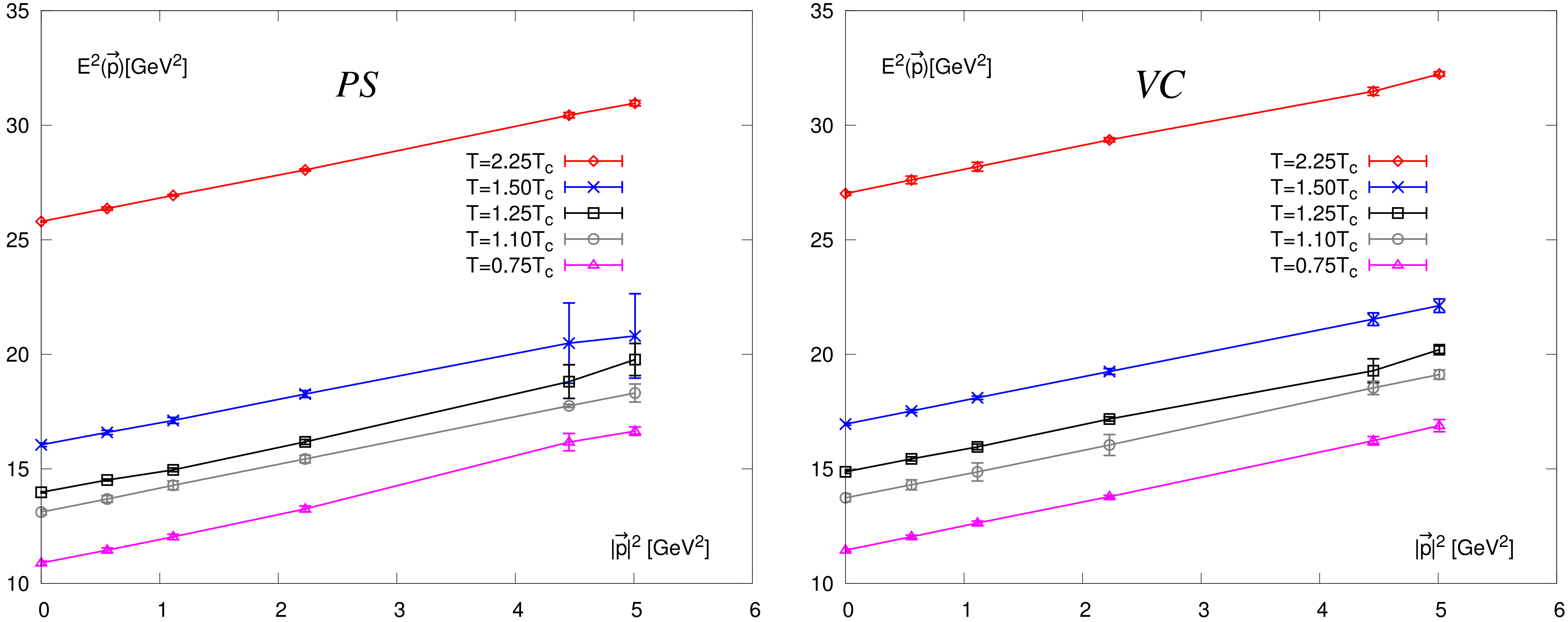
- For both bottomonia & charmonia, M_{scr} of s-wave states increases in *T*.
- For both bottomonia & charmonia, M_{scr} of p-wave states decrease first in *T* and then increase.
- More changes in M_{scr} are observed for charmonia.

Screening mass—Quenched v.s. Dynamic QCD



- Quenched and 2+1 HISQ : similar *T*-dependence.
- Different dip location for p-waves: $1.10T_c$ in quenched QCD, $1.43T_c$ in dynamic QCD.

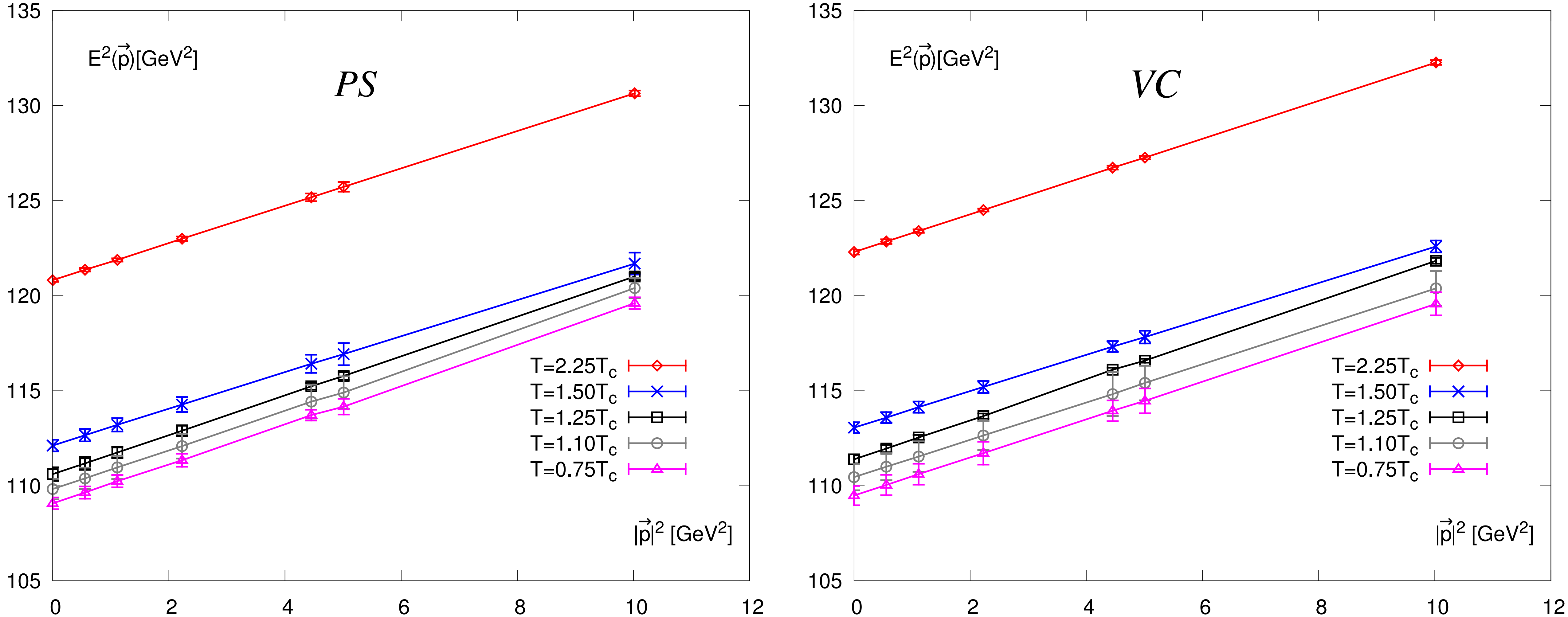
Dispersion relation of charmonia



$$E_{scr}{}^2 = A(T)\vec{p}^2 + M^2(T)$$

- Dispersion relation for s-wave states remains unmodified for charmonia as *A(T)~1*. See also: A. Ikeda, et al., PRD95. 014504
- The reason could be that the largest momentum 3.17GeV is still less than the masses of charmonia (~3.5GeV).

Dispersion relation of bottomonia



$$E_{scr}{}^2 = A(T)\vec{p}^{\,2} + M^2(T)$$

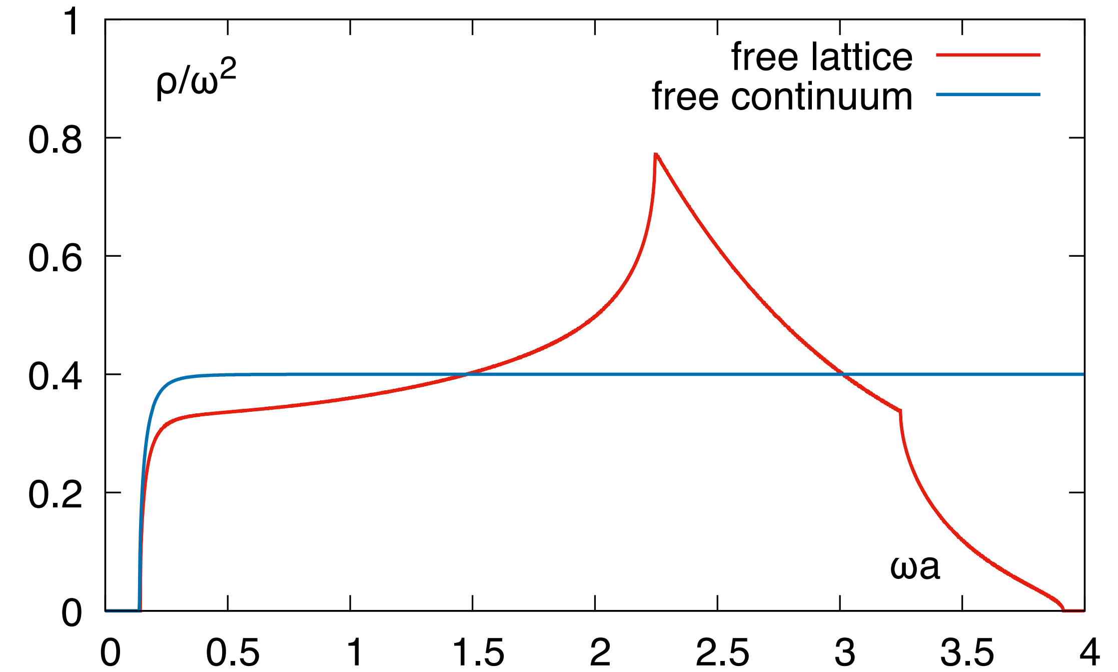
- Similar situation has been observed as in the case of charmonia.

Non-relativistic quarks: G. Aarts, et al., JHEP 1303(2013) 084

Prior information in the default model

- High frequency of the SPF :
  - ✶Free continuum SPF
  - ✶Free lattice SPF

Heng-Tong Ding, et al, arXiv:0910.3098



- Low frequency of the SPF:
  - ✶Non-interacting: F. Karsch et al., PRD68, 014504; G. Aarts et al, NPB726, 93

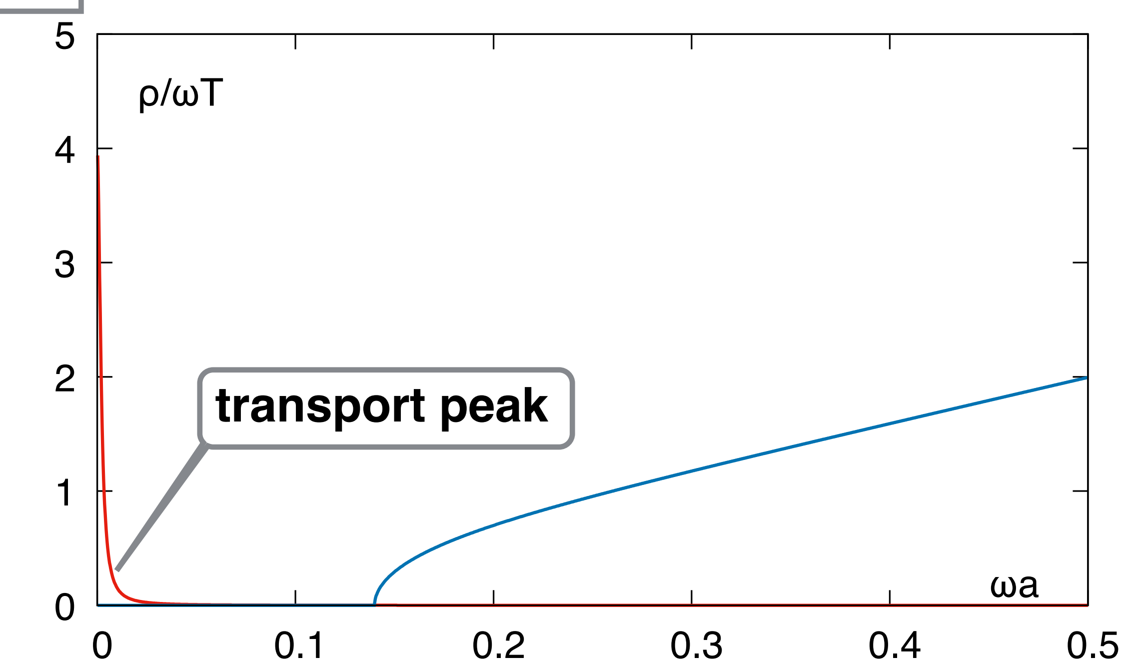
$$\rho_H(\omega) = N_c\left[(a_H^{(1)} + a_H^{(3)})I_1 + (a_H^{(2)} + a_H^{(3)})I_2\right]\omega\delta(\omega) \Longrightarrow D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^V(\omega,\mathbf{0})}{\omega} = \infty$$

$\omega\delta(\omega)$ gives infinite quark diffusion coefficient.

  - ✶Interacting: P. Petreczky and D. Teaney, PRD73,014508

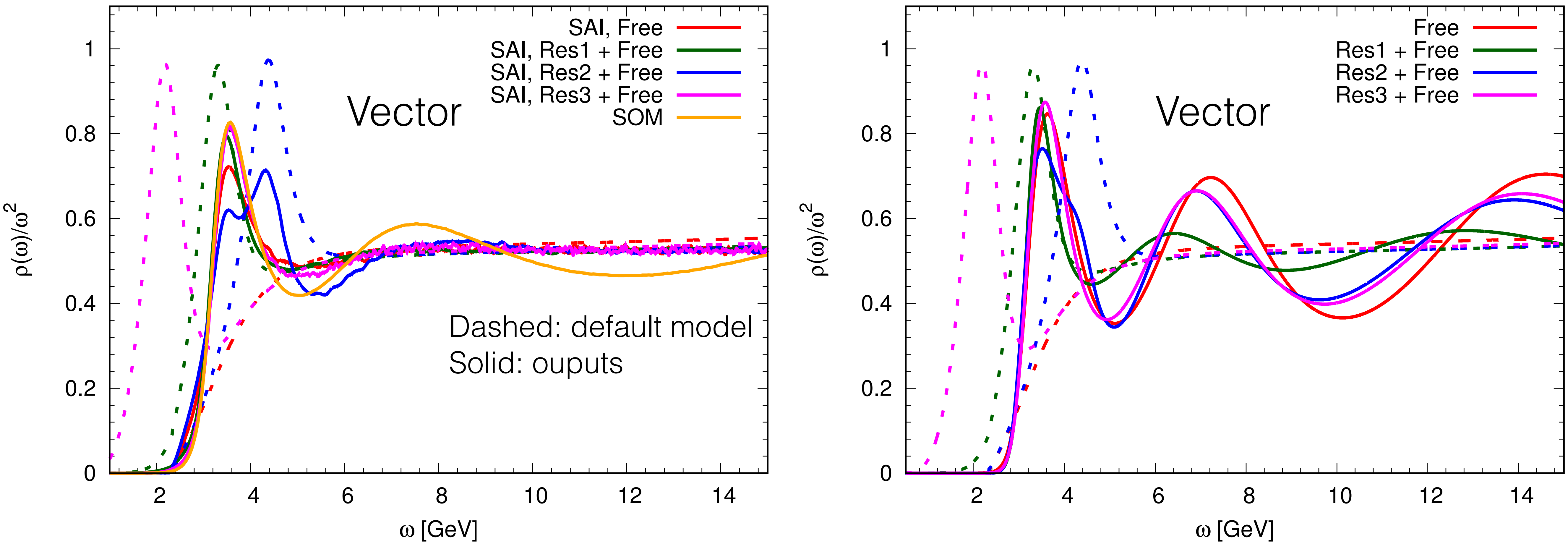
$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2} \Longrightarrow D \propto 1/\eta$$

$\delta(\omega)$ is smeared into a transport peak linear in ω at ω~0.



Default model dependence of the charmonium SPF at 0.75 T_c

Stochastic approaches | MEM

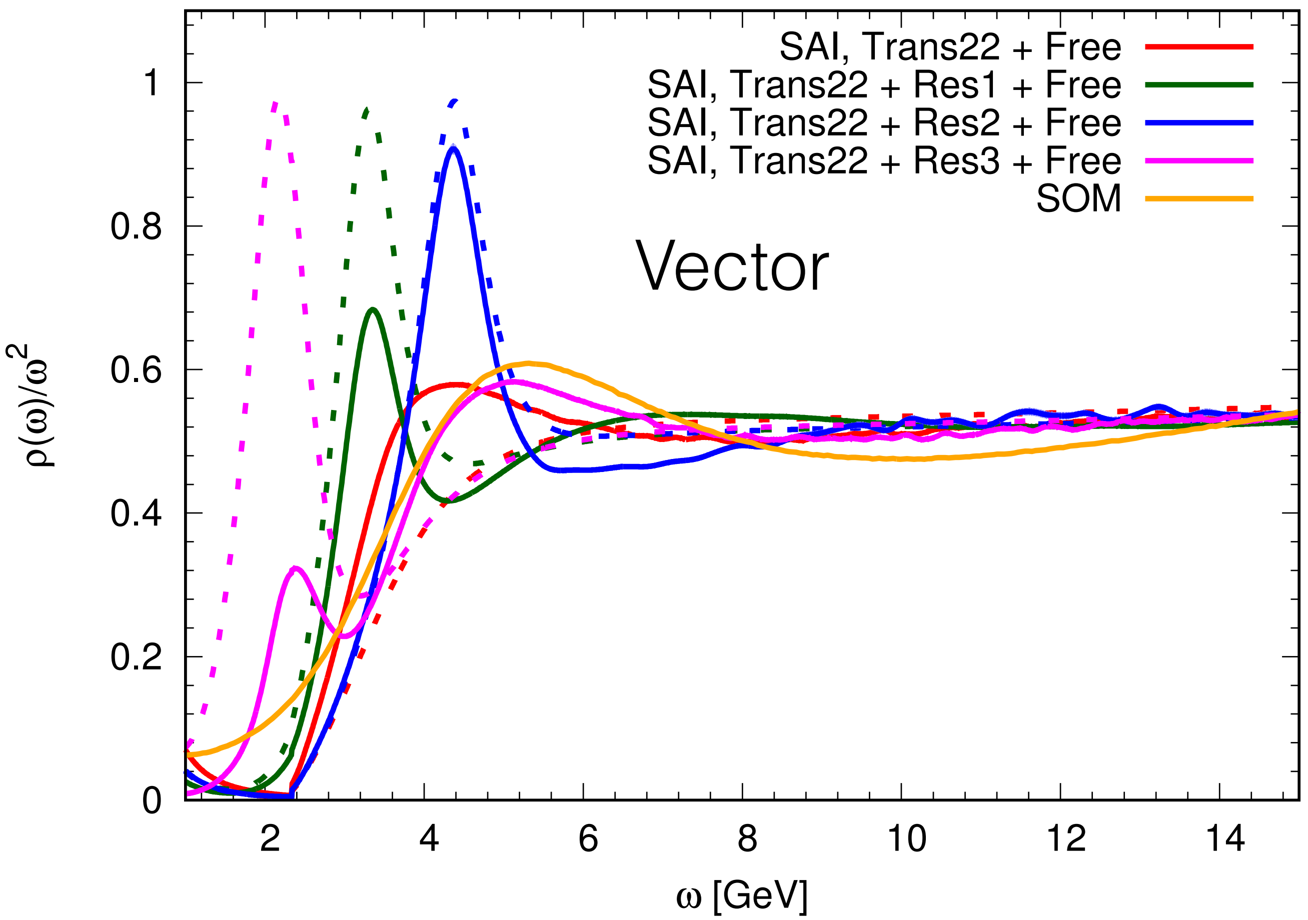


DM = Free, Res(1-3) + Free
Peak location: Res1 ~ J/Ψ mass, Res2 > J/Ψ mass, Res3 < J/Ψ mass

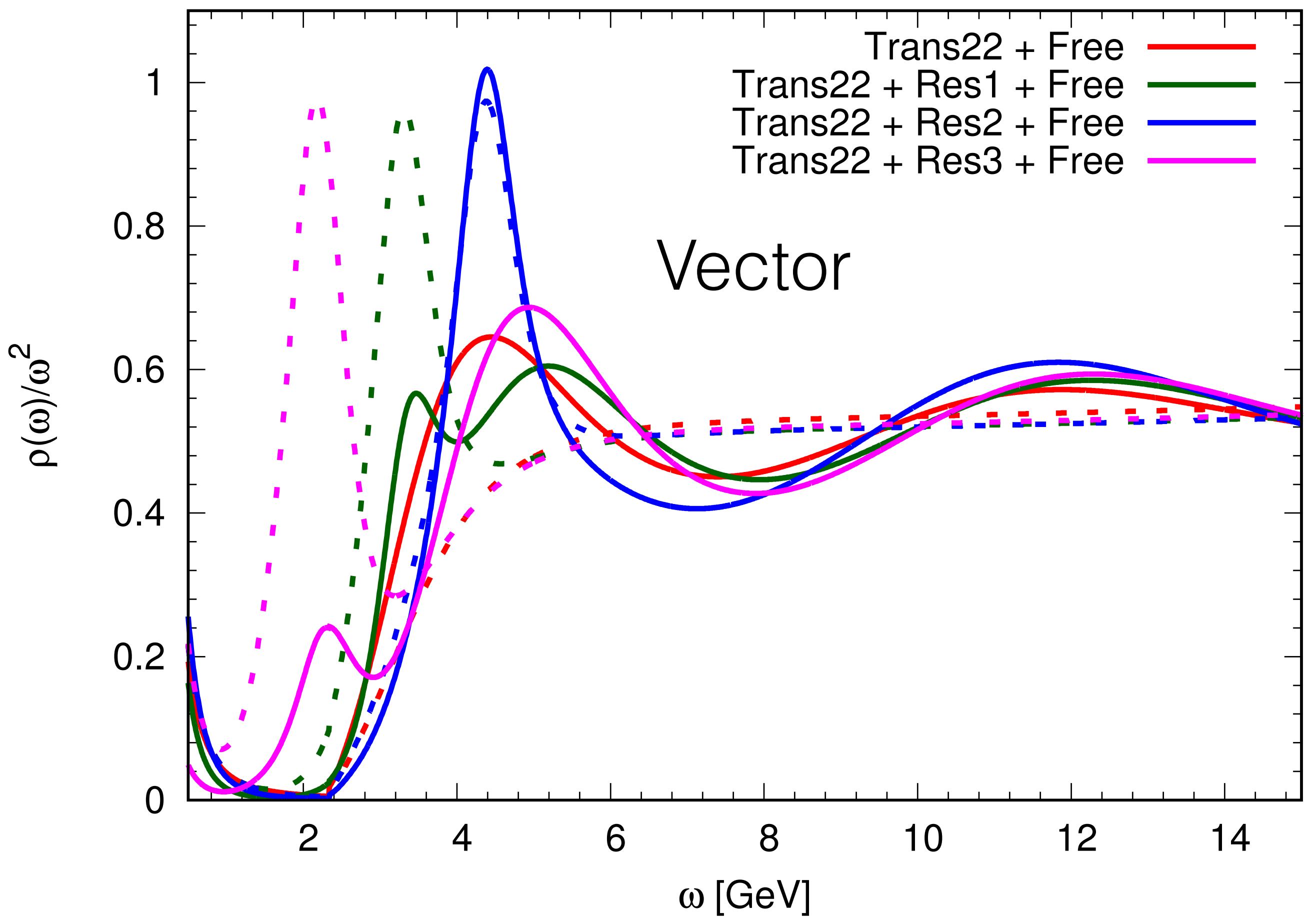
- Location of the first peak is robust, same as the screening mass

Default model dependence of the charmonium SPF at 1.5T_c (1)

Stochastic approaches



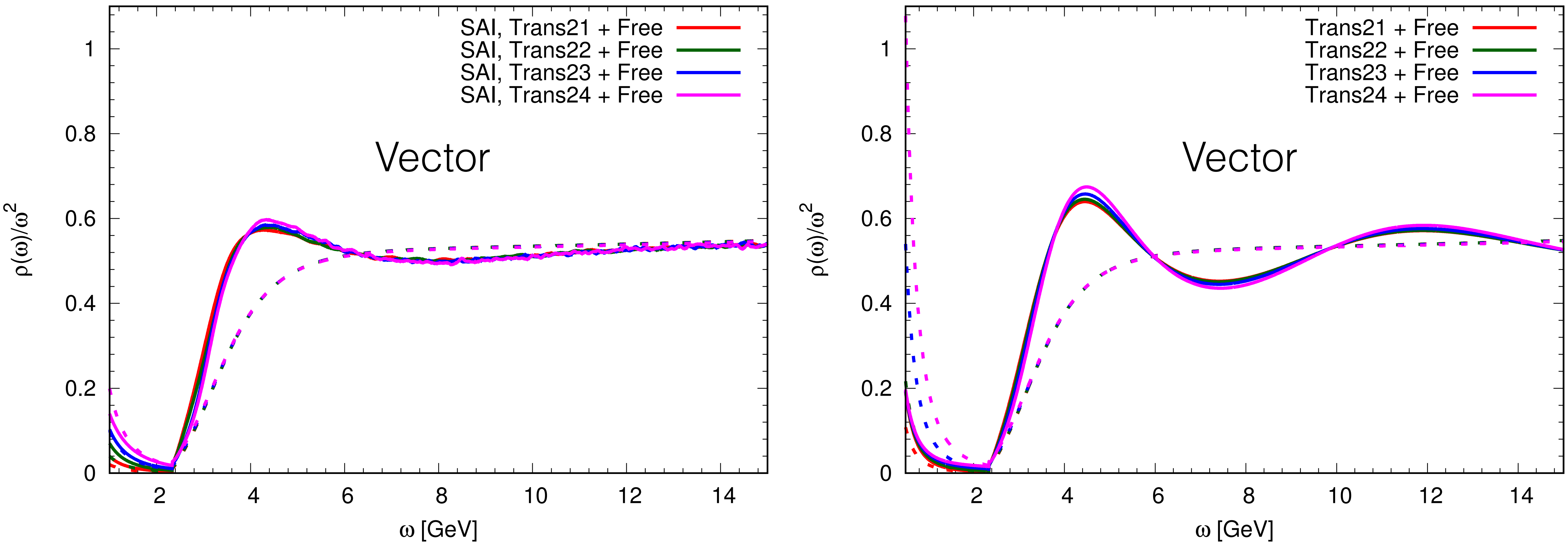
MEM



DM = Trans + Free, Trans + Res(1-3) + Free
Peak location: Res1 ~ J/Ψ mass, Res2 > J/Ψ mass, Res3 < J/Ψ mass

Default model dependence of the charmonium SPF at 1.5T_C (2)

Stochastic approaches | Vector | MEM



DM = Trans(21-24) + Free

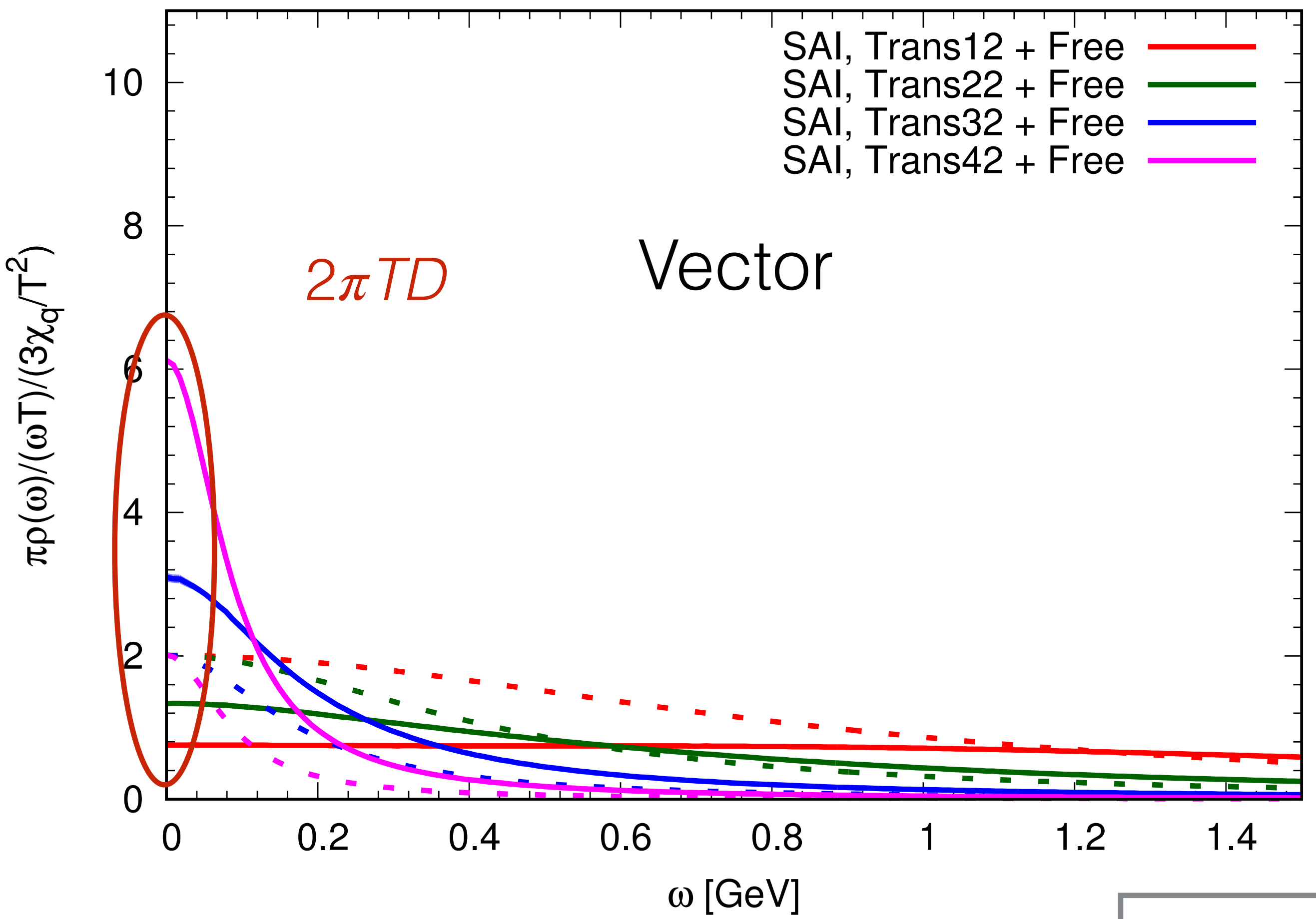
Trans(21-24): $2\pi TD$ ~ (1,2,5,10). Width of the transport peak is fixed.

- High frequency part has small DM dependence.

J/Ψ seems to melt already at T = 1.5T_C !

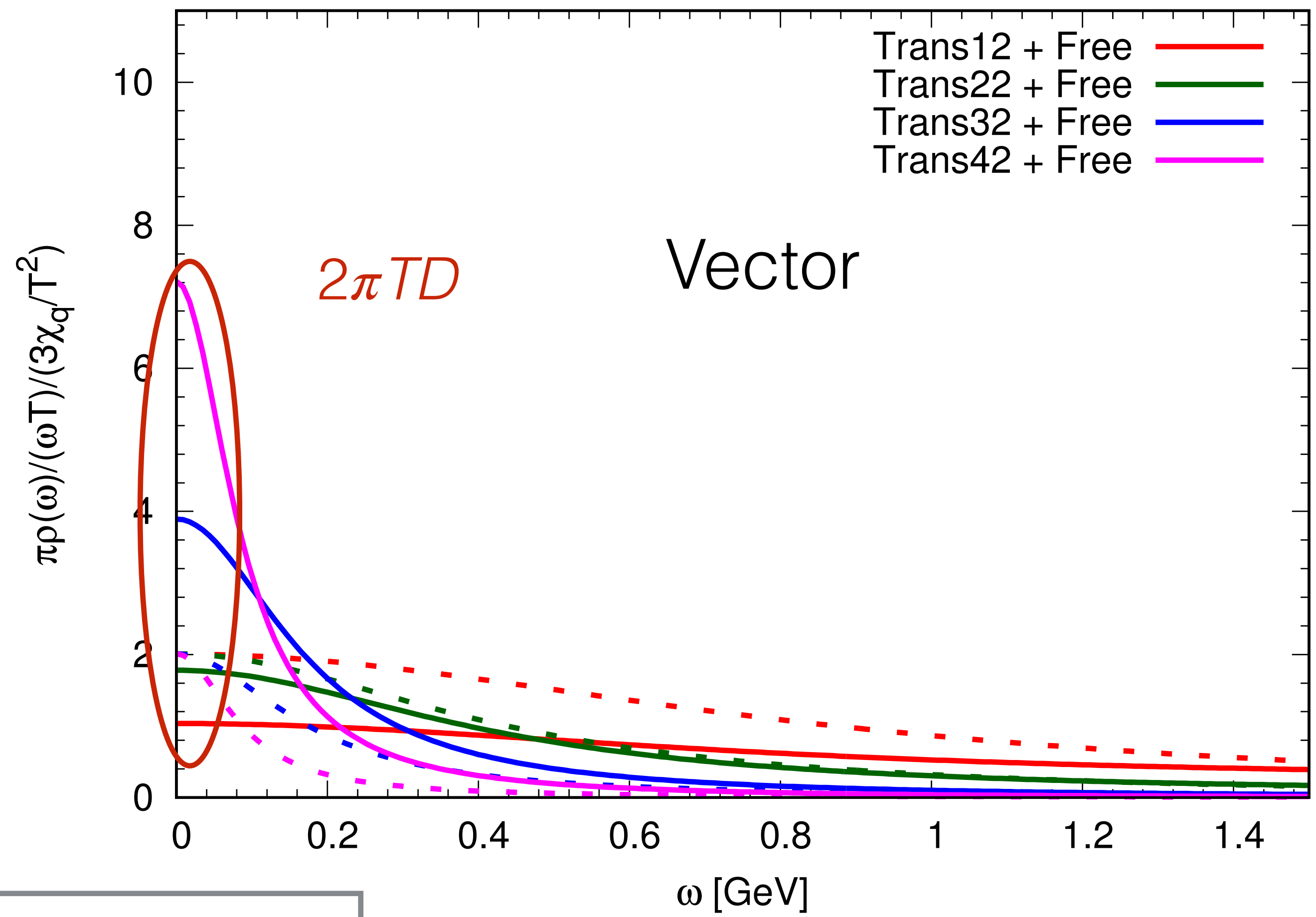
Default model dependence of the charmonium SPF at $1.5T_c$ (3)

Stochastic approaches



Vector

MEM



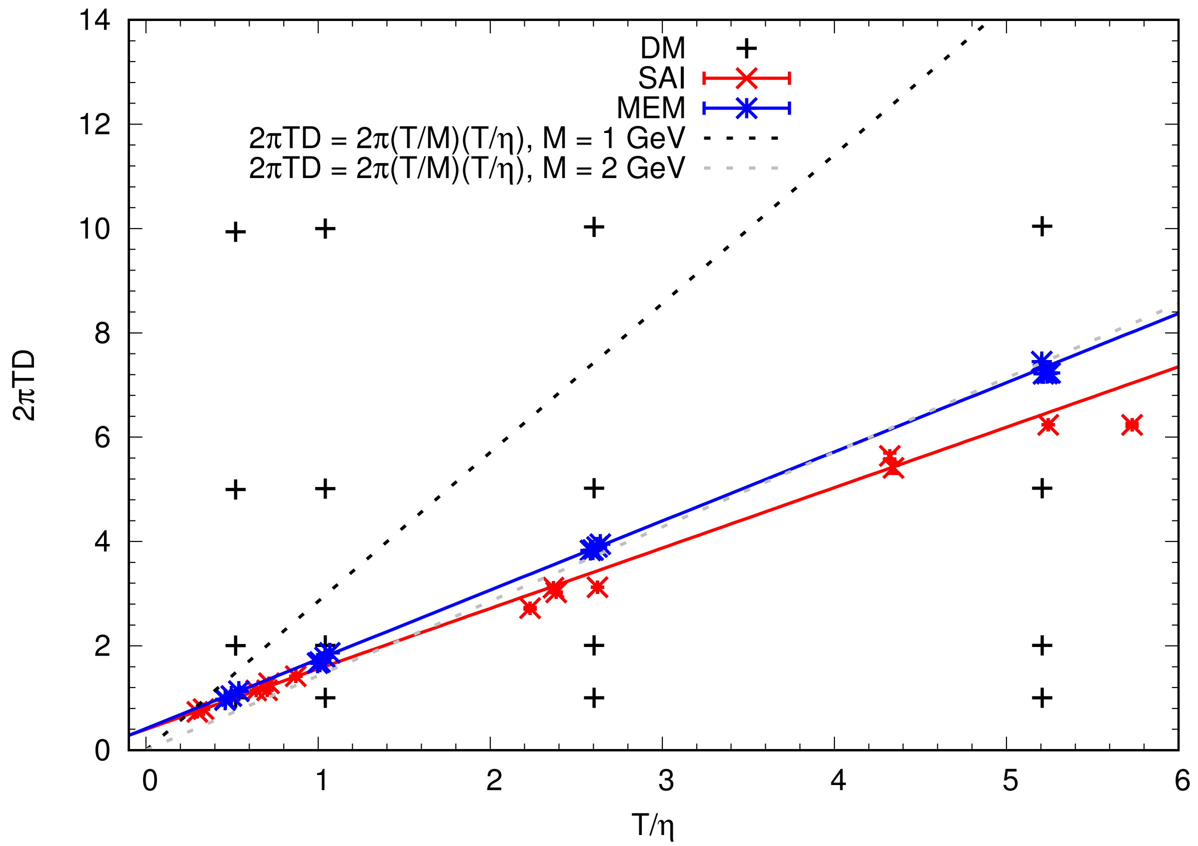
$2\pi TD \sim$ 1-7

DM = Trans(12-42) + Free

Trans(12-42): $\eta/T \sim$ (2,1,0.2,0.1). Height of the transport peak is fixed.

- Low frequency part is sensitive to DMs.

Default model dependence of the charm quark diffusion coefficient at 1.5 T_c



Output η/T is almost the same to DM η/T for MEM.

Output $2\pi TD$ varies as DM η/T changes.

It seems that $D\eta$ is always fixed.

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} c\, \frac{\eta}{\omega^2+\eta^2} \Longrightarrow c \propto D\eta$$

$$G(\tau) = \int \frac{d\omega}{2\pi}\, c\, \frac{\omega\eta}{\omega^2+\eta^2} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

$$\propto c \quad (\omega \ll T)$$

Both SAI and MEM can only determine the coefficient c or $D\eta$

→ The diffusion coefficient can be determined once η/T is fixed.

$2\pi TD$ = 1.16(4) T/η + 0.40(2) for SAI
$2\pi TD$ = 1.33(4) T/η + 0.42(2) for MEM
The Einstein relation suggests $2\pi TD \lesssim 6$.

for T/η= 1—5 ⇒

$2\pi TD$ ~ 1.6—6.2 for SAI
$2\pi TD$ ~ 1.8—7.0 for MEM

Conclusion & Outlook

We have performed simulations on large quenched lattices to calculate both the temporal & spatial Euclidean correlation functions at both zero and nonzero p.

✳Most of the results suggest that J/Ψ might melt already $T=1.5T_C$.

✳So far we observed a relation between $2\pi TD$ and T/η, which gives a range $1 \lesssim 2\pi TD \lesssim 7$ for $1 \lesssim T/\eta \lesssim 5$.

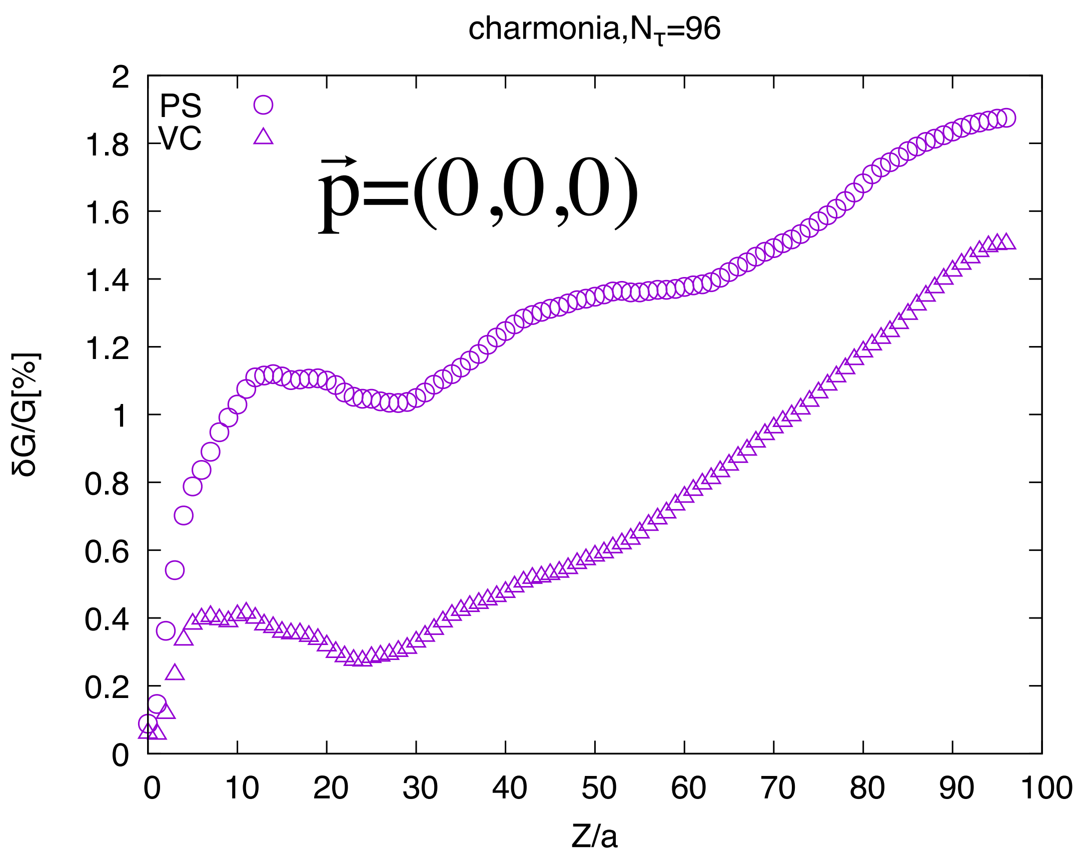
✳Charmonia suffer from more thermal effects in medium than bottomonia in screening mass.

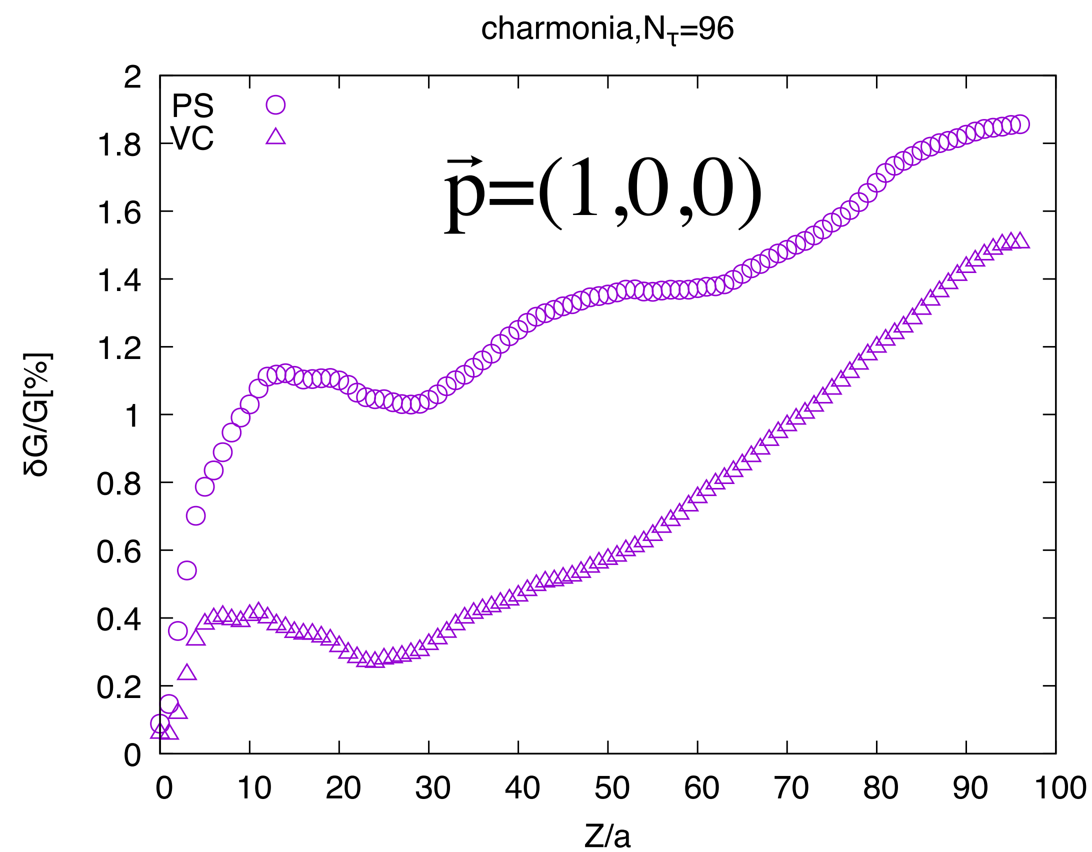
✳Dispersion relation in our quenched simulations seems to be not modified in medium when $p < M_{src}$.

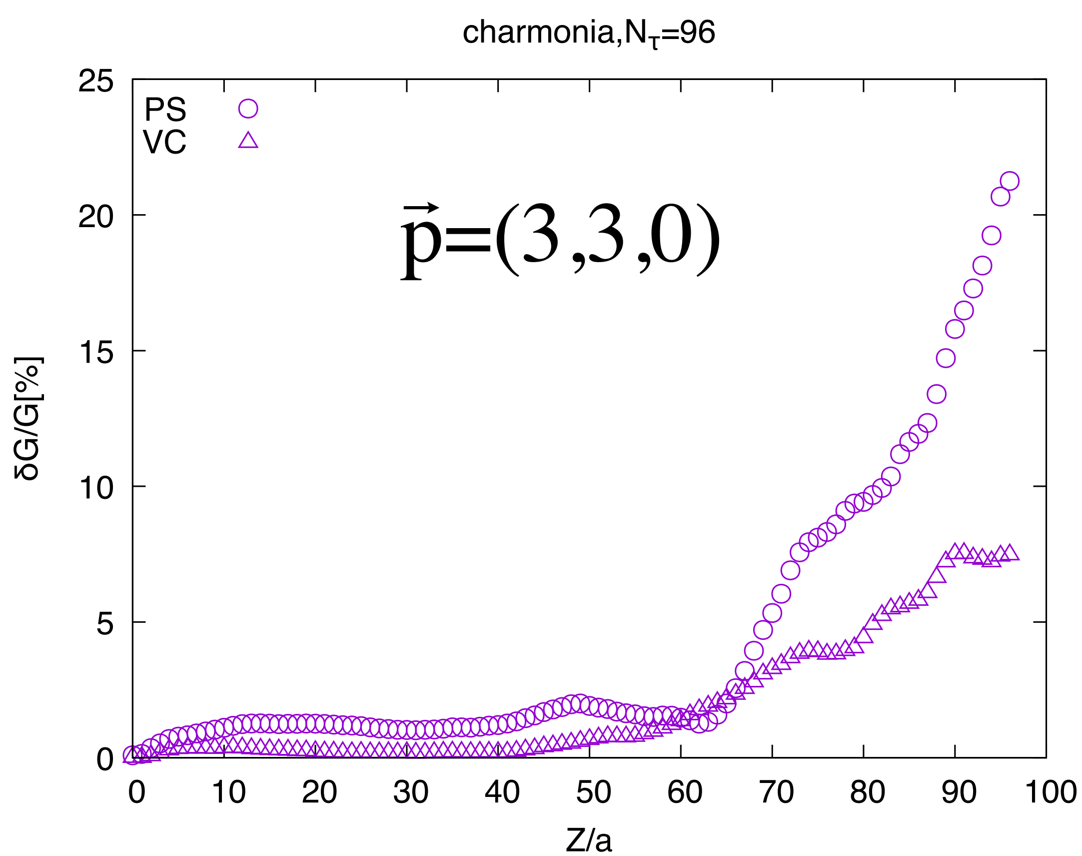
- Analysis of the temperature dependence of continuum extrapolated correlator is on the way.

Thanks!

Back-up. relative error at different momenta

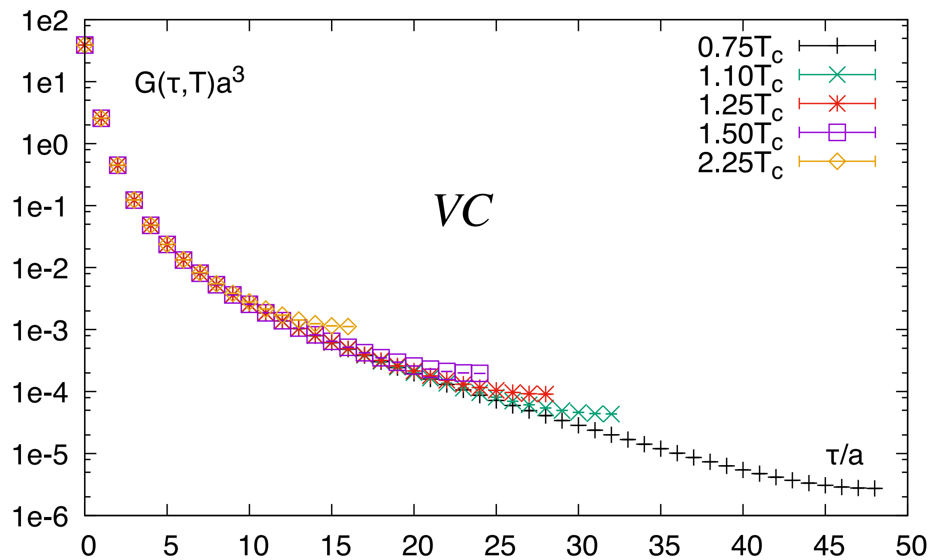


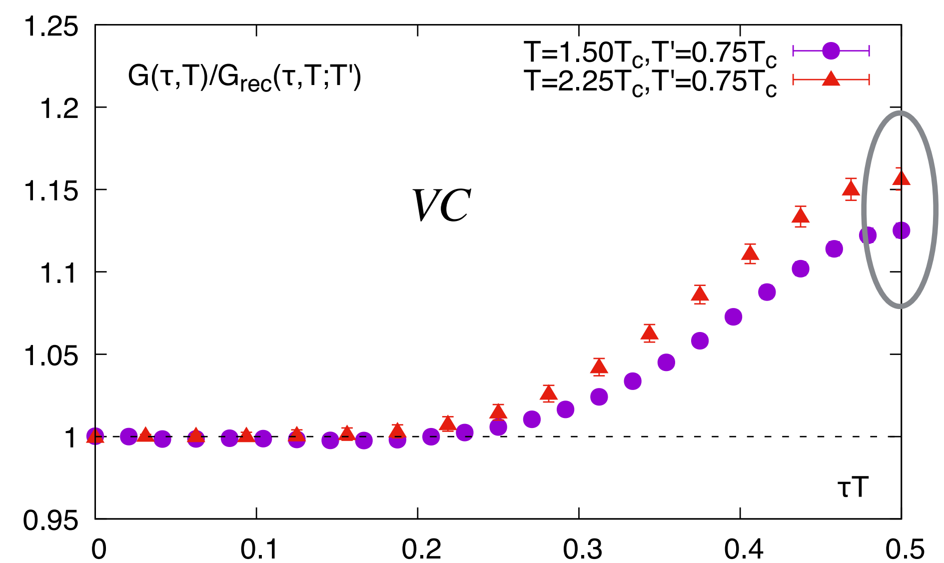




- The error is small at small momenta: around 1.5% in VC channel, 1.8% in PS channel
- The error becomes larger at the largest momenta: around 7% in VC channel, 22% in PS channel

Temperature dependence of the temporal correlation functions



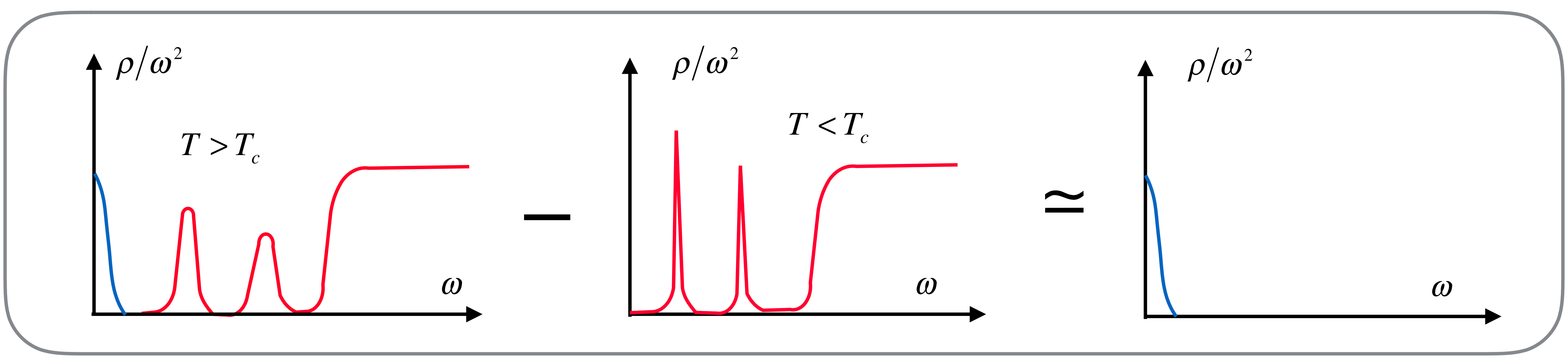


- More contribution from transport peak at higher *T*.

- Clear temperature dependence

Reconstructed correlation function:

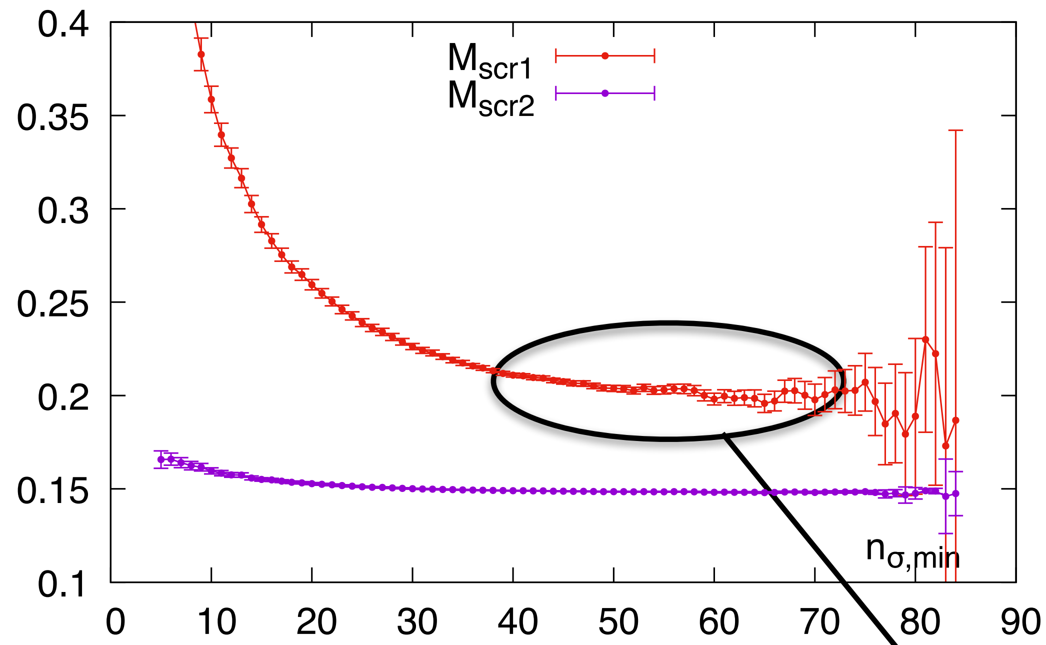
$$G_{rec}(\tau, T; T') = \int \frac{\mathrm{d}\omega}{2\pi} \rho(\omega, T') K(\omega, \tau, T)$$



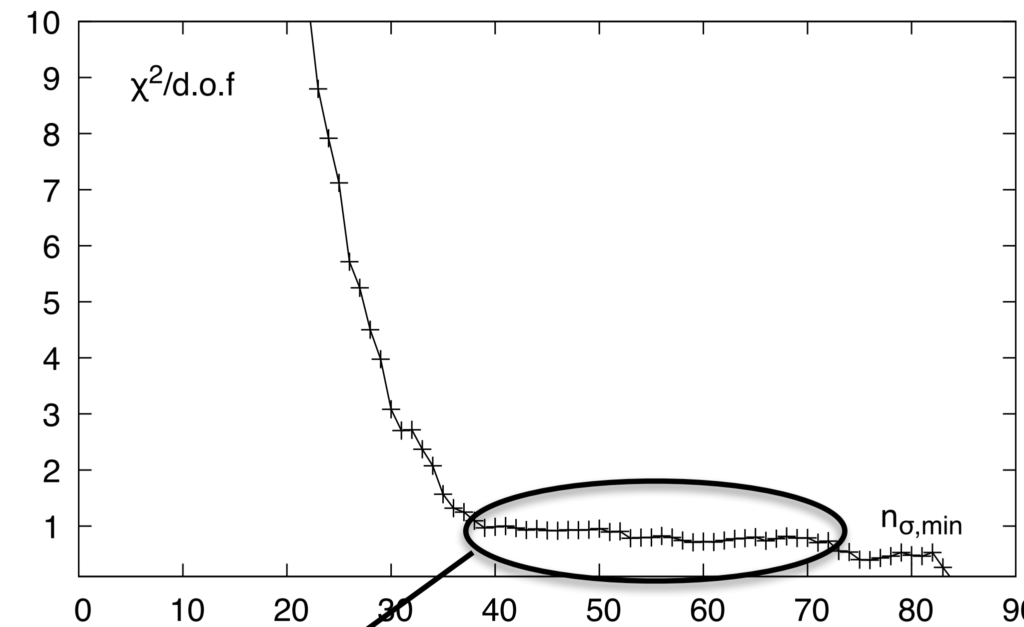
Screening mass

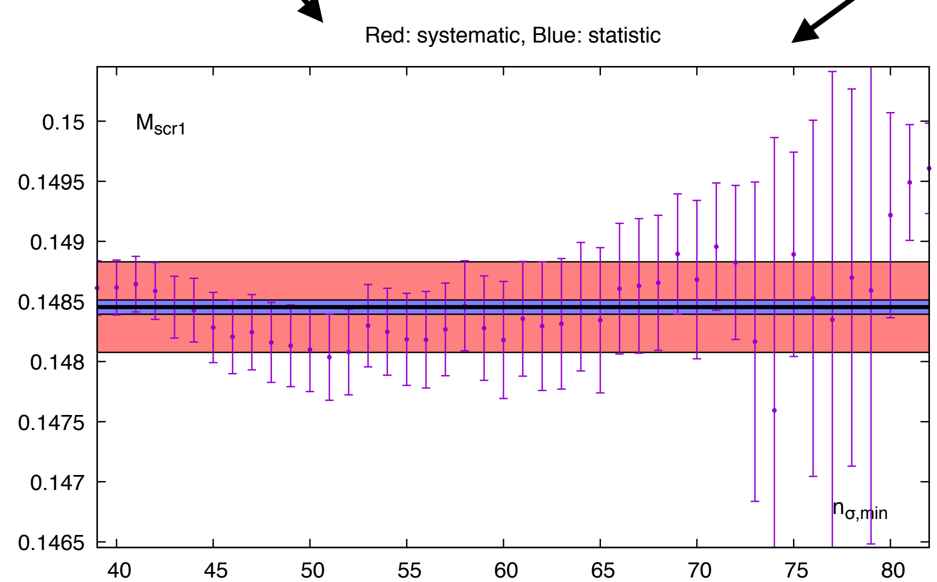
Spatial correlators: $G(n_\sigma) = A_1 \cosh\left(M_{scr1}(n_\sigma - N_\sigma/2)\right) + A_2 \cosh\left(M_{scr2}(n_\sigma - N_\sigma/2)\right) + \bullet\bullet\bullet$

Two-state ansatz:



Plateau in χ^2/d.o.f:





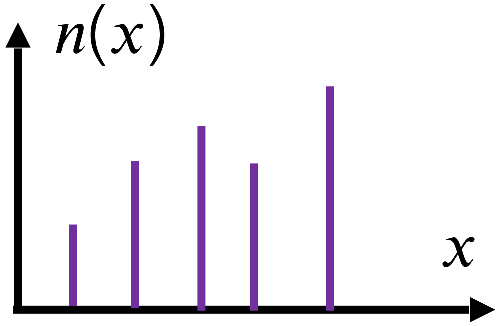
Final screening mass.

Basis of Stochastic Approaches

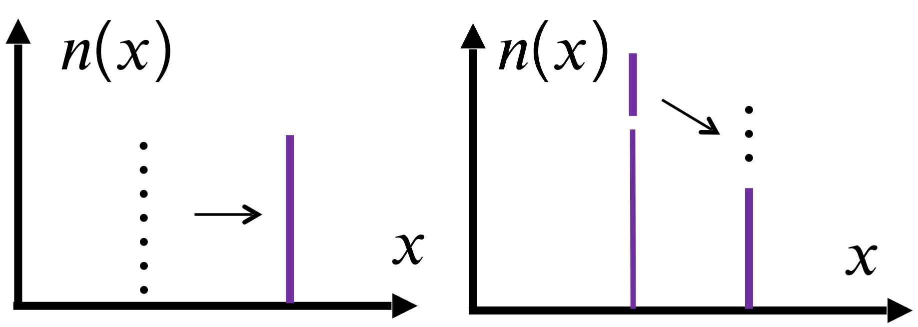
SAI

- Ingredients: δ functions

$n(x)=\sum r_i \, \delta(x\text{-}a_i)$



- Update schemes



Shift   Change residues

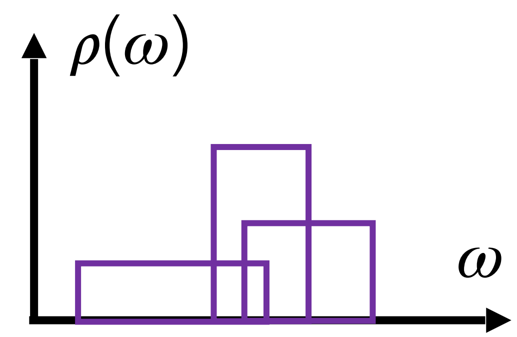
- Normalization

$\sum r_i = G(\tau_0)$

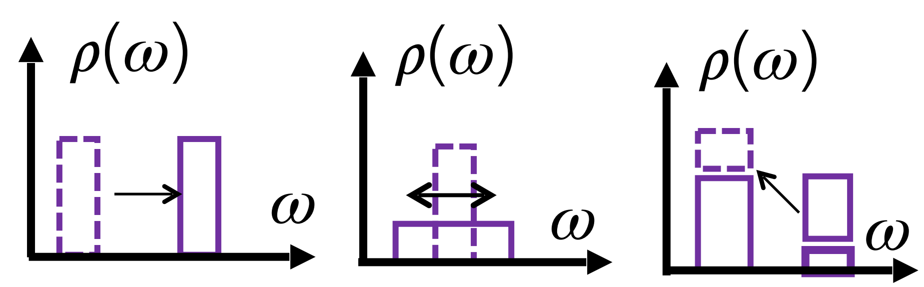
SOM

- Ingredients: boxes

$\rho(\omega)=\sum \eta_i \, (\omega)$



- Update schemes

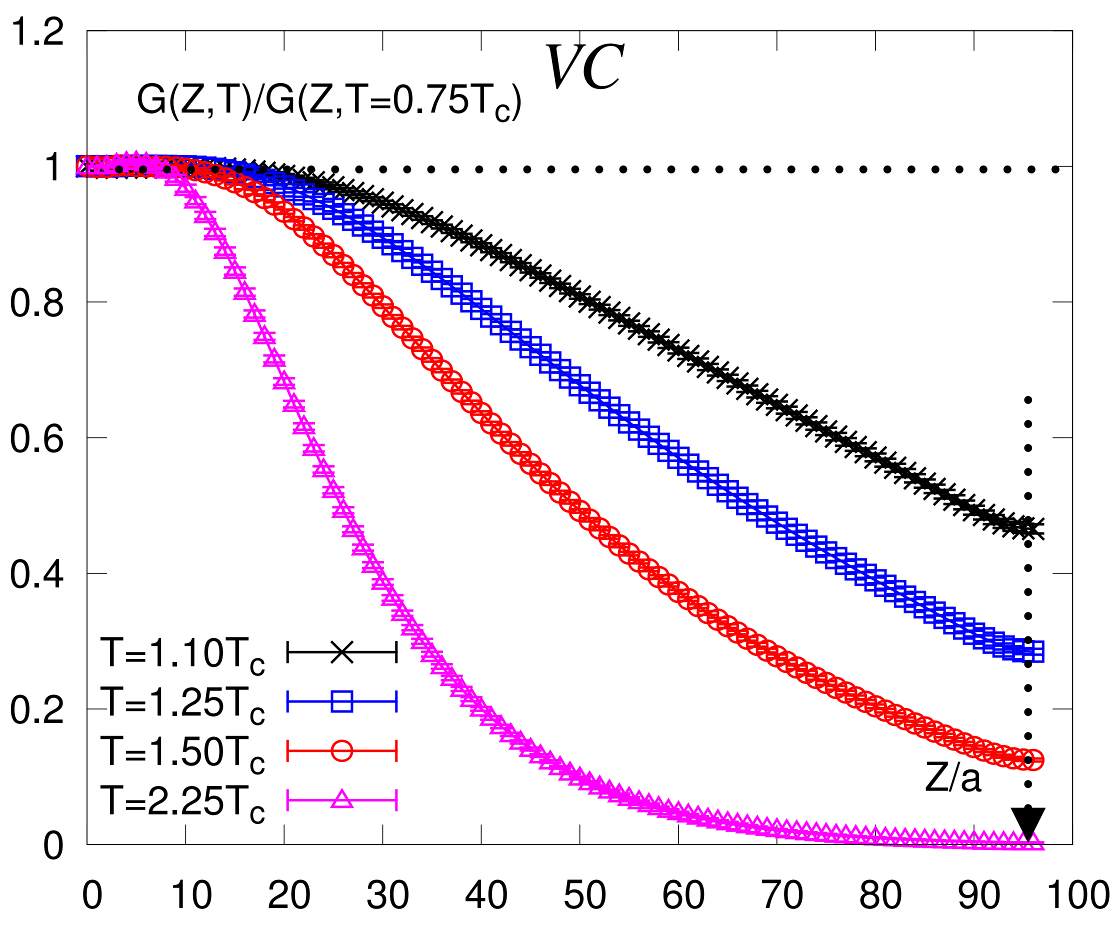


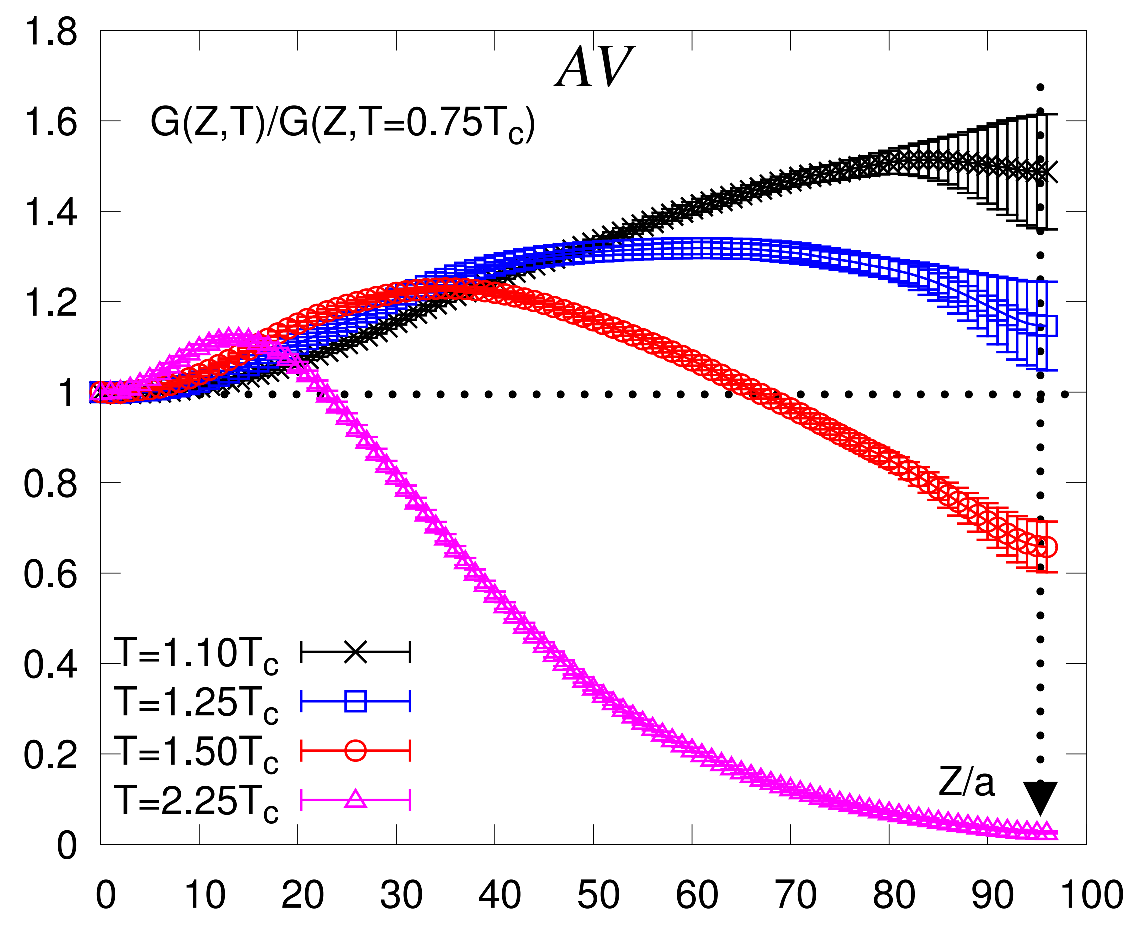
Shift   Change width   Change height

- Normalization

$\sum \eta_i = G(\tau_0)$

Ratio of correlators for $c\bar{c}$





- In both V&A channels, ratios decrease in T.
- In V channel: ratios < 1 for all T ➪ $M_{scr}(T) > M_{scr}(0.75T_c)$.
- In A channel: ratios change non-monotonically ➪ complicated situation.

Inversion Methods: Maximum Entropy Method (MEM)

A method based on Bayesian theorem to obtain the most probable solution.

[M. Jarrell, J. E. Gubernatis, Phy. Rep.269,133(1996), M. Asakawa et al., Prog.Part.Nucl.Phys. 46(2001) 459-508]

✳ Spectral function is expressed as an average with a conditional probability

$$\langle \rho \rangle = \int \mathrm{d}\alpha P[\alpha|\bar{G}] \int \boldsymbol{\mathcal{D}\rho \, P[\rho|\alpha, \bar{G}]\rho} \approx \int \mathrm{d}\alpha P[\alpha|\bar{G}] \boldsymbol{\hat{\rho}_\alpha} \ , \qquad P[\alpha|\bar{G}] \sim \exp(-F)$$

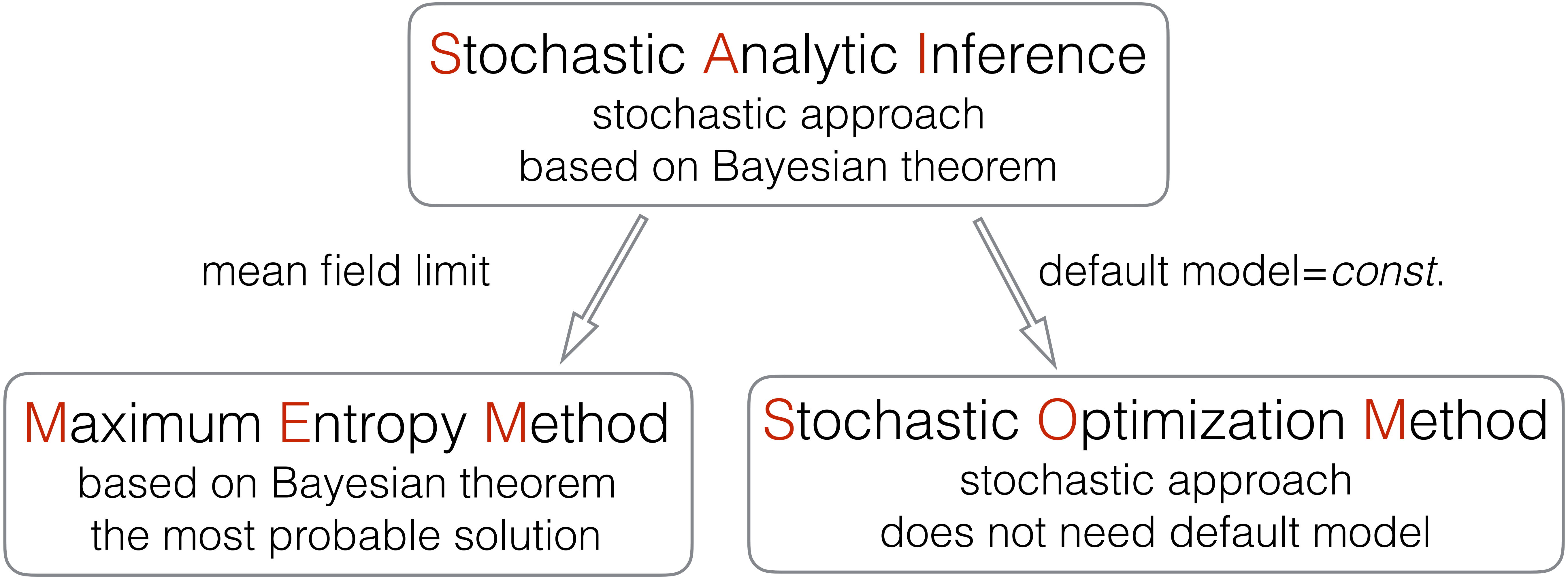
✳ Free energy:

$$F = \frac{\chi^2[\rho]}{2} - \alpha S \xrightarrow[\text{minimize } F \text{ deterministically}]{\frac{\partial F}{\partial \rho_\alpha} = 0} \boldsymbol{\hat{\rho}_\alpha} \quad \text{(the most probable solution)}$$

✳ Shanon-Jaynes entropy:

$$S[\rho] = \int \mathrm{d}\omega [\rho(\omega) - D(\omega) - \rho(\omega) \ln(\frac{\rho(\omega)}{D(\omega)})]$$

Default model *D(ω)* carries the prior information about the solution *ρ(ω)*


Stochastic Analytic Inference
stochastic approach
based on Bayesian theorem
mean field limit
default model=const.
Maximum Entropy Method
based on Bayesian theorem
the most probable solution
Stochastic Optimization Method
stochastic approach
does not need default model

Inversion Methods: Stochastic Approaches

Stochastic Analytic Inference (SAI)

[H. Ohno, PoS(LATTICE 2015)175]

✴A stochastic approach based on Bayesian theorem

$$\left\langle \langle n \rangle \right\rangle = \int \mathrm{d}\alpha \langle n \rangle_\alpha P[\alpha|\bar{G}] \qquad n(x) = \frac{\rho(\omega)}{D(\omega)}$$

✴Field treatment of *n(x)*

$$\langle n \rangle_\alpha = \frac{\int \mathcal{D}n \; n \; e^{-\chi^2/2\alpha}}{\mathcal{Z}(\alpha)}$$

mean field limit:

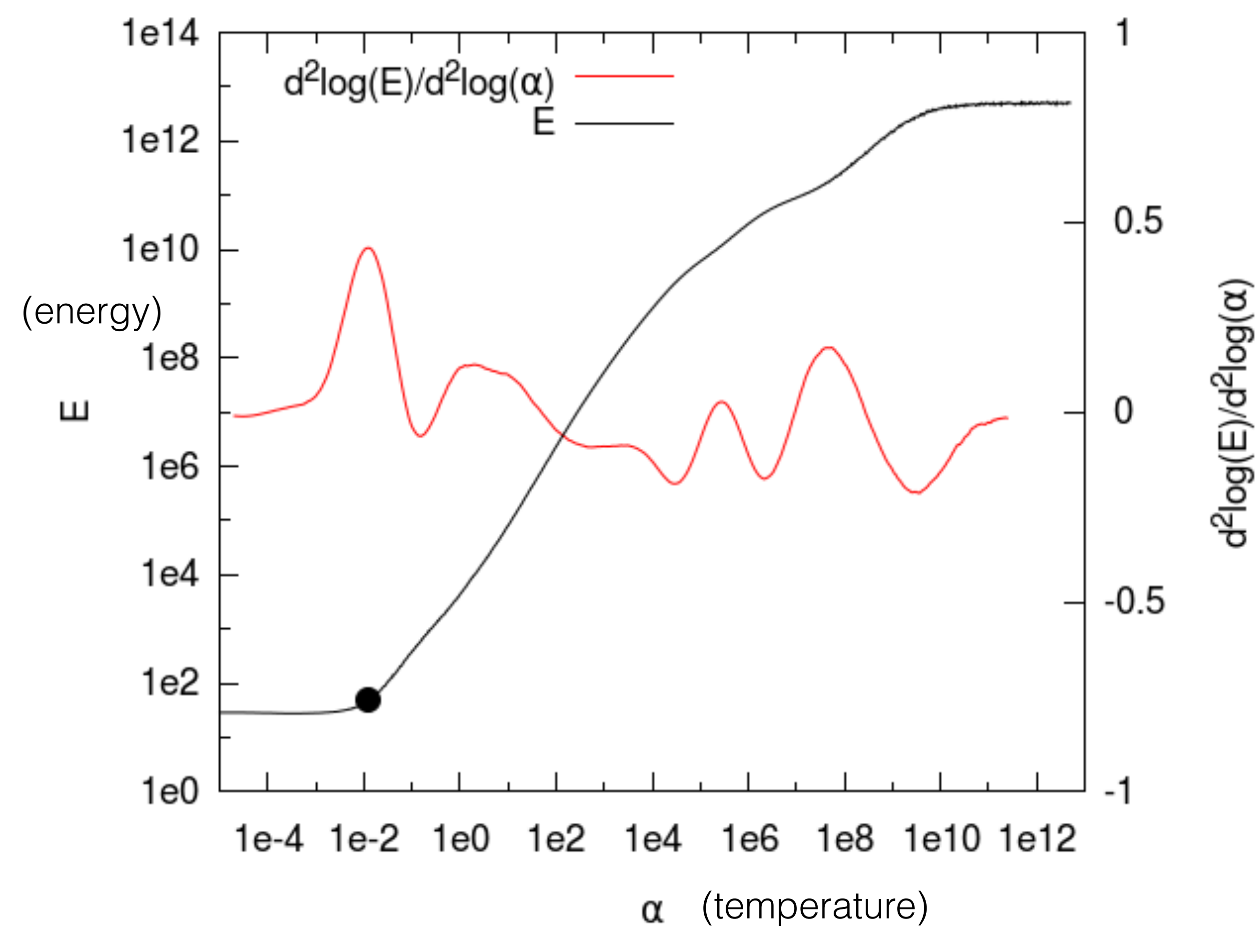
$$\big(n(x) - \hat{n}(x)\big)\big(n(y) - \hat{n}(y)\big) \approx 0$$

$$\hat{\rho}_\alpha(\omega) = \hat{n}_\alpha(\phi(\omega))D(\omega)$$

Equivalent to MEM

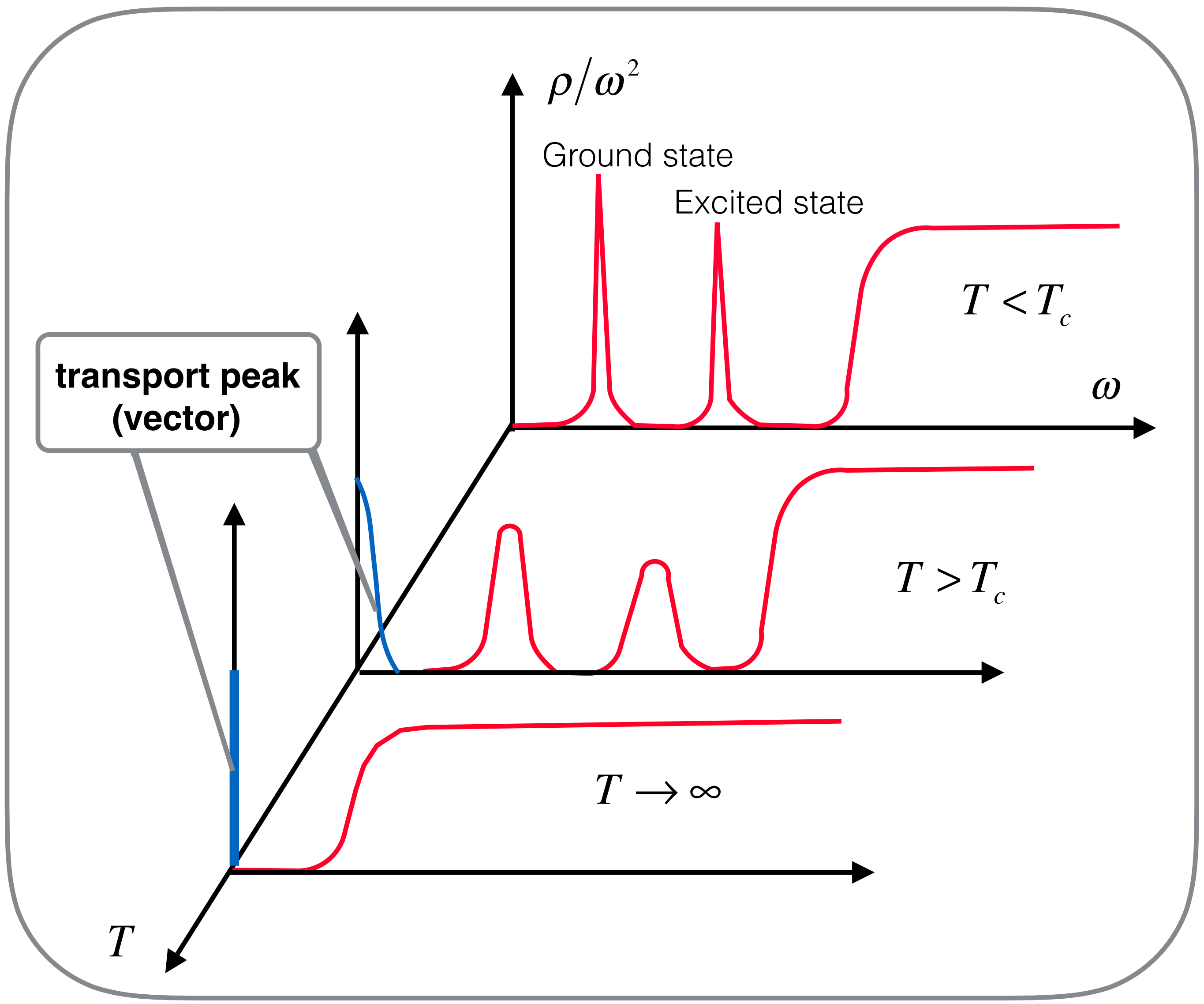
Stochastic Optimization Method (SOM)

[H.-T. Shu, et al, PoS(LATTICE 2015)180]



✴No default model is needed.

✴Equivalent to SAI using *const.* default model.


ρ/ω^2
Ground state
Excited state
$T < T_c$
ω
transport peak
(vector)
$T > T_c$
$T \to \infty$
T