Quarkonium Production and Polarization in the Improved Color Evaporation Model

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Traditional Color Evaporation Model

All quarkonium states treated like heavy quark pairs (*Q* = *c*, *b*) below heavy hadron (*H* = *D*, *B*) threshold

Color and spin are averaged over in pair cross section so color is 'evaporated' during transition from quark pair to quarkonium without changing kinematics

Distributions for quarkonium family members assumed identical

$$\sigma_Q^{\rm CEM} = F_Q \sum_{i,j} \int_{4m^2}^{4m_H^2} d\hat{s} \int dx_1 dx_2 \; f_{i/p}(x_1,\mu^2) \; f_{j/p}(x_2,\mu^2) \; \hat{\sigma}_{ij}(\hat{s})$$

Values of quark mass, m, and scale, μ, fixed from NLO calculation of heavy quark pair cross section

Scale factor F_Q fixed by comparison of $\sigma_Q^{\rm CEM}$ to energy dependence of J/ψ and Υ cross sections, $\sigma(x_F > 0)$ and $B d\sigma/dy|_{y=0}$ for J/ψ, $B d\sigma/dy|_{y=0}$ for Υ, only one F_Q for each state of quarkonium family

Spin always summed over so no previous predictions of polarization in CEM

Improved Color Evaporation Model

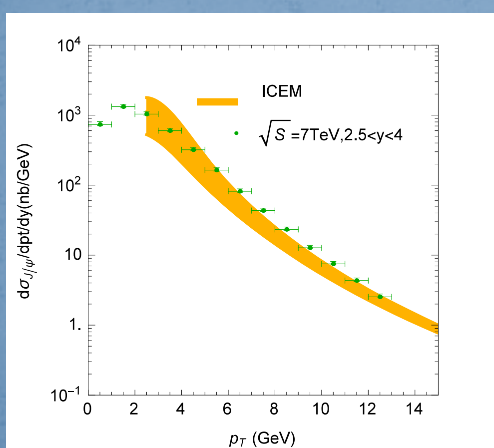
Relates average final state ψ momentum, $<p_\psi>$, to quark pair momentum p

$$\langle p_\psi \rangle = \frac{M_\psi}{M} p + \mathcal{O}(\lambda^2/m_c)$$

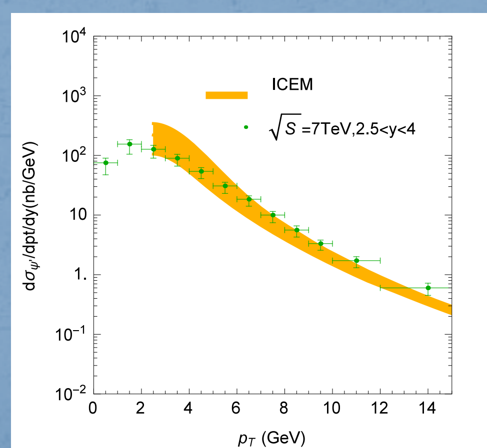
Lower limit on pair mass, M, has to be larger than $<p_\psi>$, lower limit on CEM integration has to be increased to M_ψ so that the transverse momentum distribution becomes

$$\frac{d\sigma_\psi(p)}{dp_T} = F_\psi \int_{M_\psi}^{2m_D} dM \frac{M}{M_\psi} \frac{d\sigma_{c\bar{c}}(M, p')}{dM\, dp'_T}\Big|_{p'_T=(M/M_\psi)p_T}$$

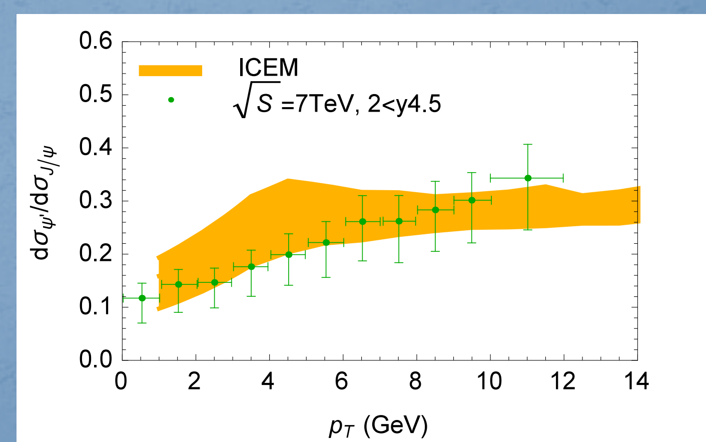
J/ψ



ψ’



ψ’/ψ ratio



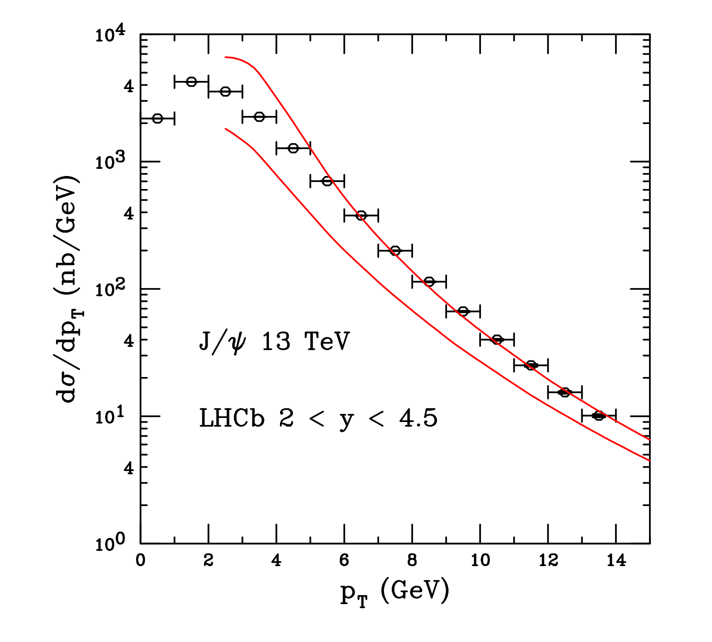
LHCb 7 TeV p+p

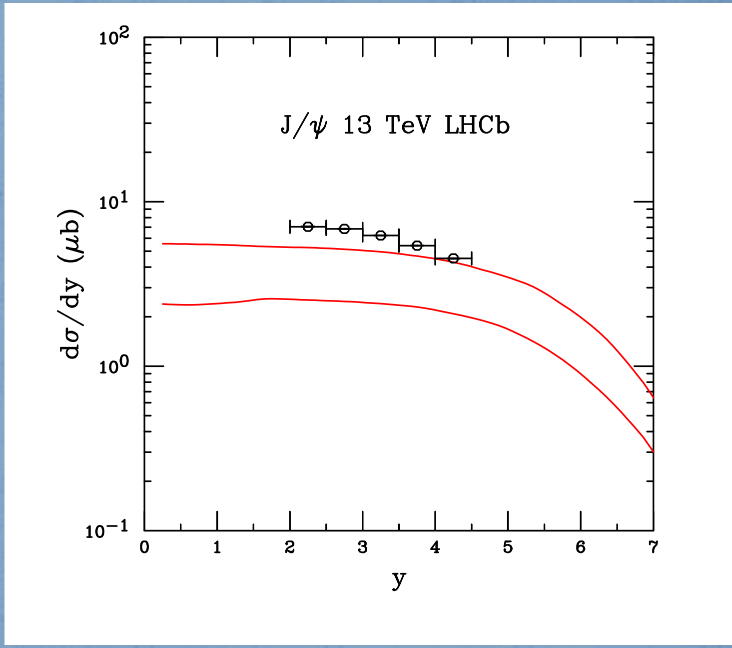
Y-Q Ma & RV, Phys. Rev. D

J/ψ at 13 TeV in the ICEM

p_T distribution is integrated over y in the measured region, $2 < y < 4.5$

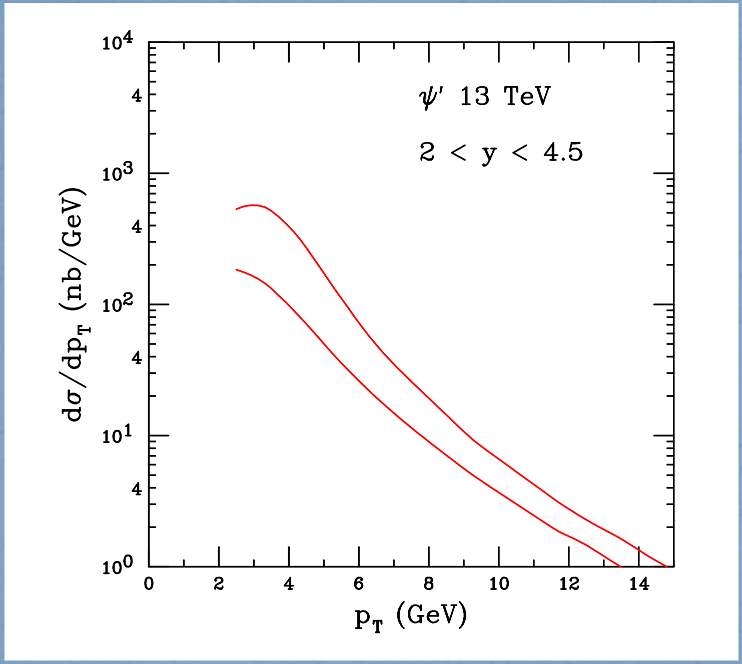
y distrbution is slightly below data because the p_T distribution is underestimated at low p_T

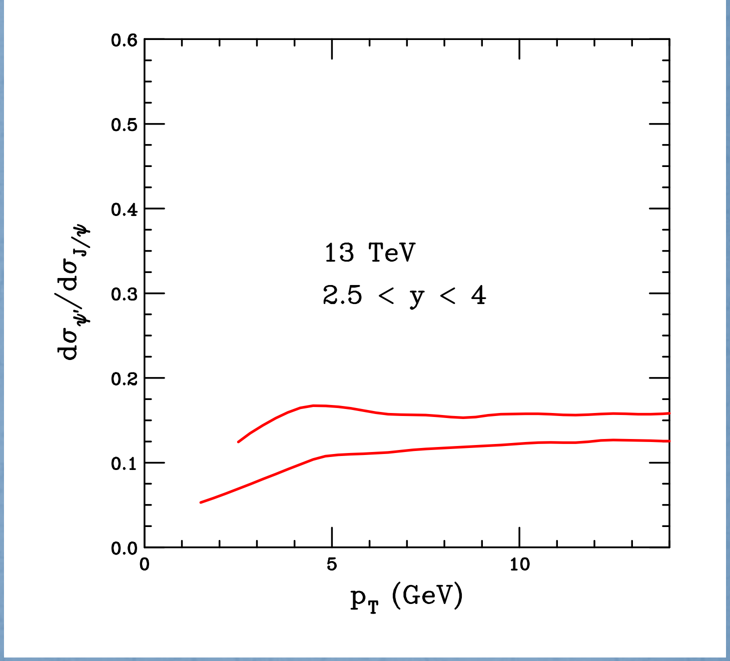




LHCb data -- arXiv:1509.00771 [hep-ex]

ψ′ and ψ′/J/ψ ratio at 13 TeV





First polarization calculation in the ICEM

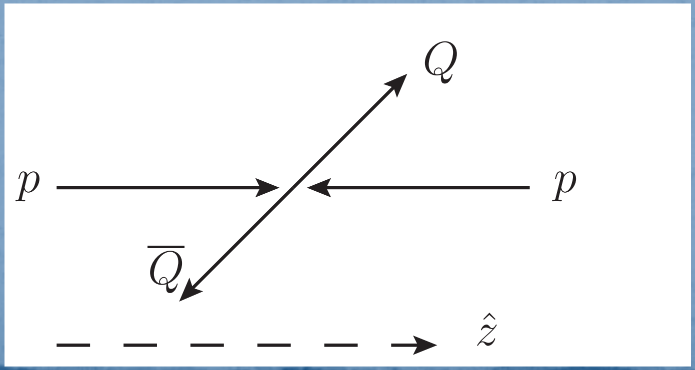
Done with UC Davis grad student Vincent Cheung, see Phys. Rev. D **96** (2017) 054014, 074021, in progress – I am representing his work here

Assume that, like kinematics (transverse momentum and rapidity), spin is ’frozen’ in and is not changed by the process of color ‘evaporation’

Sort contributions by spin but average over color as usual

Start at LO, easier obvious first step, initially just sorted longitudinal from transverse contributions, later separated S and L angular momenta

Work in helicity frame so far but at LO all frames are the same



Start with calculation of amplitudes

Take individual LO amplitudes, 1 quark-antiquark, 3 gluon-gluon and separate according to S_z of final state

$$
\begin{aligned}
\mathcal{A}_{qq} &= \frac{g_s^2}{\hat{s}}[\overline{u}(p')\gamma_\mu v(p)][\overline{v}(k)\gamma^\mu u(k')] \\
\mathcal{A}_{gg,\hat{s}} &= -\frac{g_s^2}{\hat{s}}\Big\{ -2k'\cdot\epsilon(k)[\overline{u}(p')\not{\epsilon}(k')v(p)] \\
&+ 2k\cdot\epsilon(k')[\overline{u}(p')\not{\epsilon}(k)v(p)] \\
&+ \epsilon(k)\cdot\epsilon(k')[\overline{u}(p')(\not{k}'-\not{k})v(p)]\Big\} \\
\mathcal{A}_{gg,\hat{t}} &= -\frac{g_s^2}{\hat{t}-M^2}\overline{u}(p')\not{\epsilon}(k')(\not{k}-\not{p}+M)\not{\epsilon}(k)v(p) \\
\mathcal{A}_{gg,\hat{u}} &= -\frac{g_s^2}{\hat{u}-M^2}\overline{u}(p')\not{\epsilon}(k)(\not{k}'-\not{p}+M)\not{\epsilon}(k')v(p)
\end{aligned}
$$

Calculate projection onto L = 0 or 1

At LO there is no dependence on azimuthal angle so that $L_z = 0$

To extract the projection onto a state with orbital angular momentum L = 0 or 1, calculate corresponding Legendre component $\mathcal{A}_L$ in the amplitudes as

$$\begin{aligned}
\mathcal{A}_{L=0} &= \frac{1}{2}\int_{-1}^{1} dx \mathcal{A}(x=\cos\theta) \\
\mathcal{A}_{L=1} &= \frac{3}{2}\int_{-1}^{1} dx\; x\mathcal{A}(x=\cos\theta)
\end{aligned}$$

Amplitudes and cross sections for |J,J_z>

Two helicity combinations resulting in S_z = 0 are added and normalized to give contributions to the spin triplet state S = 1, calculate amplitudes for J = 0, 1 and 2

S states (J = 1, S = 1, L = 0):

$$\begin{aligned}\mathcal{A}_{J=1,J_z=\pm1} &= \mathcal{A}_{L=0,L_z=0;S=1,S_z=\pm1}\\ \mathcal{A}_{J=1,J_z=0} &= \mathcal{A}_{L=0,L_z=0;S=1,S_z=0}\end{aligned}$$

P states (J = 0, 1, 2; L = S = 1):

$$\begin{aligned}\mathcal{A}_{J=0,J_z=0} &= -\sqrt{\frac{1}{3}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=0},\\ \mathcal{A}_{J=1,J_z=\pm1} &= \mp\frac{1}{\sqrt{2}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=\pm1}\\ \mathcal{A}_{J=1,J_z=0} &= 0\\ \mathcal{A}_{J=2,J_z=\pm2} &= 0\\ \mathcal{A}_{J=2,J_z=\pm1} &= \frac{1}{\sqrt{2}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=\pm1}\\ \mathcal{A}_{J=2,J_z=0} &= \sqrt{\frac{2}{3}}\mathcal{A}_{L=1,L_z=0;S=1,S_z=0}\end{aligned}$$

Calculation of Partonic Cross Sections

Partonic cross sections obtained by weighing amplitudes by appropriate color factors and summing contributions for given states

Contributions from different spins are added with appropriate Clebsch-Gordon coefficients

Feed down fractions give appropriate contribution to prompt J/ψ and Υ(1S)

$$
\begin{aligned}
|\mathcal{M}_{qq}^{J,J_z}|^2 &= |C_{qq}|^2|\mathcal{A}_{qq}|^2 \\
|\mathcal{M}_{gg}^{J,J_z}|^2 &= |C_{gg,\hat{s}}|^2|\mathcal{A}_{gg,\hat{s}}|^2 + |C_{gg,\hat{t}}|^2|\mathcal{A}_{gg,\hat{t}}|^2 \\
&+ |C_{gg,\hat{u}}|^2|\mathcal{A}_{gg,\hat{u}}|^2 + 2C^*_{gg,\hat{s}}C_{gg,\hat{t}}\mathcal{A}^*_{gg,\hat{s}}\mathcal{A}_{gg,\hat{t}} \\
&+ 2C^*_{gg,\hat{s}}C_{gg,\hat{u}}\mathcal{A}^*_{gg,\hat{s}}\mathcal{A}_{gg,\hat{u}} \\
&+ 2C^*_{gg,\hat{t}}C_{gg,\hat{u}}\mathcal{A}^*_{gg,\hat{t}}\mathcal{A}_{gg,\hat{u}}
\end{aligned}
$$

$$
\hat{\sigma}_{ij}^{J,J_z} = \int d\Omega \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}_{ij}^{J,J_z}|^2}{\hat{s}} \sqrt{1 - \frac{4M^2}{\hat{s}}}
$$

Cross sections for specific states I:

$J^P = 1^-$ (S state)

$$\hat{\sigma}_{q\bar{q}}^{J_z=0}(\hat{s}) = 0$$

$$\hat{\sigma}_{q\bar{q}}^{J_z=\pm 1}(\hat{s}) = \frac{\pi\alpha_s^2}{9\hat{s}}\chi$$

$$\hat{\sigma}_{gg}^{J_z=0}(\hat{s}) = \frac{7\pi\alpha_s^2}{48\hat{s}}\frac{M^2}{\hat{s}\chi}\left(\ln\frac{1+\chi}{1-\chi}\right)^2$$

$$\hat{\sigma}_{gg}^{J_z=\pm 1}(\hat{s}) = \frac{\pi^3\alpha_s^2}{1536\hat{s}}\chi\frac{(\sqrt{\hat{s}}-2M)(37\sqrt{\hat{s}}+38M)}{(2M+\sqrt{\hat{s}})^2}$$

$J^P = 0^+$ (χ_0 P state)

$$\hat{\sigma}_{q\bar{q}}^{J_z=0}(\hat{s}) = 0$$

$$\hat{\sigma}_{gg}^{J_z=0}(\hat{s}) = \frac{9\pi\alpha_s^2}{16\hat{s}}\frac{M^2}{\hat{s}\chi^3}\left(2\chi-\ln\frac{1+\chi}{1-\chi}\right)^2$$

$$\chi = \sqrt{1-4M^2/\hat{s}} \qquad M = \{m_c, m_b\}$$

Cross sections for specific states II:

$J^P = 1^+$ (χ_1 P state)

$$\begin{aligned}
\hat{\sigma}_{q\bar{q}}^{J_z=0}(\hat{s}) &= 0 \\
\hat{\sigma}_{q\bar{q}}^{J_z=\pm1}(\hat{s}) &= \frac{\pi\alpha_s^2}{18\hat{s}}\chi \\
\hat{\sigma}_{gg}^{J_z=0}(\hat{s}) &= 0 \\
\hat{\sigma}_{gg}^{J_z=\pm1}(\hat{s}) &= \frac{3\pi^3\alpha_s^2}{256\hat{s}}\chi\frac{(\sqrt{\hat{s}}-2M)(4\hat{s}-9M^2)}{(2M+\sqrt{\hat{s}})^3}
\end{aligned}$$

$$\chi = \sqrt{1-4M^2/\hat{s}} \quad M = \{m_c, m_b\}$$

Cross sections for specific states III:

$J^P = 2^+$ (χ_2 P state)

$$
\begin{aligned}
\hat{\sigma}_{q\bar{q}}^{J_z=0}(\hat{s}) &= 0 \\
\hat{\sigma}_{q\bar{q}}^{J_z=\pm1}(\hat{s}) &= \frac{\pi\alpha_s^2}{18\hat{s}}\chi \\
\hat{\sigma}_{gg}^{J_z=0}(\hat{s}) &= \frac{9\pi\alpha_s^2}{8\hat{s}}\frac{M^2}{\hat{s}\chi^3}\left(2\chi - \ln\frac{1+\chi}{1-\chi}\right)^2 \\
\hat{\sigma}_{gg}^{J_z=\pm1}(\hat{s}) &= \frac{3\pi^3\alpha_s^2}{256\hat{s}}\chi\frac{(\sqrt{\hat{s}}-2M)(4\hat{s}-9M^2)}{(2M+\sqrt{s})^3}
\end{aligned}
$$

$$\chi = \sqrt{1-4M^2/\hat{s}} \qquad M = \{m_c, m_b\}$$

Contributions from feed down:

$$R^{J_z=0}_{J/\psi} = \sum_{\psi,J_z} c_\psi S^{J_z}_\psi R^{J_z}_\psi$$

$$R^{J_z=0}_{\Upsilon(1\mathrm{S})} = \sum_{\Upsilon,J_z} c_\Upsilon S^{J_z}_\Upsilon R^{J_z}_\Upsilon$$

Q	M_Q (GeV)	c_Q	$S_Q^{J_z=0}$	$S_Q^{J_z=\pm1}$
J/ψ	3.10	0.62	1	0
$\psi(2\mathrm{S})$	3.69	0.08	1	0
$\chi_{c1}(1\mathrm{P})$	3.51	0.16	0	1/2
$\chi_{c2}(1\mathrm{P})$	3.56	0.14	2/3	1/2
$\Upsilon(1\mathrm{S})$	9.46	0.52	1	0
$\Upsilon(2\mathrm{S})$	10.0	0.1	1	0
$\Upsilon(3\mathrm{S})$	10.4	0.02	1	0
$\chi_{b1}(1\mathrm{P})$	9.89	0.13	0	1/2
$\chi_{b2}(1\mathrm{P})$	9.91	0.13	2/3	1/2
$\chi_{b1}(2\mathrm{P})$	10.3	0.05	0	1/2
$\chi_{b2}(2\mathrm{P})$	10.3	0.05	2/3	1/2

R^{Jz} is the J_z = 0 to unpolarized ratios for individual states, c is the feed down Fraction and S^{Jz} are Clebsch-Gordon coefficients

Calculation of polarization parameters:

Use ratio of $J_z = 0$ to unpolarized ratios to calculate the polarization parameter λ_θ

$J^P = 1^-$ (S state)

$$\lambda_\vartheta = \frac{1 - 3R^{J_z=0}}{1 + R^{J_z=0}}$$

$J^P = 1^+$ (χ_1 P state)

$$\lambda_\vartheta = \frac{-1 + 3R^{J_z=0}}{3 - R^{J_z=0}}$$

$J^P = 2^+$ (χ_2 P state)

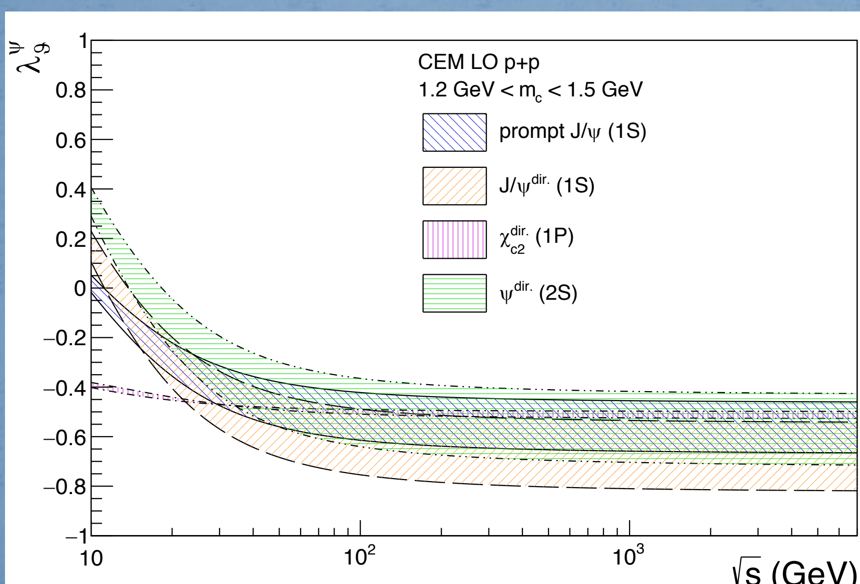
$$\lambda_\vartheta = \frac{-3 - 3R^{J_z=0}}{9 + R^{J_z=0}}$$

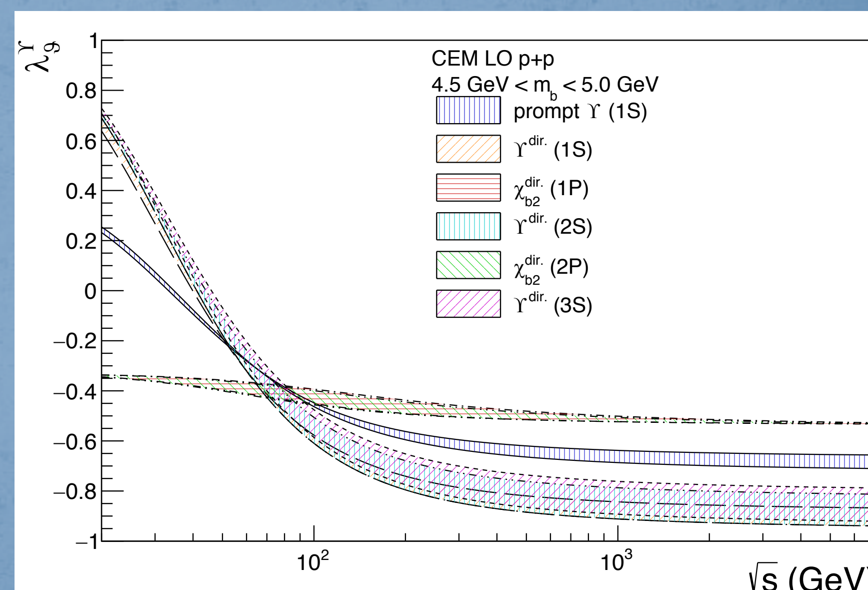
(Faccioli et al.)

Polarization in the CEM: energy dep

Dependence of λ on center of mass energy, bands show quark mass dependence, largest source of uncertainty in calculation

χ_c and χ_b states show smallest variation with energy and quark mass





V Cheung & RV, Phys. Rev. D & in preparation

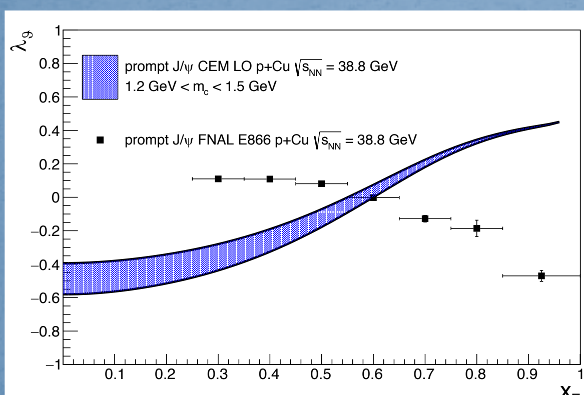
Polarization in the CEM: x_F dep

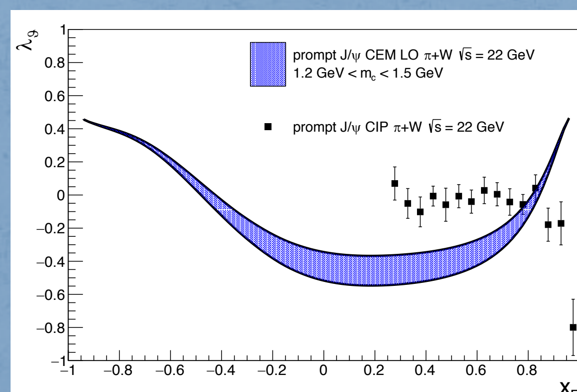
Comparison of calculations with E866 p+Cu J/ψ and Υ data at 38.8 GeV and CIP π + W J/ψ data at 22 GeV

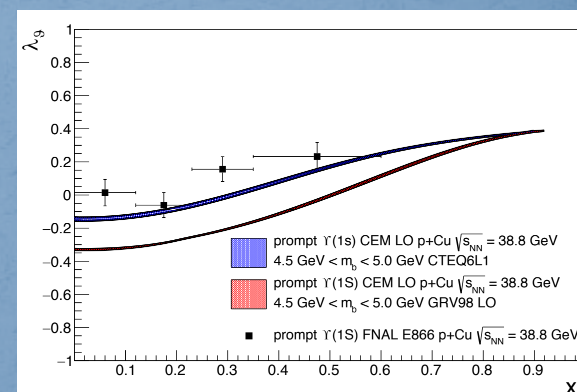
Calculations are LO CEM so no p_T is included thus there is a small kinematic mismatch between calculations

π+A and p+A are quantitatively different at forward x_F, depend on PDF and energy

Υ agreement is rather good







V Cheung & RV, Phys. Rev. D &in preparation

Going to the next level: k_T factorization

- First step toward obtaining full p_T dependence of quarkonium polarization in ICEM
- Cross section is dominated by t and u channel gluon exchange, keep only $\mathcal{A}_{gg,t}$ and $\mathcal{A}_{gg,u}$
- Use Reggeized gluons, off shell matrix elements and unintegrated gluon distribution

$$\begin{aligned}\sigma &= F_Q \int_{m_Q^2}^{4m_H^2} d\hat{s} \int dx_1 \int dx_2 \int dk_{1T}^2 \int dk_{2T}^2 \int \frac{d\phi_1}{2\pi} \int \frac{d\phi_2}{2\pi} \\ &\times \Phi_1(x_1, k_{1T}, \mu_1)\Phi_2(x_2, k_{2T}, \mu_2)\hat{\sigma}(\mathcal{R}+\mathcal{R} \to Q\overline{Q})\delta(\hat{s} - x_1 x_2 s + |\vec{k}_{1T} + \vec{k}_{2T}|^2)\end{aligned}$$

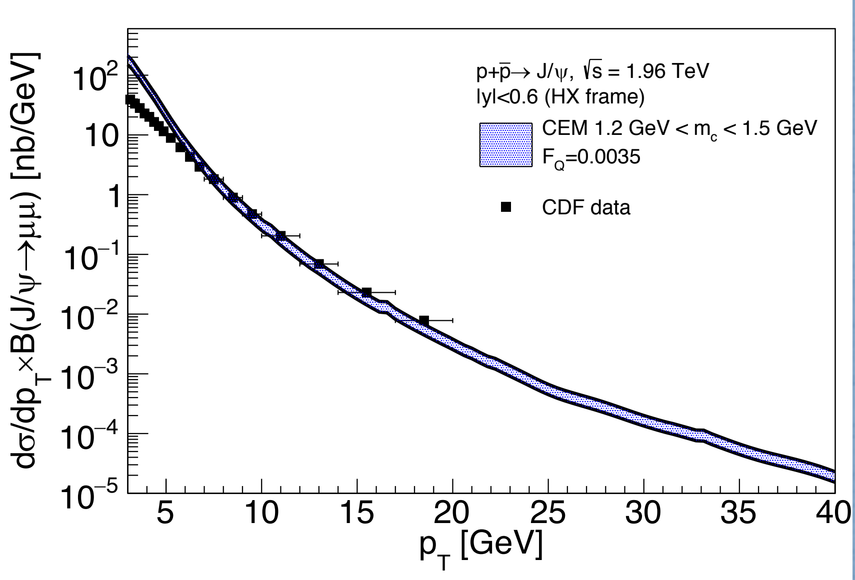
$$\begin{aligned}\mathcal{A}(\mathcal{R}+\mathcal{R} \to Q\overline{Q}) &= \epsilon^\mu(k_1)\epsilon^\nu(k_2)\mathcal{A}_{\mu,\nu}(g+g \to Q\overline{Q}) \\ \mathcal{A}_{L=0} &= \frac{1}{2}\int_{-1}^{1} dx \mathcal{A}(x = \cos\theta)\end{aligned}$$

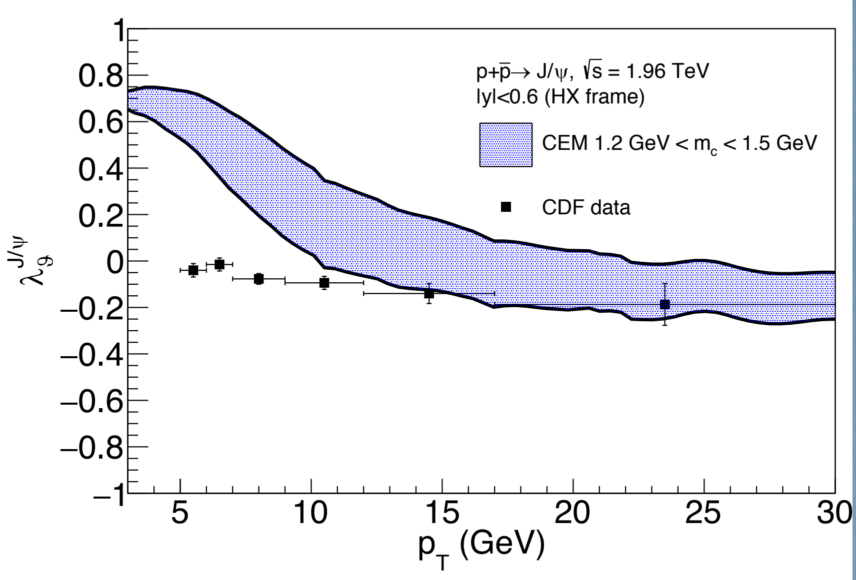
$$k_1 = (x_1, s, \vec{k}_{1T}, x_1 s) \qquad k_2 = (x_2 s, \vec{k}_{2T}, -x_2 s)$$

$$\epsilon(k_1) = (0, \vec{k}_{1T}/|k_{1T}|, 0) \qquad \epsilon(k_2) = (0, \vec{k}_{2T}/|k_{2T}|, 0)$$

J/ψ p_T distribution and polarization in ICEM, CDF Run II

First results in k_T factorization approach, more to come





RV and V Cheung, in progress

Summary

- Quarkonium production mechanism still not settled after more than 40 years
- New recent work on color evaporation may be helpful
- We have shown for the first time that the ICEM produces nonzero polarization at LO
- Taken first steps to calculate the p_T dependence, started with k_T factorization before trying to tackle full NLO