

Recent results from the KEDR experiment

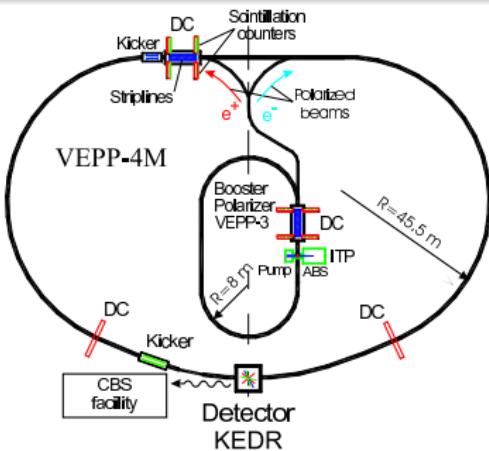
Korneliy Todyshev
KEDR collaboration

Quarkonium 2017

The 12th International Workshop on Heavy Quarkonium

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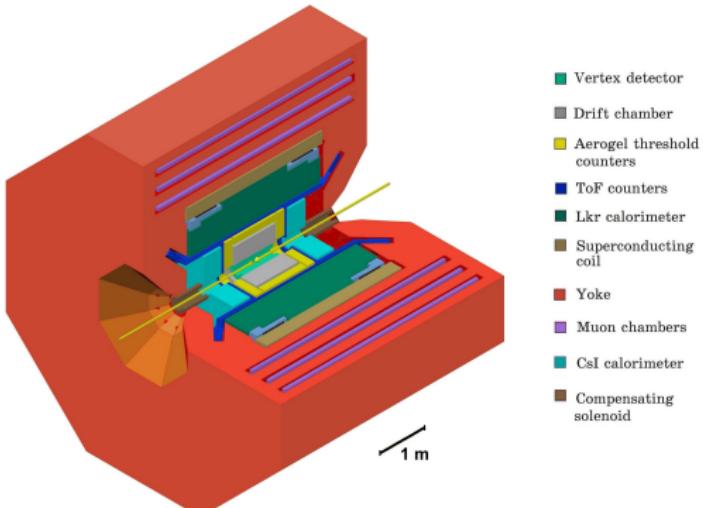
VEPP-4M and KEDR



Beam energy $1 \div 5 \text{ GeV}$
Number of bunches 2×2
Luminosity 1.8 GeV $1.5 \times 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$

Energy measurement:

- Resonant depolarization method:
Instant measurement accuracy $\sim 1 \text{ keV}$
Energy interpolation accuracy $10 \div 30 \text{ keV}$
- Compton backscattering method $\sim 100 \text{ keV}$



$\sigma^{e^+ e^- \rightarrow \text{hadrons}}$ and $\sigma^{e^+ e^- \rightarrow e^+ e^-}$ nearby a narrow resonance

In the soft photon approximation analytical expression for the annihilation cross section nearby a narrow resonance.

Ya.I. Azimov et al. JETP Lett. 21 (1975) 172. With up-today modifications one has

$$\sigma^{e^+ e^- \rightarrow \text{hadr}}(s) = \sigma_{\text{continuum}}^{e^+ e^- \rightarrow \text{hadr}} + \frac{12\pi}{s} (1 + \delta_{sf}) \left[\frac{\Gamma_{ee} \tilde{\Gamma}_h}{\Gamma M} \text{Im } f(s) - \frac{2\alpha \sqrt{R \Gamma_{ee} \tilde{\Gamma}_h}}{3\sqrt{s}} \lambda \text{Re} \frac{f^*(s)}{1 - \Pi_0} \right],$$

$$\left(\frac{d\sigma}{d\Omega} \right)^{ee \rightarrow ee} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{QED}}^{ee \rightarrow ee} + \frac{1}{s} (1 + \delta_{sf}) \left\{ \frac{9}{4} \frac{\Gamma_{ee}^2}{\Gamma M} (1 + \cos^2 \theta) \text{Im } f - \frac{3\alpha}{2} \frac{\Gamma_{ee}}{M} \left[(1 + \cos^2 \theta) \text{Re} \frac{f^*}{1 - \Pi_0(s)} - \frac{(1 + \cos \theta)^2}{(1 - \cos \theta)} \text{Re} \frac{f^*}{1 - \Pi_0(t)} \right] \right\},$$

Recently it was verified in the work X. Y. Zhou, Y. D. Wang and L. G. Xia, Chin. Phys. C 41 (2017) no.8, 083001

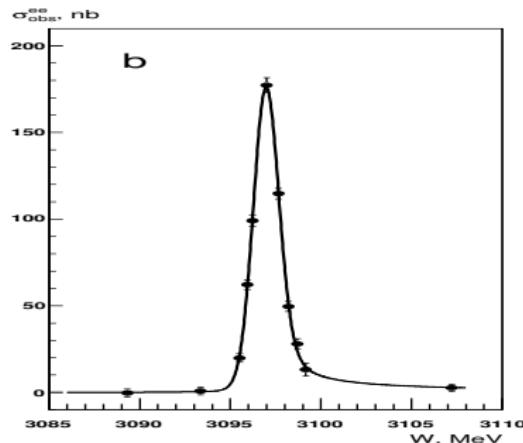
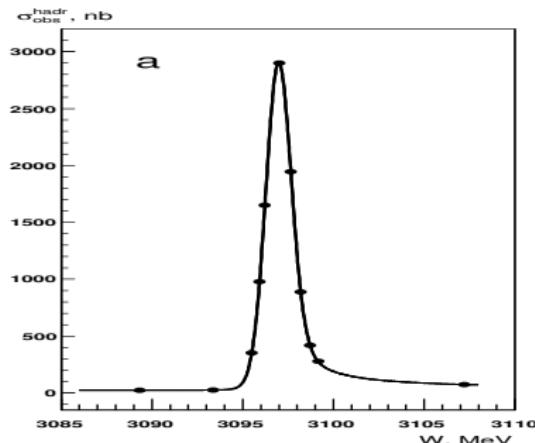
$$\delta = \frac{3}{4} \beta + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) + \beta^2 \left(\frac{37}{96} - \frac{\pi^2}{12} - \frac{L}{72} \right), \quad L = \ln \left(s/m_e^2 \right),$$

$$\beta = \frac{2\alpha}{\pi} (L - 1), \quad f(s) = \frac{\pi\beta}{\sin \pi\beta} \left(\frac{s}{M^2 - s - iM\Gamma} \right)^{1-\beta}$$

Γ_{ee} , Γ , M – 'dressed' parameters including corrections to the vacuum polarization,
 $\Gamma_{ee} = \Gamma_{ee}^{(0)} / |1 - \Pi_0|^2$, λ -parameter controls the resonance–continuum interference, $\tilde{\Gamma}_h \neq \Gamma_h$
 Numerical convolution with the collision energy distribution is used to fit resonance.



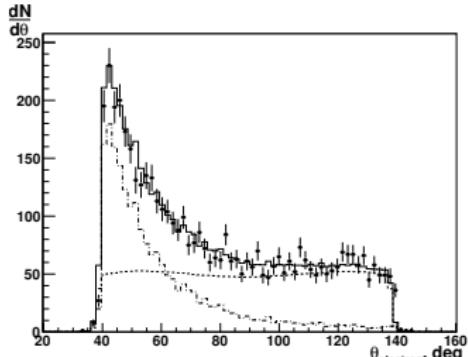
Measurement $\Gamma_{e^+e^-}(J/\psi)$ and $\Gamma_{e^+e^-}(J/\psi) \times \mathcal{B}(J/\psi \rightarrow \text{hadrons})$



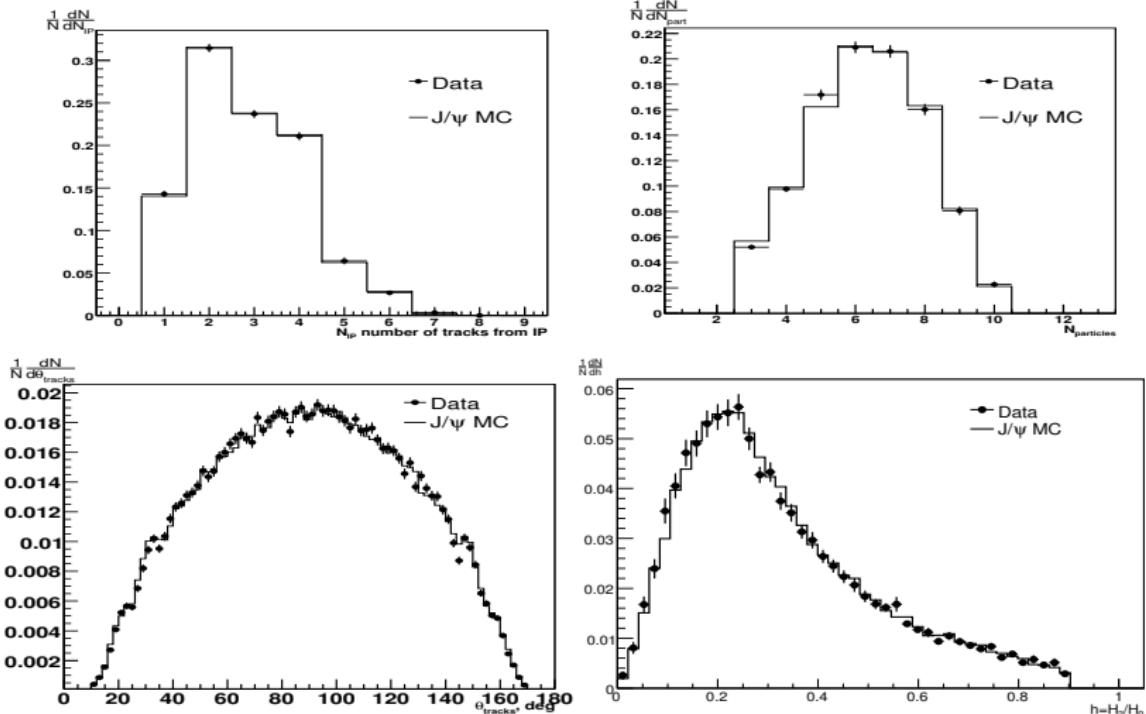
11-point scan J/ψ , $\int \mathcal{L} dt \simeq 0.23 \text{ pb}^{-1}$. Joint fit of hadronic and e^+e^- cross section in few polar angle bin. For the absolute luminosity determination e^+e^- events used.

Relative luminosity was obtained by single bremsstrahlung.

Distribution of electron polar angle in the energy range of J/ψ meson peak. The points show experimental data. The histograms correspond to MC simulation: dashed-dotted – Bhabha scattering, dashed – J/ψ resonance and their interference, solid-line – sum of the contributions.



Properties of hadronic events produced in the vicinity of J/ψ resonance



The points represent experimental data, the histograms correspond to J/ψ meson decays simulation. Number of tracks from interaction point N_{IP} , total number of particles N_{part} , θ_{tracks} distribution and the ratio of Fox-Wolfram moments H_2/H_0 .



Results and systematic uncertainties

$\Gamma_{ee}(\Gamma_{ee} \times \mathcal{B}_{\text{hadrons}})$ and $\Gamma_{ee} \times \mathcal{B}_{ee}$ values, absolute luminosity calibration factor, resonance mass, beam energy spread and continuum contribution σ_0 were considered as free fit parameters. For both cases, the ratio

$\mathcal{B}_{ee}(J/\psi)/\mathcal{B}_{\mu\mu}(J/\psi) = 1.0022 \pm 0.0065$ was fixed according to KEDR result (PLB 731(2014))

List of systematic uncertainties, %

| Source | Γ_{ee} | $\Gamma_{ee} \times \mathcal{B}_{\text{hadrons}}$ | $\Gamma_{ee} \times \mathcal{B}_{ee}$ |
|-------------------------------|---------------|---|---------------------------------------|
| Luminosity measurement | 1.1 | 1.1 | 1.1 |
| Simulation of J/ψ decays | 0.6 | 0.7 | – |
| Detector response | 0.7 | 0.8 | 0.3 |
| Accelerator related effects | 0.6 | 0.6 | 0.6 |
| Theoretical uncertainties | 0.35 | 0.35 | 0.2 |
| Sum in quadrature | 1.6 | 1.7 | 1.3 |

$$\Gamma_{ee}(J/\psi) = 5.516 \pm 0.056 \pm 0.088 \text{ keV}$$

$$\Gamma_{ee}(J/\psi) \times \mathcal{B}_{\text{hadrons}}(J/\psi) = 4.853 \pm 0.048 \pm 0.083 \text{ keV}$$

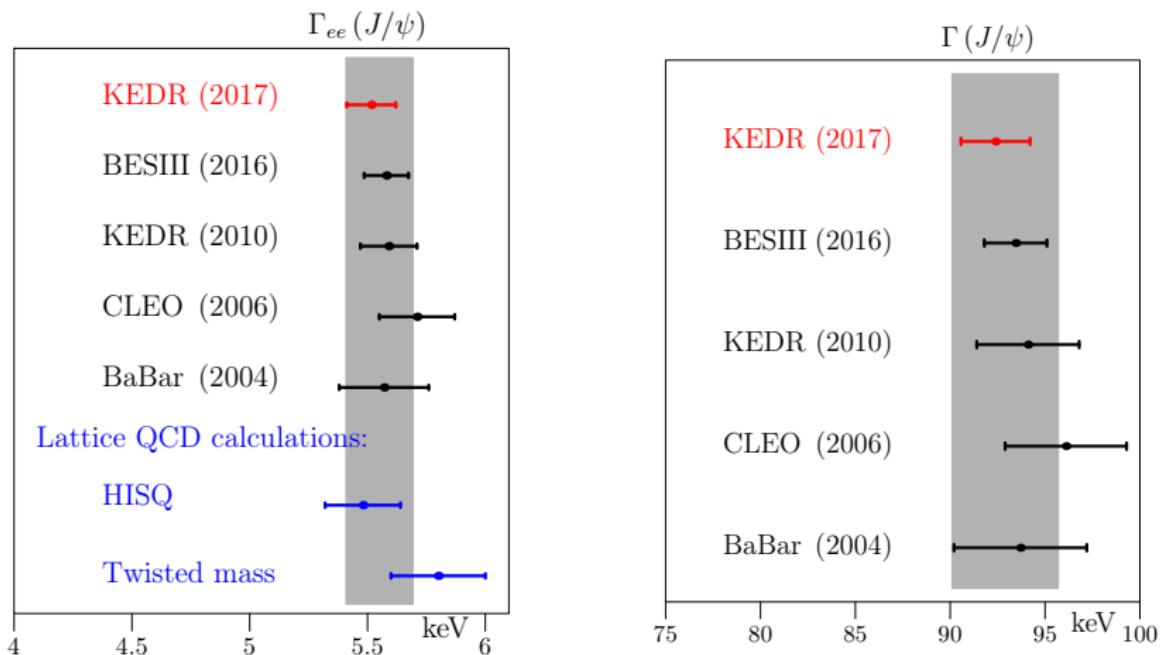
$$\Gamma_{ee}(J/\psi) \times \mathcal{B}_{ee}(J/\psi) = 0.3317 \pm 0.0066 \pm 0.0043 \text{ keV}$$

Taking into account $\mathcal{B}_{ee}(J/\psi) = (5.971 \pm 0.032) \times 10^{-2}$ from PDG2016 we determined total width of J/ψ meson:

$$\Gamma(J/\psi) = 92.38 \pm 1.82 \text{ keV.}$$



Comparison with others experiments

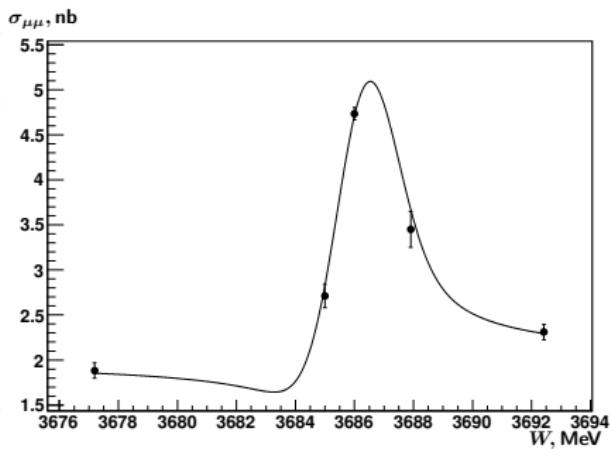


The electron and full widths obtained in our analysis agree well with world averages.

$\Gamma_{ee} \times \Gamma_{\mu\mu} / \Gamma$ determination for $\psi(2S)$

6.5 pb⁻¹ collected in $\psi(2S)$ region, corresponding $\sim 4 \times 10^6 \psi(2S)$, five peak/continuum (p/c) couples and four scans for energy spread determination:

| $\psi(2S)$ datasets | started at | \mathcal{L}_{int} nb ⁻¹ | σ_W MeV |
|------------------------|---------------|---|-------------------|
| p/c 1 | begin 2005 | 358 | 1.08 |
| p/c 2 | autumn 2005 | 222 | 0.99 |
| scan 1 | spring 2006 | 255 | 0.99 |
| p/c 3 | spring 2006 | 631 | 0.99 |
| p/c 4 | autumn 2006 | 701 | 0.99 |
| p/c 5 | autumn 2007 | 1081 | 1.01 |
| scan 2 | end 2007 | 967 | 1.01 |
| scan 3 | summer 2010 | 379 | 1.00 |
| scan 4 | end 2010 | 2005 | 0.98 |



The observed $e^+e^- \rightarrow \mu^+\mu^-$ cross section in scan 4.

$$\left(\frac{d\sigma}{d\Omega} \right)^{ee \rightarrow \mu\mu} \approx \frac{3}{4s} (1 + \delta_{fs}) (1 + \cos^2 \theta) \times \\ \left\{ \frac{3\Gamma_{ee}\Gamma_{\mu\mu}}{\Gamma M} \text{Im } f - \frac{2\alpha\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}}}{M} \text{Re } \frac{f}{(1 - \Pi_0)^*} \right\} + \left(\frac{d\sigma}{d\Omega} \right)_{\text{QED}}^{\mu\mu}$$



$\Gamma_{ee} \times \Gamma_{\mu\mu} / \Gamma$ determination for $\psi(2S)$

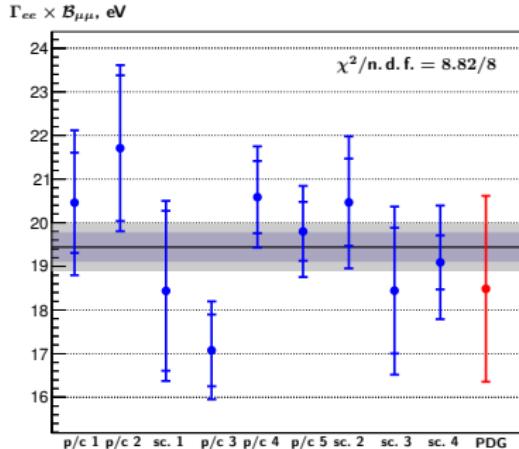
The following event selection criteria for e^+e^- and $\mu^+\mu^-$ were used:

- Two-prong central event, opposite charges, acollinearity on θ and ϕ less than 28° , energy deposit on each track less than 700 MeV for $\mu^+\mu^-$ or greater than 800 MeV for e^+e^- .
- $40^\circ < \theta < 140^\circ$ for e^+e^- , $30^\circ < \theta < 150^\circ$ for $\mu^+\mu^-$.
- Not more than one extra cluster, with energy less than 160 MeV.
- For muons - confirmation from muon system for both tracks, anti-cosmic veto from ToF.

| Systematic uncertainty source | p/c 1 | p/c 2 | sc. 1 | p/c 3 | p/c 4 | p/c 5 | sc. 2 | sc. 3 | sc. 4 | σ |
|--------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| 1 Energy spread | 2.0 | 2.2 | 1.1 | 3.0 | 2.5 | 2.5 | 1.1 | 2.8 | 2.0 | 0 |
| 2 Fixed values of M, Γ | 0.7 | 0.7 | 0.1 | 0.3 | 0.7 | 0.7 | 0.5 | 0.2 | 0.8 | 0.1 |
| 3 Energy measurement | 3.1 | 0.6 | < 0.1 | 1.7 | 0.3 | 0.5 | 0.2 | 0.9 | 2.7 | < 0.1 |
| 4 Bhabha simulation | 0.6 | 0.2 | 0.2 | 0.2 | 0.2 | 0.5 | 0.2 | 0.8 | 0.2 | 0.2 |
| 5 $\mu^+\mu^-$ scattering simulation | 0.2 | 0.2 | 0.3 | 0.2 | 0.2 | 0.2 | 0.3 | 0.2 | 0.2 | 0.2 |
| 6 Collinearity cuts | 0.2 | 1.3 | 2.0 | 1.8 | 0.9 | 1.3 | 1.4 | 3.7 | 1.5 | 0.2 |
| 7 e^+e^- polar angle range | 3.5 | 1.2 | 2.3 | 0.9 | 2.1 | 1.3 | 3.4 | 2.2 | 1.6 | 0.9 |
| 8 Charge determination | 0.1 | 0.7 | 0.8 | 0.3 | 0.2 | 1.1 | 0.8 | 0.5 | 0.9 | 0.1 |
| 9 Detector asymmetry | 0.5 | 0.3 | 0.4 | 0.5 | 0.2 | < 0.1 | 0.2 | 0.3 | 0.1 | < 0.1 |
| 10 Extra energy deposit cut | 1.3 | 1.1 | 1.8 | 0.7 | 1.3 | 1.0 | 2.3 | 1.5 | 2.1 | 0.7 |
| 11 Muon system cut | 0.7 | 1.1 | < 0.1 | 0.4 | 0.8 | 1.5 | 1.1 | 0.4 | 1.6 | 0 |
| 12 ABG thresholds | 0.1 | 0.8 | 0.6 | 0.2 | 0.3 | — | — | — | — | 0.1 |
| 13 Calo trigger thresholds | 0.5 | 0.2 | 2.1 | 0.7 | < 0.1 | 0.4 | < 0.1 | 2.2 | 2.0 | < 0.1 |
| 14 RND trigger application | 0.1 | 0.5 | 2.0 | 0.9 | 0.1 | 0.1 | 0.6 | 1.8 | 1.7 | 0.1 |
| 15 FSR accounting | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| 16 e^+e^- events θ binning | 0.6 | 0.1 | 0.4 | 0.1 | 0.3 | 0.2 | 0.3 | 0.4 | 0.2 | 0.1 |
| 17 ToF measurement efficiency | 1.9 | 2.1 | 1.6 | 1.2 | 0.9 | 0.9 | 2.8 | 3.0 | 2.2 | 0.9 |
| 18 Trigger efficiency | 0.9 | < 0.1 | 0.2 | 0.1 | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 | < 0.1 |
| 19 Theoretical accuracy | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Sum in quadrature | 5.9 | 4.2 | 5.1 | 4.5 | 3.9 | 4.0 | 5.5 | 6.9 | 6.0 | 1.5 |

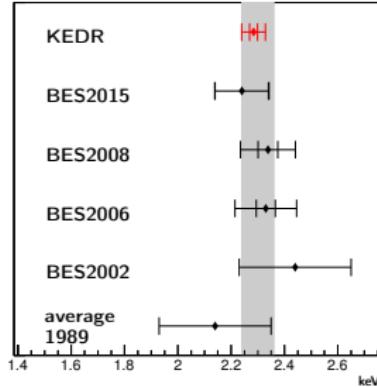


Γ_{ee} and $\Gamma_{ee} \times \Gamma_{\mu\mu}/\Gamma$ determination for $\psi(2S)$



Blue – individual KEDR measurements, Gray – weighted KEDR result, Red – product of PDG's and $\mathcal{B}_{\mu^+\mu^-}$.

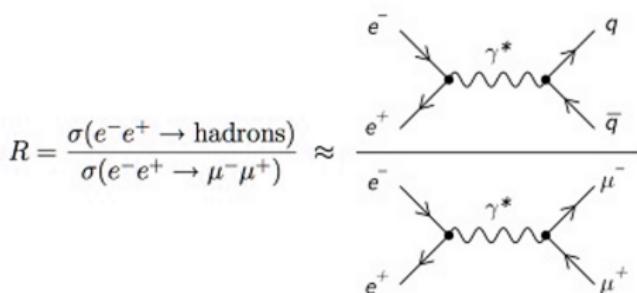
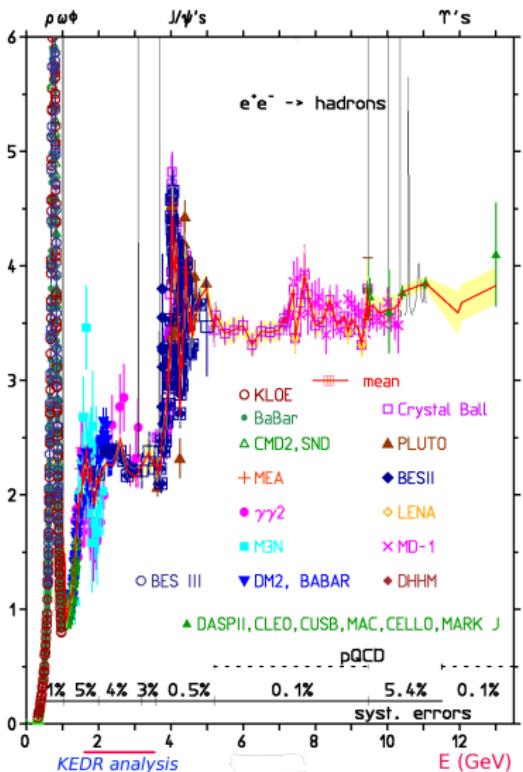
$$\Gamma_{ee} \times \Gamma_{\mu\mu}/\Gamma = 19.4 \pm 0.3 \pm 0.4 \text{ eV}$$



Although not presented as a result of this work, the $\Gamma_{ee} \times \Gamma_{ee}/\Gamma$ value was also obtained: $\Gamma_{ee} \times \Gamma_{ee}/\Gamma = 22.5 \pm 0.6 \pm 1.3 \text{ eV}$. Using KEDR results for $\Gamma_{ee} \times \mathcal{B}_{\text{hadrons}}$ (PLB 711 (2012)) and $\Gamma_{ee} \times \mathcal{B}_{\tau\tau}$ (Pis'ma v ZhETF 85 (2007)) channels we obtained the following value of the Γ_{ee} width:

$$\Gamma_{ee} = 2.284 \pm 0.015 \pm 0.042 \text{ keV}$$

$R(s)$ measurement



In first approximation:

$$R(s) \simeq 3 \sum e_q^2$$

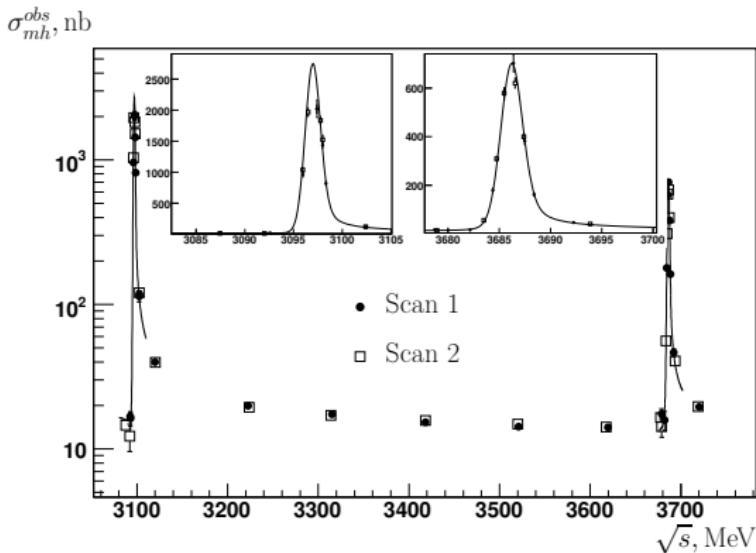
$R(s)$ is used to determine:

- $\alpha_s(s)$
- $(g_\mu - 2)/2$
- $\alpha(M_Z^2)$

F. Jegerlehner arXiv:1511.0447



R measurement between J/ψ and $\psi(2S)$



- The c.m. energy range between 3.12 and 3.72 GeV studied
- An integrated luminosity of 1.4 pb^{-1} collected at 7 equidistant points with a step of $\sim 0.1 \text{ GeV}$: 3.12, 3.22, . . . , 3.72 GeV
- $(2 - 3) \times 10^3$ events per point, $\sim 18 \times 10^3$ in total
- Simulation of the uds continuum based on the tuned JETSET generator, alternatively used LUARLW (H.M. Hu and A. Tai, hep-ex/0106017)

Analysis

The way that we are measuring R :

$$R = \frac{\sigma_{obs}(s) - \sum \varepsilon_\psi^{tail}(s)\sigma_\psi^{tail}(s) - \sum \varepsilon_{bg}^i(s)\sigma_{bg}^i(s)}{\varepsilon(s)(1 + \delta(s))\sigma_{\mu\mu}^0}$$

with $\sigma_{obs}(s) = \frac{N_{mh} - N_{res.bg.}}{\int \mathcal{L} dt}$ where N_{mh} represent all events pass hadronic selection criteria, $N_{res.bg.}$ – residual machine background

$\sum \varepsilon_\psi^{tail}(s)\sigma_\psi^{tail}(s)$ is contribution from J/ψ and $\psi(2S)$ resonances

$\sum \varepsilon_{bg}^i(s)\sigma_{bg}^i(s)$ is contribution from physical processes: $e^+e^- \rightarrow l^+l^-$, $\gamma\gamma$ -processes.

$\varepsilon(s)$ – multihadron efficiency.

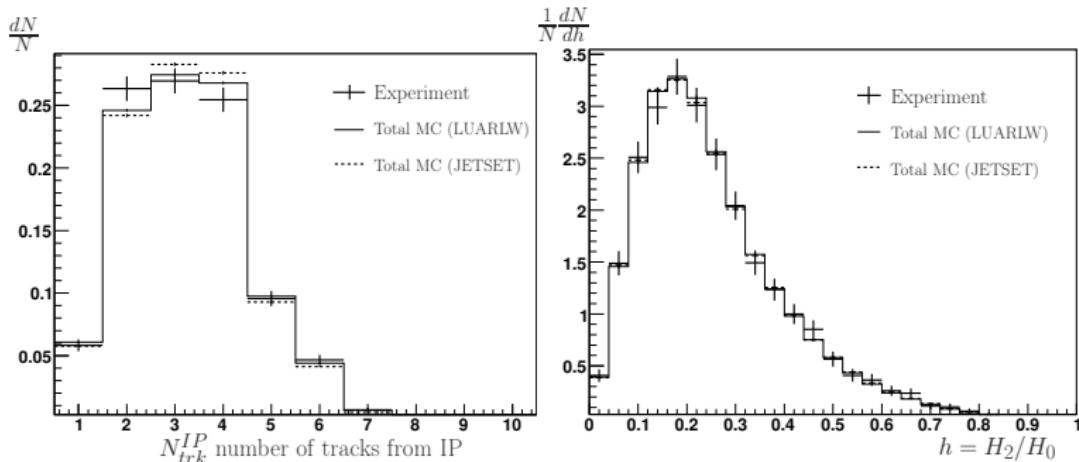
$$1 + \delta(s) = \int dx \frac{1}{1-x} \frac{\mathcal{F}(s, x)}{|1 - \tilde{\Pi}(s(1-x))|^2} \frac{\tilde{R}(s(1-x))\varepsilon(s(1-x))}{R(s)\varepsilon(s)}$$

$\mathcal{F}(s, x)$ – radiative correction kernel ([E.A.Kuraev, V.S.Fadin](#)

[Sov.J.Nucl.Phys.41\(466-472\)1985](#)) Here $\tilde{\Pi}$ and \tilde{R} does not includes J/ψ and $\psi(2S)$ resonances.



Simulation: JETSET and LUARLW

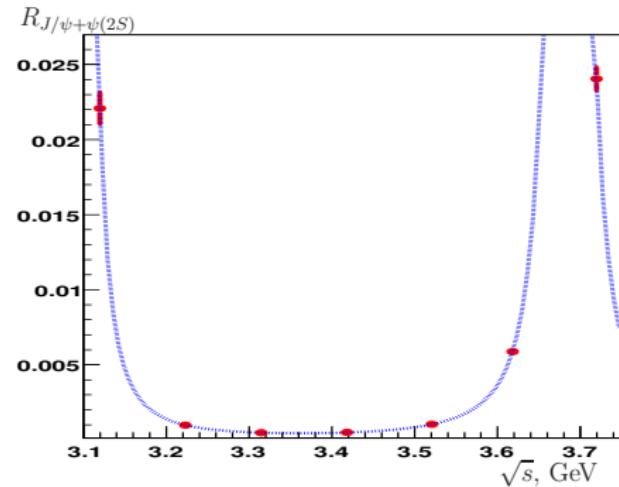


Number of tracks and ratio of Fox-Wolfram moments

Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ($\sqrt{s} = 3.12$ GeV).

R for $\sqrt{s} = 3.12 - 3.72$ GeV

| Source | Syst. uncertainty, % |
|-----------------------|----------------------|
| Luminosity | 1.1 |
| Rad. corr. | $0.4 \div 0.6$ |
| <i>uds</i> simulation | $1.4 \div 2.1$ |
| J/ψ | $0.1 \div 2.7$ |
| $\psi(2S)$ | 1.4 at 3.72 GeV |
| I^+I^- | $0.1 \div 0.2$ |
| e^+e^-X | $0.1 \div 0.2$ |
| Trigger | 0.2 |
| Nuclear interaction | 0.2 |
| Machine background | $0.7 \div 1.1$ |
| Cuts | 0.6 |
| Total | $2.1 \div 3.5$ |



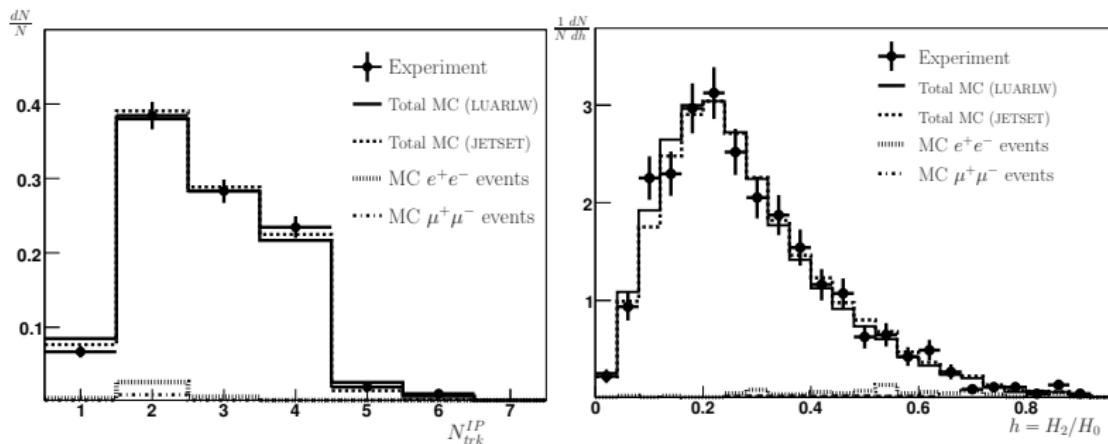
Using J/ψ and $\psi(2S)$ parameters, we obtain $R_{uds}(s) + R_{J/\psi+\psi(2S)} \implies R(s)$

| \sqrt{s} , MeV | $R_{uds}(s)$ | $R(s)$ |
|------------------|-----------------------------|-----------------------------|
| 3119.9 ± 0.2 | $2.215 \pm 0.089 \pm 0.066$ | $2.237 \pm 0.089 \pm 0.066$ |
| 3223.0 ± 0.6 | $2.172 \pm 0.057 \pm 0.045$ | $2.173 \pm 0.057 \pm 0.045$ |
| 3314.7 ± 0.7 | $2.200 \pm 0.056 \pm 0.043$ | $2.200 \pm 0.056 \pm 0.043$ |
| 3418.2 ± 0.2 | $2.168 \pm 0.050 \pm 0.042$ | $2.168 \pm 0.050 \pm 0.042$ |
| 3520.8 ± 0.4 | $2.200 \pm 0.050 \pm 0.044$ | $2.201 \pm 0.050 \pm 0.044$ |
| 3618.2 ± 1.0 | $2.201 \pm 0.059 \pm 0.044$ | $2.207 \pm 0.059 \pm 0.044$ |
| 3719.4 ± 0.7 | $2.187 \pm 0.068 \pm 0.060$ | $2.211 \pm 0.068 \pm 0.060$ |

V.V. Anashin et al., Phys.Lett. B 753, 533 (2016)

R for $\sqrt{s} = 1.84 - 3.05$ GeV

- An integrated luminosity 0.66 pb^{-1} collected at 13 equidistant points with a step $\sim 0.1 \text{ GeV}$: $1.841, 1.937 \dots 3.048 \text{ GeV}$
- $\sim 10^3$ hadronic events per point, 14.8×10^3 events in total
- Simulation of the uds continuum based on the LUARLW generator, tuned JETSET alternatively used at 6 points for a cross-check.



Number of tracks and ratio of Fox-Wolfram moments.

Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ($\sqrt{s} = 2.14 \text{ GeV}$).

R for $\sqrt{s} = 1.84 - 3.05$ GeV

Measured value of $R = \frac{\sigma_{obs}(s) - \sum \varepsilon_{bg}^i(s) \sigma_{bg}^i(s)}{\varepsilon(s)(1+\delta(s))\sigma_{\mu\mu}^0}$

| Point | \sqrt{s} , MeV | $R(s)$ |
|-------|------------------|-----------------------------|
| 1 | 1841.0 | $2.226 \pm 0.139 \pm 0.158$ |
| 2 | 1937.0 | $2.141 \pm 0.081 \pm 0.073$ |
| 3 | 2037.3 | $2.238 \pm 0.068 \pm 0.072$ |
| 4 | 2135.7 | $2.275 \pm 0.072 \pm 0.055$ |
| 5 | 2239.2 | $2.208 \pm 0.069 \pm 0.053$ |
| 6 | 2339.5 | $2.194 \pm 0.064 \pm 0.048$ |
| 7 | 2444.1 | $2.175 \pm 0.067 \pm 0.048$ |
| 8 | 2542.6 | $2.222 \pm 0.070 \pm 0.047$ |
| 9 | 2644.8 | $2.220 \pm 0.069 \pm 0.049$ |
| 10 | 2744.6 | $2.269 \pm 0.065 \pm 0.050$ |
| 11 | 2849.7 | $2.223 \pm 0.065 \pm 0.047$ |
| 12 | 2948.9 | $2.234 \pm 0.064 \pm 0.051$ |
| 13 | 3048.1 | $2.278 \pm 0.075 \pm 0.048$ |

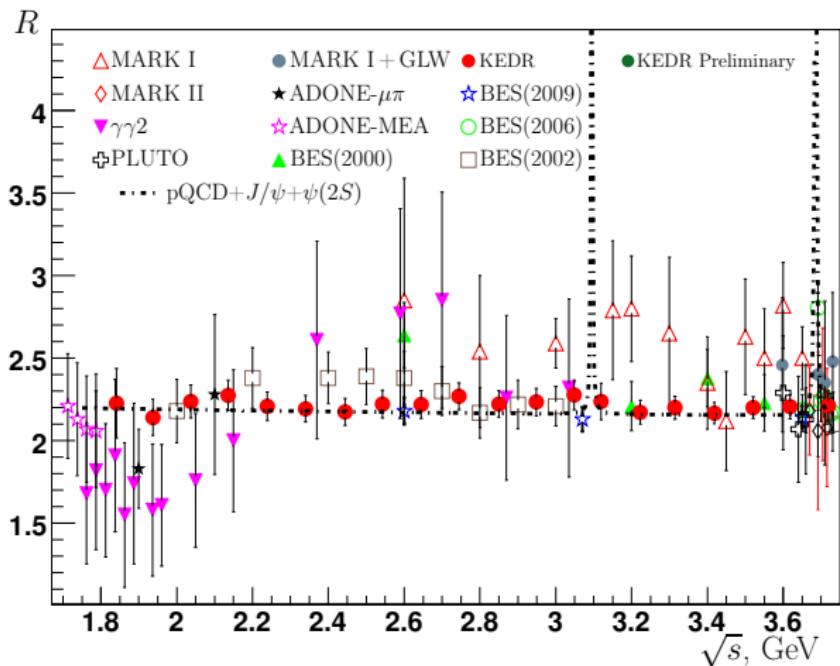
The main systematic uncertainties in the R :

| Source | Error, % |
|-----------------------|----------------|
| Luminosity | 1.2 |
| Rad. corr. | $0.5 \div 2.0$ |
| <i>uds</i> simulation | $1.2 \div 6.6$ |
| I^+I^- | $0.3 \div 0.6$ |
| e^+e^-X | 0.2 |
| Trigger | 0.3 |
| Nuclear interaction | 0.4 |
| Machine background | $0.4 \div 0.9$ |
| Cuts | 0.7 |
| Total | $2.1 \div 7.1$ |

V.V. Anashin et al., Phys.Lett. B 770C, 174 (2017)



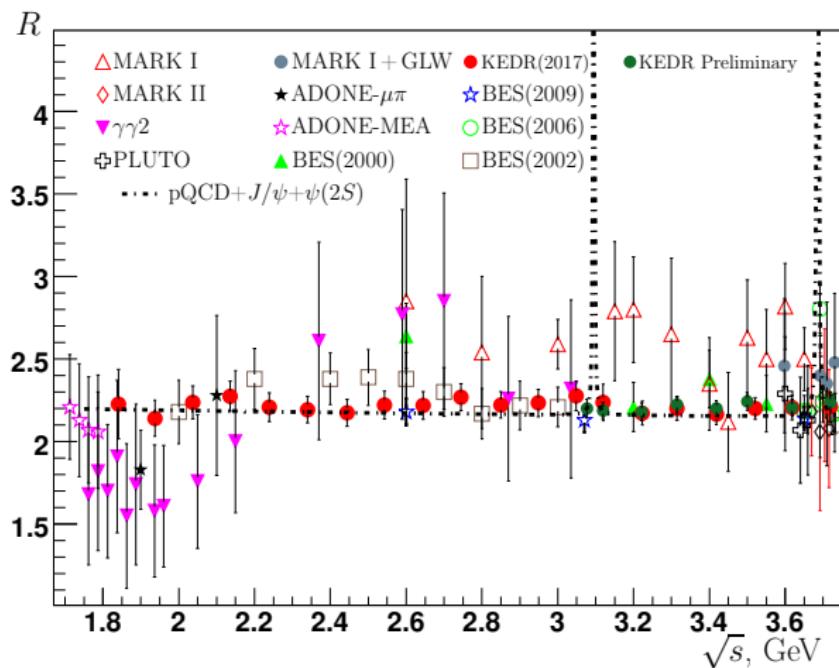
Comparison with others experiments



The quantity R versus the c.m. energy and the sum of the prediction of perturbative QCD and a contribution of narrow resonances.

New preliminary result for $\sqrt{s} = 3.077 - 3.72$ GeV

New data in the energy range $3.077 - 3.72$ GeV: $\int \mathcal{L} dt \simeq 1.4 \text{ pb}^{-1}$ collected at 8 energy points + scan of narrow resonances.



The dark green circles (only statistical errors are shown) are new KEDR measurements of R (preliminary).

Conclusion

- J/ψ results:

$$\Gamma_{ee} = 5.516 \pm 0.056 \pm 0.088 \text{ keV}$$

$$\Gamma_{ee} \times \mathcal{B}_{\text{hadrons}} = 4.853 \pm 0.048 \pm 0.083 \text{ keV}$$

$$\Gamma_{ee} \times \mathcal{B}_{ee} = 0.3317 \pm 0.0066 \pm 0.0043 \text{ keV}$$

- $\psi(2S)$ results:

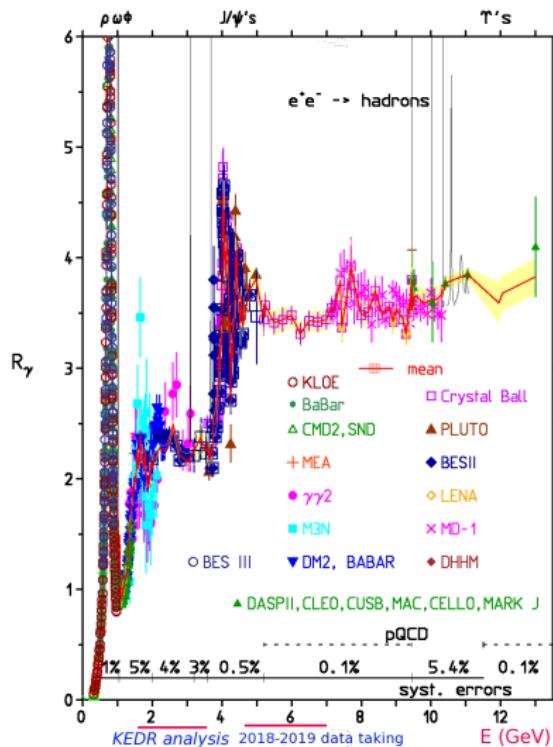
$$\Gamma_{ee} = 2.284 \pm 0.015 \pm 0.042 \text{ keV}$$

$$\Gamma_{ee} \times \Gamma_{\mu\mu} / \Gamma = 19.4 \pm 0.3 \pm 0.4 \text{ eV}$$

- We have determined the values of R at thirteen points of the center-of-mass energy between 1.84 and 3.05 GeV. The achieved accuracy is about or better than 3.9% at most of energy points with a systematic uncertainty less than 2.4%.
- We measured the values of R at seven points of the center-of-mass energy between 3.12 and 3.72 GeV. The total achieved accuracy is about or better than 3.3% at most of energy points with a systematic uncertainty of about 2.1%.
- R measurement in the energy range 3.077 – 3.72 GeV after detector repair and upgrade: analysis in progress.



Plans



F. Jegerlehner arXiv:1511.0447

- We plan to measure R value in the energy range 5-7 GeV

Thank you for your attention

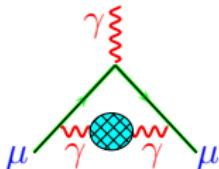
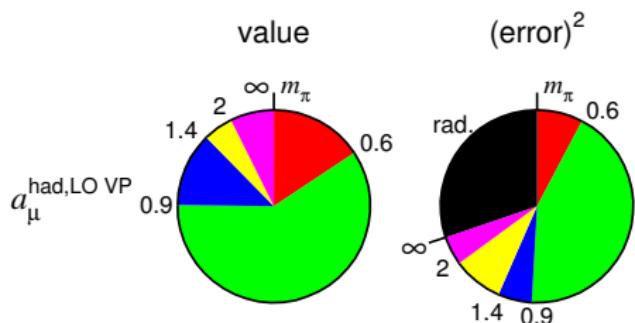


BACKUP SLIDES

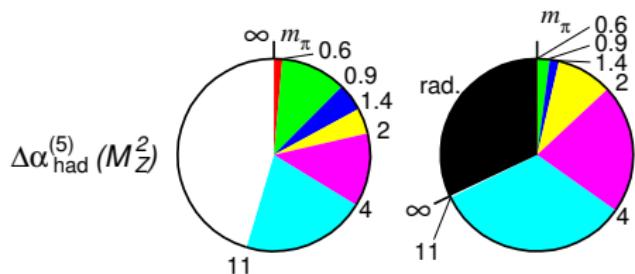


R contribution in a_μ and $\alpha(M_Z^2)$

$$a_\mu^{\text{exp}} = (g_\mu - 2)/2$$



$$a_\mu^{LO \ VP} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^\infty \frac{K(s)R(s)}{s} ds$$



$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

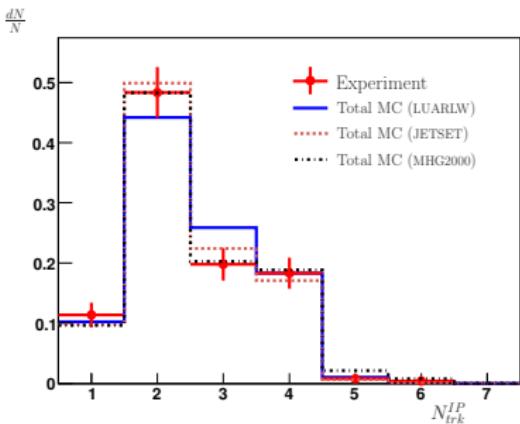
$$\Delta\alpha = \sum_f \gamma \text{---} \textcirclearrowleft \text{---} \gamma = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}(s)$$

$$\Delta\alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \operatorname{Re} \int_{m_\pi^2}^\infty \frac{R(s)ds}{s(s-M_z^2-i\epsilon)}$$

K.Hagiwara et al. arxiv:1105.3149

Detection efficiency uncertainty

- Used two essentially different MC generators (LUARLW and tuned JETSET)
- We validated our estimate of the systematic uncertainty related to simulation of the uds continuum using an unfolding method (Chinese Physics C Vol. 37, No. 6 (2013) 063001).
- The estimate at the most problematic energy point 1.84 GeV was additionally verified using the exclusive generator MHG2000.



Detection efficiency uncertainties obtained by different methods

| | LUARLW/JETSET | Unfolding method | LUARLW/MHG2000 |
|------------|---------------|------------------|----------------|
| point 1 | 6.6% | 3.6% | 3.8% |
| point 2-3 | 2.5% | 1.9% | - |
| point 4-13 | 1.2% | 0.5% | - |

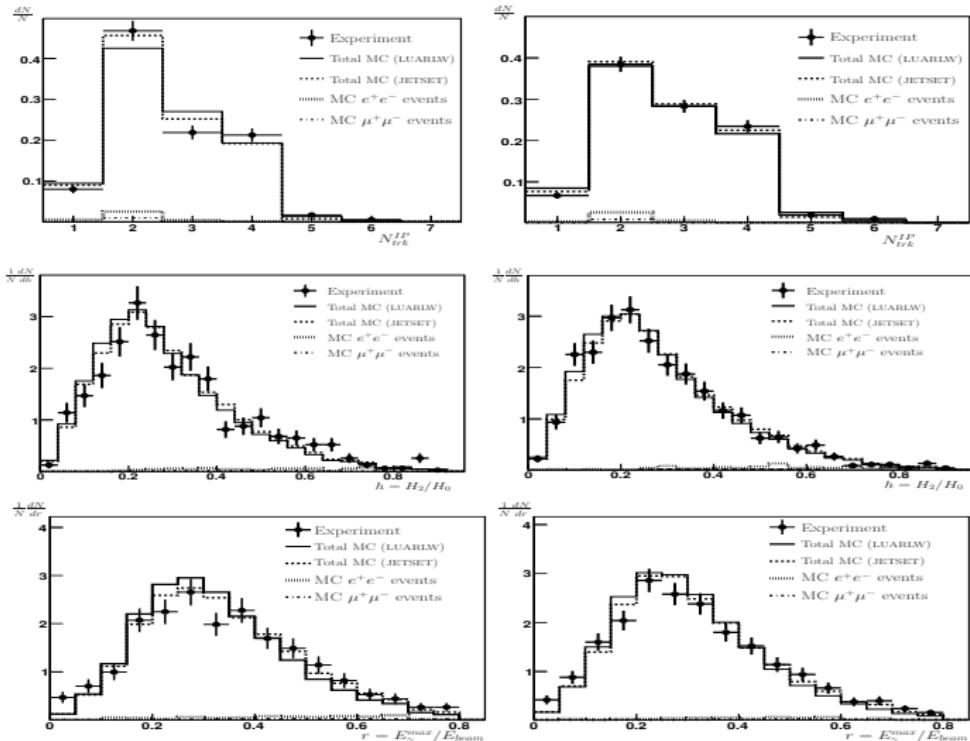
Selection criteria

Selection criteria for hadronic events which were used by AND.

| Variable | Allowed range | |
|--|---|---|
| | 3.12-3.72 GeV | 1.84 - 3.05 GeV |
| $N_{\text{track}}^{\text{IP}}$ | ≥ 1 | ≥ 1 |
| E_{obs} | $> 1.6 \text{ GeV}$ | $> 1.4 \text{ GeV} \left(> 1.3 \text{ GeV if } E_{\text{beam}} < 1.05 \text{ GeV} \right)$ |
| $E_{\gamma}^{\max}/E_{\text{beam}}$ | < 0.8 | < 0.8 |
| $E_{\text{obs}} - E_{\gamma}^{\max}$ | | $> 1.2 \text{ GeV} \left(> 1.1 \text{ GeV if } E_{\text{beam}} < 1.05 \text{ GeV} \right)$ |
| E_{cal} | $> 0.75 \text{ GeV}$ | $> 0.55 \text{ GeV}$ |
| H_2/H_0 | < 0.85 | < 0.9 |
| $ P_z^{\text{miss}}/E_{\text{obs}} $ | < 0.6 | < 0.7 |
| $E_{\text{LKr}}/E_{\text{cal}}^{\text{tot}}$ | > 0.15 | > 0.15 |
| $ Z_{\text{vertex}} $ | $< 20.0 \text{ cm}$ | $< 15.0 \text{ cm}$ |
| | $N_{\text{particles}} \geq 4 \text{ or } \tilde{N}_{\text{track}}^{\text{IP}} \geq 2$ | $N_{\text{particles}} \geq 3 \text{ or } \tilde{N}_{\text{track}}^{\text{IP}} \geq 2$ |

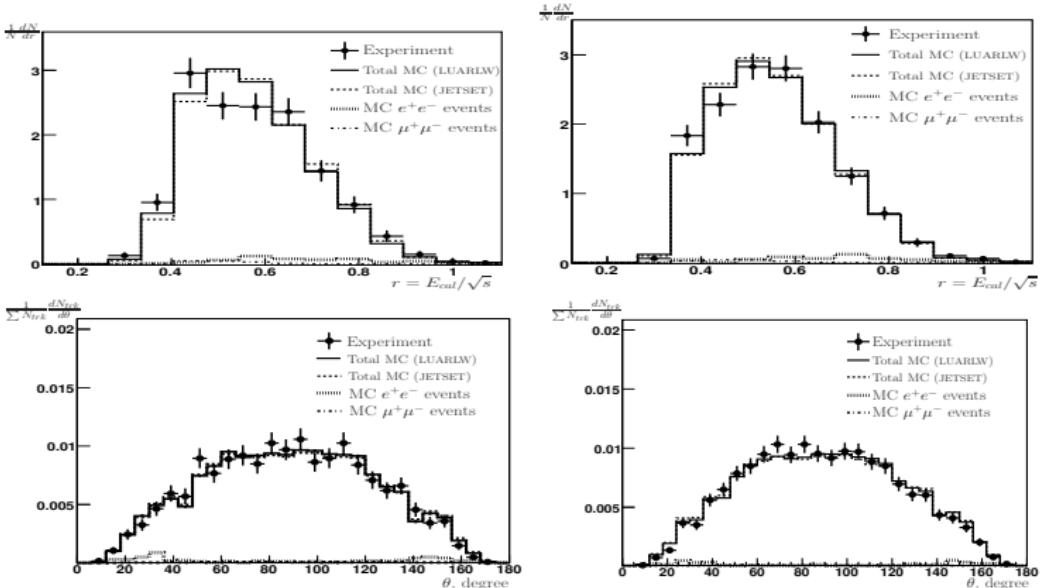


Simulation at 1.94 and 2.14 GeV: JETSET and LUARLW



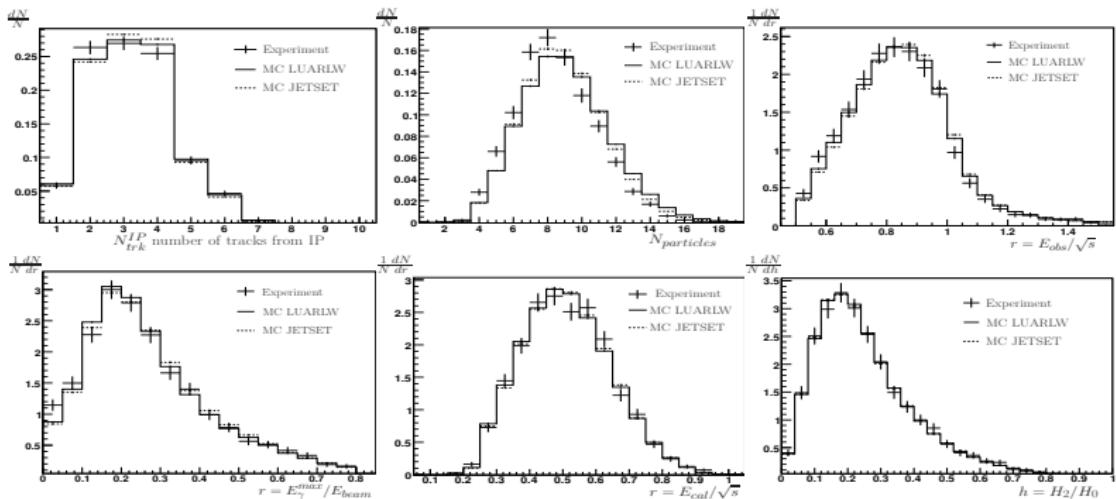
Properties of hadronic events produced in uds continuum at 1.94 GeV (left) and 2.14 GeV (right). Here, N is the number of events, H_2 and H_0 are Fox-Wolfram moments, E_γ^{\max} is energy of the most energetic photon, N_{trk} is the number of tracks in event.

Simulation at 1.94 and 2.14 GeV: JETSET and LUARLW

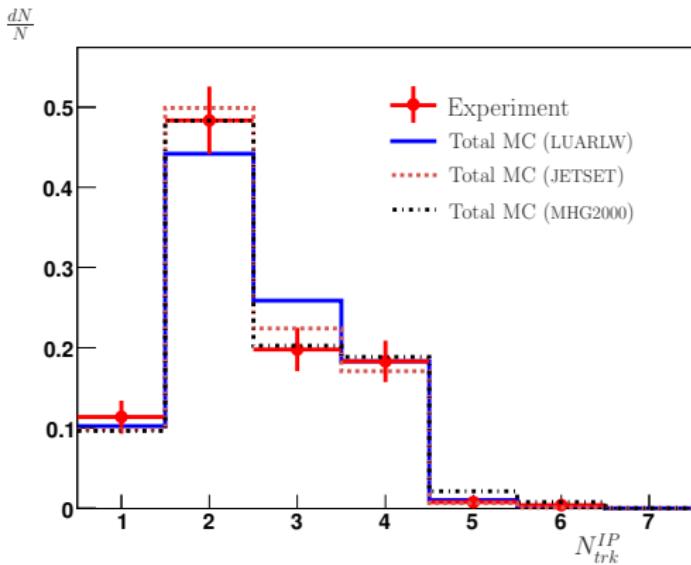


Properties of hadronic events produced in uds continuum at 1.94 GeV (left) and 2.14 GeV (right). Here, N is the number of events, E_{cal} is energy deposited in the calorimeter, θ is polar angle, N_{trk} is the number of tracks in event. Integrals of all distributions are normalized to unity.

Simulation at 3.12 GeV: JETSET and LUARLW



Properties of hadronic events produced in uds continuum at 3.12 GeV. Here N is the number of events, H_2 and H_0 are Fox-Wolfram moments. Integrals of all distributions are normalized to unity.



To obtain the detection efficiency required for calculation of the radiative correction, we performed simulation of the hadronic events using LUARLW and the event generator MHG2000 developed by the CMD-3 collaboration. MGH2000 generates about 30 exclusive channels accounting for the resonance production below 1.9 GeV.

Detection efficiency: JETSET and LUARLW

Detection efficiency for the uds continuum in % (statistical errors only).

| \sqrt{s} , MeV | ϵ_{LUARLW} | ϵ_{JETSET} | $\delta\epsilon/\epsilon$ |
|------------------|---------------------|---------------------|---------------------------|
| 1841.0 | 42.2 ± 0.1 | 45.0 ± 0.1 | -6.6 ± 0.3 |
| 1937.0 | 47.2 ± 0.1 | 46.0 ± 0.1 | -2.5 ± 0.3 |
| 2037.3 | 53.4 ± 0.1 | | |
| 2135.7 | 52.5 ± 0.1 | 51.3 ± 0.1 | -1.2 ± 0.3 |
| 2239.2 | 57.0 ± 0.1 | | |
| 2339.5 | 61.6 ± 0.1 | | |
| 2444.1 | 64.3 ± 0.1 | | |
| 2542.6 | 66.7 ± 0.1 | | |
| 2644.8 | 68.2 ± 0.1 | 68.0 ± 0.1 | -0.2 ± 0.2 |
| 2744.6 | 70.3 ± 0.1 | 70.6 ± 0.1 | $+0.4 \pm 0.2$ |
| 2849.7 | 71.6 ± 0.1 | | |
| 2948.9 | 73.0 ± 0.1 | | |
| 3048.1 | 72.4 ± 0.1 | 73.2 ± 0.1 | $+1.1 \pm 0.2$ |



Detection efficiency: JETSET and LUARLW

| \sqrt{s} , MeV | ϵ_{JETSET} | ϵ_{LUARLW} | $\delta\epsilon/\epsilon$ |
|------------------|---------------------|---------------------|---------------------------|
| Scan 1 | | | |
| 3119.9 | 75.5 ± 0.1 | 75.0 ± 0.1 | -0.7 ± 0.2 |
| 3222.4 | 76.9 ± 0.1 | 76.2 ± 0.1 | -0.9 ± 0.2 |
| 3315.2 | 77.0 ± 0.1 | 77.0 ± 0.1 | 0.0 ± 0.2 |
| 3418.1 | 78.1 ± 0.1 | 77.4 ± 0.1 | -0.9 ± 0.2 |
| 3521.0 | 78.3 ± 0.1 | 78.2 ± 0.1 | -0.1 ± 0.2 |
| 3619.7 | 79.6 ± 0.1 | 78.6 ± 0.1 | -1.3 ± 0.2 |
| 3720.4 | 80.8 ± 0.1 | 79.2 ± 0.1 | -2.0 ± 0.2 |
| Scan 2 | | | |
| 3120.1 | 75.3 ± 0.1 | 74.9 ± 0.1 | -0.5 ± 0.2 |
| 3223.6 | 75.9 ± 0.1 | 75.1 ± 0.1 | -1.1 ± 0.2 |
| 3313.9 | 77.5 ± 0.1 | 77.3 ± 0.1 | -0.3 ± 0.2 |
| 3418.4 | 78.7 ± 0.1 | 78.0 ± 0.1 | -0.9 ± 0.2 |
| 3520.3 | 78.8 ± 0.1 | 78.7 ± 0.1 | -0.1 ± 0.2 |
| 3617.6 | 80.0 ± 0.1 | 79.0 ± 0.1 | -1.3 ± 0.2 |
| 3718.9 | 80.9 ± 0.1 | 79.4 ± 0.1 | -1.9 ± 0.2 |



Luminosity determination: 3.12-3.72 GeV

$e^+e^- \rightarrow e^+e^-(\gamma)$ events detected by the LKr calorimeter $41^\circ < \theta < 159^\circ$ and CsI calorimeter $20^\circ < \theta < 32^\circ$ and $148^\circ < \theta < 160^\circ$

Systematic uncertainties of the luminosity determination in %.

| Source | Uncertainty, % |
|---------------------------|----------------|
| Calorimeter response | 0.7 |
| Calorimeter alignment | 0.2 |
| Polar angle resolution | 0.2 |
| Cross section calculation | 0.5 |
| Background | 0.1 |
| MC statistics | 0.1 |
| Variation of cuts | 0.6 |
| Sum in quadrature | 1.1 |

Differences of an integrated luminosities obtained using the LKr and CsI calorimeters in two scans are $0.5 \pm 0.5\%$ and $0.0 \pm 0.5\%$, respectively.

- The contribution of residual machine background was estimated using runs with separated e^+ and e^- bunches.
- The residual background was evaluated and subtracted using the number of events which passed selection criteria in the background runs in the assumption that the background rate is proportional to the beam current and the measured vacuum pressure.
- As alternative we assumed that background rate is proportional to the current only. The difference between the numbers of background events obtained with the two assumption was considered as the uncertainty estimate for given energy point.

The residual machine background in % of observed cross section

| Point | Scan 1 | Scan 2 |
|-------|-----------------------|-----------------------|
| 1 | $1.3 \pm 0.2 \pm 0.4$ | $1.3 \pm 0.2 \pm 0.4$ |
| 2 | $2.4 \pm 0.4 \pm 0.5$ | $2.7 \pm 0.4 \pm 0.5$ |
| 3 | $2.7 \pm 0.5 \pm 0.4$ | $3.0 \pm 0.5 \pm 0.4$ |
| 4 | $2.9 \pm 0.5 \pm 0.4$ | $3.6 \pm 0.6 \pm 0.4$ |
| 5 | $3.1 \pm 0.6 \pm 0.5$ | $3.3 \pm 0.5 \pm 0.5$ |
| 6 | $2.7 \pm 0.5 \pm 0.4$ | $3.7 \pm 0.6 \pm 0.4$ |
| 7 | $2.1 \pm 0.4 \pm 0.2$ | $2.2 \pm 0.3 \pm 0.2$ |



Unfolding method

- An efficiency matrix ϵ_{ij} describes the efficiency of an event generated with j charged tracks to be reconstructed with i charged tracks.
- The distribution of the number of observed charged track events in data, N_i^{obs} , is known. The true multiplicity distribution in data can be estimated from the observed multiplicity distribution in data and the efficiency matrix by minimizing the χ^2 .
-

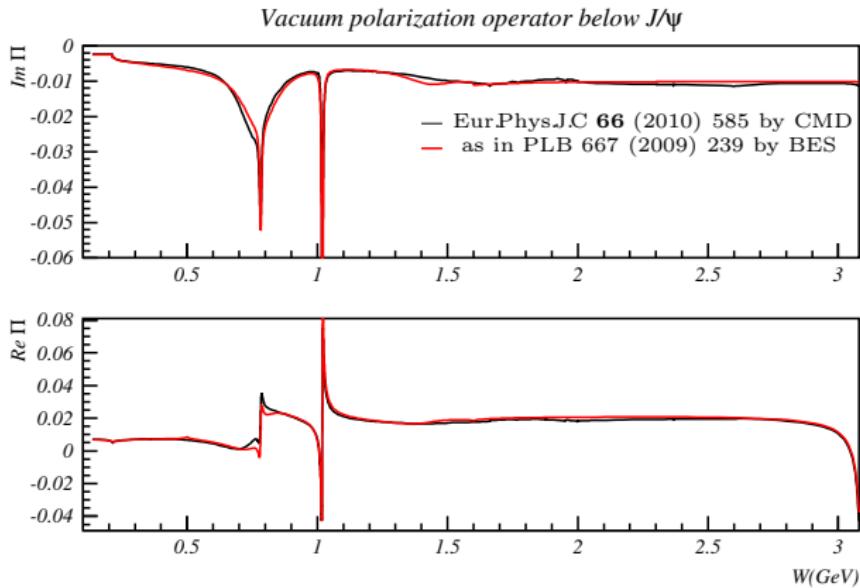
$$\chi^2 = \sum_{i=1}^{i=8} \frac{N_i^{obs} - \sum_{j=1}^{i=8} \epsilon_{ij} \times N_j}{N_i^{obs}}$$

where the N_j ($j = 0, 2, 4, 6, 8$) describe the true multiplicity distribution in data and are taken as floating parameters in the fit.

- The total «true» number of events in data can be obtained by summing all fitted N_j .



$\Pi(s)$ calculation



Luminosity determination

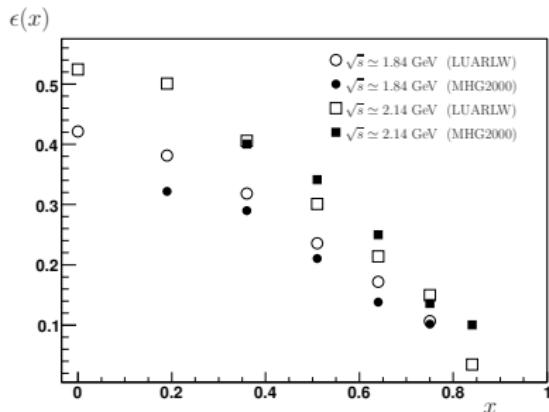
$e^+e^- \rightarrow e^+e^-(\gamma)$ events detected by the LKr calorimeter $41^\circ < \theta < 159^\circ$ and CsI calorimeter $20^\circ < \theta < 32^\circ$ and $148^\circ < \theta < 160^\circ$

Systematic uncertainties of the luminosity determination in %.

| Source | Uncertainty, % |
|---------------------------|----------------|
| Calorimeter response | 0.7 |
| Calorimeter alignment | 0.2 |
| Polar angle resolution | 0.2 |
| Cross section calculation | 0.5 |
| Background | 0.1 |
| MC statistics | 0.1 |
| Variation of cuts | 0.6 |
| Sum in quadrature | 1.1 |

Differences of an integrated luminosities obtained using the LKr and CsI calorimeters in two scans are $0.5 \pm 0.5\%$ and $0.0 \pm 0.5\%$, respectively.

Radiation correction calculation in the energy range 1.84 – 3.05 GeV



Detection efficiency vs variable x at 1.84 and 2.14 GeV.

$$1 + \delta(s) = \int \frac{dx}{1 - x} \frac{\mathcal{F}(s, x)}{|1 - \Pi((1 - x)s)|^2} \frac{R((1 - x)s)\varepsilon((1 - x)s)}{R(s)\varepsilon(s)}$$

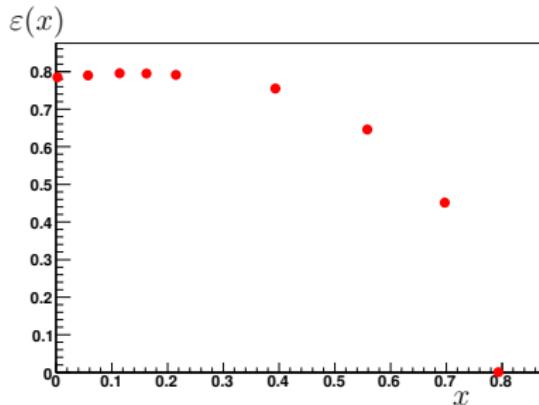
$$R(s) = -\frac{3}{\alpha} \operatorname{Im} \Pi_{\text{hadr}}(s)$$

Vacuum polarization according to
CMD-2 data compilation:
Eur. Phys. J. C66 (2010) 585

Radiative correction factor $1 + \delta$

| \sqrt{s} , MeV | $1 + \delta$ | \sqrt{s} , MeV | $1 + \delta$ |
|------------------|---------------------|------------------|---------------------|
| 1841.0 | 1.0423 ± 0.0208 | 2542.6 | 1.0739 ± 0.0054 |
| 1937.0 | 1.0429 ± 0.0156 | 2644.8 | 1.0796 ± 0.0054 |
| 2037.3 | 1.0515 ± 0.0126 | 2744.6 | 1.0809 ± 0.0054 |
| 2135.7 | 1.0634 ± 0.0106 | 2849.7 | 1.0823 ± 0.0054 |
| 2239.2 | 1.0645 ± 0.0096 | 2948.9 | 1.0774 ± 0.0054 |
| 2339.5 | 1.0664 ± 0.0075 | 3048.1 | 1.0584 ± 0.0053 |
| 2444.1 | 1.0684 ± 0.0064 | | |

Radiation correction calculation in the energy range 3.12 – 3.72 GeV



$$1 + \delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s, x)}{|1 - \tilde{\Pi}((1-x)s)|^2} \frac{\tilde{R}((1-x)s)\varepsilon((1-x)s)}{R(s)\varepsilon(s)}$$

$$R(s) = -\frac{3}{\alpha} \operatorname{Im} \Pi_{\text{hadr}}(s)$$

Vacuum polarization according to
CMD-2 data compilation:
Eur. Phys. J. C66 (2010) 585

| \sqrt{s} , MeV | Scan 1 | | Scan 2 | | Uncertainty, % | | | | Total |
|------------------|---------------------|--|---------------------|--|----------------|----------------------|-------------------------|-----|-------|
| | $1 + \delta$ | | $\Pi(s)$ | | δR | $\delta \varepsilon$ | $\delta_{\text{calc.}}$ | | |
| 3119.9 | 1.0941 ± 0.0066 | | 1.1074 ± 0.0066 | | 0.3 | 0.5 | 0.2 | 0.2 | 0.6 |
| 3223.0 | 1.0949 ± 0.0055 | | 1.1049 ± 0.0055 | | 0.1 | 0.4 | 0.2 | 0.2 | 0.5 |
| 3314.7 | 1.0959 ± 0.0055 | | 1.1100 ± 0.0056 | | 0.1 | 0.4 | 0.2 | 0.2 | 0.5 |
| 3418.2 | 1.0982 ± 0.0044 | | 1.1094 ± 0.0044 | | 0.1 | 0.3 | 0.2 | 0.2 | 0.4 |
| 3520.8 | 1.1032 ± 0.0044 | | 1.1102 ± 0.0044 | | 0.1 | 0.3 | 0.2 | 0.2 | 0.4 |
| 3618.2 | 1.1021 ± 0.0044 | | 1.1098 ± 0.0044 | | 0.1 | 0.3 | 0.2 | 0.2 | 0.4 |
| 3719.4 | 1.1049 ± 0.0055 | | 1.1067 ± 0.0055 | | 0.4 | 0.3 | 0.2 | 0.2 | 0.5 |



List of systematic uncertainties in the energy range 1.84-3.05 GeV

R systematic uncertainties (in %) assigned to each energy point.

| | 1841.0 | 1937.0 | 2037.3 | 2135.7 | 2239.2 | 2339.5 | 2444.1 |
|--------------------------|--------|--------|--------|--------|--------|--------|--------|
| Luminosity | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 |
| Radiative correction | 2.0 | 1.5 | 1.2 | 1.0 | 0.9 | 0.7 | 0.6 |
| Continuum simulation | 6.6 | 2.5 | 2.5 | 1.2 | 1.2 | 1.2 | 1.2 |
| Track reconstruction | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| J/ψ contribution | 0.6 | 0.5 | 0.4 | 0.4 | 0.4 | 0.4 | 0.3 |
| $e^+ e^- X$ contribution | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Trigger efficiency | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| Nuclear interaction | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| Neutral events | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Cuts variation | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| Machine background | 0.6 | 0.5 | 0.4 | 0.7 | 0.8 | 0.6 | 0.8 |
| Energy determination | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Sum in quadrature | 7.1 | 3.4 | 3.2 | 2.4 | 2.4 | 2.2 | 2.2 |
| | 2542.6 | 2644.8 | 2744.6 | 2849.7 | 2948.9 | 3048.1 | |
| Luminosity | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 |
| Radiative correction | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| Continuum simulation | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 |
| Track reconstruction | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| J/ψ contribution | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| $e^+ e^- X$ contribution | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Trigger efficiency | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| Nuclear interaction | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| Neutral events | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Cuts variation | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| Machine background | 0.4 | 0.6 | 0.8 | 0.4 | 0.9 | 0.5 | |
| Energy determination | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | |
| Sum in quadrature | 2.1 | 2.2 | 2.2 | 2.1 | 2.3 | 2.1 | |



List of systematic uncertainties in the energy range 3.12-3.72 GeV

R_{uds} systematic uncertainties (in %) assigned to each energy point.

| | 3119.9 | 3223.0 | 3314.7 | 3418.2 | 3520.8 | 3618.2 | 3719.4 |
|--------------------------|------------|------------|------------|------------|------------|------------|------------|
| <i>Scan 1</i> | | | | | | | |
| Luminosity | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| Radiative correction | 0.6 | 0.5 | 0.5 | 0.4 | 0.4 | 0.4 | 0.5 |
| Continuum simulation | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 2.1 |
| J/ψ contribution | 2.7 | 0.5 | 0.3 | 0.2 | 0.2 | 0.1 | 0.1 |
| $\psi(2S)$ contribution | | | | | | | 1.4 |
| e^+e^-X contribution | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.2 | 0.2 |
| I^+I^- contribution | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 |
| Trigger efficiency | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Nuclear interaction | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Cuts variation | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| Machine background | 1.1 | 0.8 | 0.7 | 0.7 | 0.9 | 0.7 | 0.7 |
| Sum in quadrature | 3.5 | 2.2 | 2.1 | 2.1 | 2.2 | 2.1 | 3.0 |
| <i>Scan 2</i> | | | | | | | |
| Luminosity | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| Radiative correction | 0.6 | 0.5 | 0.5 | 0.4 | 0.4 | 0.4 | 0.5 |
| Continuum simulation | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 2.1 |
| J/ψ contribution | 2.8 | 0.6 | 0.3 | 0.2 | 0.2 | 0.1 | 0.1 |
| $\psi(2S)$ contribution | | | | | | | 1.3 |
| e^+e^-X contribution | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.2 | 0.2 |
| I^+I^- contribution | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 |
| Trigger efficiency | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Nuclear interaction | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Cuts variation | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| Machine background | 1.1 | 0.8 | 0.7 | 0.8 | 0.8 | 0.7 | 0.5 |
| Sum in quadrature | 3.6 | 2.2 | 2.1 | 2.1 | 2.1 | 2.1 | 2.9 |



pQCD calculation

$R(s)$, obtained in:

P.A.Baikov *et al.* Nucl. and Part. Phys. Proceed. 261-262(2015):

$$R^{n_f=3}(s) = 2 \left[1 + \frac{\alpha_s}{\pi} + 1.6398 \left(\frac{\alpha_s}{\pi} \right)^2 - 10.2839 \left(\frac{\alpha_s}{\pi} \right)^3 - 106.8798 \left(\frac{\alpha_s}{\pi} \right)^4 \right].$$

α_s obtained in K.G.Chetyrkin, B.A.Kniehl, M.Steinhauser PRL 79 (1997)

$$\begin{aligned} \alpha_s &= \frac{1}{\beta_0 L} - \frac{1}{(\beta_0 L)^2} \frac{\beta_1}{\beta_0} \ln L + \frac{1}{(\beta_0 L)^3} \left[\left(\frac{\beta_1}{\beta_0} \right)^2 (\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0} \right] \\ &\quad + \frac{1}{(\beta_0 L)^4} \left[\left(\frac{\beta_1}{\beta_0} \right)^3 \left(-\ln^3 L + \frac{5}{2} \ln^2 L + 2 \ln L - \frac{1}{2} \right) - 3 \frac{\beta_1 \beta_2}{\beta_0^2} \ln L + \frac{\beta_3}{2 \beta_0} \right] \end{aligned}$$

$$\text{for } n_f = 3, \beta_0 = \frac{9}{4}, \beta_1 = 4, \beta_2 = \frac{3863}{384}, \beta_3 = \frac{445}{32} \zeta(3) + \frac{140599}{4608}, L = \ln^2 \frac{Q^2}{M^2}$$

$\alpha_s(m_\tau^2) = 0.331 \pm 0.013$ (A.Pich Nucl. and Part. Phys. Proceed. 260 (2015) 61-69) allow to get $R_{uds}^{pQCD} = 2.16 \pm 0.01$ in energy range $3.1 \div 3.7$ GeV.



R for $\sqrt{s} = 3.048 - 3.72$ GeV

| \sqrt{s} , MeV | 2016-2017 $R(s)$ | Preliminary new $R(s)$ |
|---------------------------------|-----------------------------|------------------------|
| 3048.1 ± 2 | $2.278 \pm 0.075 \pm 0.048$ | – |
| 3076.7 ± 1 | – | 2.200 ± 0.056 |
| $3119.9 \pm 0.2 (3119.3 \pm 1)$ | $2.237 \pm 0.089 \pm 0.066$ | 2.189 ± 0.050 |
| $3223.0 \pm 0.6 (3221.8 \pm 1)$ | $2.173 \pm 0.057 \pm 0.045$ | 2.180 ± 0.050 |
| $3314.7 \pm 0.7 (3314.7 \pm 1)$ | $2.200 \pm 0.056 \pm 0.043$ | 2.225 ± 0.054 |
| $3418.2 \pm 0.2 (3418.3 \pm 1)$ | $2.168 \pm 0.050 \pm 0.042$ | 2.201 ± 0.049 |
| 3499.7 ± 1.0 | – | 2.244 ± 0.055 |
| 3520.8 ± 0.4 | $2.201 \pm 0.050 \pm 0.044$ | |
| $3618.2 \pm 1.0 (3618.1 \pm 1)$ | $2.207 \pm 0.059 \pm 0.044$ | 2.207 ± 0.049 |
| $3719.4 \pm 0.7 (3719.6 \pm 1)$ | $2.211 \pm 0.068 \pm 0.060$ | 2.235 ± 0.049 |

