Search for Graviton in BESSIII:
$J/\Psi \to \gamma + \text{graviton}$

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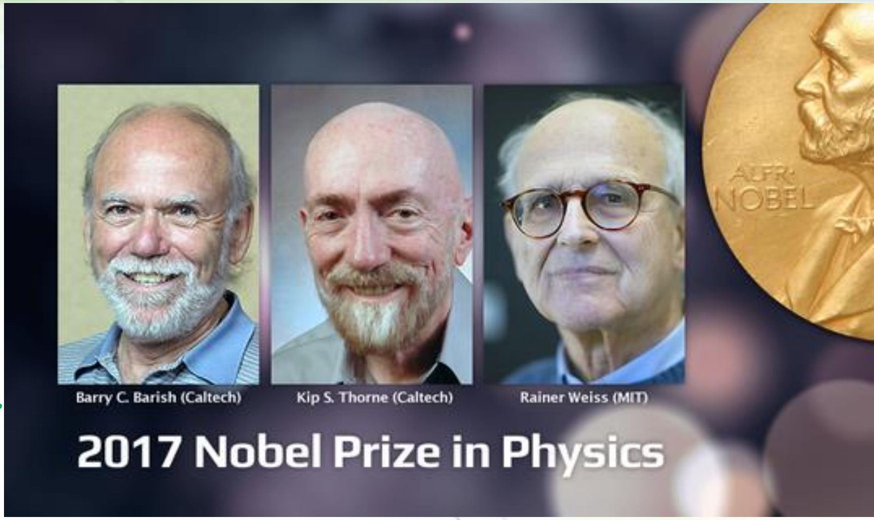
Motivation



- *Gravitational waves from binary black hole mergers and the binary neutron star inspiral are recently observed directly on earth by LIGO Scientific and Virgo collaboration, which initiates a new era for fundamental physics and astronomy. (Abbott et al., PRL (2016), Abbott et al., PRL (2017))*
- *Physical properties of graviton remain untouched.*
- *A full theory of quantum gravity is far from available.*
- *General relativity as an effective theory can be used in making predictions. (Donoghue, PRD (1994) .)*
- *Various predictions based on perturbative quantum gravity are reported: QG corrections to Newton's law of universal gravitation, QG corrections to bending of light, etc. (Donoghue, PRL (1994), Bjerrum-Bohr, et al., PRL(2015))*

$$\theta_\gamma - \theta_\varphi = \frac{8(\mathrm{bu}^\gamma - \mathrm{bu}^\varphi)}{\pi} \frac{G^2 \hbar M}{b^3} \qquad \mathrm{bu}^\gamma - \mathrm{bu}^\varphi = -\frac{43}{20}$$

- ***J/Ψ→γ+graviton** provides a possible channel to study the quantum nature of graviton.*

General Relativity as an Effective Field Theory

background filed method: DeWitt, PR (1967)

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h^{\nu}_{\lambda} + \cdots$$

classical background ($\eta_{\mu\nu}$) — *quantum fluctuation* ($h_{\mu\nu}$)

fermion-gravity coupling:

$$\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi \Longrightarrow \bar{\Psi}\gamma^{\mu}D_{\mu}\Psi \qquad D_{\mu}\Psi \equiv \partial_{\mu}\Psi + \frac{1}{2}\omega_{\mu ab}\sigma^{ab}\Psi,$$

spin connection: $\omega_{\mu}^{ab} = \frac{1}{2}(e^{[a\nu}\partial_{[\mu}e_{\nu}^{b}) + e^{a\rho}e^{b\sigma}\partial_{[\sigma}e_{c\rho]}e_{\mu}^{c}$

$$g_{\mu\nu} = e_{\mu a}e_{\nu}^{a}$$
$$e_{\mu a} = \bar{e}_{\mu a} + \kappa c_{\mu a}$$
$$\Longrightarrow \quad c_{\{\mu\nu\}} \approx h_{\nu\nu} + \cdots$$

Effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{VK} + \mathcal{L}_{GGH} + \mathcal{L}_{AAH} + \mathcal{L}_{FK} + \mathcal{L}_{FFAH} + \mathcal{L}_{FFGH} + \mathcal{L}_{FFH} + \mathcal{L}_{HK} + \cdots$$

$$\mathcal{L}_{VK} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu},$$

$$\mathcal{L}_{GGH} = \kappa\Big\{ -\frac{1}{4}h^\alpha_\alpha\left(\partial_\mu G^a_\nu \partial^\mu G^{a\nu} - \partial_\nu G^a_\mu \partial^\mu G^{a\nu}\right)$$
$$+\frac{1}{2}h_{\rho\sigma}\left(\partial_\alpha G^{a\sigma}\partial^\alpha G^{a\rho} + \partial^\sigma G^a_\alpha \partial^\rho G^{a\alpha} - \partial_\alpha G^{a\sigma}\partial^\rho G^{a\alpha} - \partial^\sigma G^a_\alpha \partial^\alpha G^{a\rho}\right)\Big\}$$

$$\mathcal{L}_{AAH} = \kappa\Big\{ -\frac{1}{4}h^\alpha_\alpha\left(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\nu A_\mu \partial^\mu A^\nu\right)$$
$$+\frac{1}{2}h_{\rho\sigma}\left(\partial_\alpha A^\sigma \partial^\alpha A^\rho + \partial^\sigma A_\alpha \partial^\rho A^\alpha - \partial_\alpha A^\sigma \partial^\rho A^\alpha - \partial^\sigma A_\alpha \partial^\alpha A^\rho\right)\Big\},$$

$$\mathcal{L}_{FK} = i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi,$$

$$\mathcal{L}_{FFAH} = \kappa\left\{-\frac{1}{3}e\bar{\psi}\gamma^\mu\psi\left(h_{\mu\nu} - h^\alpha_\alpha\eta_{\mu\nu}\right)A^\nu\right\},$$

$$\mathcal{L}_{FFGH} = \kappa\left\{-\frac{1}{3}e\bar{\psi}\gamma^\mu T^a\psi\left(h_{\mu\nu} - h^\alpha_\alpha\eta_{\mu\nu}\right)G^{a\nu}\right\},$$

$$\mathcal{L}_{FFH} = \kappa\Big\{\frac{1}{2}h^\alpha_\alpha\left(i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi\right) - \frac{i}{2}\bar{\psi}\gamma^\nu\partial^\mu\psi h_{\mu\nu}$$
$$+\frac{i}{2}\bar{\psi}\left(\frac{1}{2}\gamma^\rho\eta^{\mu\nu} - \frac{1}{2}\gamma^\nu\eta^{\mu\rho}\right)\psi\partial_\rho h_{\mu\nu}\Big\},$$

Calculations

NRQCD factorization:

$$\mathcal{M}(J/\Psi \to \gamma h) = \left(\mathcal{A}_\mu + \langle v^2\rangle \mathcal{B}_\mu\right) \langle 0|\, \chi^\dagger \sigma^\mu \psi \,|J/\Psi\rangle = \epsilon_\psi^\mu \left(\mathcal{A}_\mu + \langle v^2\rangle \mathcal{B}_\mu\right) \sqrt{\langle \mathcal{O}\rangle_\psi} \equiv \mathcal{C}\sqrt{\langle \mathcal{O}\rangle_\psi},$$

perturbative matching:

$$\mathcal{M}(c\bar{c} \to \gamma h) = \sqrt{2N_c}\epsilon^*_{h\mu\nu}\Big(C^{(a)}\epsilon_\gamma^{*\mu}\epsilon_\psi^\nu + C^{(b)}P\cdot\epsilon_\gamma^* k\cdot\epsilon_\psi P^\mu P^\nu + C^{(c)}P\cdot\epsilon_\gamma^* P^\mu \epsilon_\psi^\nu + C^{(d)}k\cdot\epsilon_\psi P^\mu \epsilon_\gamma^{*\nu} + C^{(e)}\epsilon_\gamma^*\cdot\epsilon_\psi P^\mu P^\nu\Big),$$

Ward identities leads to:

$$C^{(c)} = \frac{2C^{(a)}}{M^2}, \qquad C^{(b)} = -\frac{4C^{(a)}}{M^2}, \qquad C^{(d)} = \frac{2C^{(a)}}{M^2}, \qquad C^{(e)} = 0.$$

$$\mathcal{C} = C^{(a)}\epsilon^*_{h\mu\nu}\left(\epsilon_\gamma^{*\mu}\epsilon_\psi^\nu - \frac{4}{M^2}P\cdot\epsilon_\gamma^* k\cdot\epsilon_\psi P^\mu P^\nu - \frac{2}{M^2}P\cdot\epsilon_\gamma^* P^\mu\epsilon_\psi^\nu + \frac{2}{M^2}k\cdot\epsilon_\psi P^\mu\epsilon_\gamma^{*\nu}\right).$$

helicity amplitude: $(\mathcal{C}_{\lambda_\psi\lambda_\gamma\lambda_h})$

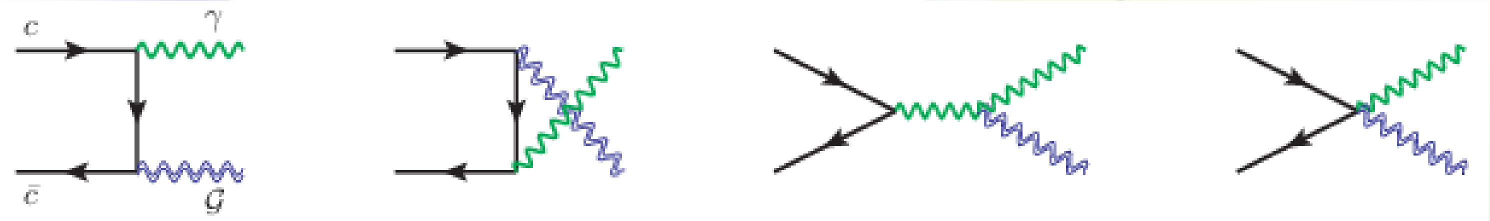
$$\mathcal{C}_{1\pm1\pm2} = \mathcal{C}_{-1\mp1\mp2} = \frac{1}{2}C^{(a)}(-1\pm\cos\theta);$$

$$\mathcal{C}_{0\pm1\pm2} = \mp\frac{1}{\sqrt{2}}C^{(a)}\sin\theta.$$

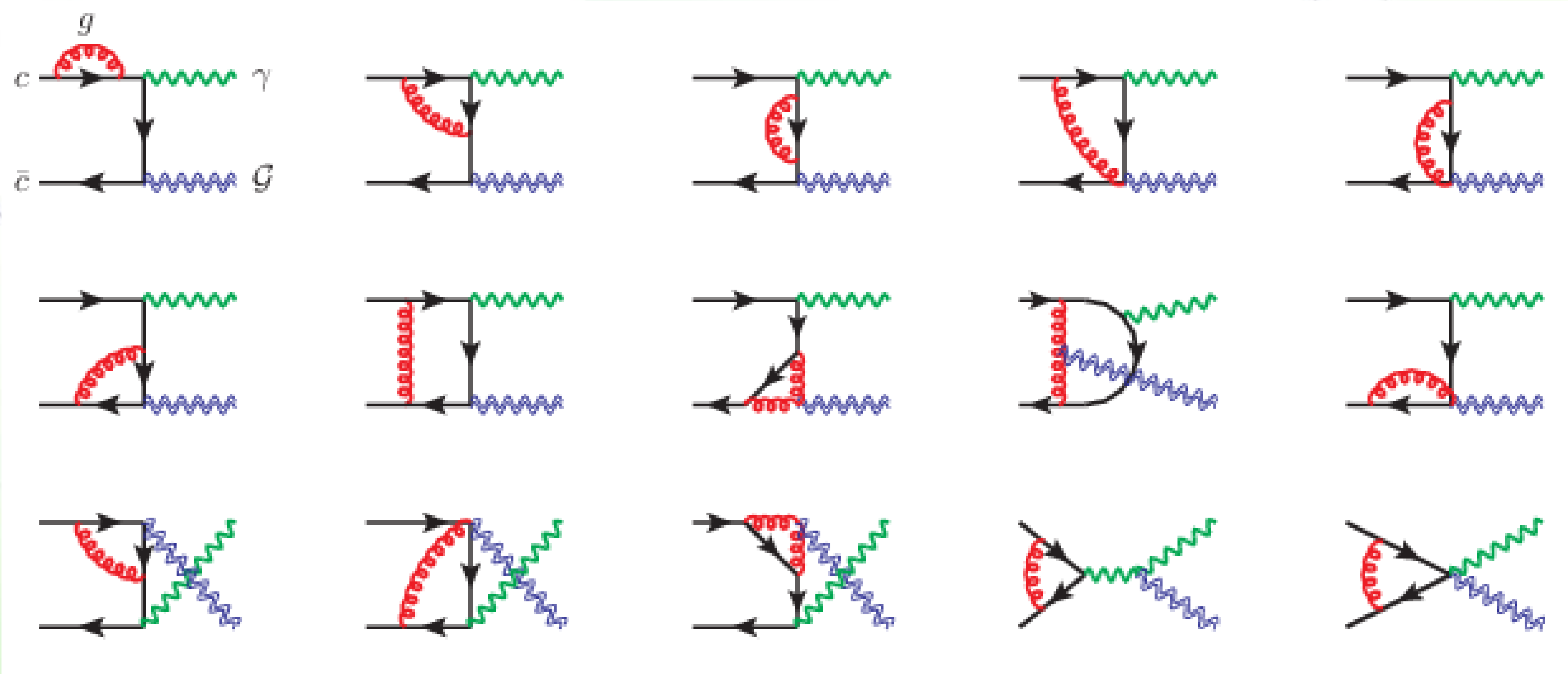
decay rate:

$$\Gamma(J/\Psi \to \gamma h) = \int d\Omega \frac{\pi}{4}\sum_{\text{pol}}|\mathcal{M}(J/\Psi \to \gamma h)|^2 = \frac{2\pi^2}{3}|C^{(a)}|^2\langle \mathcal{O}\rangle_\psi$$

Results



representative diagrams at leading order in α_s



representative diagrams at next-to-leading order in α_s

analytic results:

$$C^{(a)} = \left(-\frac{4\log 2 - 1}{8\pi} C_F \alpha_s + \frac{1}{6}\langle v^2 \rangle \right) Q_c e \kappa,$$

numerical inputs: *(PDG and Bodwin et al., PRD (2008))*

$$\alpha = 7.30 \times 10^{-3}, \qquad \alpha_s(m) = 0.31,$$

$$\kappa = 8.21 \times 10^{-19}\ \mathrm{GeV}^{-1}, \qquad M = 3.096\ \mathrm{GeV}$$

$$\Gamma_{J/\Psi} = 9.29 \times 10^{-5}\ \mathrm{GeV},$$

$$\langle \mathcal{O}\left({}^3S_1\right)\rangle_\psi = 0.440\ \mathrm{GeV}^3, \qquad \langle v \rangle_\psi^2 = 0.225.$$

numerical results:

$$\Gamma(J/\Psi \to \gamma h) = 5.5 \times 10^{-42}\mathrm{GeV}$$

$$\mathrm{Br}(J/\psi \to \gamma + \mathrm{graviton}) = 5.9 \times 10^{-38}$$

massive gravity

$$\tilde{C}_{1\pm1\pm2} = \tilde{C}_{-1\mp1\mp2} = \frac{e\kappa Q_c m_h^2}{4m_\psi^2}(1\mp\cos\theta);$$

$$\tilde{C}_{0\pm1\pm2} = \pm\frac{e\kappa Q_c m_h^2}{2\sqrt{2}m_\psi^2}\sin\theta;$$

$$\tilde{C}_{1\pm1\pm1} = -\tilde{C}_{-1\mp1\mp1} = -\frac{e\kappa Q_c m_h}{4m_\psi}\sin\theta;$$

$$\tilde{C}_{0\pm1\pm1} = -\frac{e\kappa Q_c m_h}{2\sqrt{2}m_\psi}\cos\theta;$$

$$\tilde{C}_{1\pm10} = \tilde{C}_{-1\mp10} = \frac{e\kappa Q_c}{4\sqrt{6}}(1\pm\cos\theta);$$

$$\tilde{C}_{0\pm10} = \mp\frac{e\kappa Q_c}{4\sqrt{3}}\sin\theta,$$

vDVZ discontinuity

$$\Gamma(J/\Psi\to\gamma\tilde{h}) = \frac{\pi^2}{36}e^2\kappa^2Q_c^2\langle\mathcal{O}\rangle_\psi = 3.3\times10^{-39}\mathrm{GeV}$$

Conclusion

As a application of perturbative quantum gravity and NRQCD, we calculated the NLO QCD correction and relativistic correction to the process $J/\Psi \to \gamma + \text{graviton}$ *. The branch ratio is about* 5.9×10^{-38}.

We also generalized the calculations to the case of massive gravity. It is found that the longitudinal polarizations are not decoupled at massless limit, which reveals the well-known vDVZ discontinuity.

Thanks for your attention!