A crisis of NRQCD in describing charmonium?

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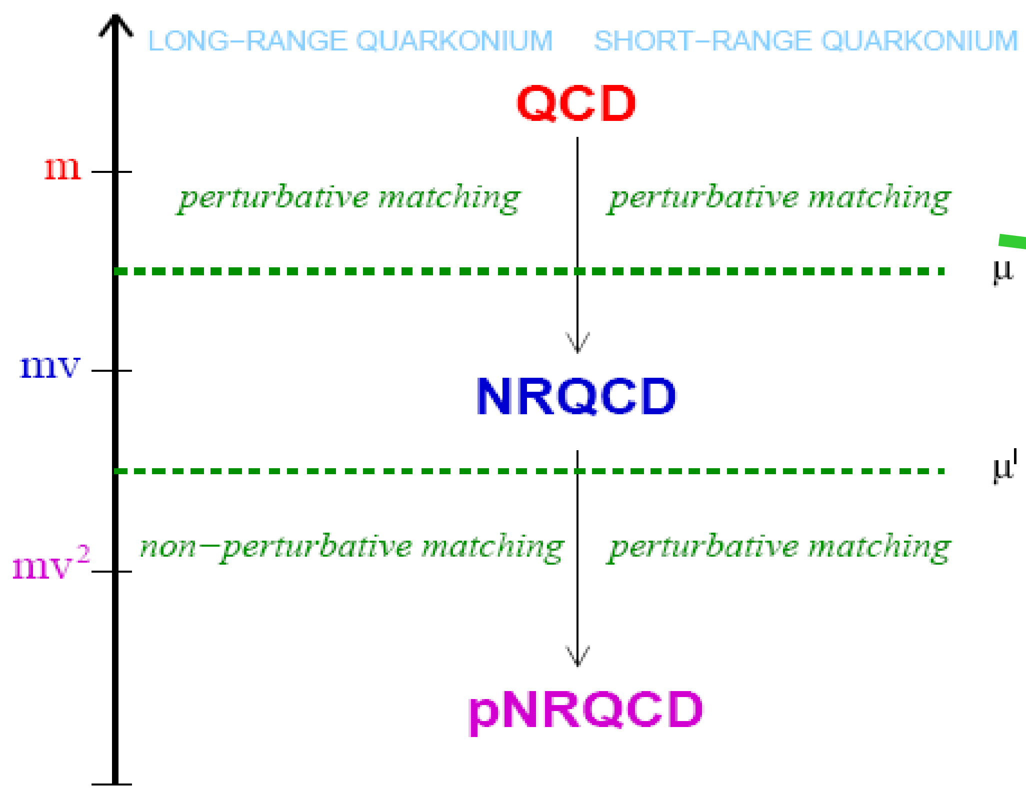
- **Brief review of NRQCD factorization to quarkonium production/decay**
- **NNLO QCD correction to $\gamma\gamma^* \to \eta_c$ form factor and confront BaBar data**
- **NNLO QCD correction to $\eta_c \to \gamma\gamma$ (including "light-by-light")**
- **NNLO QCD correction to $\eta_c \to$ light hadrons and $Br[\eta_c \to \gamma\gamma]$, then confront the PDG data**
- **Summary**

Nonrelativistic QCD (NRQCD): Paradigm of EFT, tailored for describing heavy quarkonium dynamics: exploiting NR nature of quarkonium

Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1995)



NRQCD factorization is viewed as being first principle of QCD

This scale separation is usually referred to as **NRQCD factorization.**

The NRQCD short-dist. coefficients can be computed in perturbation theory, order by order

NRQCD Lagrangian (characterized by velocity expansion)

$$\mathcal{L}_{\rm NRQCD} \;=\; \mathcal{L}_{\rm light} \;+\; \mathcal{L}_{\rm heavy} \;+\; \delta\mathcal{L}.$$

$$\mathcal{L}_{\rm light} \;=\; -\frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu} \;+\; \sum \bar{q}\, i\not{D} q,$$

$$\mathcal{L}_{\rm heavy} \;=\; \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi \;+\; \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi,$$

$$\begin{aligned}
\delta\mathcal{L}_{\rm bilinear} = {} & \frac{c_1}{8M^3} \left(\psi^\dagger (\mathbf{D}^2)^2 \psi \;-\; \chi^\dagger (\mathbf{D}^2)^2 \chi \right) \\
& + \frac{c_2}{8M^2} \left(\psi^\dagger (\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D}) \psi \;+\; \chi^\dagger (\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D}) \chi \right) \\
& + \frac{c_3}{8M^2} \left(\psi^\dagger (i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D}) \cdot \boldsymbol{\sigma} \psi \;+\; \chi^\dagger (i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D}) \cdot \boldsymbol{\sigma} \chi \right) \\
& + \frac{c_4}{2M} \left(\psi^\dagger (g\mathbf{B}\cdot\boldsymbol{\sigma}) \psi \;-\; \chi^\dagger (g\mathbf{B}\cdot\boldsymbol{\sigma}) \chi \right),
\end{aligned}$$

Identical to HQET, but with different power counting

NRQCD is the mainstream tool in studying quarkonium (see Brambilla et al. EPJC 2011 for a review)

Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes:

charmonium: $v^2/c^2 \sim 0.3$ not truly non-relativistic to some extent

bottomonium: $v^2/c^2 \sim 0.1$ a better "non-relativistic" system

Exemplified by

$e^+e^- \to J/\psi + \eta_c$ at B factories (**exclusive charmonium production**)

Unpolarized/polarized J/ψ production at hadron colliders (**inclusive**)

Very active field in recent years (Chao's group, Kniehl's group, Wang's group, Bodwin's group, Qiu's group …) marked by a plenty of PRLs

The ubiquitous symptom of NRQCD factorization: often plagued with huge QCD radiative correction

Most of the NRQCD successes based on the NLO QCD predictions.

However, the NLO QCD corrections are often large:

$e^+e^- \to J/\psi + \eta_c$	K factor: $1.8 \sim 2.1$	Zhang *et.al.*
$e^+e^- \to J/\psi + J/\psi$	K factor: $-0.31 \sim 0.25$	Gong *et.al.*
$p + p \to J/\psi + X$	K factor: ~ 2	Campbell *et.al.*
$J/\psi \to \gamma\gamma\gamma$	K factor: ≤ 0	Mackenzie *et.al.*

… …

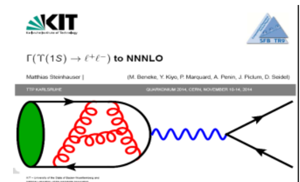
The existing NNLO corrections are rather **few**: all related to S-wave quarkonium **decay**

1. $\Upsilon(J/\Psi) \rightarrow e^+ e^-$

NNLO corrections were first computed by two groups in 1997:

Czarnecki and Melkinov; Beneke, Smirnov, and Signer;

NNNLO correction available very recently: Steinhausser et al. (2013)



2. $\eta_c \rightarrow \gamma\gamma$

NNLO correction was computed by Czarnecki and Melkinov (2001) : (neglecting light-by-light)

3. $B_c \rightarrow l\,\nu$:

NNLO correction computed by Onishchenko, Veretin (2003);

Chen and Qiao, (2015)

Perturbative convergence of these decay processes appears to be rather poor

$$\Gamma(J/\psi \to \ell\ell) = \Gamma^{(0)} \left[1 - \frac{8}{3}\frac{\alpha_s}{\pi} - (44.55 - 0.41\, n_f) \left(\frac{\alpha_s}{\pi}\right)^2 \right]^2$$

$$+(-2091 + 120.66\, n_f - 0.82\, n f^2) \left(\frac{\alpha_s}{\pi}\right)^3$$

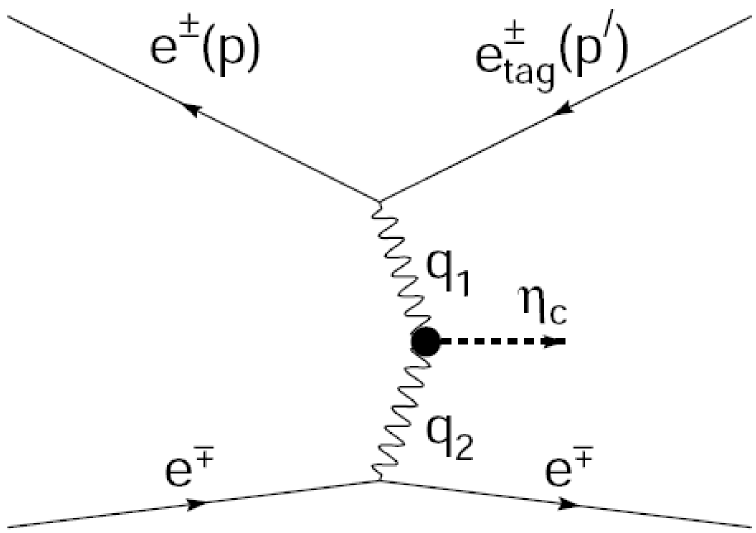
$$\Gamma(B_c \to \ell\nu) = \Gamma^{(0)} \left[1 - 1.39\frac{\alpha_s}{\pi} - 23.7 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3) \right]^2$$

$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma^{(0)} \left[1 - 1.69\frac{\alpha_s}{\pi} - 56.52 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3) \right]^2$$

So calculating the higher order QCD correction is imperative to test the usefulness of NRQCD factorization!

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Experiment

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**

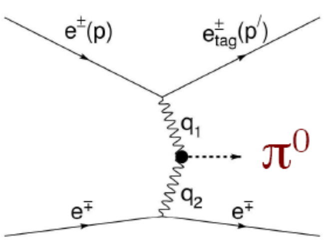


$$q_2^2 \approx 0$$

$$q_1^2 = -Q^2 = (p' - p)^2$$

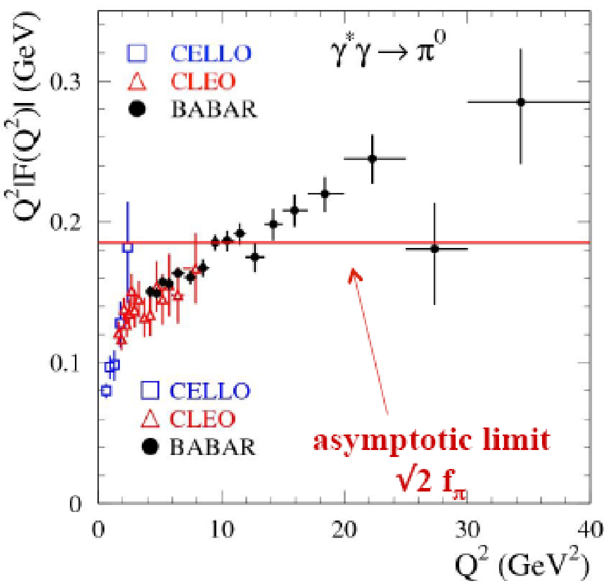
Babar measures the $\gamma\gamma^* \rightarrow \eta_c$ transition form factor in the momentum transfer range from **2 to 50 GeV^2**.

Digression: recall the surprise brought by BaBar two-photon experiment on $\gamma\gamma^* \rightarrow \pi^0$



The π^0 Transition Form Factor

Comparison of the result of experiment to the QCD limit

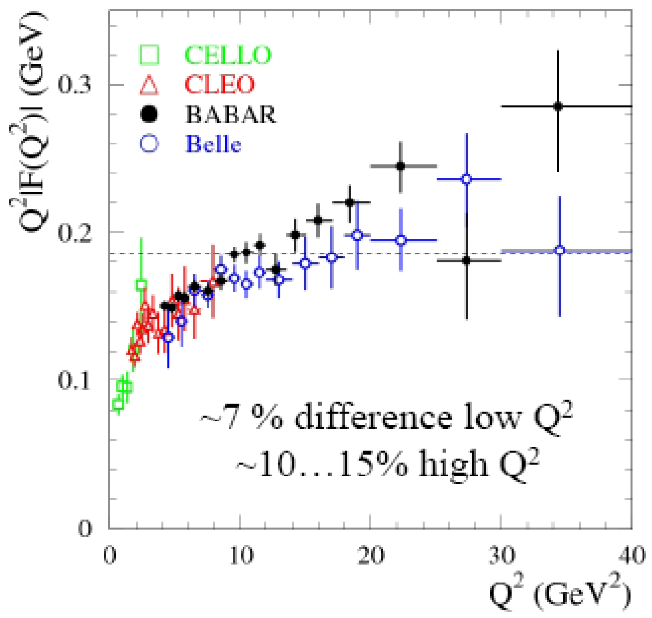


- Experiment:
  In Q^2 range 4-9 GeV^2 CLEO results are consistent with more precise BaBar data
- QCD prediction (Brodsky-Lepage '79):
  at high Q^2 data should reach asymptotic limit (either from below or from above)

  $Q^2 F(Q^2) = \sqrt{2} f_\pi = 0.185$ GeV assuming the asymptotic DA

Belle did not confirm BaBar measurement on $\gamma\gamma^* \rightarrow \pi^0$! Situation needs clarification

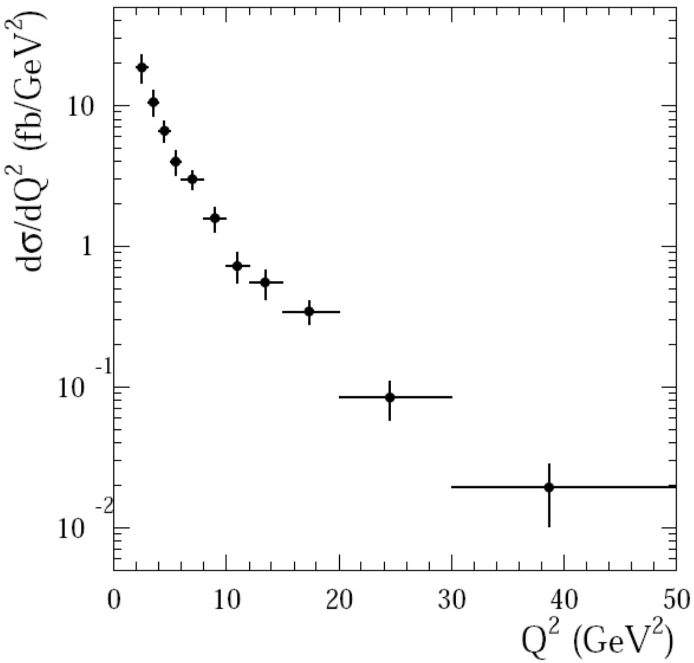
Comparison with BELLE, arXiv:1205.3249



- Difference BABAR – BELLE ~$2\sigma_{syst}$
- BELLE has lower detection efficiency (~factor 2)
- BELLE has higher systematic uncertainties

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor: There also exists BaBar measurements!

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**



$$\frac{d\sigma(e^+e^-\to\eta_c e^+e^-)}{dQ^2} \times \mathcal{B}(\eta_c \to K\bar{K}\pi)$$

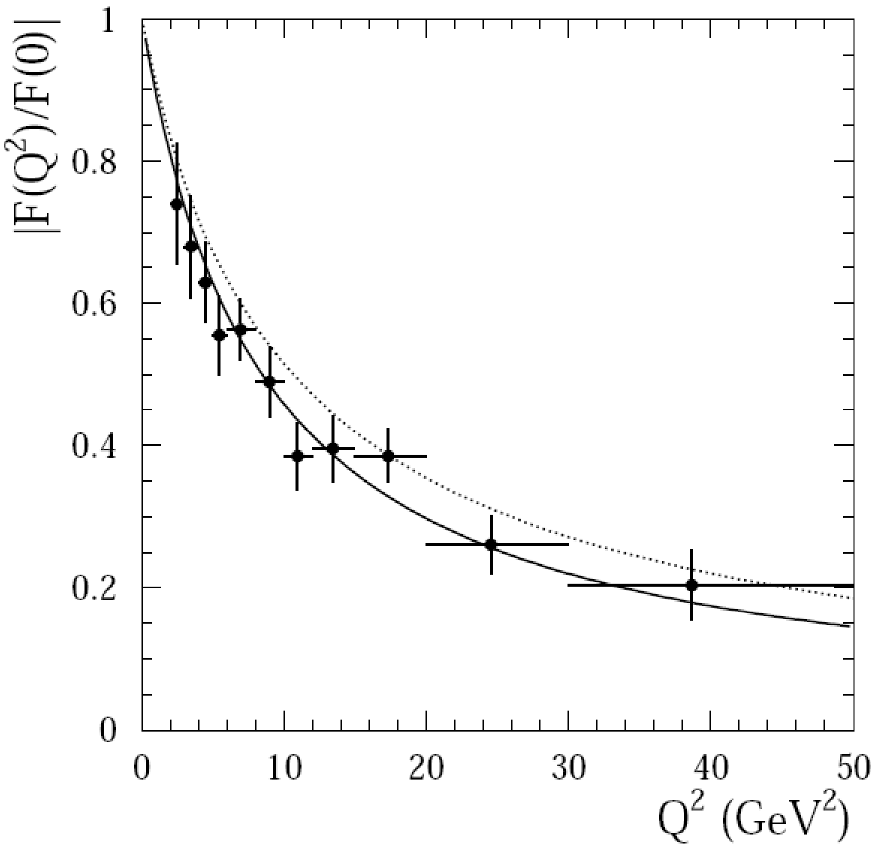
Q^2 interval (GeV^2)	$\overline{Q^2}$ (GeV^2)	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV^2)	$\lvert F(\overline{Q^2})/F(0)\rvert$
2–3	2.49	$18.7 \pm 4.2 \pm 0.8$	0.740 ± 0.085
3–4	3.49	$10.6 \pm 2.1 \pm 0.8$	0.680 ± 0.073
4–5	4.49	$6.62 \pm 1.18 \pm 0.19$	0.629 ± 0.057
5–6	5.49	$4.00 \pm 0.80 \pm 0.10$	0.555 ± 0.056
6–8	6.96	$3.00 \pm 0.43 \pm 0.17$	0.563 ± 0.043
8–10	8.97	$1.58 \pm 0.30 \pm 0.08$	0.490 ± 0.049
10–12	10.97	$0.72 \pm 0.17 \pm 0.05$	0.385 ± 0.048
12–15	13.44	$0.55 \pm 0.13 \pm 0.03$	0.395 ± 0.047
15–20	17.35	$0.34 \pm 0.07 \pm 0.01$	0.385 ± 0.038
20–30	24.53	$0.084 \pm 0.026 \pm 0.004$	0.261 ± 0.041
30–50	38.68	$0.019 \pm 0.009 \pm 0.001$	0.204 ± 0.049

$F(Q^2)$: $\gamma^*\gamma \to \eta_c$ form factor

$F(0)$: $\eta_c \to \gamma\gamma$ form factor

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Experiment

BaBar Collaboration: **Phys. Rev. D81 (2010) 052010**



The solid curve is from a simple monopole fit:

$$|F(Q^2)/F(0)| = \frac{1}{1 + Q^2/\Lambda}$$

with $\Lambda = 8.5 \pm 0.6 \pm 0.7\ \mathrm{GeV}^2$

The dotted curve is from pQCD prediction

Feldmann and Kroll, Phys. Lett. B 413, 410 (1997)

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Previous investigation

- $k_\perp$ factorization: Feldmann *et.al.*, Cao *and Huang*
- Lattice QCD: Dudek *et.al.*,
- **J/ψ** -pole-dominance: Lees *et.al.*,
- QCD sum rules: Lucha *et.al.*,
- light-front quark model: Geng *et.al.*,
- Dyson-Schwinger approach: Chang, Chen, Ding, Liu, Roberts, 2016

All yield predictions compatible with the data, at least in the small Q^2 range.

So far, so good. Unlike $\gamma\gamma^* \rightarrow \pi^0$, there is no open puzzle here

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

Motivation

- Model-independent method is always welcome.

  (NRQCD is the first principle approach from QCD)

- In the normalized form factor, nonperturbative NRQCD matrix element cancels out. Therefore, our predictions are free from any freely adjustable parameters!

- Is LO/NLO NRQCD prediction sufficient?

- The momentum transfer is not large enough, we are not bothered by resumming the large collinear logarithms.

The first NNLO calculation for (exclusive) quarkonium production process

PRL 115, 222001 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?

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Unlike the bewildering situation in the $\gamma\gamma^* \to \pi$ form factor, a widespread view is that perturbative QCD can decently account for the recent *BABAR* measurement of the $\gamma\gamma^* \to \eta_c$ transition form factor. The next-to-next-to-leading-order perturbative correction to the $\gamma\gamma^* \to \eta_{c,b}$ form factor, is investigated in the nonrelativistic QCD (NRQCD) factorization framework for the first time. As a byproduct, we obtain, by far, the most precise order-α_s^2 NRQCD matching coefficient for the $\eta_{c,b} \to \gamma\gamma$ process. After including the substantial negative order-α_s^2 correction, the good agreement between NRQCD prediction and the measured $\gamma\gamma^* \to \eta_c$ form factor is completely ruined over a wide range of momentum transfer squared. This eminent discrepancy casts some doubts on the applicability of the NRQCD approach to hard exclusive reactions involving charmonium.

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Investigation on $\gamma\gamma^* \to \eta_c$ form factor

Definition for form factor:

$$\langle \eta_c(p) | J^\mu | \gamma(k,\varepsilon) \rangle = i e^2 \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu q_\rho k_\sigma F(Q^2)$$

NRQCD factorization demands:

Factorization scale

$$F(Q^2) = C(Q, m, \mu_R, \mu_\Lambda) \frac{\langle \eta_c | \psi^\dagger \chi(\mu_\Lambda) | 0 \rangle}{\sqrt{m}} + \mathcal{O}(v^2)$$

Short-distance coefficient (SDC)
We are going to compute it to NNLO

$$\overline{R_{\eta_c}}(\Lambda) \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \psi(\Lambda) | \eta_c \rangle,$$

$$\overline{R_\psi}(\Lambda)\, \epsilon \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \boldsymbol{\sigma} \psi(\Lambda) | \psi(\epsilon) \rangle\ ,$$

Investigation on $\gamma\gamma^* \to \eta_c$ form factor
Perturbative series for NRQCD SDCs

Upon general consideration, the SDC can be written as

$$C(Q,m,\mu_R,\mu_\Lambda)=C^{(0)}(Q,m)\Bigg\{1+C_F\frac{\alpha_s(\mu_R)}{\pi}f^{(1)}(\tau)$$
$$+\frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2+m^2}C_F f^{(1)}(\tau)-\pi^2 C_F\left(C_F+\frac{C_A}{2}\right)\right.$$
$$\left.\times\ln\frac{\mu_\Lambda}{m}+f^{(2)}(\tau)\right]+\mathcal{O}(\alpha_s^3)\Bigg\},$$

RG invariance

IR pole matches **anomalous dimension** of NRQCD pseudo-scalar density

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Theoretical calculation

$$C^{(0)}(Q,m) = \frac{4e_c^2}{Q^2+4m^2}$$

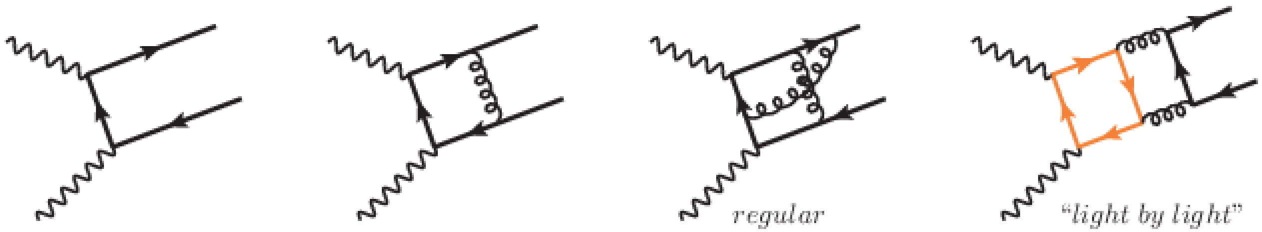
Tree-level SDC

$$f^{(1)}(\tau) = \frac{\pi^2(3-\tau)}{6(4+\tau)} - \frac{20+9\tau}{4(2+\tau)} - \frac{\tau(8+3\tau)}{4(2+\tau)^2}\ln\frac{4+\tau}{2} + 3\sqrt{\frac{\tau}{4+\tau}}\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}} + \frac{2-\tau}{4+\tau}\left(\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}}\right)^2 - \frac{\tau}{2(4+\tau)}\mathrm{Li}_2\left(-\frac{2+\tau}{2}\right),$$

$$\tau \equiv \frac{Q^2}{m^2}$$

NLO QCD correction

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Feynman diagrams



LO NLO NNLO

Numer of diagrams

LO	NLO	NNLO (regular)	NNLO (“light by light”)
2	**8**	**108**	**12**

Investigation on $\gamma\gamma^* \to \eta_c$ form factor
NNLO corrections

$$f^{(2)}(\tau) = f^{(2)}_{\rm reg}(\tau) + f^{(2)}_{\rm lbl}(\tau).$$

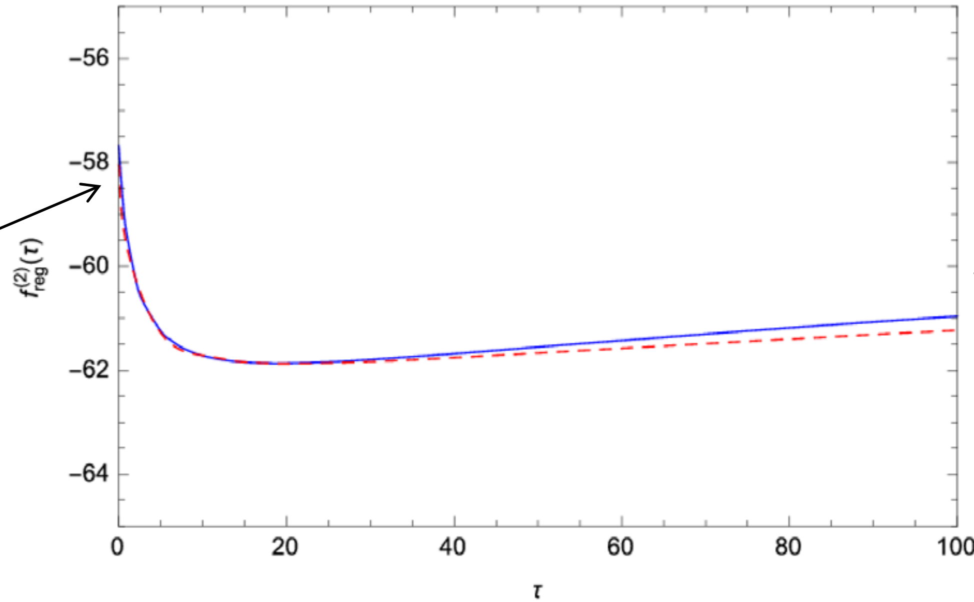
regular

Light-by-light
UV/IR finite

At $\tau \gg 0$, the value of $f^{(2)}_{\rm reg}(\tau)$ is compatible with asymptotic behavior $\ln^2\tau$ solving ERBL equation by Yang, NPB 2009

Reproduce known NNLO corr. to ηc->γγ

Czarnecki et al. 2001



Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

NNLO corrections

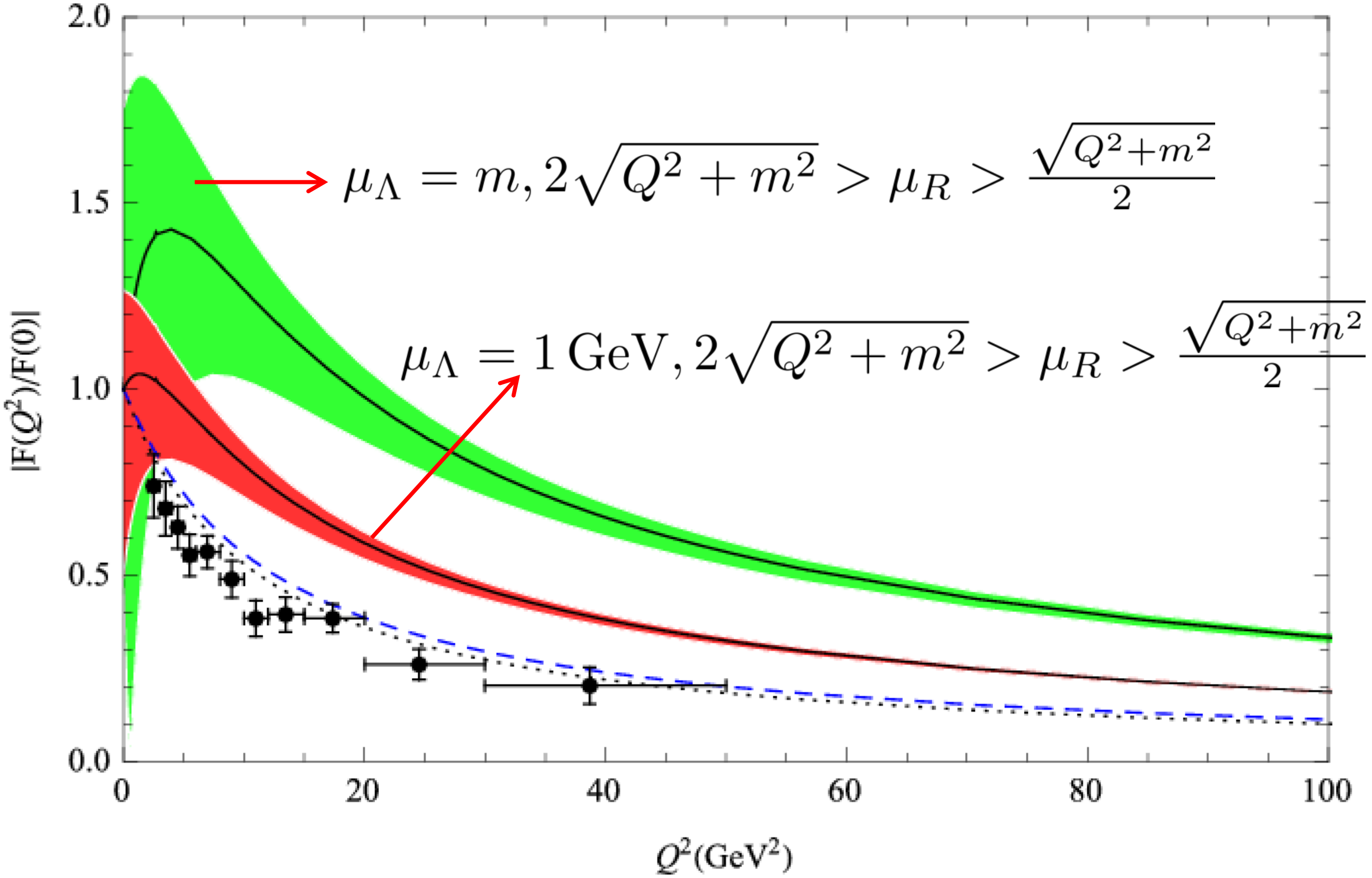
τ	1	5	10	25	50
$f_{\rm reg}^{(2)}$	-59.420(6)	-61.242(6)	-61.721(7)	-61.843(8)	-61.553(8)
$f_{\rm lbl}^{(2)}$	$0.49(1)$ $-0.65(1)i$	$-0.48(1)$ $-0.72(1)i$	$-1.10(1)$ $-0.71(1)i$	$-2.13(1)$ $-0.69(1)i$	$-3.07(1)$ $-0.68(1)i$
$f_{\rm reg}^{(2)}$	-59.636(6)	-61.278(6)	-61.716(7)	-61.864(8)	-61.668(8)
$f_{\rm lbl}^{(2)}$	$0.79(1)$ $-12.45(1)i$	$-5.61(1)$ $-13.55(1)i$	$-9.45(1)$ $-13.83(1)i$	$-15.32(1)$ $-14.03(1)i$	$-20.26(1)$ $-14.10(1)i$

Table 1: $f_{\rm reg}^{(2)}(\tau)$ and $f_{\rm lbl}^{(2)}(\tau)$ at some typical values of τ. The first two rows for η_c and the last two for η_b.

Contribution from light-by-light is not always negligible!

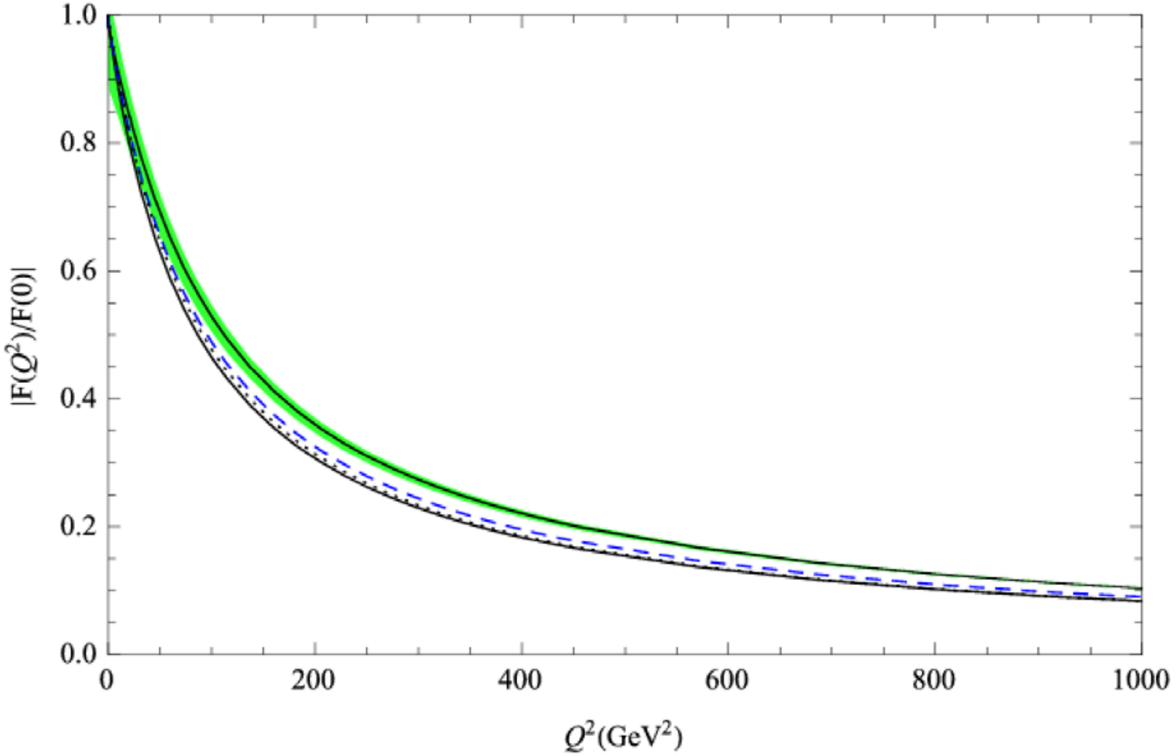
Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Theory vs Experiment

Our Prediction is free of any nonperturbative parameters!



$\gamma\gamma^* \rightarrow \eta_c$: **NNLO predictions seriously fails to describe data!**

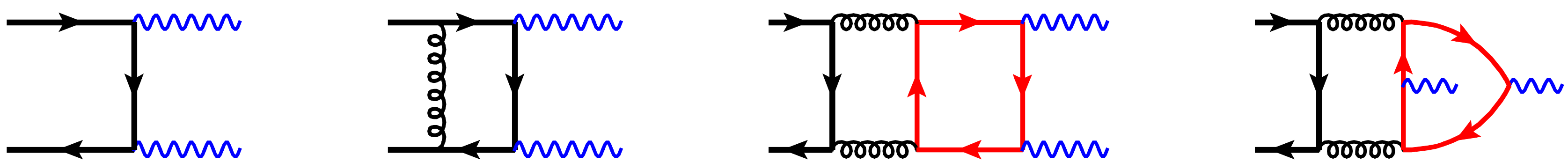
Prediction to $\gamma\gamma^* \rightarrow \eta_b$ form factor



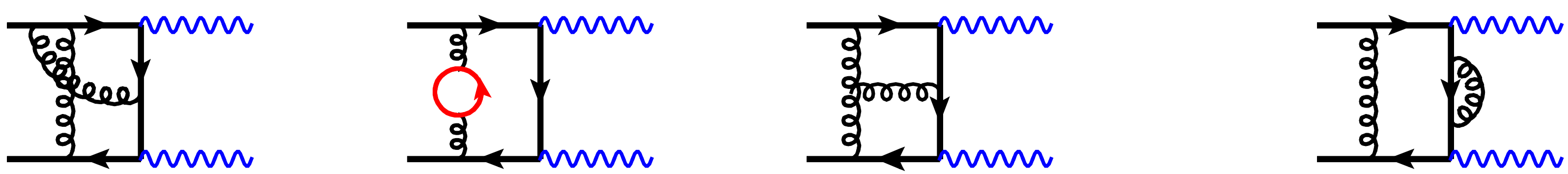
Convergence of perturbation series is reasonably well.
Await **CEPC/ILC** to test our predictions?

As a by-product, we also have a complete NNLO prediction for $\eta_c \rightarrow 2\gamma$ (including "light-by-light" diagrams)

We can focus on form factor at $Q^2 = 0$:



LO NLO NNLO ("*light by light*")



NNLO (*regular*)

Updated NNLO predictions to $\eta_c \to 2\gamma$

NNLO correction was previously computed by Czarnecki and Melkinov (2001) (neglecting light-by-light);

Here we present a complete/highly precise NNLO predictions

Form factor at Q^2 =0:

$$F(0) = \frac{e_c^2}{m^{5/2}} \langle \eta_c | \psi^\dagger \chi(\mu_\Lambda)|0\rangle \left\{ 1 + C_F \frac{\alpha_s(\mu_R)}{\pi}\left(\frac{\pi^2}{8} - \frac{5}{2}\right) + \frac{\alpha_s^2}{\pi^2}\left[C_F\left(\frac{\pi^2}{8} - \frac{5}{2}\right)\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m^2} - \pi^2 C_F\left(C_F + \frac{C_A}{2}\right)\ln\frac{\mu_\Lambda}{m} + f_{\rm reg}^{(2)}(0) + f_{\rm lbl}^{(2)}(0)\right] + \mathcal{O}(\alpha_s^3)\right\},$$

NRQCD factorization scale dependence

$$f_{\rm reg}^{(2)}(0) = -21.107\,897\,97(4) C_F^2 - 4.792\,980\,00(3) C_F C_A - \left(\frac{13\pi^2}{144} + \frac{2}{3}\ln 2 + \frac{7}{24}\zeta(3) - \frac{41}{36}\right) C_F T_F n_L + 0.223\,672\,013(2) C_F T_F n_H, \quad (8)$$

$$f_{\rm lbl}^{(2)}(0) = \left(0.731\,284\,59 + i\pi\left(\frac{\pi^2}{9} - \frac{5}{3}\right)\right) C_F T_F \sum_i^{n_L} \frac{e_i^2}{e_Q^2} + (0.646\,965\,57 + 2.073\,575\,56 i) C_F T_F n_H, \quad (9)$$

$$\Gamma(\eta_c \to 2\gamma) = (\pi\alpha^2/4)|F(0)|^2 M_{\eta_c}^3 .$$

Complete NNLO correction to η_c → light hadrons (first NNLO calculation for inclusive process involving quarkonium)

arXiv:1707.05758 [hep-ph]

Next-to-next-to-leading-order QCD corrections to hadronic width of pseudoscalar quarkonium

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(Dated: July 19, 2017)

We compute the next-to-next-to-leading order (NNLO) QCD corrections to the hadronic decay width of the pseudoscalar quarkonium, at the lowest order in velocity expansion. The validity of NRQCD factorization for inclusive quarkonium decay process, for the first time, is verified to relative order α_s^2. As a byproduct, the renormalization group equation (RGE) of the leading NRQCD 4-fermion operator $\mathcal{O}_1(^1S_0)$ is also deduced to this perturbative order. By incorporating this new piece of correction together with available relativistic corrections, we find that there still exists severe tension between the state-of-the-art NRQCD predictions and the measured η_c hadronic width, and in particular the branching fraction of $\eta_c \to \gamma\gamma$. NRQCD appears to be capable of accounting for η_b hadronic decay to a satisfactory degree, and our most refined prediction is $\mathrm{Br}(\eta_b \to \gamma\gamma) = (4.6 \pm 0.3) \times 10^{-5}$.

PACS numbers: *12.38.Bx, 13.25.Gv, 14.40.Pq*

Heavy quarkonium decay has historically played a preeminent role in establishing asymptotic freedom of QCD [1, 2]. Due to the nonrelativistic nature of heavy quark inside a quarkonium, the decay rates are traditionally expressed as the squared bound-state wave function at the origin multiplying the short-distance quark-antiquark annihilation decay rates. With the advent of modern nonrelativistic effective field theory, the nonrelativistic QCD (NRQCD), this factorization picture has been put on a firmer ground, and one is allowed to systematically include the QCD radiative and relativistic corrections when tackling various quarkonium decay processes [3].

The aim of this Letter is to critically scrutinize one of the most basic quantities in the area of quarkonium physics, *i.e.*, the hadronic width of $^1S_0$ charmonia and bottomonia. The latest Particle Data Group (PDG) compilation lists the total widths $\Gamma_{\rm had}(\eta_c) = 31.8 \pm 0.8$ MeV, and $\Gamma_{\rm had}(\eta_b) = 10^{+5}_{-4}$ MeV [4]. These simple observables naturally constitute the ideal candidates to critically examine the validity of NRQCD factorization approach.

According to NRQCD factorization [3], through the relative order v^2, the inclusive hadronic decay rate of the pseudoscalar quarkonium, say, η_c, can be written as

$$\Gamma(\eta_c \to \mathrm{LH}) = \frac{F_1(^1S_0)}{m^2}\langle\eta_c|\mathcal{O}_1(^1S_0)|\eta_c\rangle + \frac{G_1(^1S_0)}{m^4}\langle\eta_c|\mathcal{P}_1(^1S_0)|\eta_c\rangle + \mathcal{O}(v^3\Gamma), \quad (1)$$

where $\mathcal{O}_1(^1S_0) = \psi^\dagger\chi\chi^\dagger\psi$, $\mathcal{P}_1(^1S_0) = \frac{1}{2}[\psi^\dagger\chi\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi + \mathrm{h.c.}]$. Here ψ, χ represent the quark and anti-quark Pauli spinor fields in NRQCD, and $\mathbf{D}$ denotes the spatial part of the gauge covariant derivative. In Refs. [5, 6], more complete NRQCD factorization formulae are presented through the relative order v^4. Since the explosion of the number of poorly-constrained operator matrix elements severely hampers the predictive power of NRQCD, in this Letter we will be contented with the accuracy of the velocity expansion as prescribed in (1). Some crude power-counting argument estimates that the uncalculated terms in (1) might yield a contribution as large as 25% [5].

It is convenient to organize these short-distance coefficients in terms of perturbative series expansion:

$$F_1(^1S_0) = \frac{\pi\alpha_s^2 C_F}{N_c}\left\{1 + \frac{\alpha_s}{\pi}f_1 + \frac{\alpha_s^2}{\pi^2}f_2 + \cdots\right\}, \quad (2a)$$

$$G_1(^1S_0) = -\frac{4\pi\alpha_s^2 C_F}{3N_c}\left\{1 + \frac{\alpha_s}{\pi}g_1 + \cdots\right\}. \quad (2b)$$

The $\mathcal{O}(\alpha_s)$ correction to the short-distance coefficient f_1 was first computed in Refs. [7, 8]. The tree-level contri-

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NLO perturbative corr. 1979/1980

[7] R. Barbieri, E. d'Emilio, G. Curci and E. Remiddi, Nucl. Phys. B **154**, 535 (1979).

[8] K. Hagiwara, C. B. Kim and T. Yoshino, Nucl. Phys. B **177**, 461 (1981).

40 years lapsed from NLO to NNLO;

Another ??? years to transition into NNNLO?

Promising only if Alpha-Loop takes over?

NRQCD factorization for $\eta_c \rightarrow$ light hadrons – up to relative order-v^4 corrections

Bodwin, Petrelli PRD (2002)

$$\Gamma({}^1S_0 \to \mathrm{LH}) = \frac{F_1({}^1S_0)}{m^2}\langle {}^1S_0|\mathcal{O}_1({}^1S_0)|{}^1S_0\rangle + \frac{G_1({}^1S_0)}{m^4}\langle {}^1S_0|\mathcal{P}_1({}^1S_0)|{}^1S_0\rangle + \frac{F_8({}^3S_1)}{m^2}\langle {}^1S_0|\mathcal{O}_8({}^3S_1)|{}^1S_0\rangle + \frac{F_8({}^1S_0)}{m^2}\langle {}^1S_0|\mathcal{O}_8({}^1S_0)|{}^1S_0\rangle + \frac{F_8({}^1P_1)}{m^4}\langle {}^1S_0|\mathcal{O}_8({}^1P_1)|{}^1S_0\rangle + \frac{H_1^1({}^1S_0)}{m^6}\langle {}^1S_0|\mathcal{Q}_1^1({}^1S_0)|{}^1S_0\rangle + \frac{H_1^2({}^1S_0)}{m^6}\langle {}^1S_0|\mathcal{Q}_1^2({}^1S_0)|{}^1S_0\rangle .$$

$$\mathcal{O}_1({}^1S_0) = \psi^\dagger\chi\chi^\dagger\psi, \tag{2.2a}$$

$$\mathcal{P}_1({}^1S_0) = \frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi + \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi\chi^\dagger\psi\right], \tag{2.2b}$$

$$\mathcal{O}_8({}^3S_1) = \psi^\dagger\boldsymbol{\sigma}T_a\chi\cdot\chi^\dagger\boldsymbol{\sigma}T_a\psi, \tag{2.2c}$$

$$\mathcal{O}_8({}^1S_0) = \psi^\dagger T_a\chi\chi^\dagger T_a\psi, \tag{2.2d}$$

$$\mathcal{O}_8({}^1P_1) = \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)T_a\chi\cdot\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)T_a\psi, \tag{2.2e}$$

$$\mathcal{Q}_1^1({}^1S_0) = \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi, \tag{2.2f}$$

$$\mathcal{Q}_1^2({}^1S_0) = \frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^4\psi + \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^4\chi\chi^\dagger\psi\right], \tag{2.2g}$$

$$\mathcal{Q}_1^3({}^1S_0) = \frac{1}{2}[\psi^\dagger\chi\chi^\dagger(\overleftrightarrow{\mathbf{D}}\cdot g\mathbf{E} + g\mathbf{E}\cdot\overleftrightarrow{\mathbf{D}})\psi - \psi^\dagger(\overleftrightarrow{\mathbf{D}}\cdot g\mathbf{E} + g\mathbf{E}\cdot\overleftrightarrow{\mathbf{D}})\chi\chi^\dagger\psi], \tag{2.2h}$$

NRQCD factorization for $\eta_c \rightarrow$ light hadrons – up to relative order-v^4 corrections

Brambilla, Mereghetti, Vairo, 0810.2259

$$
\begin{aligned}
\Gamma(^1S_0 \to \text{l.h.}) &= \frac{2\,\mathrm{Im}\, f_1(^1S_0)}{M^2}\langle H(^1S_0)|\mathcal{O}_1(^1S_0)|H(^1S_0)\rangle \\
&+\frac{2\,\mathrm{Im}\, g_1(^1S_0)}{M^4}\langle H(^1S_0)|\mathcal{P}_1(^1S_0)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f_8(^3S_1)}{M^2}\langle H(^1S_0)|\mathcal{O}_8(^3S_1)|H(^1S_0)\rangle \\
&+\frac{2\,\mathrm{Im}\, f_8(^1S_0)}{M^2}\langle H(^1S_0)|\mathcal{O}_8(^1S_0)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f_8(^1P_1)}{M^4}\langle H(^1S_0)|\mathcal{O}_8(^1P_1)|H(^1S_0)\rangle \\
&+\frac{2\,\mathrm{Im}\, s_{1\text{-}8}(^1S_0,{}^3S_1)}{M^4}\langle H(^1S_0)|\mathcal{S}_{1\text{-}8}(^1S_0,{}^3S_1)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f'_{8\,\mathrm{cm}}}{M^4}\langle H(^1S_0)|\mathcal{O}'_{8\,\mathrm{cm}}|H(^1S_0)\rangle \\
&+\frac{2\,\mathrm{Im}\, g_{8a\,\mathrm{cm}}}{M^4}\langle H(^1S_0)|\mathcal{P}_{8a\,\mathrm{cm}}|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, f_{1\,\mathrm{cm}}}{M^4}\langle H(^1S_0)|\mathcal{O}_{1\,\mathrm{cm}}|H(^1S_0)\rangle \\
&+\frac{2\,\mathrm{Im}\, h'_1(^1S_0)}{M^6}\langle H(^1S_0)|\mathcal{Q}'_1(^1S_0)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, h''_1(^1S_0)}{M^6}\langle H(^1S_0)|\mathcal{Q}''_1(^1S_0)|H(^1S_0)\rangle \\
&+\frac{2\,\mathrm{Im}\, g_8(^3S_1)}{M^4}\langle H(^1S_0)|\mathcal{P}_8(^3S_1)|H(^1S_0)\rangle + \frac{2\,\mathrm{Im}\, g_8(^1S_0)}{M^4}\langle H(^1S_0)|\mathcal{P}_8(^1S_0)|H(^1S_0)\rangle \\
&+\frac{2\mathrm{Im}\, g_8(^1P_1)}{M^6}\langle H(^1S_0)|\mathcal{P}_8(^1P_1)|H(^1S_0)\rangle + \frac{2\mathrm{Im}\, h'_8(^1S_0)}{M^6}\langle H(^1S_0)|\mathcal{Q}'_8(^1S_0)|H(^1S_0)\rangle \\
&+\frac{2\mathrm{Im}\, h_8(^1D_2)}{M^6}\langle H(^1S_0)|\mathcal{Q}_8(^1D_2)|H(^1S_0)\rangle + \frac{2\mathrm{Im}\, h_1(^1D_2)}{M^6}\langle H(^1S_0)|\mathcal{Q}_1(^1D_2)|H(^1S_0)\rangle \\
&+\frac{2\mathrm{Im}\, d_8(^1S_0,{}^1P_1)}{M^5}\langle H(^1S_0)|\mathcal{D}_{8\text{-}8}(^1S_0,{}^1P_1)|H(^1S_0)\rangle,
\end{aligned}
$$

Notice the explosion of number of higher-dimensional operators!

NRQCD factorization for $\eta_c \to$ light hadrons – Current status of radiative corrections

$$\Gamma(\eta_c \to \mathrm{LH}) = \frac{F_1({}^1S_0)}{m^2}\langle\eta_c|\mathcal{O}_1({}^1S_0)|\eta_c\rangle + \frac{G_1({}^1S_0)}{m^4}\langle\eta_c|\mathcal{P}_1({}^1S_0)|\eta_c\rangle + \mathcal{O}(v^3\Gamma),$$

To warrant predictive power, we only retain terms through relative order-v^2

$$F_1({}^1S_0) = \frac{\pi\alpha_s^2 C_F}{N_c}\left\{1 + \frac{\alpha_s}{\pi}f_1 + \frac{\alpha_s^2}{\pi^2}f_2 + \cdots\right\}$$

$$G_1({}^1S_0) = -\frac{4\pi\alpha_s^2 C_F}{3N_c}\left\{1 + \frac{\alpha_s}{\pi}g_1 + \cdots\right\}.$$

W.Y.Keung, I. Muzinich, 1983

$$f_1 = \frac{\beta_0}{2}\ln\frac{\mu_R^2}{4m^2} + \left(\frac{\pi^2}{4} - 5\right)C_F + \left(\frac{199}{18} - \frac{13\pi^2}{24}\right)C_A - \frac{8}{9}n_L - \frac{2n_H}{3}\ln 2, \quad (3a)$$

Barbieri et al., 1979

Hagiwara et al., 1980

$$g_1 = \frac{\beta_0}{2}\ln\frac{\mu_R^2}{4m^2} - C_F\ln\frac{\mu_\Lambda^2}{m^2} - \left(\frac{49}{12} - \frac{5\pi^2}{16} - 2\ln 2\right)C_F + \left(\frac{479}{36} - \frac{11\pi^2}{16}\right)C_A - \frac{41}{36}n_L - \frac{2n_H}{3}\ln 2. \quad (3b)$$

Guo,Ma,Chao, 2011

Our calculation of short-distance coefficient utilizes **Method of Region (Beneke and Smirnov 1998)** to directly extract the hard region contribution from multi-loop diagrams

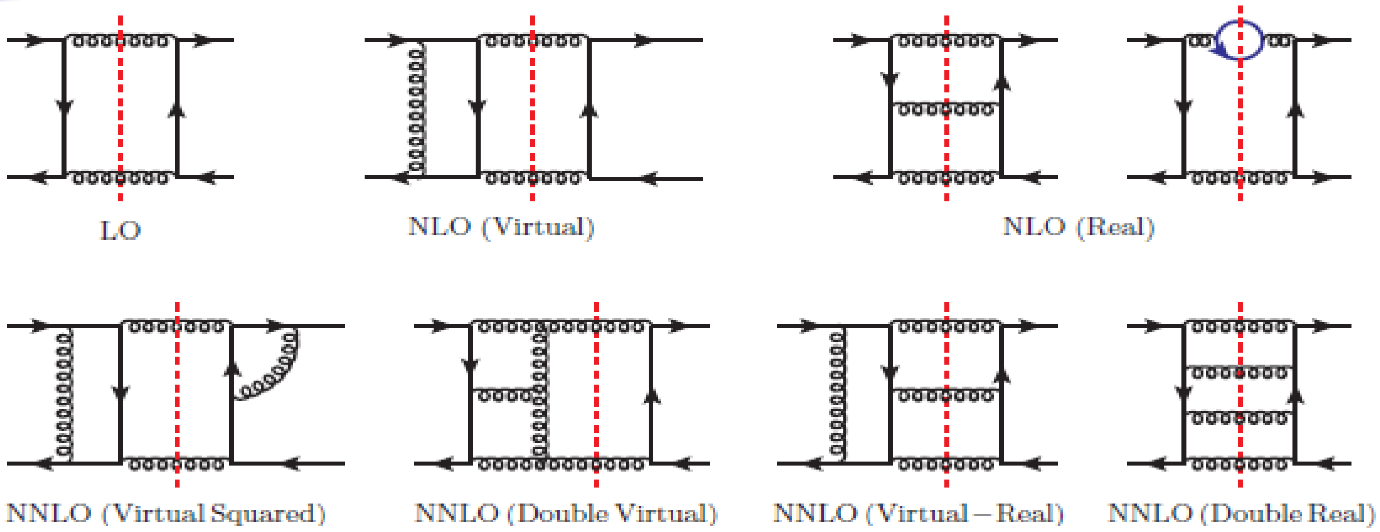


FIG. 1: Representative cut Feynman diagrams responsible for the quark reaction $c\bar{c}(^1S_0^{(1)}) \to c\bar{c}(^1S_0^{(1)})$ through NNLO in α_s. The vertical dashed line denotes the Cutkosky cut.

Roughly 1700 3-loop forward-scattering diagrams, divided into 4 distinct cut topologies; Cutkosky rule is imposed

Employ a well-known trick to deal with phase-space type integrals

Key technique: using IBP to deal with phase-space integral

$$\int \frac{d^D p_i}{(2\pi)^D} 2\pi\, i\, \delta^+(p_i^2) = \int \frac{d^D p_i}{(2\pi)^D} \left(\frac{1}{p_i^2 + i\varepsilon} - \frac{1}{p_i^2 - i\varepsilon} \right).$$

duction. Finally, we end up with 93 MIs for the "Double Virtual" type of diagrams, 89 MIs for the "Virtual-Real" type of diagrams, and 32 MIs for "Double Real" type of diagrams, respectively. To the best of our knowledge, this work represents the first application of the trick (4) in higher-order calculation involving quarkonium.

The nontrivial aspects of the calculation

Encounter some rather time-consuming MIs using sector decomposition method (Fiesta)

Roughly speaking, **10^5 CPU core hour is expensed;** Run numerical integration at the GuangZhou Tianhe Supercomputer Center/China Grid.

Explicitly verify the cancellation of IR poles among the 4 types of cut diagrams. Starting from the **$1/\varepsilon^4$ poles**, observe the exquisite cancelation until $1/\varepsilon$

Our key results

$$f_2 = \hat{f}_2 + \frac{3\beta_0^2}{16}\ln^2\frac{\mu_R^2}{4m^2} + \left(\frac{\beta_1}{8} + \frac{3}{4}\beta_0\hat{f}_1\right)\ln\frac{\mu_R^2}{4m^2} - \pi^2\left(C_F^2 + \frac{C_A C_F}{2}\right)\ln\frac{\mu_\Lambda^2}{m^2}, \quad (5)$$

→ NNLO SDC

Same IR divergence as η_c → 2γ!

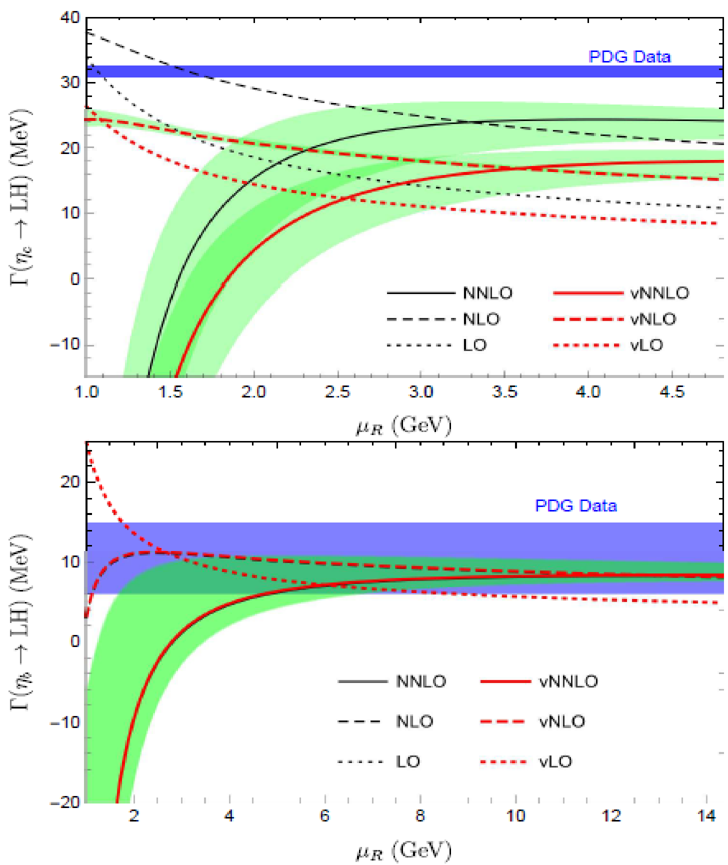
$$\hat{f}_2 = -0.799(13)N_c^2 - 7.4412(5)n_L N_c - 3.6482(2)N_c + 0.37581(3)n_L^2 + 0.56165(5)n_L + 32.131(5) - 0.8248(3)\frac{n_L}{N_c} - \frac{0.67105(3)}{N_c} - \frac{9.9475(2)}{N_c^2}. \quad (6)$$

grals. Concretely, $\hat{f}_2 = -50.1(1)$ for η_c hadronic decay, and $-69.5(1)$ for η_b decay. For completeness, here we also enumerate the numerical values of the non-logarithmic parts of f_1 and g_1 in (3): $\hat{f}_1 = 10.62$, $\hat{g}_1 = 16.20$ for η_c hadronic decay; $\hat{f}_1 = 9.73$, $\hat{g}_1 = 15.06$ for η_b decay.

Validate the NRQCD factorization for S-wave onium inclusive decay at NNLO!
We also obtain the following RGE for the leading 4-fermion NRQCD operator:

$$\frac{d\langle\mathcal{O}_1(^1S_0)\rangle_{\eta_c}}{d\ln\mu_\Lambda^2} = \alpha_s^2\left(C_F^2 + \frac{C_A C_F}{2}\right)\langle\mathcal{O}_1(^1S_0)\rangle_{\eta_c} - \frac{4}{3}\frac{\alpha_s}{\pi}C_F\frac{\langle\mathcal{P}_1(^1S_0)\rangle_{\eta_c}}{m^2} + \cdots, \quad (7)$$

Phenomenological study: hadronic width



Input parameters:

$$\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_c} = 0.470\,\mathrm{GeV}^3,\ \langle v^2\rangle_{\eta_c} = \frac{0.430\,\mathrm{GeV}^2}{m_c^2},$$
$$\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_b} = 3.069\,\mathrm{GeV}^3,\ \langle v^2\rangle_{\eta_b} = -0.009. \qquad (9)$$

PDG values:

$$\Gamma_{\mathrm{had}}(\eta_c) = 31.8 \pm 0.8\ \mathrm{MeV},$$
$$\Gamma_{\mathrm{had}}(\eta_b) = 10^{+5}_{-4}\ \mathrm{MeV}$$

FIG. 2: The predicted hadronic widths of η_c (top) and η_b (bottom) as functions of μ_R, at various level of accuracy in α_s and v expansion. The horizontal blue bands correspond to the measured hadronic widths taken from PDG 2016 [4], with $\Gamma_{\mathrm{had}}(\eta_c) = 31.8 \pm 0.8$ MeV and $\Gamma_{\mathrm{had}}(\eta_b) = 10^{-4}_{+5}$ MeV. The label "LO" represents the NRQCD prediction at the lowest-order α_s and v, and the label "NLO" denotes the "LO" prediction plus the $\mathcal{O}(\alpha_s)$ perturbative correction, while the label "NNLO" signifies the "NLO" prediction plus the $\mathcal{O}(\alpha_s^2)$ perturbative correction. The label "vLO" represents the "LO" prediction together with the tree-level order-v^2 correction, and the label "vNLO" designates the "vLO" prediction supplemented with the relative order-α_s and order-$\alpha_s v^2$ correction, while the label "vNNLO" refers to the "vNLO" prediction further supplemented with the order-α_s^2 correction. The green bands are obtained by varying μ_Λ from 1 GeV to twice heavy quark mass, and the central curve inside the bands are obtained by setting μ_Λ equal to heavy quark mass.

Phenomenological study of Br($\eta_{c,b} \to \gamma\gamma$), Non-Perturbative matrix elements cancel out

For η_c more than 10σ discrepancy!

$$\mathrm{Br}(\eta_c \to \gamma\gamma) = \frac{8\alpha^2}{9\alpha_s^2}\left\{1 - \frac{\alpha_s}{\pi}\left[4.17\ln\frac{\mu_R^2}{4m_c^2} + 14.00\right] + \frac{\alpha_s^2}{\pi^2}\left[4.34\ln^2\frac{\mu_R^2}{4m_c^2} + 22.75\ln\frac{\mu_R^2}{4m_c^2} + 78.8\right] + 2.24\langle v^2\rangle_{\eta_c}\frac{\alpha_s}{\pi}\right\}, \tag{10a}$$

$$\mathrm{Br}(\eta_b \to \gamma\gamma) = \frac{\alpha^2}{18\alpha_s^2}\left\{1 - \frac{\alpha_s}{\pi}\left[3.83\ln\frac{\mu_R^2}{4m_b^2} + 13.11\right] + \frac{\alpha_s^2}{\pi^2}\left[3.67\ln^2\frac{\mu_R^2}{4m_b^2} + 20.30\ln\frac{\mu_R^2}{4m_b^2} + 85.5\right] + 1.91\langle v^2\rangle_{\eta_b}\frac{\alpha_s}{\pi}\right\}. \tag{10b}$$

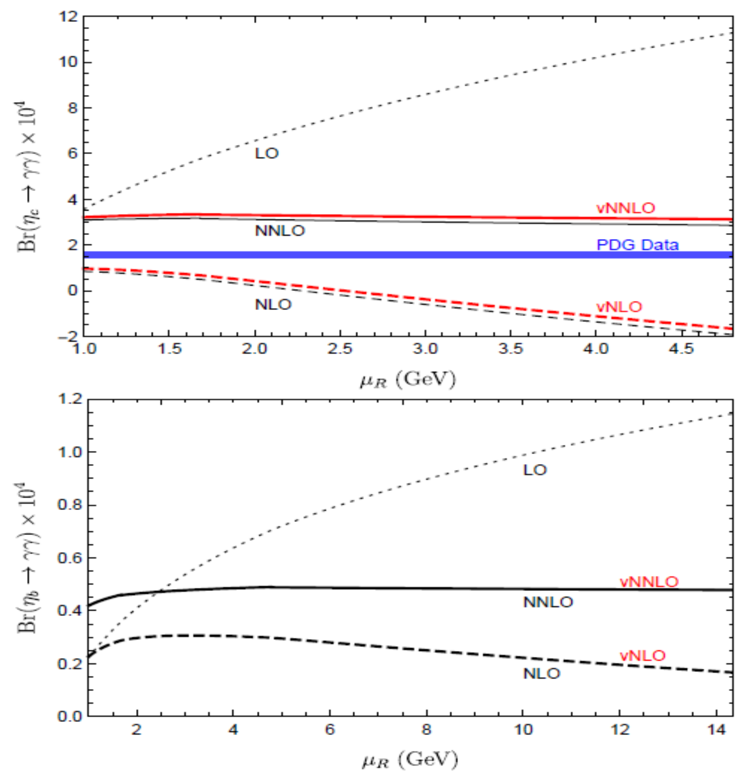


FIG. 3: The predicted branching fractions of $\eta_c \to \gamma\gamma$ (top) and $\eta_b \to \gamma\gamma$ (bottom) as functions of μ_R, at various level of accuracy in α_s and v. The blue band corresponds to the measured branching ratio for $\eta_c \to \gamma\gamma$ taken from PDG 2016 [4], with $\mathrm{Br}(\eta_c \to \gamma\gamma) = (1.59 \pm 0.13) \times 10^{-4}$. The labels characterizing different curves are the same as in Fig. 2.

To date most refined predictior for $\eta_b \to \gamma\gamma$

$$\mathrm{Br}(\eta_b \to \gamma\gamma) = (4.8 \pm 0.7) \times 10^{-5},$$

Summary

- **Investigated NNLO QCD corrections to $\gamma\gamma^* \rightarrow \eta_c$ ， ($\chi_{c0,2} \rightarrow$ 2γ)， $\eta_c \rightarrow$ LH。** Observe significant NNLO corrections. Alarming discrepancy with the existing measurements.

- Perturbative expansion seems to have poor convergence behavior for charmonium

- Perturbative expansion bears much better behavior for bottomonium

Personal biased perspectives

Maybe Nature is just not so mercy to us:

The charm quark is simply not heavy enough to warrant the reliable application of NRQCD to charmonium, just like one cannot fully trust HQET to cope with charmed hadron

Symptom: mc is not much greater than Λ_QCD

Bigger value of α_s at charm mass scale

But we should still trust NRQCD to be capable of rendering qualitatively correct phenomenology for charmonium

We may need be less ambitious for soliciting precision predictions

Thanks for your attention!

Thanks PKU for warm hospitality!