Quasi-two-body decays $B_{(s)} \to D(\rho,\rho',\rho'' \to)\pi\pi$ in PQCD approach

Ai-Jun Ma（马爱军）

Ya Li, Wen-Fei Wang, and Zhen-Jun Xiao

Nanjing Normal University

Nucl. Phys. B 923, 54 (2017); arXiv:1708.01889[hep-ph]

HFCPV, CCNU

2017.10

Outline

- **Motivation and introduction**
- **Framework**
- **Results and discussion**
- **Summary**

Motivation and introduction

Physics Letters B 763 (2016) 29–39

Contents lists available at ScienceDirect

Physics Letters B

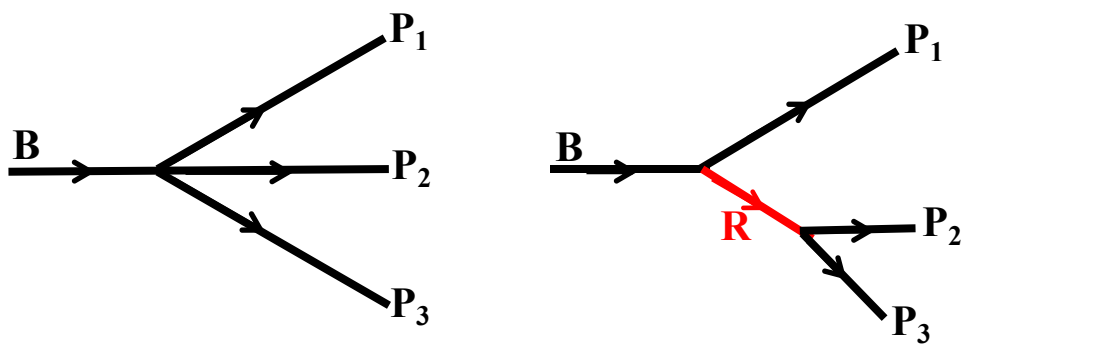
www.elsevier.com/locate/physletb

Quasi-two-body decays $B \to K\rho \to K\pi\pi$ in perturbative QCD approach

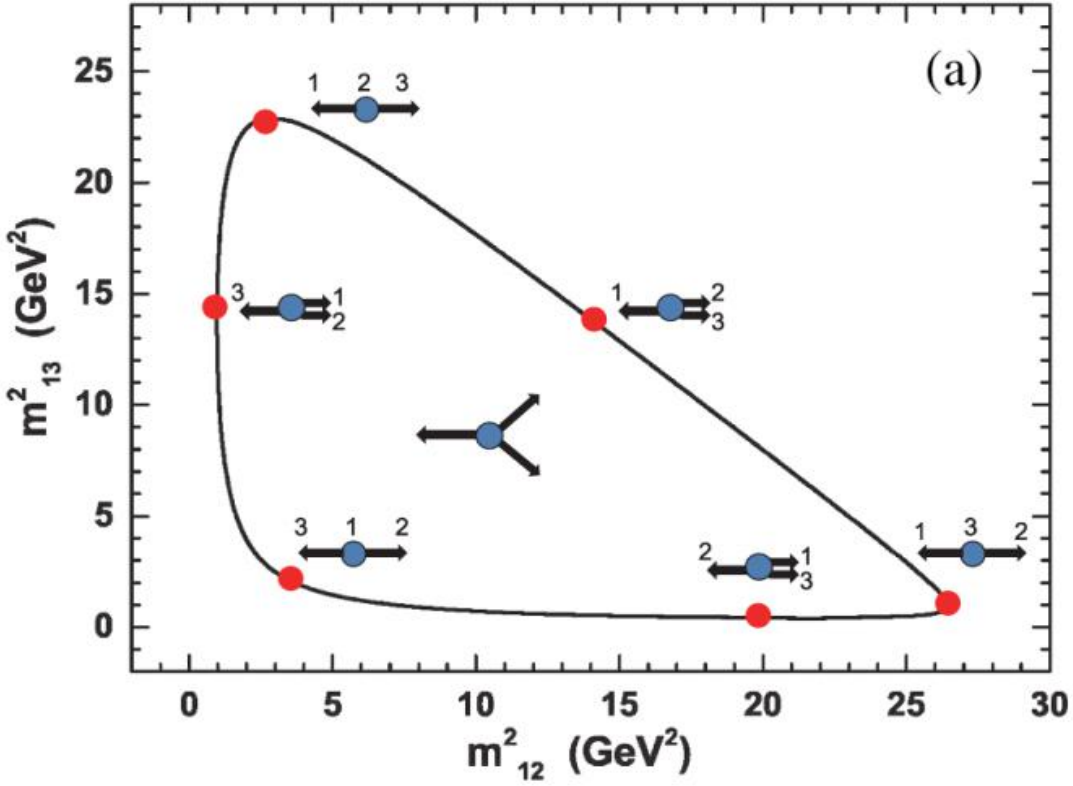
Wen-Fei Wang [a,b], Hsiang-nan Li [a,*]

D *P* **Ya Li's talk**

Three-body decays have non-trivial kinematics and the phase space distributions contain far more information than the two-body decays.



Quasi-two-body decays



Phys. Rev. D 94, 094015(2016)

- The study of *CP* violation (*CPV*) in charmless three-body *B* decays is one of the important topics in contemporary particle physics.
- **For $B \to Dhh'$: study spectroscopy in the DK, $D\pi$, Dp, $K\pi$, $\pi\pi$ and $p\pi$ systems and understand such resonant states; measure the CKM angle...**

Belle: ***Phys. Lett. B542, 171 (2002), Phys. Rev. D69, 112002 (2004), Phys. Rev. D76, 012006 (2007), Phys. Rev. D80, 052005 (2009)…***

BABAR*:** ***Phys. Rev. Lett. 95, 171802 (2005), Phys. Rev. Lett. 96, 011803 (2006), Phys. Rev. D79, 112004 (2009)…

LHCb: ***Phys. Rev. D90, 072003 (2014), Phys. Rev. D91, 092002 (2015), Phys. Rev. D92, 032002 (2015), Phys. Rev. D92, 012012 (2015), Phys. Rev. D94, 072001(2016) …***

$B_{(s)} \to D(\rho \to)\pi\pi$

	PDG	HFAG
$B^+ \to \bar{D}^0 \rho^+$	$[1.34 \pm 0.18] \times 10^{-2}$	
$B^+ \to D_s^+ \rho^0$	$< 3.0 \times 10^{-4}$	
$B^0 \to D^- \rho^+$	$[7.9 \pm 1.3] \times 10^{-3}$	
$B^0 \to \bar{D}^0 \rho^0$	$[3.21 \pm 0.21] \times 10^{-4}$	$[2.9 \pm 1.1] \times 10^{-4}$
$B^0 \to D_s^+ \rho^-$	$< 2.4 \times 10^{-5}$	$[0.11 \pm 0.09] \times 10^{-4}$
$B_s^0 \to D_s^- \rho^+$	$[6.9 \pm 1.4] \times 10^{-3}$	$[8.5 \pm 2.1] \times 10^{-3}$

PDG2016

$\rho(770)$ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
$\pi\pi$		~ 100 %		363
$\rho(770)^\pm$ decays				
$\pi^\pm\gamma$		$(4.5 \pm 0.5) \times 10^{-4}$	S=2.2	375
$\pi^\pm\eta$		$< 6 \times 10^{-3}$	CL=84%	152
$\pi^\pm\pi^+\pi^-\pi^0$		$< 2.0 \times 10^{-3}$	CL=84%	254
$\rho(770)^0$ decays				
$\pi^+\pi^-\gamma$		$(9.9 \pm 1.6) \times 10^{-3}$		362
$\pi^0\gamma$		$(6.0 \pm 0.8) \times 10^{-4}$		376
$\eta\gamma$		$(3.00 \pm 0.20) \times 10^{-4}$		194
$\pi^0\pi^0\gamma$		$(4.5 \pm 0.8) \times 10^{-5}$		363
$\mu^+\mu^-$	[i]	$(4.55 \pm 0.28) \times 10^{-5}$		373
e^+e^-	[i]	$(4.72 \pm 0.05) \times 10^{-5}$		388
$\pi^+\pi^-\pi^0$		$(1.01^{+0.54}_{-0.36} \pm 0.34) \times 10^{-4}$		323
$\pi^+\pi^-\pi^+\pi^-$		$(1.8 \pm 0.9) \times 10^{-5}$		251
$\pi^+\pi^-\pi^0\pi^0$		$(1.6 \pm 0.8) \times 10^{-5}$		257
$\pi^0 e^+ e^-$		$< 1.2 \times 10^{-5}$	CL=90%	376

$B_{(s)} \to D\rho$ in 2 body framework

SU(3) symmetry:
Phys. Rev. D 75, 074021 (2007)...

Factorization-Assisted Topological-Amplitude Approach (FAT):
Phys. Rev. D 92, 094016 (2015) ...

Perturbative QCD factorization approach(PQCD):
Phys. Rev. D 69, 094018 (2004), Phys. Rev. D 78, 014018 (2008),
J. Phys. G 37, 015002 (2010)...

.....

Phys.Rev. D76, 012006(2007) *Belle*

[arXiv:1007.4464] *BABAR*

PHYSICAL REVIEW D **92,** 032002 (2015)

Dalitz plot analysis of $B^0 \to \bar{D}^0\pi^+\pi^-$ decays

R. Aaij *et al.*[*]
(LHCb Collaboration)

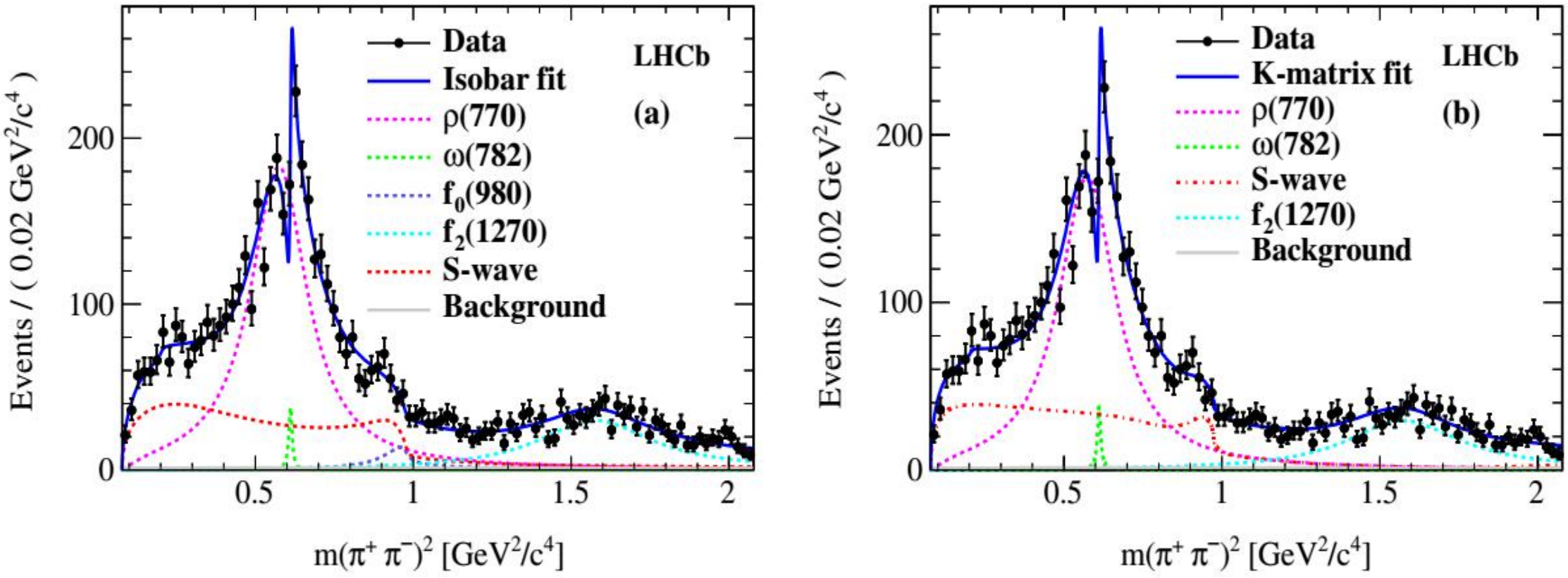


FIG. 9 (color online). Distributions of $m^2(\pi^+\pi^-)$ in the $\rho(770)$ mass region. The different fit components are described in the legend. Results from (a) the isobar model and (b) the K-matrix model are shown.

PHYSICAL REVIEW D **92,** 032002 (2015)

Dalitz plot analysis of $B^0 \to \bar{D}^0\pi^+\pi^-$ decays

The first observation of the decays
$B^0 \to \bar{D}^0\rho(1450)$

R. Aaij *et al.*[*]
(LHCb Collaboration)

TABLE XI. Measured branching fractions of $\mathcal{B}(B^0 \to rh_3) \times \mathcal{B}(r \to h_1h_2)$ for the isobar and K-matrix models. The first uncertainty is statistical, the second the experimental systematic, the third the model-dependent systematic, and the fourth the uncertainty from the normalization $B^0 \to D^*(2010)^-\pi^+$ channel.

Resonance	Isobar ($\times 10^{-5}$)	K-matrix ($\times 10^{-5}$)
$f_0(500)$	$11.2 \pm 0.8 \pm 0.5 \pm 2.1 \pm 0.5$	n/a
$f_0(980)$	$1.34 \pm 0.25 \pm 0.10 \pm 0.46 \pm 0.06$	n/a
$f_0(2020)$	$1.35 \pm 0.31 \pm 0.14 \pm 0.85 \pm 0.06$	n/a
S-wave	$14.1 \pm 0.5 \pm 0.6 \pm 1.3 \pm 0.7$	$14.2 \pm 0.6 \pm 1.5 \pm 0.9 \pm 0.7$
$\rho(770)$	$32.1 \pm 1.0 \pm 1.2 \pm 0.9 \pm 1.5$	$31.0 \pm 1.0 \pm 2.1 \pm 0.7 \pm 1.5$
$\omega(782)$	$0.42 \pm 0.11 \pm 0.02 \pm 0.03 \pm 0.02$	$0.43 \pm 0.11 \pm 0.02 \pm 0.02 \pm 0.02$
$\rho(1450)$	$1.36 \pm 0.28 \pm 0.08 \pm 0.19 \pm 0.06$	$1.91 \pm 0.37 \pm 0.73 \pm 0.19 \pm 0.09$
$\rho(1700)$	$0.33 \pm 0.11 \pm 0.06 \pm 0.05 \pm 0.02$	$0.73 \pm 0.18 \pm 0.53 \pm 0.10 \pm 0.03$
$f_2(1270)$	$9.5 \pm 0.5 \pm 0.4 \pm 1.0 \pm 0.4$	$9.1 \pm 0.6 \pm 0.8 \pm 0.5 \pm 0.4$
$D_0^*(2400)^-$	$7.7 \pm 0.5 \pm 0.3 \pm 0.3 \pm 0.4$	$8.0 \pm 0.5 \pm 0.8 \pm 0.4 \pm 0.4$
$D_2^*(2460)^-$	$24.4 \pm 0.7 \pm 1.0 \pm 0.4 \pm 1.2$	$23.8 \pm 0.7 \pm 1.2 \pm 0.5 \pm 1.1$
$D_3^*(2760)^-$	$1.03 \pm 0.16 \pm 0.07 \pm 0.08 \pm 0.05$	$1.34 \pm 0.19 \pm 0.16 \pm 0.06 \pm 0.06$

Framework

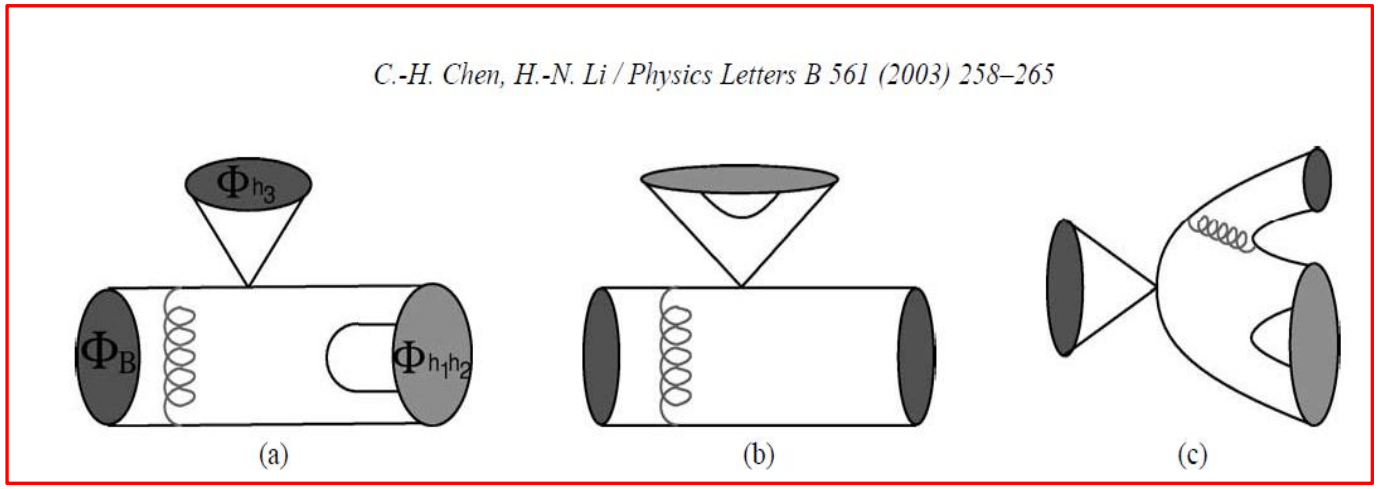
PQCD approach based on k_T factorization **Qi-An's talk**

- **Phys. Lett. B561, 258-265 (2003)** ***Three body nonleptonic B decays in perturbative QCD***
  (Chuan-Hung Chen and Hsiang-nan Li)

A new input is necessary in order to catch dominant contributions to three-body decays in a simple manner, the idea is to introduce two-meson distribution amplitudes.

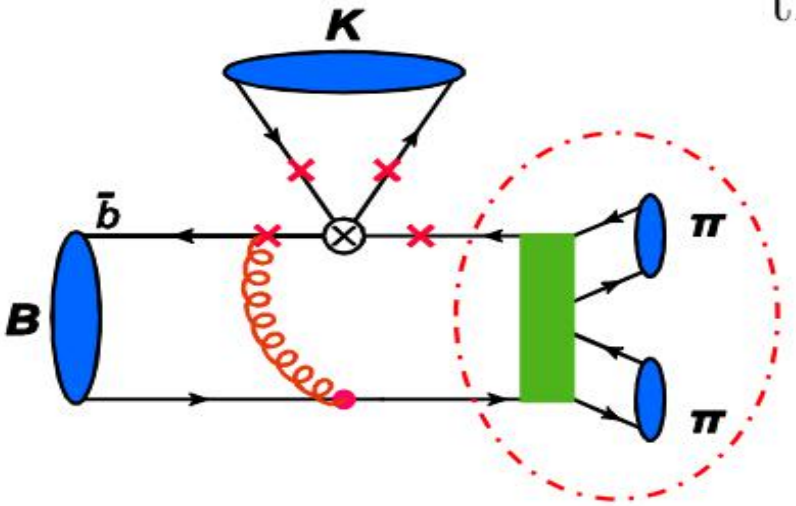
A factorization formula for a $B \to h_1 h_2 h_3$ decay amplitude is written as:

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}.$$



- **Phys. Lett. B 763, 29 (2016)** ***Quasi-two-body decays $B \to K\rho \to K\pi\pi$ in perturbative QCD approach***

(Wen-Fei Wang and Hsiang-nan Li)



From the definition of the vector current time-like form factor F_π we have

$$\frac{g_{\rho\pi\pi} w f_\rho}{D_\rho} = F_\pi^\rho(s),$$

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_K \otimes \left[\Phi_\rho \frac{F_\pi^\rho(s)}{f_\rho}(2\zeta - 1)\right]$$

Wen-Fei's Talk @NKU

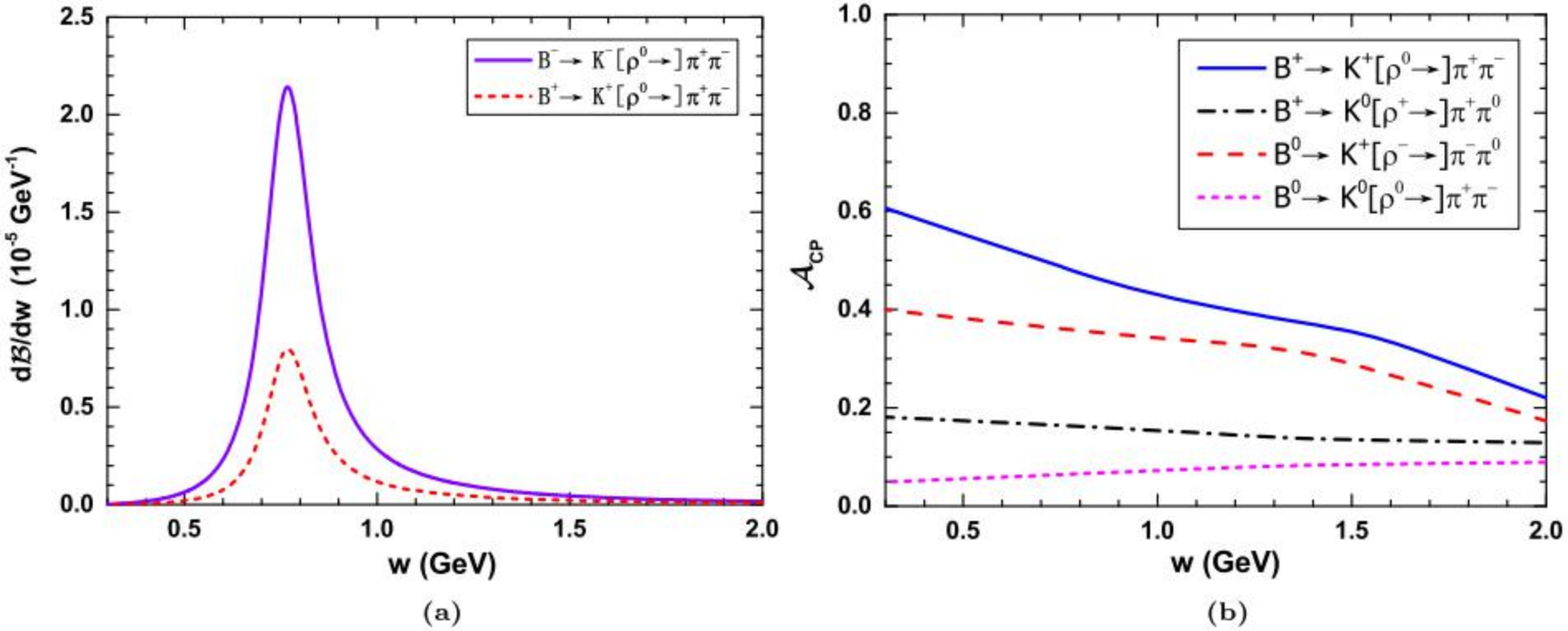


Fig. 2. (a) Differential branching ratios for the $B^{\pm} \to K^{\pm}\rho^0 \to K^{\pm}\pi^+\pi^-$ decays, and (b) differential distributions of $\mathcal{A}_{CP}$ in w for the $B \to K\rho \to K\pi\pi$ decays.

$B_{(s)} \to D(\rho \to)\pi\pi$

The momenta can be chosen as:

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad p = \frac{m_B}{\sqrt{2}}(1 - r^2, \eta, 0_T), \quad p_3 = \frac{m_B}{\sqrt{2}}(r^2, 1 - \eta, 0_T),$$

$$k_B = (0, x_B \frac{m_B}{\sqrt{2}}, k_{BT}), \quad k = (z \frac{(1 - r^2)m_B}{\sqrt{2}}, 0, k_T), \quad k_3 = (0, x_3 \frac{(1 - \eta)m_B}{\sqrt{2}}, k_{3_T}),$$

$$p_1^+ = \zeta p^+, \quad p_2^+ = (1 - \zeta)p^+, \quad p_1^- = (1 - \zeta)p^-, \quad p_2^- = \zeta p^-,$$

$$\eta = w^2/[(1 - r^2)m_B^2]$$

$$2m_\pi \leq w \leq m_B - m_D$$

The P-wave two-pion distribution amplitudes are organized into:

$$\phi_{\pi\pi}^{I=1} = \frac{1}{\sqrt{2N_c}}\left[\not{p}\phi_{v\nu=-}^{I=1}(z,\zeta,w^2) + w\phi_s^{I=1}(z,\zeta,w^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{w(2\zeta-1)}\phi_{t\nu=+}^{I=1}(z,\zeta,w^2)\right],$$

$$\phi_{v\nu=-}^{I=1}(z,\zeta,w^2) \equiv \phi^0(z,\zeta,w^2) = \frac{3F_\pi(w^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a_2^0 C_2^{3/2}(1-2z)\right] P_1(2\zeta-1),$$

$$\phi_s^{I=1}(z,\zeta,w^2) \equiv \phi^s(z,\zeta,w^2) = \frac{3F_s(w^2)}{2\sqrt{2N_c}}(1-2z)\left[1 + a_2^s\left(1 - 10z + 10z^2\right)\right] P_1(2\zeta-1),$$

$$\phi_{t\nu=+}^{I=1}(z,\zeta,w^2) \equiv \phi^t(z,\zeta,w^2) = \frac{3F_t(w^2)}{2\sqrt{2N_c}}(1-2z)^2\left[1 + a_2^t C_2^{3/2}(1-2z)\right] P_1(2\zeta-1),$$

the Legendre polynomial $P_1(2\zeta-1) = 2\zeta - 1$

PHYSICAL REVIEW D **86,** 032013 (2012)

Precise measurement of the $e^+e^- \to \pi^+\pi^-(\gamma)$ cross section with the initial-state radiation method at *BABAR*

$$F_\pi(w^2) = \left[\mathrm{GS}_\rho(w^2, m_\rho, \Gamma_\rho) \frac{1 + c_\omega \mathrm{BW}_\omega(w^2, m_\omega, \Gamma_\omega)}{1 + c_\omega} + \sum c_i \mathrm{GS}_i(w^2, m_i, \Gamma_i) \right] \left(1 + \sum c_i\right)^{-1}$$

the Gounaris-Sakurai (GS) model

$$\mathrm{BW}^{\mathrm{GS}}(s, m, \Gamma) = \frac{m^2(1 + d(m)\Gamma/m)}{m^2 - s + f(s, m, \Gamma) - im\Gamma(s, m, \Gamma)},$$

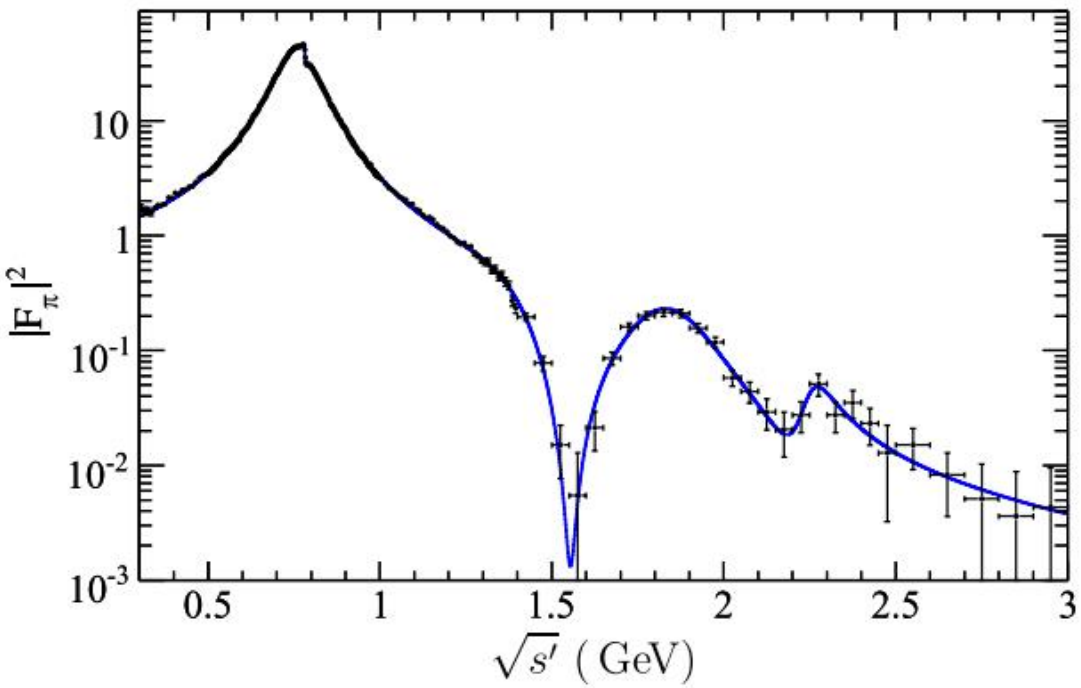
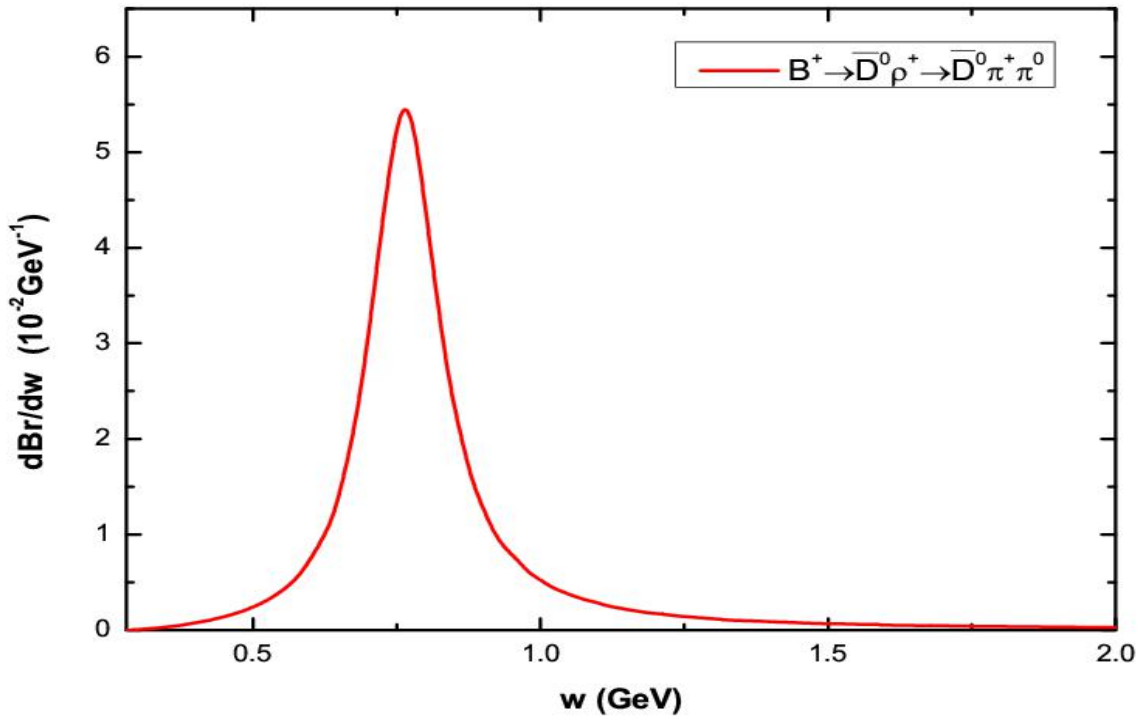


FIG. 45 (color online). The pion form factor-squared measured by *BABAR* as a function of $\sqrt{s'}$ from 0.3 to 3 GeV and the VDM fit described in the text.

Results and discussion



$$\mathcal{B}(B^+ \to \bar{D}^0 \rho^+ \to \bar{D}^0 \pi^+ \pi^0) = \begin{cases} 89, & \text{for} \quad w = [m_\rho - \Gamma_\rho, m_\rho + \Gamma_\rho], \\ 109, & \text{for} \quad w = [m_\rho - 3\Gamma_\rho, m_\rho + 3\Gamma_\rho], \\ 115, & \text{for} \quad 2m_\pi \leq w \leq m_B - m_D. \end{cases}$$

10^{-2}

$$\Gamma = 149.1 \pm 0.8 \text{ MeV}$$

Phys. Rev. D 78, 014008 (2008)

Phys. Rev. D 92, 094016(2015)

Table 1

The PQCD predictions for the branching ratios (in units of 10^{-4}) of $B_{(s)} \to \bar{D}_{(s)}\rho \to \bar{D}_{(s)}\pi\pi$ decays in the quasi-two-body (second column) and the two-body (third column) framework. We also list those currently available measured values [64,65] of the two-body cases and the central values of the theoretical predictions as given in Ref. [60] and Ref. [52].

Decays	Quasi-two-body ≈	Two-body	Data [64,65]	Two-body [60]	FAT [52]
$\mathcal{B}(B^+ \to \bar{D}^0\rho^+ \to \bar{D}^0\pi^+\pi^0)$	115^{+59}_{-38}	116^{+56}_{-37}	134 ± 18	111	105
$\mathcal{B}(B^0 \to D^-\rho^+ \to D^-\pi^+\pi^0)$	$82.3^{+49.2}_{-29.0}$	$88.2^{+49.7}_{-30.7}$	79 ± 13	67.0	65.3
$\mathcal{B}(B^0 \to \bar{D}^0\rho^0 \to \bar{D}^0\pi^+\pi^-)$	$1.39^{+1.24}_{-0.90}$	$1.23^{+0.90}_{-0.64}$	2.9 ± 1.1	1.99	2.60
$\mathcal{B}(B_s^0 \to \bar{D}^0\rho^0 \to \bar{D}^0\pi^+\pi^-)$	$0.026^{+0.010}_{-0.006}$	$0.022^{+0.006}_{-0.005}$	–	0.042	0.010
$\mathcal{B}(B_s^0 \to D^-\rho^+ \to D^-\pi^+\pi^0)$	$0.051^{+0.022}_{-0.014}$	$0.044^{+0.012}_{-0.011}$	–	0.079	0.019
$\mathcal{B}(B_s^0 \to D_s^-\rho^+ \to D_s^-\pi^+\pi^0)$	$77.2^{+40.2}_{-25.6}$	$79.5^{+40.6}_{-26.3}$	85 ± 21	47.0	78.6

Table 2
The PQCD predictions for the branching ratios of the CKM suppressed $B_{(s)} \to D_{(s)}\rho \to D_{(s)}\pi\pi$ decays in the quasi-two-body (second column) and the two-body (third column) framework. We also list those currently available measured values [64,65] of the two-body cases and the central values of the theoretical predictions as given in Ref. [61] and Ref. [52].

Decays	Quasi-two-body	Two-body	Data [64,65]	Two-body [61]	FAT [52]
$\mathcal{B}(B^+ \to D^0\rho^+ \to D^0\pi^+\pi^0)(10^{-7})$	$0.50^{+0.22}_{-0.14}$	$0.53^{+0.26}_{-0.14}$	–	0.93	4.80
$\mathcal{B}(B^0 \to D^+\rho^- \to D^+\pi^-\pi^0)(10^{-7})$	$7.63^{+5.92}_{-3.08}$	$9.45^{+6.48}_{-4.89}$	–	12.7	9.40
$\mathcal{B}(B^0 \to D^0\rho^0 \to D^0\pi^+\pi^-)(10^{-7})$	$0.13^{+0.09}_{-0.08}$	$0.13^{+0.10}_{-0.05}$	–	0.34	1.20
$\mathcal{B}(B^+ \to D^+\rho^0 \to D^+\pi^+\pi^-)(10^{-7})$	$5.33^{+3.60}_{-2.65}$	$5.99^{+3.93}_{-2.91}$	–	7.50	3.30
$\mathcal{B}(B_s^0 \to D^0\rho^0 \to D^0\pi^+\pi^-)(10^{-7})$	$3.41^{+1.03}_{-0.75}$	$3.13^{+0.98}_{-0.64}$	–	1.90	1.30
$\mathcal{B}(B_s^0 \to D^+\rho^- \to D^+\pi^-\pi^0)(10^{-7})$	$6.88^{+1.98}_{-1.58}$	$6.30^{+1.96}_{-1.29}$	–	3.70	2.50
$\mathcal{B}(B^+ \to D_s^+\rho^0 \to D_s^+\pi^+\pi^-)(10^{-5})$	$1.52^{+1.11}_{-0.82}$	$1.82^{+1.19}_{-0.91}$	< 30	1.94	1.68
$\mathcal{B}(B^0 \to D_s^+\rho^- \to D_s^+\pi^-\pi^0)(10^{-5})$	$2.82^{+2.04}_{-1.53}$	$3.37^{+2.19}_{-1.63}$	1.1 ± 0.9	3.59	3.12

PDG2016

$\rho(1450)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	seen	720
4π	seen	669
e^+e^-	seen	732
$\eta\rho$	seen	311
$a_2(1320)\pi$	not seen	54
$K\overline{K}$	not seen	541
$K\overline{K}^*(892)+$ c.c.	possibly seen	229
$\eta\gamma$	seen	630
$f_0(500)\gamma$	not seen	–
$f_0(980)\gamma$	not seen	398
$f_0(1370)\gamma$	not seen	92
$f_2(1270)\gamma$	not seen	177

$\rho(1700)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$2(\pi^+\pi^-)$	large	803
$\rho\pi\pi$	dominant	653
$\rho^0\pi^+\pi^-$	large	651
$\rho^\pm\pi^\mp\pi^0$	large	652
$a_1(1260)\pi$	seen	404
$h_1(1170)\pi$	seen	447
$\pi(1300)\pi$	seen	349
$\rho\rho$	seen	372
$\pi^+\pi^-$	seen	849
$\pi\pi$	seen	849
$K\overline{K}^*(892)+$ c.c.	seen	496
$\eta\rho$	seen	545
$a_2(1320)\pi$	not seen	334
$K\overline{K}$	seen	704
e^+e^-	seen	860
$\pi^0\omega$	seen	674

$$\Gamma_{\rho'\to\pi\pi} = \frac{g^2_{\rho'\pi\pi}}{6\pi}\frac{|\overrightarrow{p_\pi}(m^2_{\rho'})|^3}{m^2_{\rho'}}.$$

$$\mathcal{B}(\rho' \to \pi\pi) = 10.04^{+5.23}_{-2.61}\%$$

Phys. Lett. B 763, 29 (2016)

$$\mathcal{B}(\rho'' \to \pi\pi) = 8.11^{+2.22}_{-1.47}\%$$

Phys. Rev. D 96, 036014 (2017)

Quasi-two body ↓ **Two body** ↓

$$\mathcal{B}(B_{(s)} \to D(\rho', \rho'') \to D\pi\pi) = \mathcal{B}(B_{(s)} \to D(\rho', \rho'')) \cdot \mathcal{B}((\rho', \rho'') \to \pi\pi).$$

Decay modes	Quasi-two-body decays	Two-body decays
$B_{(s)} \to \bar{D}_{(s)}\rho' \to \bar{D}_{(s)}\pi\pi$	$\mathcal{B}$	$\mathcal{B}$
$B^+ \to \bar{D}^0\rho'^+ \to \bar{D}^0\pi^+\pi^0$	$(8.68^{+4.84}_{-2.91}(\omega_B)^{+0.42}_{-0.33}(a_2^t)^{+0.11}_{-0.09}(a_2^0)^{+0.04}_{-0.05}(a_2^s)^{+0.65}_{-0.58}(C_D)) \times 10^{-4}$	$(8.65^{+4.88}_{-2.98}) \times 10^{-3}$
$B^0 \to D^-\rho'^+ \to D^-\pi^+\pi^0$	$(6.80^{+4.27}_{-2.49}(\omega_B)^{+0.17}_{-0.12}(a_2^t)^{+0.09}_{-0.03}(a_2^0)^{+0.08}_{-0.08}(a_2^s)^{+0.59}_{-0.50}(C_D)) \times 10^{-4}$	$(6.77^{+4.30}_{-2.53}) \times 10^{-3}$
$B^0 \to \bar{D}^0\rho'^0 \to \bar{D}^0\pi^+\pi^-$	$(9.04^{+3.71}_{-2.75}(\omega_B)^{+4.83}_{-4.26}(a_2^t)^{+0.45}_{-0.59}(a_2^0)^{+0.04}_{-0.07}(a_2^s)^{+0.14}_{-0.10}(C_D)) \times 10^{-6}$	$(9.00^{+6.08}_{-5.18}) \times 10^{-5}$
$B_s^0 \to D^-\rho'^+ \to D^-\pi^+\pi^0$	$(4.21^{+0.55}_{-0.61}(\omega_B)^{+1.10}_{-0.81}(a_2^t)^{+0.27}_{-0.25}(a_2^0)^{+0.46}_{-0.39}(a_2^s)^{+0.11}_{-0.19}(C_D)) \times 10^{-7}$	$(4.19^{+1.34}_{-1.13}) \times 10^{-6}$
$B_s^0 \to \bar{D}^0\rho'^0 \to \bar{D}^0\pi^+\pi^-$	$(1.88^{+0.48}_{-0.20}(\omega_B)^{+0.57}_{-0.34}(a_2^t)^{+0.12}_{-0.11}(a_2^0)^{+0.25}_{-0.17}(a_2^s)^{+0.10}_{-0.08}(C_D)) \times 10^{-7}$	$(1.87^{+0.80}_{-0.45}) \times 10^{-6}$
$B_s^0 \to D_s^-\rho'^+ \to D_s^-\pi^+\pi^0$	$(5.33^{+2.96}_{-1.80}(\omega_B)^{+0.00}_{-0.00}(a_2^t)^{+0.02}_{-0.01}(a_2^0)^{+0.00}_{-0.00}(a_2^s)^{+0.41}_{-0.40}(C_D)) \times 10^{-4}$	$(5.31^{+2.98}_{-1.83}) \times 10^{-3}$
$B_{(s)} \to D_{(s)}\rho' \to D_{(s)}\pi\pi$	$\mathcal{B}$	$\mathcal{B}$
$B^+ \to D^0\rho'^+ \to D^0\pi^+\pi^0$	$(1.51^{+0.33}_{-0.29}(\omega_B)^{+0.14}_{-0.05}(a_2^t)^{+0.13}_{-0.07}(a_2^0)^{+0.29}_{-0.25}(a_2^s)^{+0.04}_{-0.04}(C_D)) \times 10^{-8}$	$(1.50^{+0.48}_{-0.39}) \times 10^{-7}$
$B^+ \to D^+\rho'^0 \to D^+\pi^+\pi^-$	$(5.88^{+0.90}_{-0.82}(\omega_B)^{+1.46}_{-1.17}(a_2^t)^{+0.07}_{-0.06}(a_2^0)^{+0.88}_{-0.82}(a_2^s)^{+0.05}_{-0.04}(C_D)) \times 10^{-8}$	$(5.86^{+1.92}_{-1.65}) \times 10^{-7}$
$B^0 \to D^0\rho'^0 \to D^0\pi^+\pi^-$	$(9.75^{+3.30}_{-3.18}(\omega_B)^{+4.05}_{-2.36}(a_2^t)^{+1.25}_{-1.26}(a_2^0)^{+5.19}_{-3.71}(a_2^s)^{+1.22}_{-0.81}(C_D)) \times 10^{-10}$	$(9.71^{+7.53}_{-5.61}) \times 10^{-9}$
$B^0 \to D^+\rho'^- \to D^+\pi^-\pi^0$	$(7.10^{+1.06}_{-1.02}(\omega_B)^{+2.61}_{-2.03}(a_2^t)^{+0.03}_{-0.01}(a_2^0)^{+1.32}_{-1.22}(a_2^s)^{+0.13}_{-0.12}(C_D)) \times 10^{-8}$	$(7.07^{+3.10}_{-2.57}) \times 10^{-7}$
$B^+ \to D_s^+\rho'^0 \to D_s^+\pi^+\pi^-$	$(1.38^{+0.20}_{-0.20}(\omega_B)^{+0.42}_{-0.34}(a_2^t)^{+0.04}_{-0.04}(a_2^0)^{+0.22}_{-0.20}(a_2^s)^{+0.01}_{-0.01}(C_D)) \times 10^{-6}$	$(1.37^{+0.51}_{-0.44}) \times 10^{-5}$
$B^0 \to D_s^+\rho'^- \to D_s^+\pi^-\pi^0$	$(2.56^{+0.38}_{-0.36}(\omega_B)^{+0.79}_{-0.60}(a_2^t)^{+0.08}_{-0.08}(a_2^0)^{+0.38}_{-0.40}(a_2^s)^{+0.02}_{-0.02}(C_D)) \times 10^{-6}$	$(2.55^{+0.95}_{-0.81}) \times 10^{-5}$
$B_s^0 \to D^0\rho'^0 \to D^0\pi^+\pi^-$	$(3.26^{+0.47}_{-0.51}(\omega_B)^{+0.29}_{-0.31}(a_2^t)^{+0.21}_{-0.25}(a_2^0)^{+0.07}_{-0.08}(a_2^s)^{+0.19}_{-0.19}(C_D)) \times 10^{-8}$	$(3.25^{+0.62}_{-0.68}) \times 10^{-7}$
$B_s^0 \to D^+\rho'^- \to D^+\pi^-\pi^0$	$(6.56^{+0.93}_{-1.03}(\omega_B)^{+0.56}_{-0.66}(a_2^t)^{+0.39}_{-0.53}(a_2^0)^{+0.14}_{-0.18}(a_2^s)^{+0.38}_{-0.38}(C_D)) \times 10^{-8}$	$(6.53^{+1.22}_{-1.40}) \times 10^{-7}$

$$\mathcal{B}(B^0 \to \bar{D}^0\rho^0(1450) \to \bar{D}^0\pi^+\pi^-) = \begin{cases} 1.36 \pm 0.28 \pm 0.08 \pm 0.19 \pm 0.06 \times 10^{-5} \text{ (Isobar)}, \\ 1.91 \pm 0.37 \pm 0.73 \pm 0.19 \pm 0.09 \times 10^{-5} \text{ (K} - \text{matrix)} \end{cases}$$

Decay modes	Quasi-two-body decays	Two-body decays
$B_{(s)} \to \bar{D}_{(s)}\rho' \to \bar{D}_{(s)}\pi\pi$	$\mathcal{B}$	$\mathcal{B}$
$B^+ \to \bar{D}^0\rho'^+ \to \bar{D}^0\pi^+\pi^0$	$(8.68^{+4.84}_{-2.91}(\omega_B)^{+0.42}_{-0.33}(a_2^t)^{+0.11}_{-0.09}(a_2^0)^{+0.04}_{-0.05}(a_2^s)^{+0.65}_{-0.58}(C_D)) \times 10^{-4}$	$(8.65^{+4.88}_{-2.98}) \times 10^{-3}$
$B^0 \to D^-\rho'^+ \to D^-\pi^+\pi^0$	$(6.80^{+4.27}_{-2.49}(\omega_B)^{+0.17}_{-0.12}(a_2^t)^{+0.09}_{-0.03}(a_2^0)^{+0.08}_{-0.08}(a_2^s)^{+0.59}_{-0.50}(C_D)) \times 10^{-4}$	$(6.77^{+4.30}_{-2.53}) \times 10^{-3}$
$B^0 \to \bar{D}^0\rho'^0 \to \bar{D}^0\pi^+\pi^-$	$(9.04^{+3.71}_{-2.75}(\omega_B)^{+4.83}_{-4.26}(a_2^t)^{+0.45}_{-0.59}(a_2^0)^{+0.04}_{-0.07}(a_2^s)^{+0.14}_{-0.10}(C_D)) \times 10^{-6}$	$(9.00^{+6.08}_{-5.18}) \times 10^{-5}$
$B_s^0 \to D^-\rho'^+ \to D^-\pi^+\pi^0$	$(4.21^{+0.55}_{-0.61}(\omega_B)^{+1.10}_{-0.81}(a_2^t)^{+0.27}_{-0.25}(a_2^0)^{+0.46}_{-0.39}(a_2^s)^{+0.11}_{-0.19}(C_D)) \times 10^{-7}$	$(4.19^{+1.34}_{-1.13}) \times 10^{-6}$
$B_s^0 \to \bar{D}^0\rho'^0 \to \bar{D}^0\pi^+\pi^-$	$(1.88^{+0.48}_{-0.20}(\omega_B)^{+0.57}_{-0.34}(a_2^t)^{+0.12}_{-0.11}(a_2^0)^{+0.25}_{-0.17}(a_2^s)^{+0.10}_{-0.08}(C_D)) \times 10^{-7}$	$(1.87^{+0.80}_{-0.45}) \times 10^{-6}$
$B_s^0 \to D_s^-\rho'^+ \to D_s^-\pi^+\pi^0$	$(5.33^{+2.96}_{-1.80}(\omega_B)^{+0.00}_{-0.00}(a_2^t)^{+0.02}_{-0.01}(a_2^0)^{+0.00}_{-0.00}(a_2^s)^{+0.41}_{-0.40}(C_D)) \times 10^{-4}$	$(5.31^{+2.98}_{-1.83}) \times 10^{-3}$
$B_{(s)} \to D_{(s)}\rho' \to D_{(s)}\pi\pi$	$\mathcal{B}$	$\mathcal{B}$
$B^+ \to D^0\rho'^+ \to D^0\pi^+\pi^0$	$(1.51^{+0.33}_{-0.29}(\omega_B)^{+0.14}_{-0.05}(a_2^t)^{+0.13}_{-0.07}(a_2^0)^{+0.29}_{-0.25}(a_2^s)^{+0.04}_{-0.04}(C_D)) \times 10^{-8}$	$(1.50^{+0.48}_{-0.39}) \times 10^{-7}$
$B^+ \to D^+\rho'^0 \to D^+\pi^+\pi^-$	$(5.88^{+0.90}_{-0.82}(\omega_B)^{+1.46}_{-1.17}(a_2^t)^{+0.07}_{-0.06}(a_2^0)^{+0.88}_{-0.82}(a_2^s)^{+0.05}_{-0.04}(C_D)) \times 10^{-8}$	$(5.86^{+1.92}_{-1.65}) \times 10^{-7}$
$B^0 \to D^0\rho'^0 \to D^0\pi^+\pi^-$	$(9.75^{+3.30}_{-3.18}(\omega_B)^{+4.05}_{-2.36}(a_2^t)^{+1.25}_{-1.26}(a_2^0)^{+5.19}_{-3.71}(a_2^s)^{+1.22}_{-0.81}(C_D)) \times 10^{-10}$	$(9.71^{+7.53}_{-5.61}) \times 10^{-9}$
$B^0 \to D^+\rho'^- \to D^+\pi^-\pi^0$	$(7.10^{+1.06}_{-1.02}(\omega_B)^{+2.61}_{-2.03}(a_2^t)^{+0.03}_{-0.01}(a_2^0)^{+1.32}_{-1.22}(a_2^s)^{+0.13}_{-0.12}(C_D)) \times 10^{-8}$	$(7.07^{+3.10}_{-2.57}) \times 10^{-7}$
$B^+ \to D_s^+\rho'^0 \to D_s^+\pi^+\pi^-$	$(1.38^{+0.20}_{-0.20}(\omega_B)^{+0.42}_{-0.34}(a_2^t)^{+0.04}_{-0.04}(a_2^0)^{+0.22}_{-0.20}(a_2^s)^{+0.01}_{-0.01}(C_D)) \times 10^{-6}$	$(1.37^{+0.51}_{-0.44}) \times 10^{-5}$
$B^0 \to D_s^+\rho'^- \to D_s^+\pi^-\pi^0$	$(2.56^{+0.38}_{-0.36}(\omega_B)^{+0.79}_{-0.60}(a_2^t)^{+0.08}_{-0.08}(a_2^0)^{+0.38}_{-0.40}(a_2^s)^{+0.02}_{-0.02}(C_D)) \times 10^{-6}$	$(2.55^{+0.95}_{-0.81}) \times 10^{-5}$
$B_s^0 \to D^0\rho'^0 \to D^0\pi^+\pi^-$	$(3.26^{+0.47}_{-0.51}(\omega_B)^{+0.29}_{-0.31}(a_2^t)^{+0.21}_{-0.25}(a_2^0)^{+0.07}_{-0.08}(a_2^s)^{+0.19}_{-0.19}(C_D)) \times 10^{-8}$	$(3.25^{+0.62}_{-0.68}) \times 10^{-7}$
$B_s^0 \to D^+\rho'^- \to D^+\pi^-\pi^0$	$(6.56^{+0.93}_{-1.03}(\omega_B)^{+0.56}_{-0.66}(a_2^t)^{+0.39}_{-0.53}(a_2^0)^{+0.14}_{-0.18}(a_2^s)^{+0.38}_{-0.38}(C_D)) \times 10^{-8}$	$(6.53^{+1.22}_{-1.40}) \times 10^{-7}$

$$\mathcal{B}(B^0 \to \bar{D}^0\rho^0(1700) \to \bar{D}^0\pi^+\pi^-) = \begin{cases} 0.33 \pm 0.11 \pm 0.06 \pm 0.05 \pm 0.02 \times 10^{-5} \text{ (Isobar)}, \\ 0.73 \pm 0.18 \pm 0.53 \pm 0.10 \pm 0.03 \times 10^{-5} \text{ (K} - \text{matrix)}. \end{cases}$$

Decay modes	Quasi-two-body decays	Two-body decays
$B_{(s)} \to \bar{D}_{(s)}\rho'' \to \bar{D}_{(s)}\pi\pi$	$\mathcal{B}$	$\mathcal{B}$
$B^+ \to \bar{D}^0\rho''^+ \to \bar{D}^0\pi^+\pi^0$	$(4.58^{+2.62}_{-1.59}(\omega_B)^{+0.17}_{-0.21}(a_2^t)^{+0.06}_{-0.05}(a_2^0)^{+0.01}_{-0.01}(a_2^s)^{+0.29}_{-0.30}(C_D)) \times 10^{-4}$	$(5.65^{+3.26}_{-2.01}) \times 10^{-3}$
$B^0 \to D^-\rho''^+ \to D^-\pi^+\pi^0$	$(3.30^{+2.09}_{-1.21}(\omega_B)^{+0.08}_{-0.07}(a_2^t)^{+0.03}_{-0.02}(a_2^0)^{+0.05}_{-0.04}(a_2^s)^{+0.26}_{-0.26}(C_D)) \times 10^{-4}$	$(4.07^{+2.60}_{-1.53}) \times 10^{-3}$
$B^0 \to \bar{D}^0\rho''^0 \to \bar{D}^0\pi^+\pi^-$	$(5.68^{+2.14}_{-1.65}(\omega_B)^{+2.96}_{-2.46}(a_2^t)^{+0.09}_{-0.09}(a_2^0)^{+0.27}_{-0.34}(a_2^s)^{+0.09}_{-0.07}(C_D)) \times 10^{-6}$	$(7.00^{+4.51}_{-3.98}) \times 10^{-5}$
$B_s^0 \to D^-\rho''^+ \to D^-\pi^+\pi^0$	$(2.08^{+0.49}_{-0.43}(\omega_B)^{+0.78}_{-0.60}(a_2^t)^{+0.11}_{-0.13}(a_2^0)^{+0.34}_{-0.30}(a_2^s)^{+0.04}_{-0.03}(C_D)) \times 10^{-7}$	$(2.56^{+1.21}_{-0.97}) \times 10^{-6}$
$B_s^0 \to \bar{D}^0\rho''^0 \to \bar{D}^0\pi^+\pi^-$	$(1.04^{+0.23}_{-0.21}(\omega_B)^{+0.39}_{-0.31}(a_2^t)^{+0.06}_{-0.07}(a_2^0)^{+0.17}_{-0.16}(a_2^s)^{+0.02}_{-0.02}(C_D)) \times 10^{-7}$	$(1.28^{+0.60}_{-0.51}) \times 10^{-6}$
$B_s^0 \to D_s^-\rho''^+ \to D_s^-\pi^+\pi^0$	$(2.57^{+1.46}_{-0.89}(\omega_B)^{+0.00}_{-0.00}(a_2^t)^{+0.01}_{-0.01}(a_2^0)^{+0.00}_{-0.00}(a_2^s)^{+0.20}_{-0.19}(C_D)) \times 10^{-4}$	$(3.17^{+1.82}_{-1.11}) \times 10^{-5}$
$B_{(s)} \to D_{(s)}\rho'' \to D_{(s)}\pi\pi$	$\mathcal{B}$	$\mathcal{B}$
$B^+ \to D^0\rho''^+ \to D^0\pi^+\pi^0$	$(8.39^{+1.17}_{-1.38}(\omega_B)^{+1.41}_{-0.89}(a_2^t)^{+0.64}_{-0.55}(a_2^0)^{+1.68}_{-1.27}(a_2^s)^{+0.06}_{-0.22}(C_D)) \times 10^{-9}$	$(1.03^{+0.31}_{-0.27}) \times 10^{-7}$
$B^+ \to D^+\rho''^0 \to D^+\pi^+\pi^-$	$(1.55^{+0.07}_{-0.07}(\omega_B)^{+0.36}_{-0.17}(a_2^t)^{+0.01}_{-0.01}(a_2^0)^{+0.33}_{-0.29}(a_2^s)^{+0.02}_{-0.02}(C_D)) \times 10^{-8}$	$(1.91^{+0.61}_{-0.43}) \times 10^{-7}$
$B^0 \to D^0\rho''^0 \to D^0\pi^+\pi^-$	$(3.62^{+0.90}_{-1.18}(\omega_B)^{+1.58}_{-0.81}(a_2^t)^{+0.45}_{-0.59}(a_2^0)^{+2.46}_{-1.79}(a_2^s)^{+0.25}_{-0.42}(C_D)) \times 10^{-10}$	$(4.46^{+3.82}_{-2.97}) \times 10^{-9}$
$B^0 \to D^+\rho''^- \to D^+\pi^-\pi^0$	$(1.41^{+0.06}_{-0.04}(\omega_B)^{+0.73}_{-0.37}(a_2^t)^{+0.01}_{-0.03}(a_2^0)^{+0.36}_{-0.29}(a_2^s)^{+0.03}_{-0.04}(C_D)) \times 10^{-8}$	$(1.74^{+1.01}_{-0.59}) \times 10^{-7}$
$B^+ \to D_s^+\rho''^0 \to D_s^+\pi^+\pi^-$	$(3.25^{+0.02}_{-0.14}(\omega_B)^{+1.32}_{-0.77}(a_2^t)^{+0.08}_{-0.08}(a_2^0)^{+0.61}_{-0.55}(a_2^s)^{+0.03}_{-0.03}(C_D)) \times 10^{-7}$	$(4.01^{+1.80}_{-1.18}) \times 10^{-6}$
$B^0 \to D_s^+\rho''^- \to D_s^+\pi^-\pi^0$	$(6.03^{+0.02}_{-0.26}(\omega_B)^{+2.44}_{-1.44}(a_2^t)^{+0.15}_{-0.14}(a_2^0)^{+1.14}_{-1.02}(a_2^s)^{+0.06}_{-0.05}(C_D)) \times 10^{-7}$	$(7.44^{+3.33}_{-2.21}) \times 10^{-6}$
$B_s^0 \to D^0\rho''^0 \to D^0\pi^+\pi^-$	$(1.65^{+0.32}_{-0.26}(\omega_B)^{+0.20}_{-0.15}(a_2^t)^{+0.14}_{-0.10}(a_2^0)^{+0.06}_{-0.05}(a_2^s)^{+0.09}_{-0.08}(C_D)) \times 10^{-8}$	$(2.04^{+0.52}_{-0.41}) \times 10^{-7}$
$B_s^0 \to D^+\rho''^- \to D^+\pi^-\pi^0$	$(3.31^{+0.64}_{-0.52}(\omega_B)^{+0.40}_{-0.30}(a_2^t)^{+0.26}_{-0.20}(a_2^0)^{+0.13}_{-0.08}(a_2^s)^{+0.18}_{-0.17}(C_D)) \times 10^{-8}$	$(4.08^{+0.99}_{-0.80}) \times 10^{-7}$

Summay

We studied the quasi-two-body $B_{(s)} \to D(\rho,\rho',\rho'' \to)\pi\pi$ decays by employing the PQCD factorization approach and found that:

- **For all considered decays, the PQCD predictions based on the quasi-two-body and the two-body framework agree well with each other and most of our predictions agree well with those currently available experimental measurements;**

- **We can extract the decay rates for the two body decays from the corresponding quasi-two-body decays.**

Thank you !