## 2017 手征有效场论研讨会,XiAn,Nov 16,China

# Octet baryon magnetic moments at next-to-next-to-leading order in covariant chiral perturbation theory



北京航空航天大学

Beihang University

Speaker : Yang Xiao Supervisor : L.S. Geng Cooperators : J.X. Lu , X.L. Ren







# **Magnetic moment of particles**



- Radiative correction (electron and muon)
- Internal structure (proton and neutron)

#### SU(3) Symmetry

#### **Sheldon Lee Glashow**



$$\begin{split} \mu(\Sigma^+) &= \mu(p) \\ \mu(\Lambda) &= \frac{1}{2}\mu(n) \\ \mu(\Xi^0) &= \mu(n) \\ \mu(\Xi^-) &= \mu(\Sigma^-) = -[\mu(p) + \mu(n)] \\ \mu(\Sigma^0) &= -\frac{1}{2}\mu(n) \end{split}$$

**Sidney Coleman** 

Coleman, Sidney R. et al. Phys.Rev.Lett. 6 423 (1961)

A Leading order calculation in Chiral Perturbation Theory(ChPT)

#### SU(3) Symmetry

#### **Sheldon Lee Glashow**



2.46 = 2.793-0.61 = -0.596-1.25 = -1.193-0.65 = -1.16 = -0.8? = 0.596

**Sidney Coleman** 

Coleman, Sidney R. et al. Phys.Rev.Lett. 6 423 (1961)

A Leading order calculation in Chiral Perturbation Theory(ChPT)

#### SU(3) Symmetry

#### **Sheldon Lee Glashow**



$$2.46 = 2.793$$

These relations are consistent with experiment data

? = 0.596

**Sidney Coleman** 

Coleman, Sidney R. et al. Phys.Rev.Lett. 6 423 (1961)

A Leading order calculation in Chiral Perturbation Theory(ChPT)

# **Loop correction**

#### SU(3) symmetry breaking

- 1. D. G. Caldi and H. Pagels, Phys. Rev. D 10, 3739 (1974).
- 2. J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985).
- 3. J. Gasser, M. E. Sainio, and A. Svarc, Nucl. Phys. B307, 779 (1988).
- 4. E. E. Jenkins, M. E. Luke, A.V. Manohar, and M. J. Savage, Phys. Lett. B 302, 482 (1993); 388, 866 (1996).
- 5. S. Scherer, Adv. Nucl. Phys. 27, 277 (2003).

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6. .....
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# **The contribution of NLO turns to worsen the results**



# **Theoretical calculation**

### Nonrelativistic

 $M_K/\Lambda_{QCD} = 0.5$ 

<b>Heavy Baryon</b> chiral perturbation theory
Up to NNLO
Bad convergence at NLO
<b>Excellent convergence at NNLO</b>

$$\begin{split} \mu_p &= + 4.48(1 - 0.49 + 0.11) = +2.79, \\ \mu_n &= -2.47(1 - 0.34 + 0.12) = -1.91, \\ \mu_{\Sigma^+} &= + 4.48(1 - 0.62 + 0.17) = +2.46, \\ \mu_{\Sigma^-} &= -2.01(1 - 0.31 - 0.11) = -1.16, \\ \mu_{\Sigma^0} &= +1.24(1 - 0.87 + 0.40) = +0.65, \\ \mu_\Lambda &= -1.24(1 - 0.87 + 0.37) = -0.61, \\ \mu_{\Xi^0} &= -2.47(1 - 0.89 + 0.40) = -1.25, \\ \mu_{\Xi^-} &= -2.01(1 - 0.64 - 0.03) = -0.65, \\ \mu_{\Lambda\Sigma^0} &= +2.14(1 - 0.53 + 0.19) = +1.40. \end{split}$$

U. G. Meissner and S. Steininger, Nucl. Phys. B499, 349 (1997).

### Relativistic

- **Extended On Mass Shell ChPT**
- **Up to NLO**
- □ Nice convergence properties

$$\begin{split} \mu_p &= + 3.47(1 - 0.257) = +2.58, \\ \mu_n &= -2.55(1 - 0.175) = -2.10, \\ \mu_{\Sigma^+} &= + 3.47(1 - 0.300) = +2.43, \\ \mu_{\Sigma^-} &= -0.93(1 + 0.187) = -1.16, \\ \mu_{\Sigma^0} &= +1.27(1 - 0.482) = +0.66, \\ \mu_{\Lambda} &= -1.27(1 - 0.482) = -0.66, \\ \mu_{\Xi^0} &= -2.55(1 - 0.501) = -1.27, \\ \mu_{\Xi^-} &= -0.93(1 + 0.025) = -0.95, \\ \mu_{\Lambda\Sigma^0} &= +2.21(1 - 0.284) = +1.58. \end{split}$$

L.S. ,Geng et al. Phys.Rev.Lett. 101 222002 (2008).

# **Theoretical calculation**

### Nonrelativistic

 $M_K/\Lambda_{QCD} = 0.5$ 

<b>Heavy Baryon</b> chiral perturbation theory
Up to NNLO
Bad convergence at NLO
Excellent convergence at NNLO

$\mu_p = +4.48(1 -$	0.49	+	0.11)	= +2.79,
$\mu_n = -2.47(1 -$	0.34	+	0.12)	= -1.91,
$\mu_{\Sigma^+} = +4.48(1 -$	0.62	+	0.17)	= +2.46,
$\mu_{\Sigma^-} = -2.01(1 -$	0.31	_	0.11)	= -1.16,
$\mu_{\Sigma^0} = +1.24(1 -$	0.87	+	0.40)	= +0.65,
$\mu_{\Lambda} = -1.24(1 -$	0.87	+	0.37)	= -0.61,
$\mu_{\Xi^0} = -2.47(1 -$	0.89	+	0.40)	= -1.25,
$\mu_{\Xi^-} = -2.01(1 -$	0.64	_	0.03)	= -0.65,
$\mu_{\Lambda\Sigma^0} = +2.14(1 -$	0.53	+	0.19	= +1.40.
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U. G. Meissner and S. Steininger, Nucl. Phys. B499, 349 (1997).

### Relativistic

$\mu_{\Lambda\Sigma^0} = +2.21(1-0.284) = +1.58.$
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# **Baryon magnetic moments**

# Definition

Magnetic moments are defined through electromagnetic current

$$\langle \bar{B} | J_{\mu} | B \rangle = \bar{u}(p_f) \left[ \gamma_{\mu} F_1^B(t) + \frac{i \sigma_{\mu\nu} q^{\nu}}{2m_B} F_2^B(t) \right] u(p_i).$$
**t**=0
**Magnetic moments= Anomalous magnetic moments** + **Charge**

# Steps

- **1. Calculate all Feynman diagrams**  $\mu = \mu^{(2)} + \mu^{(3)} + \mu^{(4)} + \cdots$
- 2. Extract the  $F_2^B(0)$
- 3. Fit LECS

### **Feynman diagrams and Lagrangians**



#### Next to leading order

$$\mathcal{L}_{B}^{(1)} = \langle \bar{B}i\gamma^{\mu}D_{\mu}B\rangle,$$
$$\mathcal{L}_{MB}^{(1)} = \frac{D}{2}\langle \bar{B}\gamma^{\mu}\gamma^{5}\{u_{\mu},B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^{\mu}\gamma^{5}[u_{\mu},B]\rangle,$$
$$\mathcal{L}_{M}^{(2)} = \frac{F_{0}^{2}}{4}\langle u_{\mu}u^{\mu} + \chi^{+}\rangle,$$



## **Feynman diagrams and Lagrangians**

#### Next to next to leading order

$$\begin{aligned} \mathcal{L}_{MB}^{(4)} &= + \frac{b_{6}^{D'}}{8m} \langle \chi^{+} \rangle \langle \bar{B} \sigma^{\mu\nu} \{F_{\mu\nu}^{+}, B\} \rangle + \frac{b_{6}^{F'}}{8m} \langle \chi^{+} \rangle \langle \bar{B} \sigma^{\mu\nu} [F_{\mu\nu}^{+}, B] \rangle \\ &+ \frac{\alpha_{1}}{8m} \langle \bar{B} \sigma^{\mu\nu} [[F_{\mu\nu}^{+}, B], \chi^{+}] \rangle + \frac{\alpha_{2}}{8m} \langle \bar{B} \sigma^{\mu\nu} \{[F_{\mu\nu}^{+}, B], \chi^{+}\} \rangle \\ &+ \frac{\alpha_{3}}{8m} \langle \bar{B} \sigma^{\mu\nu} [\{F_{\mu\nu}^{+}, B\}, \chi^{+}] \rangle + \frac{\alpha_{2}}{8m} \langle \bar{B} \sigma^{\mu\nu} \{\{F_{\mu\nu}^{+}, B\}, \chi^{+}\} \rangle \\ &+ \frac{\beta_{1}}{8m} \langle \bar{B} \sigma^{\mu\nu} B \rangle \langle \chi^{+} F_{\mu\nu}^{+} \rangle. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{MB}^{(2')} &= \frac{i}{2} \left\{ b_{9} \langle \bar{B} \sigma^{\mu\nu} u_{\mu} \rangle \langle u_{\nu} B \rangle + b_{10,11} \langle \bar{B} \sigma^{\mu\nu} ([u_{\mu}, u_{\nu}], B)_{\pm} \rangle \right\} \\ \mathcal{L}_{MB}^{(2'')} &= b_{D} \langle \bar{B} \{\chi^{+}, B\} \rangle + b_{F} \langle \bar{B} [\chi^{+}, B] \rangle. \end{aligned}$$

#### Problem



### **Solution 1**

# Solution 2

Constrain  $b_6^D$  and  $b_6^F$  according to the convergence properties

# **Start point**

**>** The success of BChPT

- Baryon mass and sigma terms *PRD82:074504,2010 ;PLB766-325, 2017*
- N—N interaction arXiv:1611.08475
- π-N scattering *PRC83:055205, 2011*
- • •

## **D**Assumption

> BChPT should present good convergence properties

### Solution 2

#### **Convergence rate**

 $\mathrm{CR} = \max(\mu_B^{(3)}/\mu_B^{(2)}, \mu_B^{(4)}/\mu_B^{(3)})$ 



### **Solution 3**

# Fit to the lattice QCD data

□At physical point
> 7 data
□Large pion mass
> Lots of data

Fit to exp+LQCD !



FIG. 26: Magnetic moments of the proton and  $\Sigma^+$ . The  $\Sigma^+$  moments are offset to the right for clarity.



FIG. 27: Magnetic moments of the neutron and  $\Xi^0$ .  $\Xi^0$  moments are offset to the right for clarity.

• Phys. Rev. D 74,093005 (2006)



# Constrain the values of $b_6^D$ and $b_6^F$



CR = max $(\mu_B^{(3)}/\mu_B^{(2)}, \mu_B^{(4)}/\mu_B^{(3)})$  CR=0.6

# Constrain the values of $b_6^D$ and $b_6^F$



CR = max $(\mu_B^{(3)}/\mu_B^{(2)}, \mu_B^{(4)}/\mu_B^{(3)})$  CR=0.6

# **Convergence properties of different BChPT**

TABLE I. Contributions of differen	chiral orders of the HB, IR, and EOM	S schemes up to $\mathcal{O}(p^4)$ .
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	EO	MS	I	R	Н	IB
Baryons	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$
Р	-0.27	-0.38	-0.16	0.01	-0.44	-0.07
N	-0.19	0.02	-0.17	0.61	-0.18	0.74
Λ	-0.52	-0.08	-0.73	-0.27	-0.83	-0.32
$\Sigma^{-}$	0.18	-0.04	2.58	-0.73	-0.30	0.30
$\Sigma^+$	-0.31	-0.15	-0.05	4.20	-0.61	-0.22
$\Sigma^0$	-0.52	-0.13	-0.73	-0.31	-0.83	-0.35
$\Xi^-$	0.03	-12.88	3.10	-1.02	-0.74	-0.12
$\Xi^0$	-0.54	-0.13	-0.77	-0.32	-0.87	-0.36
$\Lambda\Sigma^0$	-0.31	0.27	-0.38	-0.11	-0.43	0.46

# **Convergence properties of different BChPT**

TABLE I. Contributions of different chiral orders of the HB, IR, and EOMS schemes up to  $\mathcal{O}(p^4)$ .



# **Check the reliability of the LECS**

# **9 LECS**

 $\boldsymbol{b_6^D}, \boldsymbol{b_6^F}, \boldsymbol{b_6^D'}, \boldsymbol{b_6^F'}, \boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \boldsymbol{\alpha_3}, \boldsymbol{\alpha_4}, \boldsymbol{\beta_1}$ 

# □ Are these LECS reliable?

- ✓ Two model independent theory
  - Chiral perturbation theory
  - Lattice QCD

✓ ChPT and Lattice QCD can verify each other

### **Pion mass dependence**



		Chi-square		
Chiral schemes	S.Boinepalli et al.[1]	NPLQCD[2]	T. Primer et al.[3]	
IR	12.01	2.48	0.69	
HB	43.45	12.49	4.64	
EOMS	3.66	1.11	0.5	

[1] Phys. Rev. D 74,093005 (2006)
[2] Phys. Rev. D 89, 034508 (2014)
[3] Phys.Rev. D 95, 114513 (2017)

# Fit to the lattice QCD data

# At NLO

Chi	ral schemes	EOMS	IR	HB
	$b_6^D$	3.83	4.82	4.73
Fit exp	$b_6^F$	1.20	-0.03	2.49
	Chi-square (exp+lattice)	1.42	28.16	12.21
Fit exp +lattice QCD	$b_6^D$	3.73	5.07	5.29
	$b_6^F$	1.00	-0.92	2.95
	Chi-square (exp+lattice)	0.64	13.18	5.75

#### **Pion mass dependence at NLO**



# **Pion mass dependence at NNLO**



#### **Chi-square between BChPT and LQCD**

#### Fit to the exp data with constrained LECs

OCTET DA DVONS	CHIDAL SCHEME		$\tilde{\chi}^2$	
OCTET BARTONS	CHIKAL SCHEME	T.Primer et al.	NPLQCD S.Bo	inepalli <i>et al</i> .
	IR	11.18	1.96	0.56
Sum	HB	27.49	8.40	3.41
	EOMS	3.98	1.24	0.53

#### Fit to the LQCD data

	$b_6^D$	$b_6^F$	$\alpha_1$	$\alpha_2$	$lpha_3$	$lpha_4$	$\beta_1$	$b_6^{D'}$	$b_6^{F'}$	$\tilde{\chi}^2$
IR	4.02	2.08	-0.20	-0.83	0.06	0.20	-2.88	-3.66	-3.59	0.14
HB	2.16	1.08	-1.47	0.28	-1.47	1.53	-2.12	0.56	0.89	0.24
EOMS	3.03	1.40	0.17	0.30	0.15	0.60	-0.56	-0.69	-0.59	0.13

#### **Contributions of different chiral orders**

#### Fit to the exp data with constrained LECs

#### Fit to the LQCD data

Domiona	EOMS		IR		HB			D	EC	MS	Ι	R	HB	
Багуопз	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	Ba	Baryons	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_{B}^{(3)}/\mu_{B}^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$
Р	-0.27	-0.38	-0.16	0.01	-0.44	-0.07		Р	-0.26	-0.12	-0.12	2.09	-0.73	-1.08
Ν	-0.19	0.02	-0.17	0.61	-0.18	0.74		Ν	-0.22	-1.09	-0.17	0.38	-0.35	2.36
$\Lambda$	-0.52	-0.08	-0.73	-0.27	-0.83	-0.32		Λ	-0.61	-0.26	-0.72	-0.15	-1.62	-0.85
$\Sigma^{-}$	0.18	-0.04	2.58	-0.73	-0.30	0.30		$\Sigma^{-}$	0.13	-1.85	1.02	-1.30	-0.41	-0.81
$\Sigma^+$	-0.31	-0.15	-0.05	4.20	-0.61	-0.22		$\Sigma^+$	-0.31	-0.01	-0.04	10.21	-1.03	-0.82
$\Sigma^0$	-0.52	-0.13	-0.73	-0.31	-0.83	-0.35		$\Sigma^0$	-0.61	-0.29	-0.72	-0.31	-1.62	-0.82
$\Xi^-$	0.03	-12.88	3.10	1.02	-0.74	-0.12		$\Xi^{-}$	0.02	-28.25	1.22	-1.53	-1.03	-0.44
$\Xi^0$	-0.54	-0.13	-0.77	-0.32	-0.87	-0.36		$\Xi^0$	-0.64	-0.38	-0.76	-0.27	-1.71	-0.91
$\Lambda\Sigma^0$	-0.31	0.27	-0.38	-0.11	-0.43	0.46		$\Lambda\Sigma^0$	-0.37	-0.33	-0.38	-0.22	-0.84	-0.78

#### **Contributions of different chiral orders**

#### Fit to the exp data with constrained LECs

#### Fit to the LQCD data

Domiona	EO	MS	Ι	R	ŀ	łB		D	EO	MS	Ι	R	Н	В
Baryons	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_{B}^{(4)}/\mu_{B}^{(3)}$		Baryons	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$
Р	-0.27	-0.38	-0.16	0.01	-0.44	-0.07		Р	-0.26	-0.12	-0.12	2.09	-0.73	-1.08
Ν	-0.19	0.	7-0 7	61	- 18	0.74		N	0.2	-1.09	-0.1	0 8	-0.35	2.36
$\Lambda$	-0.52	7.08	-0 3	.2	- 83	-0. 2	Ŋ	<b>H</b>	·0.6	- 26	0.7	-0 5	-1.62	-0.85
$\Sigma^{-}$	0.18	-0.04	2.58	-0.73	-0.30	0.30		$\Sigma^{-}$	0.13	-1.85	1.02	-1.30	-0.41	-0.81
$\Sigma^+$	-0.31	-0.15	-0.05	4.20	-0.61	-0.22		$\Sigma^+$	-0.21	-0.01	-0.04	10.21	-1.03	-0.82
$\Sigma^0$	-0.52	-0.13	-0.73	-0.31	-0.3	-0.			-0.61	-0.29	-0.72	-0.31	-1.62	-0.82
$\Xi^-$	0.03	-12.88	3.10	1.02	-0.74	-0.12		Ξ-	0.02	-28.25	1.22	-1.53	-1.03	-0.44
$\Xi^0$	-0.54	-0.13	-0.77	-0.32	-0.87	-0.36		$\Xi^0$	-0.64	-0.38	-0.76	-0.27	-1.71	-0.91
$\Lambda\Sigma^0$	-0.31	0.27	-0.38	-0.11	-0.43	0.46		$\Lambda\Sigma^0$	-0.37	-0.33	-0.38	-0.22	-0.84	-0.78

#### **Strange quark mass dependence**





- Calculated the baryon magnetic moments in EOMS
   ChPT up to next-to-next-to-leading order
- **Determined low energy constants in two different ways** 
  - Fit to experiment data with LECs constrained by
    - convergence properties
  - > Fit to the lattice QCD data
- More lattice QCD simulations are needed to reach a firmer conclusion

Thanks!