DD^* potential in ChPT and possible molecular states

许豪　　　2015级博士

兰州大学

collaborated with *bo wang*, *zhan-wei liu* and *xiang liu*

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ABSTRACT

- DD^* potential is studied within the framework of heavy meson chiral perturbation theory , at one loop level .
- The contact, one-pion and two-pion exchange are studied in detail.
- With the potential in coordinate space, we solve the schrodinger , we find there exists a bound state in isospin-0 channel with a reasonable cutoff.

- background
- DD^* potential
- potentials in coordinate space and possible molecular state
- summary

BACKGROUND

- As we know, the nucleon-nucleon interaction with ChPT is a remarkable success, the research is still active in recent. See E. Epelbaum, et al, Rev. Mod. Phys. 81, 1773 (2009); R. Machleidt, et al, Phys. Rept. 503, 1 (2011) , and

  老师们相关的精彩报告。

- However the appliacation in the heavy hadron system is not fairly developed, many discovered XYZ and other exotics demands a systematic description in terms of heavy hadrons interaction.

- XYZ: many exotic states were discovered in recen ten years especially XYZ. Molecule, tetraquarks, non-resonant effect, etc. . The common feature: near threshold. Therefore, is is nature to study the related two- heavy hadron interaction to disentangle them.
- two-pion exchange: Although there already exists many works about heavy meson system (X(3872, Zb, etc) discussing contact and one-pion exchange contribution, the higher order estimation (especially two-pion exchange) is still lacking.
- molecular model: one-boson exchanges and many other effects was developed to derive potential of heavy hadron system, the potential derived in ChPT would be a interesting topic, and can be used to solve Schrodinger equation.

- the doubly charmed DD^* potential with Weinberg's formalism,
- discuss one-pion, contact and complete two pion exchange contributions in detail,
- use it to solve the schrodinger equation to find bound state solutions.

- We follow previous work : Z. W. Liu, N. Li and S. L. Zhu, Chiral perturbation theory and the $B\,B$ strong interaction, Phys. Rev. D 89, 074015 (2014), further develop the loop integral calculation method, and investigate the DD^* potential in coordinate space.

DD^* POTENTIAL

lagrangian

- First, we show the lagrangian

$$\mathcal{L}_{H\phi}^{(1)} = -\langle (iv\cdot\partial H)\bar{H}\rangle + \langle Hv\cdot\Gamma\bar{H}\rangle + g\langle H\not{u}\gamma_5\bar{H}\rangle - \frac{1}{8}\delta\langle H\sigma^{\mu\nu}\bar{H}\sigma_{\mu\nu}\rangle.$$

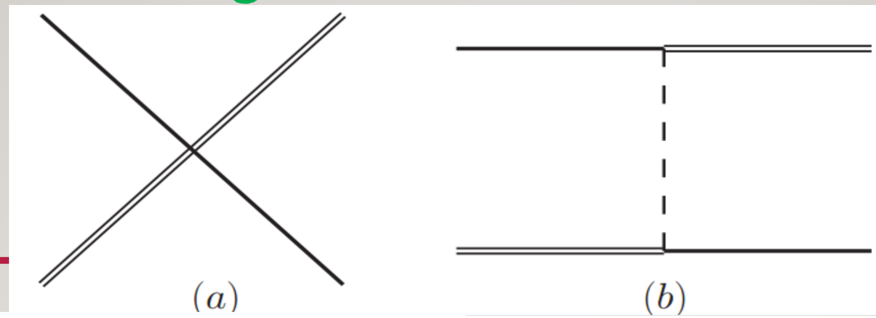
$$\mathcal{L}_{4H}^{(0)} = D_a\mathrm{Tr}[H\gamma_\mu\bar{H}]\mathrm{Tr}[H\gamma^\mu\bar{H}] + D_b\mathrm{Tr}[H\gamma_\mu\gamma_5\bar{H}]\mathrm{Tr}[H\gamma^\mu\gamma_5\bar{H}] + E_a\mathrm{Tr}[H\gamma_\mu\tau^a\bar{H}]\mathrm{Tr}[H\gamma^\mu\tau_a\bar{H}] + E_b\mathrm{Tr}[H\gamma_\mu\gamma_5\tau^a\bar{H}]\mathrm{Tr}[H\gamma^\mu\gamma_5\tau_a\bar{H}],$$

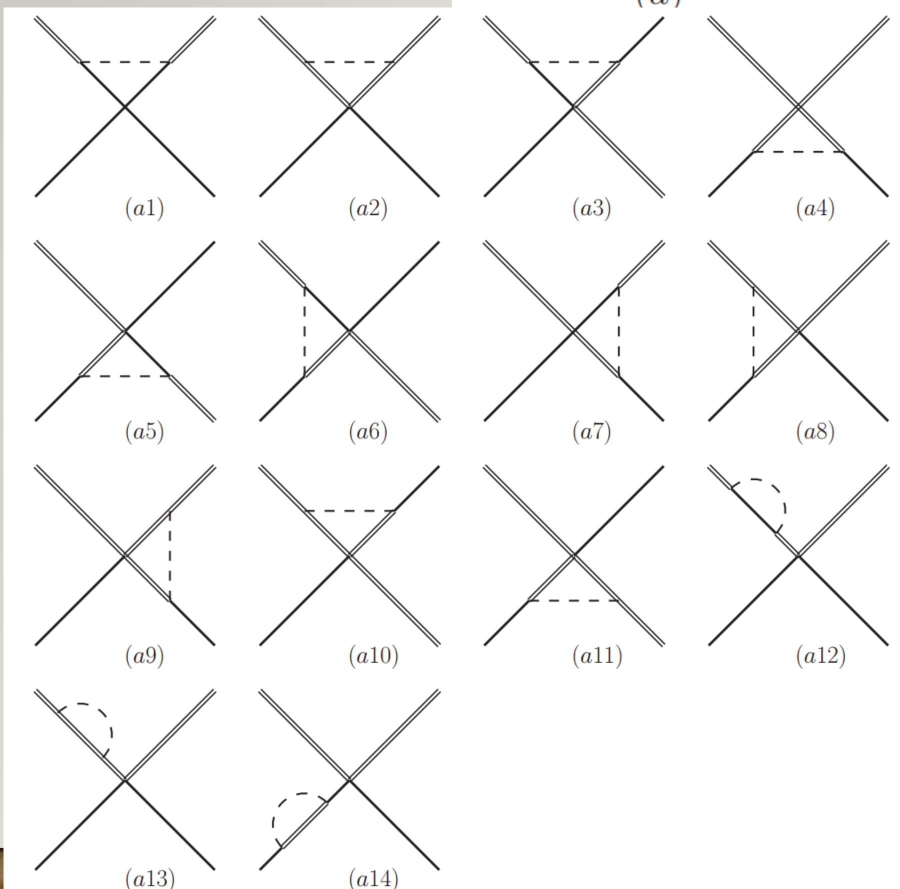
$$
\begin{aligned}
\mathcal{L}_{4H}^{(2,h)} &= D_a^h \mathrm{Tr}[H\gamma_\mu \bar{H}]\mathrm{Tr}[H\gamma^\mu \bar{H}]\mathrm{Tr}(\chi_+) \\
&\quad + D_b^h \mathrm{Tr}[H\gamma_\mu\gamma_5 \bar{H}]\mathrm{Tr}[H\gamma^\mu\gamma_5 \bar{H}]\mathrm{Tr}(\chi_+) \\
&\quad + E_a^h \mathrm{Tr}[H\gamma_\mu\tau^a \bar{H}]\mathrm{Tr}[H\gamma^\mu\tau_a \bar{H}]\mathrm{Tr}(\chi_+) \\
&\quad + E_b^h \mathrm{Tr}[H\gamma_\mu\gamma_5\tau^a \bar{H}]\mathrm{Tr}[H\gamma^\mu\gamma_5\tau_a \bar{H}]\mathrm{Tr}(\chi_+), \qquad (6)
\end{aligned}
$$

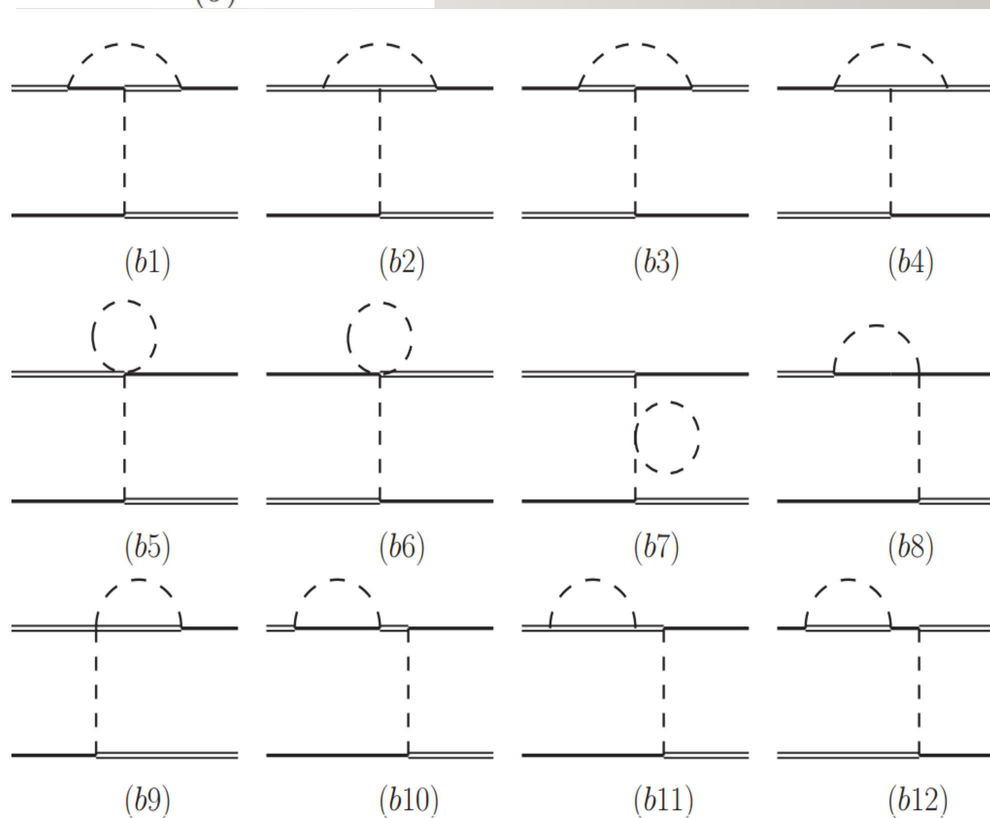
$$
\begin{aligned}
\mathcal{L}_{4H}^{(2,v)} &= \{D_{a1}^v \mathrm{Tr}[(v\cdot DH)\gamma_\mu(v\cdot D\bar{H})]\mathrm{Tr}[H\gamma^\mu \bar{H}] \\
&\quad + D_{a2}^v \mathrm{Tr}[(v\cdot DH)\gamma_\mu \bar{H}]\mathrm{Tr}[(v\cdot DH)\gamma^\mu \bar{H}] \\
&\quad + D_{a3}^v \mathrm{Tr}[(v\cdot DH)\gamma_\mu \bar{H}]\mathrm{Tr}[H\gamma^\mu(v\cdot D\bar{H})] + \\
&\quad D_{a4}^v \mathrm{Tr}[((v\cdot D)^2 H)\gamma_\mu \bar{H}]\mathrm{Tr}[H\gamma^\mu \bar{H}] \\
&\quad + D_{b1}^v \mathrm{Tr}[(v\cdot DH)\gamma_\mu\gamma_5(v\cdot D\bar{H})]\mathrm{Tr}[H\gamma^\mu\gamma_5 \bar{H}] + \ldots \\
&\quad + E_{a1}^v \mathrm{Tr}[(v\cdot DH)\gamma_\mu\tau^a(v\cdot D\bar{H})]\mathrm{Tr}[H\gamma^\mu\tau_a \bar{H}] + \ldots \\
&\quad + E_{b1}^v \mathrm{Tr}[(v\cdot DH)\gamma_\mu\gamma_5\tau^a(v\cdot D\bar{H})]\mathrm{Tr}[H\gamma^\mu\gamma_5\tau_a \bar{H}] \\
&\quad + \ldots\} + \mathrm{H.c.}, \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{4H}^{(2,q)} &= \{D_1^q \mathrm{Tr}[(D^\mu H)\gamma_\mu\gamma_5(D^\nu \bar{H})]\mathrm{Tr}[H\gamma_\nu\gamma_5 \bar{H}] \\
&\quad + D_2^q \mathrm{Tr}[(D^\mu H)\gamma_\mu\gamma_5 \bar{H}]\mathrm{Tr}[(D^\nu H)\gamma_\nu\gamma_5 \bar{H}] \\
&\quad + D_3^q \mathrm{Tr}[(D^\mu H)\gamma_\mu\gamma_5 \bar{H}]\mathrm{Tr}[H\gamma_\nu\gamma_5(D^\nu \bar{H})] \\
&\quad + D_4^q \mathrm{Tr}[(D^\mu D^\nu H)\gamma_\mu\gamma_5 \bar{H}]\mathrm{Tr}[H\gamma_\nu\gamma_5 \bar{H}] \\
&\quad + E_1^q \mathrm{Tr}[(D^\mu H)\gamma_\mu\gamma_5\tau^a(D^\nu \bar{H})]\mathrm{Tr}[H\gamma_\nu\gamma_5\tau_a \bar{H}] \\
&\quad + \ldots\} + \mathrm{H.c.}, \cdots, \qquad (8)
\end{aligned}
$$

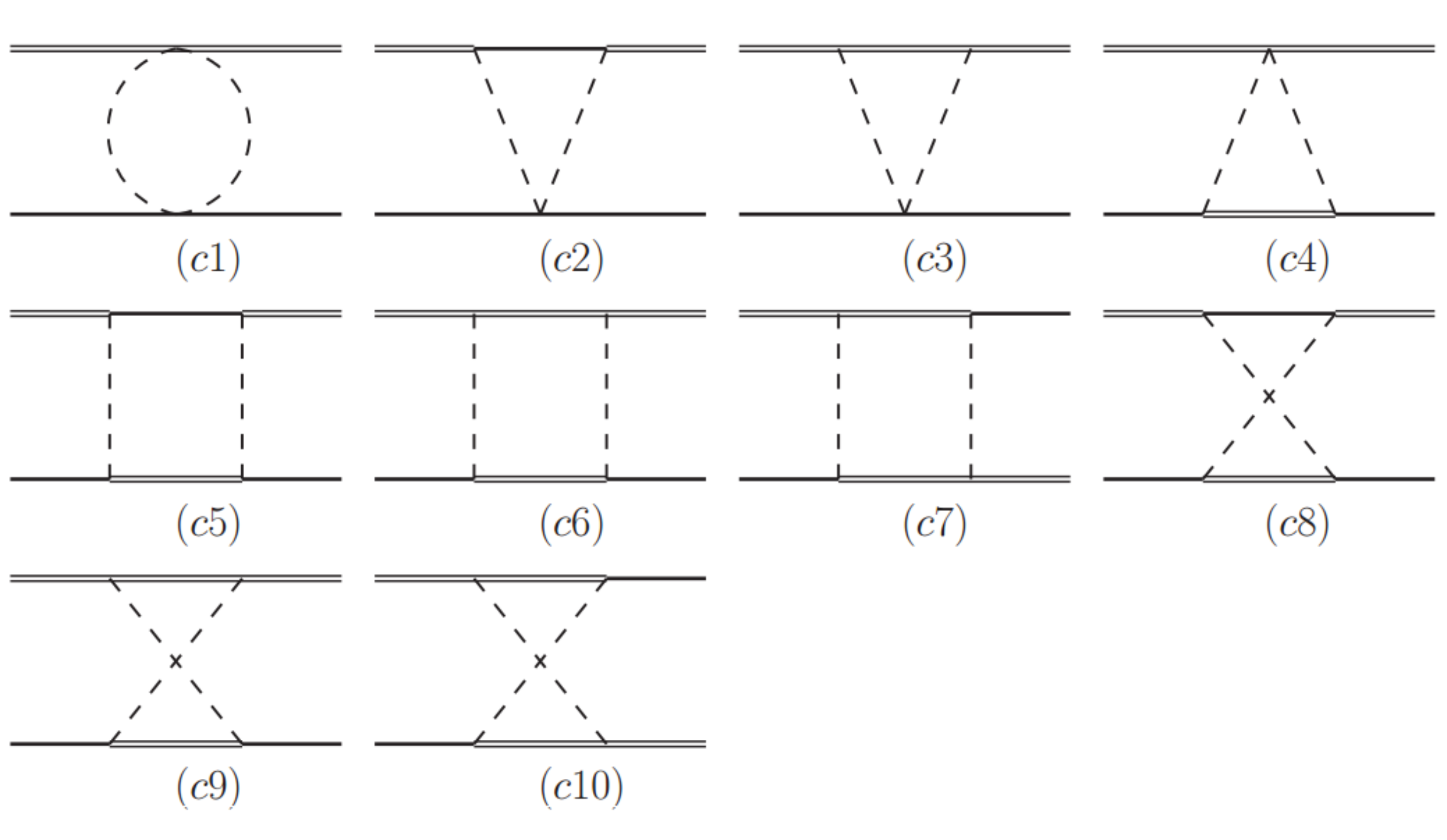
Feynmann diagram







Complete 2π exchange diagram



Power counting breaking problem

- We can not calculate the scattering matrix element directly, since two heavy hadron box digram violates the power cunting.

$$
\begin{aligned}
& i \int d^4 l \frac{i}{l^0 + P^0 + i\varepsilon} \frac{i}{-l^0 + P^0 + i\varepsilon} \times \cdots \\
& = i \int d l^0 \frac{i}{l^0 + P^0 + i\varepsilon} \frac{i}{-l^0 + P^0 + i\varepsilon} \int d^3 l \cdots \\
& = \frac{\pi}{P^0 + i\varepsilon} \int d^3 l \cdots \\
& = \frac{\pi}{\vec{P}^2/(2M_N) + i\varepsilon} \int d^3 l \cdots .
\end{aligned}
$$

- The true order is enhanced comparing to naïve estimate

- Therefore we calculate the two-particle-irreducible(2PI) part of the all diagram, i.e., the potential first. Then use the potential to obtain the physical observable.

$$\int d^4 l \frac{1}{v\cdot l + a + i\varepsilon}\frac{1}{-v\cdot l - a + i\varepsilon}\times\cdots$$

Substituted by

$$\frac{1}{v\cdot l + a + i\varepsilon}\frac{1}{-v\cdot l - a + i\varepsilon}$$
$$= \frac{1}{v\cdot l + a + i\varepsilon}\left[-\frac{1}{v\cdot l + a + i\varepsilon} + 2\pi\delta(v\cdot l + a)\right]$$

Some loop integral calculation

- Z-W Liu, Phys. Rev. D 89, 074015 (2014),

$$i\int\frac{d^D l\mu^{4-D}}{(2\pi)^D}\frac{\{1,\ l^\alpha,\ l^\alpha l^\beta,\ l^\alpha l^\beta l^\gamma,\ l^\alpha l^\beta l^\gamma l^\delta\}}{[(+/-)v\cdot l+\omega+i\varepsilon](l^2-m_1^2+i\varepsilon)[(q+l)^2-m_2^2+i\varepsilon]}$$
$$\equiv\Big\{J_0^{T/S},\ q^\alpha J_{11}^{T/S}+v^\alpha J_{12}^{T/S},\ g^{\alpha\beta}J_{21}^{T/S}+q^\alpha q^\beta J_{22}^{T/S}+v^\alpha v^\beta J_{23}^{T/S}+(q\vee v)J_{24}^{T/S},\ (g\vee q)J_{31}^{T/S}+q^\alpha q^\beta q^\gamma J_{32}^{T/S}+(q^2\vee v)J_{33}^{T/S}$$
$$+(g\vee v)J_{34}^{T/S}+(q\vee v^2)J_{35}^{T/S}+v^\alpha v^\beta v^\gamma J_{36}^{T},\ (g\vee g)J_{41}^{T/S}+(g\vee q^2)J_{42}^{T/S}+q^\alpha q^\beta q^\gamma q^\delta J_{43}^{T/S}+(g\vee v^2)J_{44}^{T/S}+v^\alpha v^\beta v^\gamma v^\delta J_{45}^{T/S}$$
$$+(q^3\vee v)J_{46}^{T/S}+(q^2\vee v^2)J_{47}^{T/S}+(q\vee v^3)J_{48}^{T/S}+(g\vee q\vee v)J_{49}^{T/S}\Big\}(m_1,m_2,\omega,q), \tag{C4}$$

$$i\int\frac{d^D l\mu^{4-D}}{(2\pi)^D}\frac{\{1,\ l^\alpha,\ l^\alpha l^\beta,\ l^\alpha l^\beta l^\gamma,\ l^\alpha l^\beta l^\gamma l^\delta\}}{(v\cdot l+\omega_1+i\varepsilon)[(+/-)v\cdot l+\omega_2+i\varepsilon](l^2-m_1^2+i\varepsilon)[(q+l)^2-m_2^2+i\varepsilon]}$$
$$\equiv\Big\{J_0^{R/B},\ q^\alpha J_{11}^{R/B}+v^\alpha J_{12}^{R/B},\ g^{\alpha\beta}J_{21}^{R/B}+q^\alpha q^\beta J_{22}^{R/B}+v^\alpha v^\beta J_{23}^{R/B}+(q\vee v)J_{24}^{R/B},\ (g\vee q)J_{31}^{R/B}+q^\alpha q^\beta q^\gamma J_{32}^{R/B}+(q^2\vee v)J_{33}^{R/B}$$
$$+(g\vee v)J_{34}^{R/B}+(q\vee v^2)J_{35}^{R/B}+v^\alpha v^\beta v^\gamma J_{36}^{R/B},(g\vee g)J_{41}^{R/B}+(g\vee q^2)J_{42}^{R/B}+q^\alpha q^\beta q^\gamma q^\delta J_{43}^{R/B}+(g\vee v^2)J_{44}^{R/B}+v^\alpha v^\beta v^\gamma v^\delta J_{45}^{R/B}$$
$$+(q^3\vee v)J_{46}^{R/B}+(q^2\vee v^2)J_{47}^{R/B}+(q\vee v^3)J_{48}^{R/B}+(g\vee q\vee v)J_{49}^{R/B}\Big\}(m_1,m_2,\omega_1,\omega_2,q), \tag{C5}$$

- 例如

$$J^R(\omega_1,\omega_2) = \frac{1}{\omega_2-\omega_1}\left[J^T(\omega_1) - J^T(\omega_2)\right],$$
$$J^B(\omega_1,\omega_2) = \frac{1}{\omega_2+\omega_1}\left[J^T(\omega_1) + J^S(\omega_2)\right].$$

$$\begin{aligned}
&J^T_{45}\\
&= 8L\int_0^1 dx_1 b(2b^2-c) + \frac{1}{4\pi^2}\int_0^1 dx_1 b^3 + \frac{1}{4\pi^2}\int_0^1 dx_1\\
&\quad \times b(2b^2-c)\left[-\log\mu^2 + \log(-b^2+c)\right] + \frac{1}{16\pi}\int_0^1 dx_1\\
&\quad \times(-b^2+c)^{\frac{3}{2}} - \frac{3}{8\pi}\int_0^1 dx_1 b^2(-b^2+c)^{\frac{1}{2}} + \frac{1}{16\pi}\int_0^1 dx_1\\
&\quad \times b^4(-b^2+c)^{-\frac{1}{2}} + \frac{1}{8\pi^2}\int_0^1 dx_1 E, \qquad \text{(C14)}
\end{aligned}$$

$$\begin{aligned}
b &= (1-x_1)v\cdot q - \omega,\\
c &= (1-x_1)^2q^2 - (1-x_1)q^2 + x_1(m_1^2 - m_2^2) + m_2^2 - i\epsilon,\\
D &= \left\{\sqrt{c-b^2}\left[(4b^2-c)\log\left(1-\frac{b^2}{c}\right) + 5b^2\right] + (8b^3-6bc)\right.\\
&\quad \left.\times\tan^{-1}\left(\frac{b}{\sqrt{c-b^2}}\right)\right\}\left(2\sqrt{c-b^2}\right)^{-1},\\
E &= \left\{b\sqrt{c-b^2}\left[6(2b^2-c)\left(\log(c) - \log\left[c-b^2\right]\right) - 16b^2\right.\right.\\
&\quad \left.\left. +3c\right] - 3(8b^4 - 8b^2c + c^2)\tan^{-1}\left(\frac{b}{\sqrt{c-b^2}}\right)\right\}\\
&\quad \times\left(3\sqrt{c-b^2}\right)^{-1},
\end{aligned}$$

Potential results

Because we just consider S-wave interaction, we have to replace the polarization vector as in OBE model

$$\vec{\varepsilon}(p_2) \cdot \vec{\varepsilon}^*(p_4) \rightarrowtail 1,$$

$$\vec{\varepsilon}(p_2) \cdot \vec{p}\, \vec{\varepsilon}^*(p_4) \cdot \vec{p} \rightarrowtail \frac{1}{3}\vec{p}^2,$$

The results of contact contributions

$$\mathcal{V}^{(0)}_{I=1} = -2D_a + 2D_b - 2E_a + 2E_b,$$

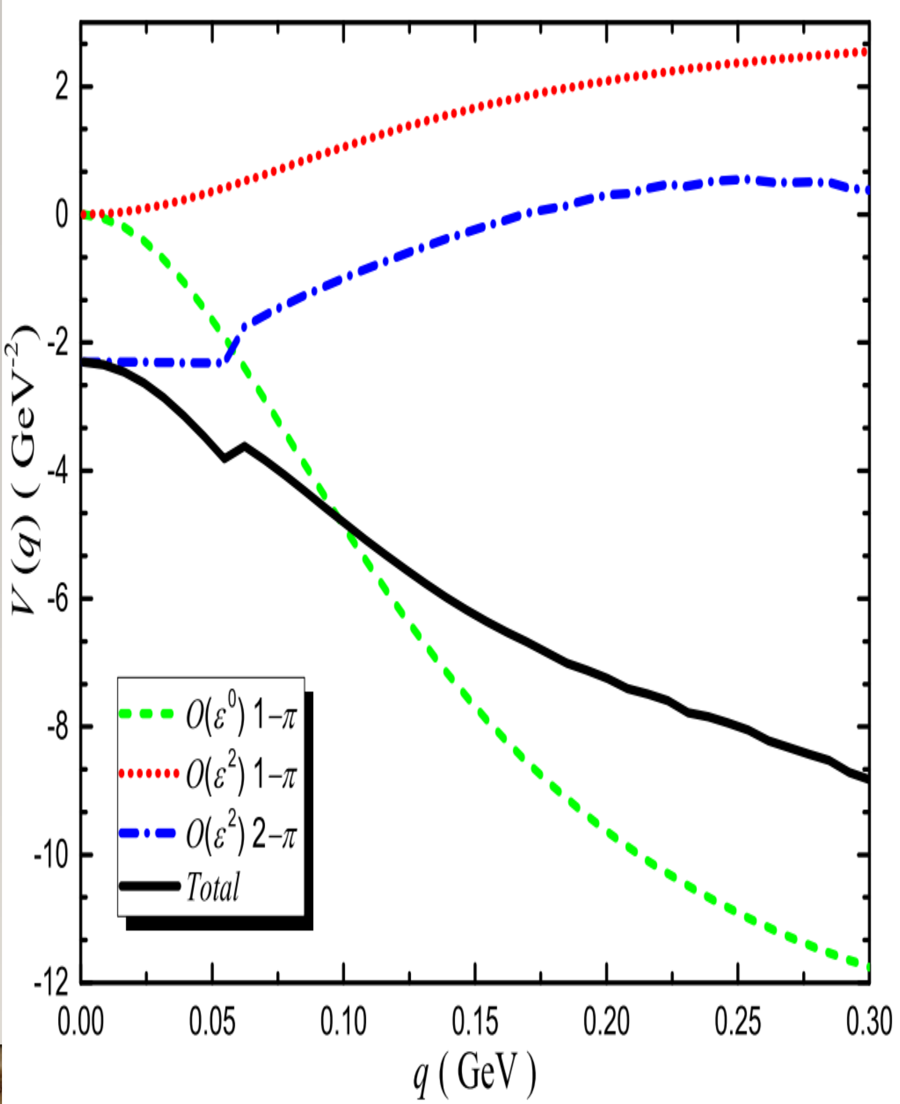
$$\mathcal{V}^{(2)}_{I=1} = -(0.237 + 0.032i)D_b + 0.056E_a -(0.124 + 0.032i)E_b.$$

And for the channel of isospin 0, we obtain

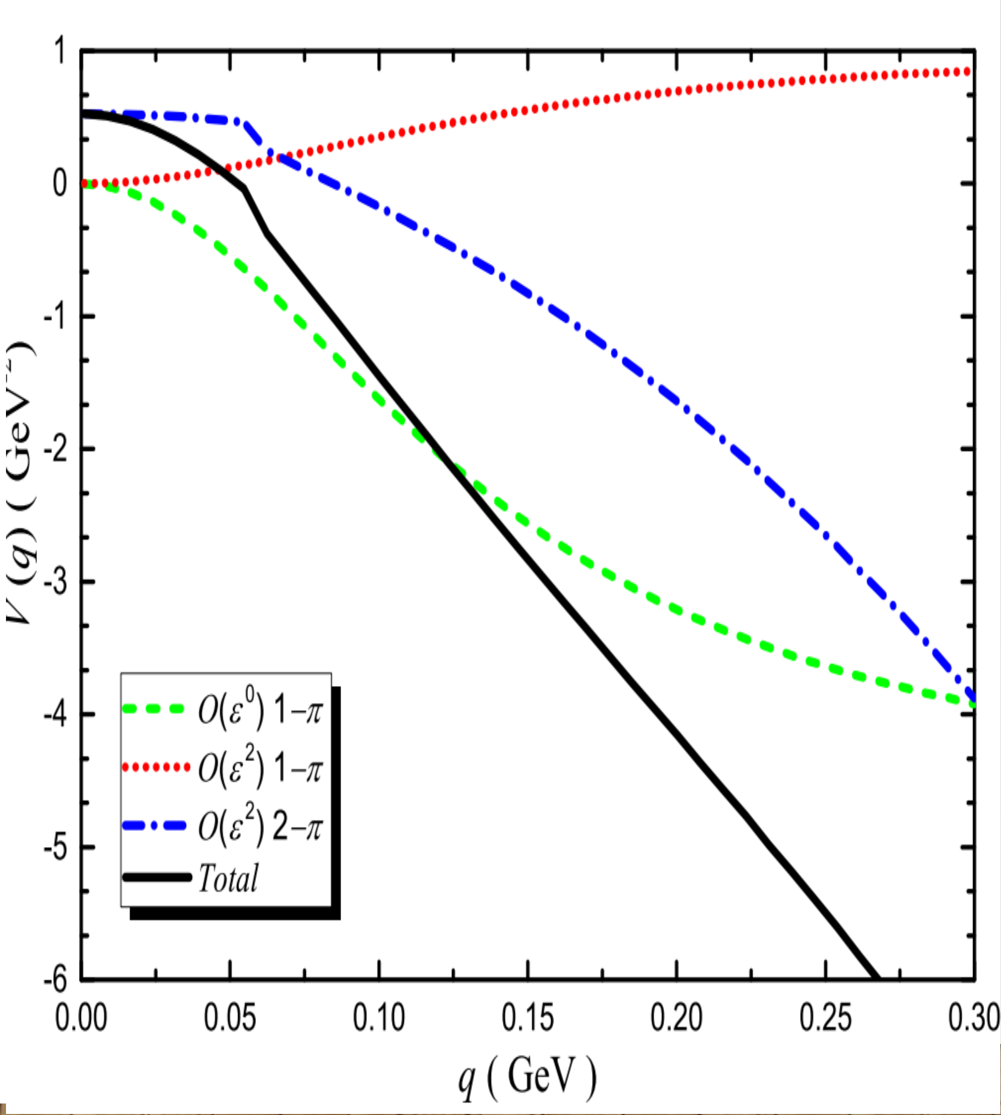
$$\mathcal{V}^{(0)}_{I=0} = -2D_a - 2D_b + 6E_a + 6E_b,$$

$$\mathcal{V}^{(2)}_{I=0} = -(1.122 + 0.196i)E_a + (0.118 + 0.048i)D_b +(0.027 - 0.148i)E_b.$$

- The results of one-pion and two-pion



I=0



I=1

POTENTIALS IN COORDINATE SPACE AND POSSIBLE MOLECULAR STATE

- The only unknown parameter now is the LECs in the contact terms, we determine them by resonance saturation method.

We assume contact contribution is saturated by heavy vector mesons.

$$\mathcal{L}_{HHV} = i\beta\langle H v_\mu (V^\mu - \rho^\mu)\bar{H}\rangle + i\lambda\langle H\sigma_{\mu\nu}F^{\mu\nu}(\rho)\bar{H}\rangle.$$

$$D_a = -\frac{\beta^2 g_v^2}{8m_\omega^2}, \quad E_a = -\frac{\beta^2 g_v^2}{8m_\rho^2}, \quad D_b = 0, \quad E_b = 0$$

- We use the Fourier Transformation

$$\mathcal{V}(\mathbf{r}) = \int \frac{d\mathbf{q}}{(2\pi)^3} \mathcal{V}(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}}.$$

- since our perturbative series is series of momentum, the F.T. is divergent, therefore we adopt a cutoff $\exp(-\vec{p}^{2n}/\Lambda^{2n})$, which is similar to the nuclear study.

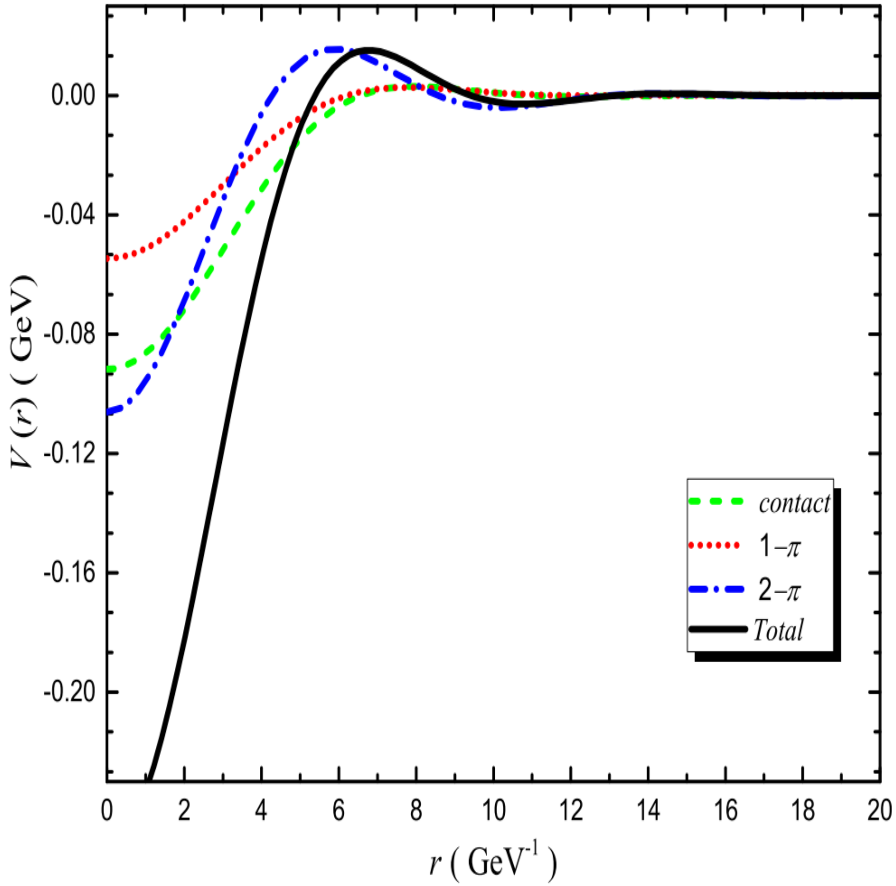


FIG. 8: (color online). S-wave potentials of the DD^* systerm with $I = 0$ in units of GeV. The green dashed, red dotted, and blue dot-

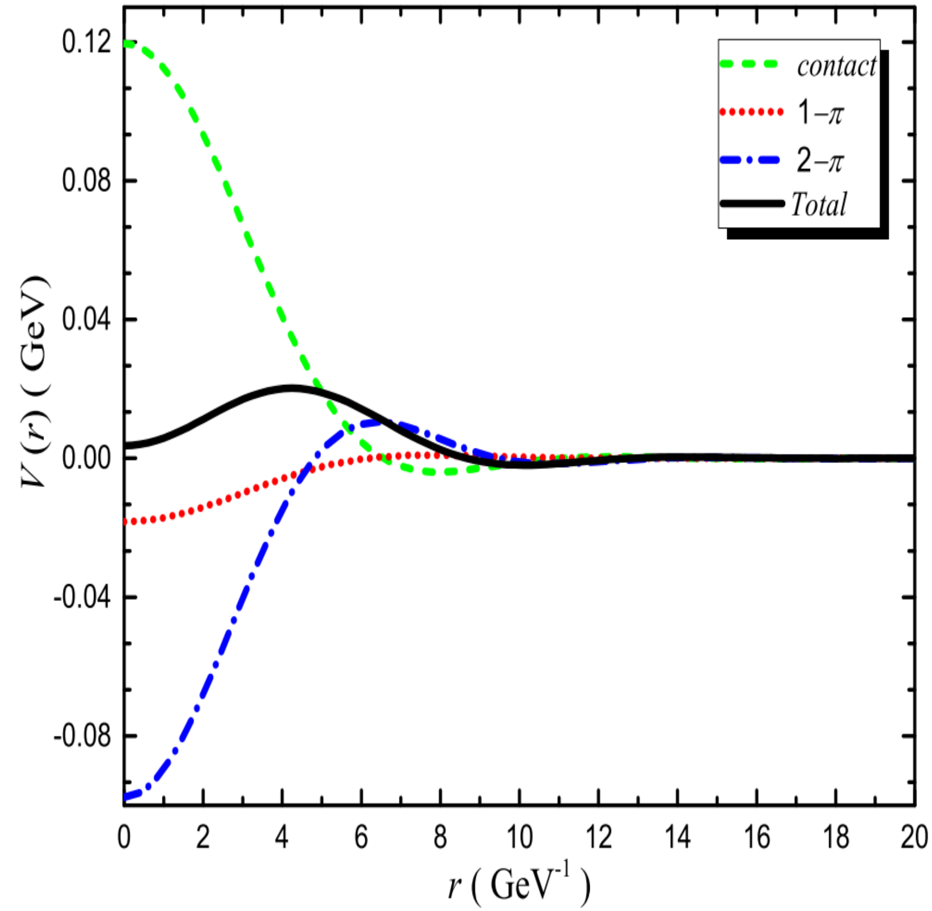
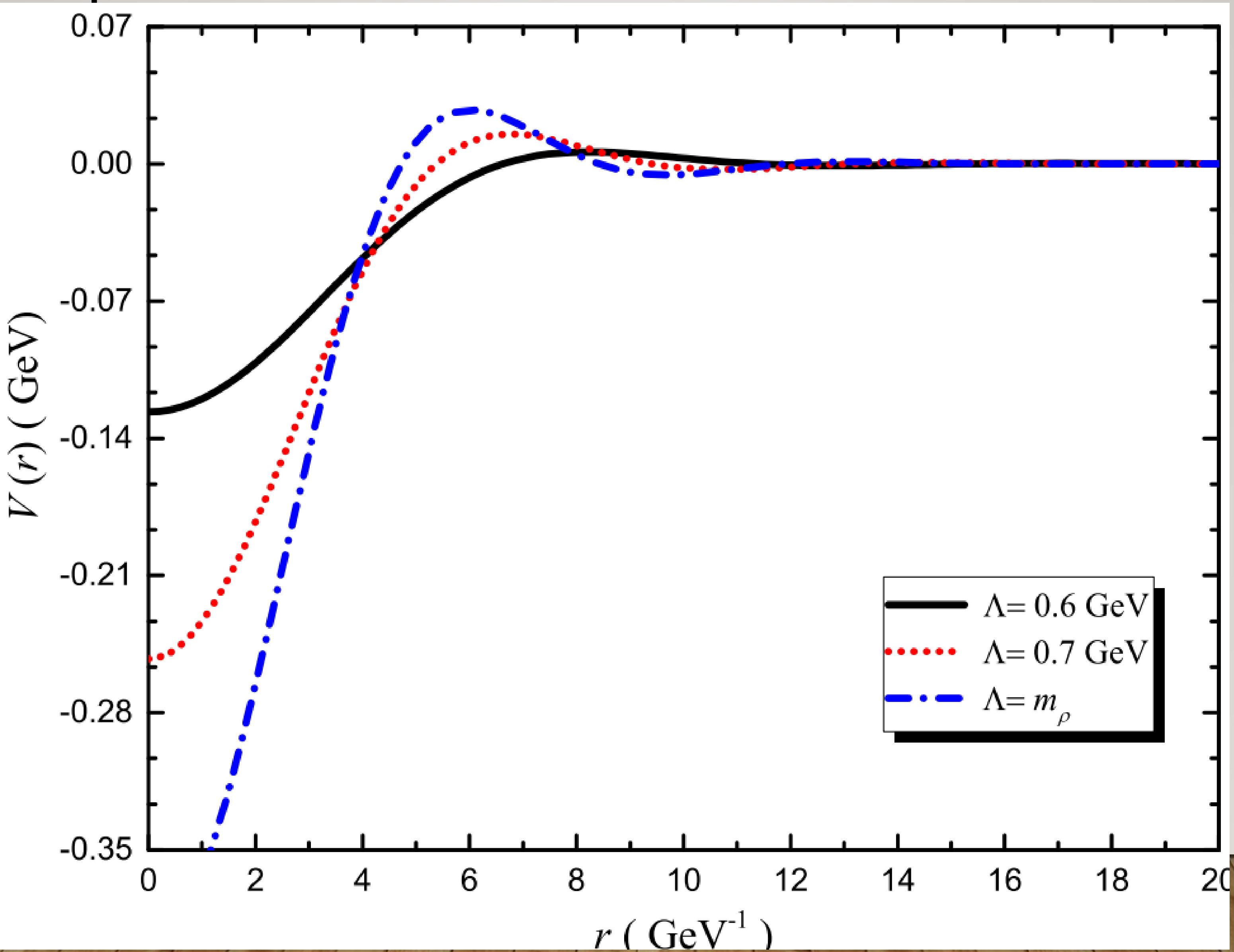


FIG. 9: (color online). S-wave potentials of the DD^* systerm with $I = 1$ in units of GeV. The line types and color schemes match those of Fig. 8.

- dependence on the cutoff



we find a bound state in I=0 channel with ~20MeV. And no bound state in I=1. We also discuss the Lambda dependence in cutoff

$\exp(-\vec{p}^{2n}/\Lambda^{2n})$,ranging from 0.6GeV, 0.7GeV and m_ρ, we obtain binding energy 2.5, 21.5, 59 MeV.

Our results is consistent with OPE model (PhysRevD.88.114008).

SUMMARY

- We investigate the DD^* potnential with ChPT, including contact, one-pion, and complete two-pion exchange.
- The I=0 and I=1 channel has quite different behavior. With the help of resonance saturation in contact term, we find I=0 exist a bound state with binding energy 21.5MeV (cutoff=1GeV).
- The effect of cut off is investigated, and the binding energy of I=1 ranges from 2.5~59.
- Our results agrees molecular model qualitatively.

- 谢谢！