On chiral effective field theories for nuclear and nucleon structures

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Outline I

1. Chiral EFT for nuclear forces
   - Weinberg (90)
   - Kaplan-Savage-Wise (98)
   - More attempts
2. Chiral EFT with one baryon
3. Facilitating χPC with covariance: two exercises
   - EFT(π) to EFT($\not\pi$) for NN scattering
   - Covariant to HB for nucleon structure
4. Summary and Prospects

Outline

- Pre-EFT(QCD) era: Yukawa (1930), OBE (1960), etc.
- EFT(QCD) era: Weinberg's idea (1990)
  - T_{NN} is nonperturbative: $1/E_N \sim M_N/Q^2 \gg 1/Q$, so
  - χPT applied to V_{NN}(2N irreducible), WPC:
    $\nu = -2 + 2A + 2(L - C) + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2\right)$
  - *NN* amplitudes from LSE (NR!): $T_{NN} = V_{NN} + V_{NN} G_0 T_{NN}$
  - FT basis for *NN* forces (RMP81(09)1773, PRp503(11)1)
- However (KSW, 98):
  - WPC formally not consistent
  - Incompatible with large $^1S_0$ scattering length

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Kaplan-Savage-Wise (98)

- Revised PC applied to *NN* amplitudes (NR!)
- Leading order resummed
- Merits:
  - PT approach, Analytical
  - Large scattering length in $^1S_0$ indeed.
- Problems
  - LET violated by actual data (Cohen etal,99)
  - Not converge above 100MeV (Fleming etal,00)
  - Other problems (Epelbaum etal, 06,09; Machleidt etal,11)

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- Failure of standard algorithm in nonperturbative regime (nucl-th/0310048, PRC71(05)034001): $T = V + VG_0T \rightarrow T^{-1} = V^{-1} - \mathcal{G}_0$
- More attempts: Beane-Bedaque-Savage-van Kolck(00), Nogga-Timmermens-vK(05), Pavon Valderrama-Ruiz Arriola(06), Soto-Tarrus(08), Long-vK(08), Yang-Elster-Phillips(08), Beane-Kaplan-Vuorinen(09), Long-Yang(11,12), etc.
- Two popular choices in literature (mostly NR!):
  - (I) Revising power counting: perturbative, stressing RG inv
  - (II) Weinberg plus Lepage: nonperturbative, finite cutoff, numerical

- Recent (Epelbaum-Gegelia,PLB716(12)338; Ren-Li-Geng-Long-Ring-Meng,1611.08475[nucl-th]): employing relativistic propagator
  - Milder UV divergence (cutoff dependence)
  - Better 'figures' at low orders already
- *A satisfactory field theory for nuclear force around 2020?*
  1930 $\Rightarrow$ 1960 $\Rightarrow$ 1990 $\Rightarrow$ 2020?

EFT approach to nucleon structures

- PDF(GPD) for baryons in χPT, Heavy Baryon (HB) formulation: Chen-Ji, PLB523(01)73,107; Arndt-Savage, NPA697(02)429
- Problems with chiral EFT involving one baryon:
  - HB: χPC preserved, incorrect near threshold
  - Covariant: Correct near threshold, χPC breaks down
  - Tang(96), Becher-Leutwyler(99), Fuchs et al(05): prescriptions to remove the ’clashes’
  - EJPA49(13)23, Moiseeva-Vladimirov: failure of HB for non-local light-cone operators
- Consensus? Covariant formalism with appropriate prescriptions seems to be winning

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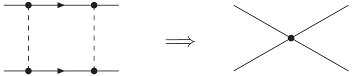
Pionfull to pionless: Box diagram

$$\mathcal{L}_{EFT(\pi)} = \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}m_\pi^2\pi^2$$
$$+\psi\left(i\not{D} - M_N + \frac{g_A}{2f_\pi}\gamma^5\tau\cdot\not\partial\pi\right)\psi - \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi + \cdots$$
$$D \equiv \partial + \frac{i}{4f_\pi^2}\tau\cdot(\pi\times\partial\pi) + \cdots$$
$$\Longrightarrow$$
$$\mathcal{L}_{EFT(\not\pi)} = \bar{N}\left(i\partial_0 + \frac{\nabla^2}{2M_N}\right)N - \frac{1}{2}C_0^{(\not\pi)}\left(\bar{N}N\right)^2 + \cdots$$



Box diagram: covariant calculation

$$T_{box}(\mathbf{0},\mathbf{0}) \sim \underbrace{\left(\Gamma_\epsilon + \ell_N + 3\right) M_N^2}_{local\ I} - 4 \underbrace{\left(\Gamma_\epsilon + \ell_N + 1\right) m_\pi^2}_{local\ II}$$

$$+ \underbrace{\frac{15}{4}\left[\Gamma_\epsilon + \ell_\pi + \frac{1}{5} + o(1/\rho)\right] m_\pi^2}_{chiral} \quad \langle \to V_{2\pi} \rangle$$

$$+ \underbrace{f(\rho)\,(\approx -3\pi)\, M_N m_\pi}_{nonlocal,\ 'IR\ enchancement'} \quad \langle \to T_{it,1\pi}, 2NR \rangle$$

$$\ell_N \equiv \ln \frac{4\pi\mu^2}{M_N^2},\ \ell_\pi \equiv \ln \frac{4\pi\mu^2}{m_\pi^2},\ \rho \equiv \frac{M_N^2}{m_\pi^2}$$

Box diagram I: covariant calculation

- In the foregoing calculation, we have used

$$\ln \rho = \ln \frac{M_N^2}{m_\pi^2} = \Gamma_\epsilon + \ell_\pi - (\Gamma_\epsilon + \ell_N)$$

- About the violation of χPC in covariant chiral EFT for nuclear force:
  - Local items could be removed in Chiral EFT for *NN*
  - Nonlocal items (pinching) must be resummed somehow
- One prescription to remove local violations:

$$\mathcal{A}(M_N, m_\pi, \cdots)_{(D)} : \quad \left(\partial_{m_\pi^2}\right)^{\omega_{\mathcal{A}}+1} \to \int_{(D)} \to \left(\int_{m_\pi^2}\right)^{\omega_{\mathcal{A}}+1}$$

Box diagram: further analysis I

Mod. Phys. Lett. **A29**(14)1450043:

$$\underbrace{T_{it,1\pi}(\mathbf{0},\mathbf{0}) \sim M_N m_\pi f(\rho)}_{nonlocal,\ definite} \to \underbrace{T^{(NR)}_{it,1\pi}(\mathbf{0},\mathbf{0}) \sim 8\pi^2 M_N I_4(\mathbf{0})}_{divergent\ due\ to\ NR\ expansion}$$

$$I_4(\mathbf{0}) \equiv \int \frac{d^3\mathbf{l}}{(2\pi)^3}\frac{\mathbf{l}^2}{E^4_{\pi;l}} = -\frac{3m_\pi}{8\pi}(DR) + \frac{\Lambda}{2\pi^2}(Cutoff)$$

$$\langle E_{\pi;l} \equiv \sqrt{\mathbf{l}^2 + m_\pi^2}\rangle$$

$$I_4^{(\not\pi)}(\mathbf{0}) \equiv \int_{\leq m_\pi} \frac{d^3\mathbf{l}}{(2\pi)^3}\frac{\mathbf{l}^2}{E^4_{\pi;l}} = -\varepsilon_{(\not\pi)}\frac{3m_\pi}{8\pi},\ \varepsilon_{(\not\pi)} = \frac{10-3\pi}{6\pi} \ll 1$$

$$|C^{(\not\pi)}_{0,it}| \sim M_N m_\pi \gg |C^{(\not\pi)}_{0,V_{2\pi}}| \sim m_\pi^2$$

Box diagram: further analysis II

$$C_0^{(\not\pi)} = \frac{4\pi}{M_N \Lambda^{(\not\pi)}}$$

Table: Various contributions to $C_0^{(\not\pi)}$ and $\Lambda^{(\not\pi)}$

	$V_{2\pi}$(KBW)	$V_{2\pi}$(EGM)	$2NR$
$C_0^{(\not\pi)}$	$\frac{3g_A^4 m_\pi^2}{16\pi^2 f_\pi^4}$	$\frac{g_A^4 m_\pi^2}{8\pi^2 f_\pi^4}$	$\frac{9g_A^4 M_N m_\pi}{128\pi f_\pi^4}$
$\Lambda^{(\not\pi)}$	$\frac{64\pi^3 f_\pi^4}{3g_A^4 M_N m_\pi^2}$	$\frac{32\pi^3 f_\pi^4}{4_A M_N m_\pi^2}$	$\frac{512\pi^2 f_\pi^4}{9g_A^4 M_N^2 m_\pi}$

Box diagram: further analysis III

Table: $\Lambda^{(\not\pi)}$ in m_π with $(f_\pi, m_\pi, M_N) = (92.4, 138, 939)$

g_A	$V_{2\pi}$(KBW)	$V_{2\pi}$(EGM)	$2NR$
1.26	$11.63 m_\pi$	$17.44 m_\pi$	$0.97 m_\pi$
1.29	$10.58 m_\pi$	$15.88 m_\pi$	$0.88 m_\pi$
1.32	$9.65 m_\pi$	$14.48 m_\pi$	$0.80 m_\pi$

Box diagram: further analysis IV

- The $2NR$ item dominates the contributions to $C_0^{(\not\pi)}$
- NR expansion 'tucks in' extra divergence, 'damaging' LSE?
  - Most NR divergences appear as $[\int_l, \check{P}_{NR}] \neq 0$
  - Reversely, most NR divergences should be replaced by definite numbers
- KSW not quite 'consistent': large C_0 and perturbative pions

'Mapping' test with BKV 'OPE'

Beane-Kaplan-Vuorinen, PRC**80**(09)011001 ($\lambda_{\rm BKV} \approx 750$MeV)

$$V^{(\rm BKV)}_{1\pi}(\mathbf{q}) \sim \left(\frac{\sigma_1\cdot\mathbf{q}\ \sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2} - \frac{\sigma_1\cdot\mathbf{q}\ \sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+\lambda^2_{\rm BKV}}\right) + \frac{\lambda^2_{\rm BKV}}{\mathbf{q}^2+\lambda^2_{\rm BKV}},$$

$$T^{(\rm BKV)}_{it,1\pi}(\mathbf{0},\mathbf{0}) \sim \pi M_N m_\pi\left[\frac{2\theta^2-\theta+1}{8(1+\theta)} + \frac{\sigma_1\cdot\sigma_2}{6(1+\theta)}\right],\ \theta \equiv \frac{\lambda_{\rm BKV}}{m_\pi}$$

$$I^{(\not\pi)}_{4;(\rm BKV)}/I_{4;(\rm BKV)} \approx 15.6\%$$

- $C_0^{(\not\pi)}$ is still dominated by the $2NR$ item[1]
- BKV still not quite 'consistent'
- The reorganization of *NN*EFT is a complicated issue
- 'Mapping' to EFT($\not\pi$) may serve as additional 'test'

[1] MPLA29(14)1450043

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Cov to HB

$$\mathcal{L}_{Cov} = \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}m_\pi^2\pi^2 + \bar{\psi}\left(i\not{D} - M_N + \frac{g_A}{2f_\pi}\gamma^5\tau\cdot\not{\partial}\pi\right)\psi + \cdots$$

$$\Longrightarrow$$

$$\mathcal{L}_{HB} = \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}m_\pi^2\pi^2 + \bar{N}\left(iv\cdot D + \frac{g_A}{f_\pi}S_\mu\tau\cdot\partial^\mu\pi\right)N + \cdots$$

$$S_\mu \equiv \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu,\ v_\mu \equiv P_\mu/M_N,\ S\cdot v = 0$$

Leading non-singlet twist-2 operators (nucleon) in χEFT

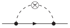
$$\mathcal{O}^{(a;cov)}_{\mu_1\cdots\mu_n} = \tilde{A}^{(n)}\bar{\psi}\gamma_{\mu_1}\left[\tau^a + \frac{\pi^a\tau\pi - \tau^a\pi^2}{2f_\pi^2} + \frac{g_A(\tau\times\pi)^a}{f_\pi}\gamma^5\right]\partial_{\mu_2}\cdots\partial_{\mu_n}\psi$$

$$\mathcal{O}^{(a;HB)}_{\mu_1\cdots\mu_n} = A^{(n)}v_{\mu_1}\cdots v_{\mu_n}\bar{N}\tau^a\left[\tau^a + \frac{\pi^a\tau\pi - \tau^a\pi^2}{2f_\pi^2}\right]N$$

Covariant calculation: n=1

$\langle N|\mathcal{O}^{(a;cov)}_{\mu_1\cdots\mu_n}|N\rangle$: one-loop diagrams

$$\sim -\frac{\Gamma_\epsilon+\ell_N+2}{2}m_\pi^2+\frac{3\Gamma_\epsilon+3\ell_\pi+1}{4}m_\pi^2+o(\frac{1}{\rho})$$



$$\sim 2(\Gamma_\epsilon+\ell_N+1)M_N^2+4(\Gamma_\epsilon+\ell_N+2)m_\pi^2 -(3\Gamma_\epsilon+3\ell_\pi+1)m_\pi^2+o(\frac{1}{\rho})$$

$$\sim -\frac{3(\Gamma_\epsilon+\ell_N+2)}{2}m_\pi^2+\frac{3(3\Gamma_\epsilon+3\ell_\pi+1)}{4}m_\pi^2+o(\frac{1}{\rho})$$

$$\sim -2(\Gamma_\epsilon+\ell_N+1)M_N^2-2(\Gamma_\epsilon+\ell_N+2)m_\pi^2 +o(\frac{1}{\rho})$$

Mixed calculation: n=1

$\langle N|\mathcal{O}^{(a;HB)}_{\mu_1\cdots\mu_n}|N\rangle$: one-loop diagrams

$$\sim -\frac{3\Gamma_\epsilon+3\ell_N+1}{2}M_N^2-(\Gamma_\epsilon+\ell_N+1)m_\pi^2 +\frac{3\Gamma_\epsilon+3\ell_\pi+1}{4}m_\pi^2+\tilde{o}(\tfrac{1}{\rho})$$

$$\sim 2(\Gamma_\epsilon+\ell_N+1)M_N^2+4(\Gamma_\epsilon+\ell_N+2)m_\pi^2 -(3\Gamma_\epsilon+3\ell_\pi+1)m_\pi^2+\tilde{o}(\tfrac{1}{\rho})$$

$$\sim -\frac{3(\Gamma_\epsilon+\ell_N+2)}{2}m_\pi^2+\frac{3(3\Gamma_\epsilon+3\ell_\pi+1)}{4}m_\pi^2+\tilde{o}(\tfrac{1}{\rho})$$

$$\sim 0$$

Heavy-Baryon calculation: n=1

$\langle \tilde{N} | \mathcal{O}^{(a;HB)}_{\mu_1\cdots\mu_n} | \tilde{N} \rangle$: one-loop diagrams

 $\sim \; +\frac{3\Gamma_\epsilon+3\ell_\pi+1}{4} m_\pi^2$

 $\sim \; -(3\Gamma_\epsilon + 3\ell_\pi + 1) m_\pi^2$

 $\sim \; +\frac{3(3\Gamma_\epsilon+3\ell_\pi+1)}{4} m_\pi^2$

 $\sim \; 0$

Table: I NN scattering

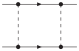
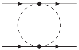
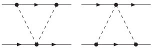
Items	$m_\pi^2(\Gamma_\epsilon + \ell_\pi)$	$M_N^2(\Gamma_\epsilon + \ell_N)$	$m_\pi^2(\Gamma_\epsilon + \ell_N)$	$M_N m_\pi$
	$\surd$	$\surd$	$\surd$	$\surd$
	$\surd$	$\times$	$\times$	$\times$
	$\surd$	$\surd$	$\surd$	$\times$

Table: II Nucleon structures

Items	$m_\pi^2(\Gamma_\epsilon + \ell_\pi)$	$M_N^2(\Gamma_\epsilon + \ell_N)$	$m_\pi^2(\Gamma_\epsilon + \ell_N)$	$M_N m_\pi$
	$\surd$	$\times$	$\surd$	$\times$
	$\surd$	$\surd$	$\surd$	$\times$
	$\surd$	$\times$	$\surd$	$\times$
	$\times$	$\surd$	$\surd$	$\times$

Summary and Prospects

- In chiral effective field theories for nucleon systems
  - Clash between covariance and chiral power counting is removable
  - Better to do in covariant form (at least with relativistic propagators)
  - Or try to avoid non-relativistic expansion as far as possible
- *Prospective studies*:
  - Elaborate organization or treatment of chiral EFT for nuclear force
  - Nonperturbative parametrization of UV divergences (with novel techniques?)

Thank you !