$\eta' \to \eta\pi\pi$ within one-loop U(3) Resonance Chiral Theory and its unitarisation

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Outline

Chiral Perturbation Theory

Gasser and Leutwyler, Nucl. Phys. B 250, 465 (1985)

- Low-energy EFT of QCD for light mesons i.e. $\pi^{\pm,0}$, $K^{\pm}$, K^0, $\bar{K}^0, \eta_8$ associated to $SU(3)_L \otimes SU(3)_R \xrightarrow{\text{SCSB}} SU(3)_V$ exhibited by QCD
- Perturbative expansion in terms of p^2 and m_q: $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots$

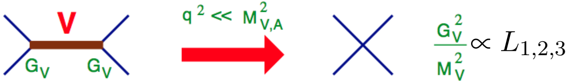
$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle \,, \quad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix},$$

$$u^2 = e^{i\frac{\sqrt{2}\phi}{F}} \,, \quad \chi = 2B\mathcal{M} \,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,, \quad u_\mu = i u^\dagger D_\mu U u^\dagger \,,$$

$$\mathcal{L}_4 = L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u^\nu \rangle \langle u^\mu u_\nu \rangle + L_3 \langle u_\mu u^\mu u_\nu u^\nu \rangle + \dots$$

⚠ η_1 not included due to the axial anomaly

⚠ Valid $\frac{p^2}{M_R^2} < 1$: polynomial cannot reproduce resonance poles

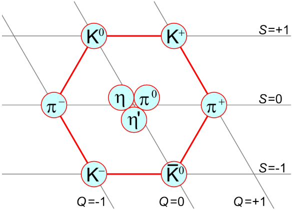


Large-N_C $U(3)$ ChPT

Kaiser and Leutwyler, EPJC 17, 623 (2000)

- Axial Anomaly is absent; η_1 as the ninth Goldstone boson
- Degrees of freedom: $\pi^{\pm,0}$, $K^{\pm}$, K^0, $\bar{K}^0$ and the η and η'

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_1\end{pmatrix}$$



- Simultaneous triple expansion in terms of $\delta \sim p^2 \sim m_q \sim 1/N_C$

$$\mathcal{L}^{\delta^0} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle\chi_+\rangle + \frac{F^2}{3}m_1^2 \ln^2 \det u\,,$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}.$$

⚠ η' heavier than some resonances $\frac{M^2_{\eta'}}{M^2_R} > 1$

Resonance Chiral Theory

Ecker, Gasser, Pich and de Rafael, Nucl. Phys. B 321, 311 (1989)

- Resonance as explicit degrees of freedom

$$\boxed{\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}^{\delta^0} + \mathcal{L}_S + \mathcal{L}^S_{\mathrm{kin}}}$$

$$\mathcal{L}_S = c_d \langle S_8 u_\mu u^\mu \rangle + c_m \langle S_8 \chi_+ \rangle + \tilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \tilde{c}_m S_1 \langle \chi_+ \rangle \,,$$

$$S_8 = \begin{pmatrix} \frac{1}{\sqrt{2}} a_0^0 + \frac{1}{\sqrt{6}} \sigma_8 & a_0^+ & \kappa^+ \\ a_0^- & -\frac{1}{\sqrt{2}} a_0^0 + \frac{1}{\sqrt{6}} \sigma_8 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -\frac{2}{\sqrt{6}} \sigma_8 \end{pmatrix}, \qquad S_1 = \sigma_1$$

$$\mathcal{L}_S = \frac{2c_d}{f^2} \langle S_8 (\partial_\mu \Phi)(\partial^\mu \Phi) \rangle + 4 B_0 c_m [\langle S_8 \mathcal{M} \rangle - \frac{1}{4f^2} \langle S_8 (\Phi^2 \mathcal{M} + \mathcal{M} \Phi^2 + 2 \Phi \mathcal{M} \Phi) \rangle]$$
$$+ \frac{2\tilde{c}_d}{f^2} S_1 \langle (\partial_\mu \Phi)(\partial^\mu \Phi) \rangle + 4 B_0 \tilde{c}_m S_1 [\langle \mathcal{M} \rangle - \frac{1}{4f^2} \langle (\phi^2 \mathcal{M} + \mathcal{M} \Phi^2 + 2 \Phi \mathcal{M} \Phi) \rangle]$$

⚠ Resonances spoils power counting, no systematic EFT but a model based on the Large-N_C limit as a guideline

Recent experimental activity on η and η' physics

- Phenomenology of η and η' among their main objectives

$\eta \to e^+e^-\gamma$ [1]	$\eta \to e^+e^-\gamma$ [3]	$\eta' \to e^+e^-\gamma$ [4]	$\eta^{(\prime)} \to \pi^+\pi^-$ [10]
$\eta \to \pi^0\gamma\gamma$ [2]	$\eta \to \pi^+\pi^-\gamma$ [3]	$\eta' \to \pi^0\gamma\gamma$ [5]	
$\eta' \to \eta\pi^0\pi^0$ [3]	$\eta \to e^+e^-e^+e^-$ [3]	$\eta' \to \omega e^+e^-$ [6]	
	$\eta \to \pi^+\pi^-e^+e^-$ [3]	$\eta' \to 4\pi$ [7]	
		$\eta' \to 3\pi$ [8]	
		$\eta' \to \pi^+\pi^-e^+e^-$ [9]	

- Experimental precision for η-η' observables is increasing
- Better theoretical predictions are demanded
- To have a better and more complete knowledge of QCD at low-energies

[1] Adlarson et.al. Phys.Rev. C 94 6 065206 (2016) ; [2] Nefknes et.al. Phys.Rev. C 90 2 025206 (2014)
[3] Adlarson et.al. Phys.Rev. C 94 6 065206 (2016) ; [4] Ablikim et.al. Phys.Rev. D 92 1 012001 (2015)
[5] Ablikim et.al. ArXiv: 1612.05721 ; [6] Ablikim et.al. Phys.Rev. D 92 5 051101 (2015)
[7] Ablikim et.al. Phys.Rev.Lett. 112 251801 (2014) ; [8] Ablikim et.al. Phys.Rev.Lett. 118 1 012001 (2017)
[9] Ablikim et.al. Phys.Rev. D 87 9 092011 (2013) ; [10] Aaji et.al. Phys.Lett. B 764 233 (2017)

Motivation for $\eta' \to \eta\pi\pi$

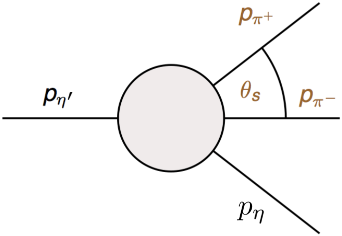
- Main decay channel of the η': $\mathrm{BR}(\eta' \to \eta\pi^0\pi^0) = 22.3(8)\%$, $\mathrm{BR}(\eta' \to \eta\pi^+\pi^-) = 42.9(7)\%$ PDG [2017]
- Cannot be described within $SU(3)$ ChPT
- Advantageous laboratory to test any of its extensions Large-N_C $U(3)$ ChPT and Resonance Chiral Theory
- G-parity conservation prevents vectors to contribute: analysis of the properties of scalar resonances i.e. $\sigma, f_0(980), a_0(980)$
- Study of the η-η' mixing
- Access $\pi\eta$ scattering and phase-shift
- New data very recently released the A2 and BESIII collaborations

Kinematics and Dalitz plot variables

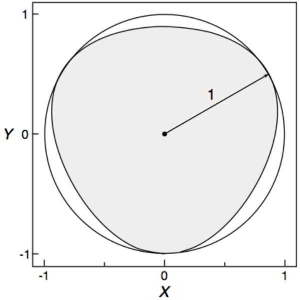
- $s = (p_{\eta'} - p_\eta)^2$
- $t = (p_{\eta'} - p_{\pi^+})^2$
- $u = (p_{\eta'} - p_{\pi^-})^2$
- $s + t + u = m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2$

  $\Rightarrow$ only two independent variables,

  e.g. s and $t - u \propto \cos\theta_s$



- $X = \frac{\sqrt{3}}{Q}(u - t)$
- $Y = \frac{m_\eta + 2m_\pi}{m_\pi} \frac{(m_{\eta'} - m_\eta)^2 - s}{2m_{\eta'}Q} - 1$
- $Q = m_{\eta'} - m_\eta - 2m_\pi$



Dalitz plot parameters: current state-of-the-art

- Dalitz plot to compare experiment and theory

$$|M(X,Y)|^2 = |N|^2\left(1 + aY + bY^2 + cX + dX^2 + \ldots\right)$$

- a, b, c, d are the Dalitz plot parameters

$\eta' \to \eta\pi^0\pi^0$	$a[Y]$	$b[Y^2]$	$c[X]$	$d[X^2]$
GAMS-4π	$-0.067(16)(4)$	$-0.064(29)(5)$	$=0$	$-0.067(20)(3)$
GAMS-4π	$-0.066(16)(4)$	$-0.063(28)(4)$	$-0.107(96)(3)$	$0.018(78)(6)$
A2	$-0.074(8)(6)$	$-0.063(14)(5)$	—	$-0.050(9)(5)$
BESIII	$-0.087(9)(6)$	$-0.073(14)(5)$	0	$-0.074(9)(4)$
Borasoy $et.al.$'05	$-0.127(9)$	$-0.049(36)$	0	$0.011(21)$
Fariborz $et.al.$'14	-0.024	0.0001	0	-0.029

$\eta' \to \eta\pi^+\pi^-$	$a[Y]$	$b[Y^2]$	$c[X]$	$d[X^2]$
VES	$-0.127(16)(8)$	$-0.106(28)(14)$	$0.015(11)(14)$	$-0.082(17)(8)$
BESIII	$-0.047(11)(3)$	$-0.069(19)(9)$	$0.019(11)(3)$	$-0.073(12)(3)$
BESIII	$-0.056(4)(3)$	$-0.049(6)(6)$	$2.7(2.4)(1.8)\cdot 10^{-3}$	$-0.063(4)(4)$
Borasoy $et.al.$'05	$-0.116(11)$	$-0.042(34)$	0	$0.010(19)$
Escribano $et.al.$'10	$-0.098(48)$	$-0.050(1)$	0	$-0.092(8)$
Escribano $et.al.$'10	$-0.098(48)$	$-0.033(1)$	0	$-0.072(1)$

$\eta' \to \eta\pi\pi$: Leading order

- ChPT Lagrangian at $\mathcal{O}(p^2)$

$$\mathcal{L}^{\delta^0} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{3} m_1^2 \ln^2 \det u$$

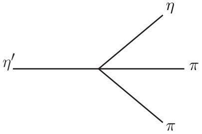
- Expanding in powers of Φ

$$\mathcal{L}^{\delta^0} = \frac{1}{2}\langle\partial_\mu\Phi\partial^\mu\Phi\rangle + \frac{1}{12f^2}\underline{\langle(\Phi(\partial_\mu\Phi)-(\partial_\mu\Phi)\Phi)(\Phi(\partial^\mu\Phi)-(\partial^\mu\Phi)\Phi)\rangle}$$

$$+B_0\left\{-\langle\mathcal{M}\Phi^2\rangle + \underline{(1/6f^2)\langle\mathcal{M}\Phi^4\rangle}\right\} + \mathcal{O}\left(\frac{\Phi^6}{f^4}\right)$$

no contribution

$\Downarrow$

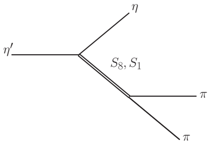
- 

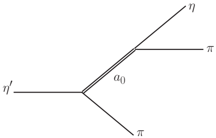
$$= \boxed{\mathcal{M}^{\mathrm{LO}}_{\eta'\to\eta\pi\pi} = \frac{M_\pi^2}{6F_\pi^2}\left(2\sqrt{2}\cos 2\theta - \sin 2\theta\right)}$$

	$BR(\eta' \to \eta\pi^+\pi^-)$	$BR(\eta' \to \eta\pi^0\pi^0)$
Leading Order	1.1%	0.6%
PDG 2016	$42.9(7)$	$22.3(8)\%$

- Reason for this difference: amplitude is chirally suppressed (vanishes when $M_\pi^2 \to 0$)
- Higher order effects? • Resonances exchanges (a_0, f_0, σ) • $\pi\pi$, $\pi\eta$ final state interactions

Scalar Resonance and loop contributions

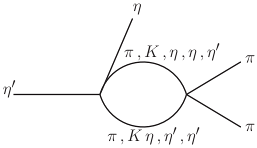




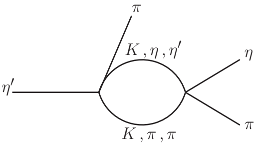
$$\mathcal{M} \propto \frac{\text{mixing}}{F_\pi^2}\Bigg[\text{LO} + \frac{4c_d c_m m_\pi^4}{F_\pi^2 M_S^2} + \frac{1}{F_\pi^2}\frac{[c_d(t-m_\eta^2-m_\pi^2)+2{c_m}^2 m_\pi^2][c_d(t-m_{\eta'}^2-m_\pi^2)+2{c_m}^2 m_\pi^2]}{M_{a_0}^2-t}$$

$$+(t\leftrightarrow u) + \frac{1}{F_\pi^2}\frac{[c_d(s-m_{\eta'}^2-m_\eta^2)+2{c_m}^2 m_\pi^2][c_d(s-m_\pi^2-m_\pi^2)+2{c_m}^2 m_\pi^2]}{M_{\sigma,f_0}^2-s}\Bigg]$$

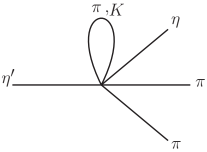
c_d: dominant
c_m: suppressed
$(\propto m_\pi)$



s-channel

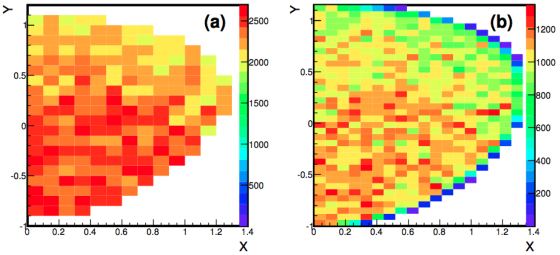


t,u-channels



Fits to experimental data

A2 Coll. 1709.04230



- We relate the experimental Dalitz plot data with the differential decay distribution from theory through

$$\frac{d^2 N_{\text{events}}}{dX dY} = \frac{d\Gamma(\eta' \to \eta\pi^0\pi^0)}{dX dY} \frac{N_{\text{events}}}{\Gamma_{\eta'} \bar{B}(\eta' \to \eta\pi^0\pi^0)} \Delta X \Delta Y \,,$$

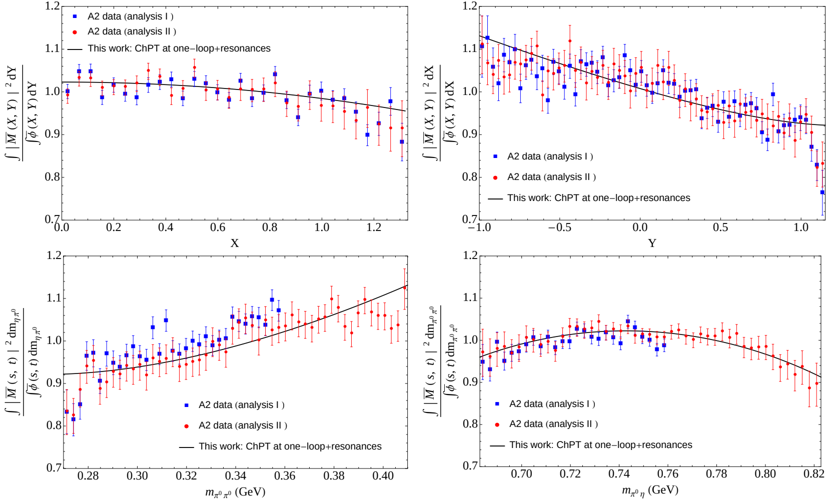
- $N_{\text{events}} = 463066$ (analysis I) and 473044 (analysis II)
- $\Delta X = \Delta Y = 0.10$

Fits to experimental data

- Fit 1: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d = c_m$

$$\text{Fit parameters}: \quad M_S = 972(6)\,\text{MeV}\,, \quad c_d = c_m = 29.9(4)\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 1.22$$

$$\text{Dalitz parameters}: \quad a = -0.095\,, \quad b = 0.005\,, \quad d = -0.037\,.$$



Fits to experimental data

- Fit 2: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d \neq c_m$

$$\text{Fit parameters}: \quad M_S = 953\,\text{MeV}\,, \quad c_d = 27.5\,\text{MeV}\,, \quad c_m = 52.9\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 1.23$$
$$\text{Dalitz parameters}: \quad a = -0.093\,, \quad b = 0.004\,, \quad d = -0.039\,.$$

- Fit 3: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ taking c_m from $4c_dc_m = f^2$

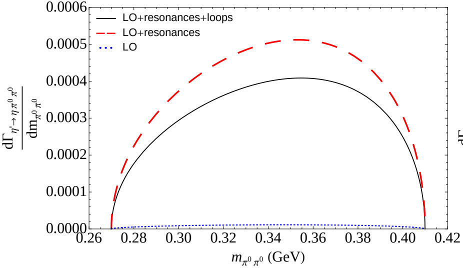
$$\text{Fit parameters}: \quad M_S = 989(6)\,\text{MeV}\,, \quad c_d = 32.5(4)\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 1.23$$
$$\text{Dalitz parameters}: \quad a = -0.098\,, \quad b = 0.005\,, \quad d = -0.033\,.$$

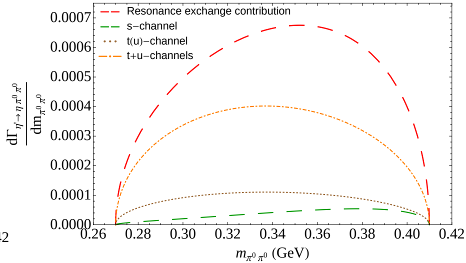
- Fit 4: $M_{S_8} = M_{a_0} = M_S$, while letting M_{S_1} to float and imposing the constraint $c_d = c_m$

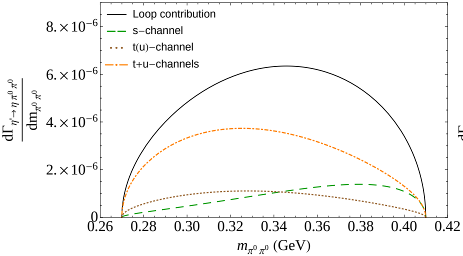
$$\text{Fit parameters}: \quad M_S = 978(27)\,\text{MeV}\,, M_{S_1} = 1017(128)\,\text{MeV}\,, c_{d,m} = 31.0(2.3)\text{MeV}$$
$$\text{Dalitz parameters}: \quad a = -0.091\,, \quad b = 0.003\,, \quad d = -0.042\,.$$

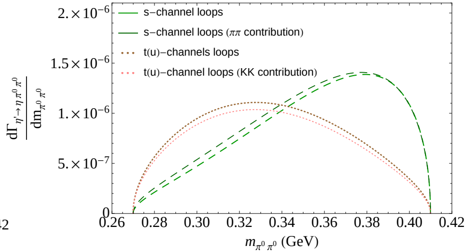
- We also have tried a fit letting all couplings to float i.e. $c_m, \tilde{c}_m, c_d$ and $\tilde{c}_d$, but in this case the fit becomes unstable since there are too many parameters to fit.
- In summary, the associated Dalitz plots remains quite stable independently of the allowed parameters to fit

Hierarchy of the contributions





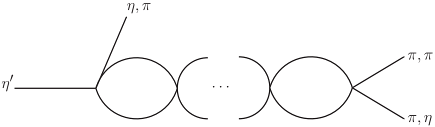


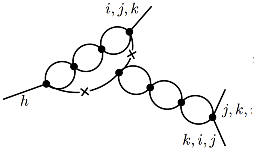


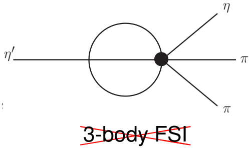
Unitarity

- Unitarity relation

$$\mathrm{Im}\mathcal{M}_{\eta'\to\eta\pi\pi} = \frac{1}{2}\sum_n (2\pi)^4\,\delta^4\left(p_\eta + p_1 + p_2 - p_n\right)\mathcal{T}^*_{n\to\eta\pi\pi}\mathcal{M}_{\eta'\to n}$$







- Restrict to 2-particle rescattering

$$\mathrm{Im}\mathcal{M}^I_{\eta'\to\eta\pi\pi}(s,t,u) = \frac{1}{2(2\pi)^2}\sum_{b,c}\int \frac{dq_b^3}{2q_b^0}\frac{dq_c^3}{2q_c^0}\delta^4(q_b + q_c - p_1 - p_2)\mathcal{T}^I_{bc\to\pi\pi}(s,\theta'_s)^*\mathcal{M}^I_{\eta'\to\eta bc}(s,\theta''_s,\phi''_s)$$

$$+\frac{1}{2(2\pi)^2}\sum_{a,b}\int \frac{dq_a^3}{2q_a^0}\frac{dq_b^3}{2q_b^0}\delta^4(q_a + q_b - p_1 - p_\eta)\mathcal{T}^I_{ab\to\pi\eta}(t,\theta'_t)^*\mathcal{M}^I_{\eta'\to ab\pi}(s,\theta''_t,\phi''_t)$$

$$+\frac{1}{2(2\pi)^2}\sum_{a,c}\int \frac{dq_a^3}{2q_a^0}\frac{dq_c^3}{2q_c^0}\delta^4(q_a + q_c - p_2 - p_\eta)\mathcal{T}^I_{ac\to\pi\eta}(u,\theta'_u)^*\mathcal{M}^I_{\eta'\to a\pi c}(u,\theta''_u,\phi''_u)$$

Unitarity

- Partial waves decompostion ($\mathcal{A} = \mathcal{T}, \mathcal{M}$)

$$\mathcal{A}^I(s, \cos\theta) = \sum_J 32\pi(2J+1)P_J(\cos\theta)a^{IJ}(s)\,,$$

- Integrating over the momentum and then using the relation

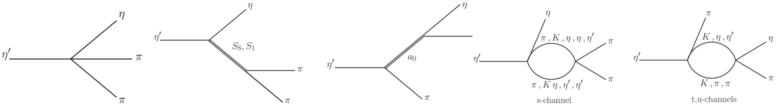
$$\int d\Omega' P_J(\cos\theta')P_{J'}(\cos\theta'') = \frac{4\pi}{2J+1}\delta_{JJ'}P_J(\cos\theta)$$

- Two-particle unitarity relation for the partial-wave decay amplitude

$$\begin{aligned}\mathrm{Im}\left(m^{IJ}_{\eta'\to\eta\pi\pi}(s,t,u)\right) \quad = \quad & \frac{\theta(s-(m_b+m_c)^2)\lambda^{1/2}(s,m_b^2,m_c^2)}{16\pi s}t^{IJ}_{bc\to\pi\pi}(s)^* m^{IJ}_{\eta'\to\eta bc}(s) \\ & + \frac{\theta(t-(m_a+m_b)^2)\lambda^{1/2}(t,m_a^2,m_b^2)}{16\pi t}t^{IJ}_{ab\to\pi\eta}(t)^* m^{IJ}_{\eta'\to ab\pi}(t) \\ & + \frac{\theta(u-(m_a+m_c)^2)\lambda^{1/2}(u,m_a^2,m_c^2)}{16\pi u}t^{IJ}_{ac\to\pi\eta}(u)^* m^{IJ}_{\eta'\to a\pi c}(u)\end{aligned}$$

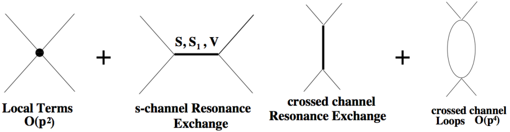
N/D unitarisation method applied to $\eta' \to \eta\pi\pi$

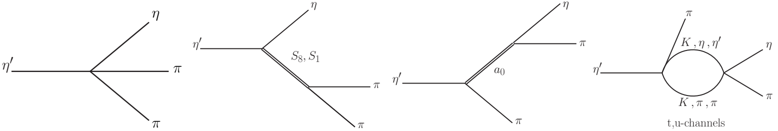
- Amplitude at one-loop in Large-N_C $U(3)$ ChPT with resonances

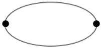
$\mathcal{M}^{\eta' \to \eta\pi\pi}(s) =$ 

- N/D representation of $\mathcal{M}^{\eta' \to \eta\pi\pi}(s)$

$$\mathcal{M}^{\eta' \to \eta\pi\pi}(s) = [1 + N(s)g(s)]^{-1} R(s)$$

$N_{\pi\pi \to \pi\pi}(s) =$ 

$R^{\eta' \to \eta\pi\pi}(s) =$ 

$g(s) = a(\mu) - 16\pi$ 

N/D applied to $\eta' \to \eta\pi\pi$

- Amplitude at one-loop in Large-N_C $U(3)$ ChPT with resonances

$$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = \mathcal{M}(s)^{(2)} + \mathcal{M}(s)^{\mathrm{Res(s,t,u)}} + \mathcal{M}(s)^{\mathrm{Loop(s,t,u)}} \quad (2)$$

- N/D representation of Eq. (2)

$$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = [1 + N(s)g(s)]^{-1} R(s) \quad (3)$$

where

$$\begin{aligned} N_{\pi\pi\to\pi\pi}(s) &= \mathcal{T}(s)^{(2)+\mathrm{Res(s,t,u)}+\mathrm{Loop(t,u)}}, \\ R^{\eta'\to\eta\pi\pi}(s) &= \mathcal{M}(s)^{(2)+\mathrm{Res(s,t,u)}+\mathrm{Loop(t,u)}}, \\ g(s) &= -16\pi \bar{J}_{\pi\pi}(s) + a, \end{aligned}$$

- Chiral expansion of Eq. (3) leads

$$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = \mathcal{M}(s)^{(2)+\mathrm{Res+Loop}} - \mathcal{M}(s)^{(2)} g(s) \mathcal{T}(s)^{(2)} + \text{higher orders}$$

$$\boxed{\mathrm{Im}\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = \sigma(s)\mathcal{M}(s)^{(2)}\mathcal{T}(s)^2}$$

Partial waves

- Unitarized amplitude in terms of the S-and-D-waves

$$\mathcal{M}^{I=0}_{\eta'\to\eta\pi\pi}(s,\cos\theta_s) = \sum_J 32\pi(2J+1)P_J(\cos\theta_s)m^{IJ}(s)$$

$$= 32\pi P_0(\cos\theta_s)\frac{m^{00}(s)}{1+16\pi g_{\pi\pi}(s)t^{00}_{\pi\pi}(s)} + 160\pi P_2(\cos\theta_s)\frac{m^{02}(s)}{1+16\pi g_{\pi\pi}(s)t^{02}_{\pi\pi}(s)}$$

$$m^{IJ}(s) = \frac{1}{32\pi}\frac{s}{\lambda(s,m^2_{\eta'},m^2_\eta)^{1/2}\lambda(s,m^2_\pi,m^2_\pi)^{1/2}}\int_{t_{\min}}^{t_{\max}} dt P_J(\cos\theta_s)\mathcal{M}^I(s,t,u)$$

$$\cos\theta_s = -\frac{s\left(m^2_{\eta'}+m^2_\eta+2m^2_\pi-s-2t\right)}{\lambda(s,m^2_{\eta'},m^2_\eta)^{1/2}\lambda(s,m^2_\pi,m^2_\pi)^{1/2}},$$

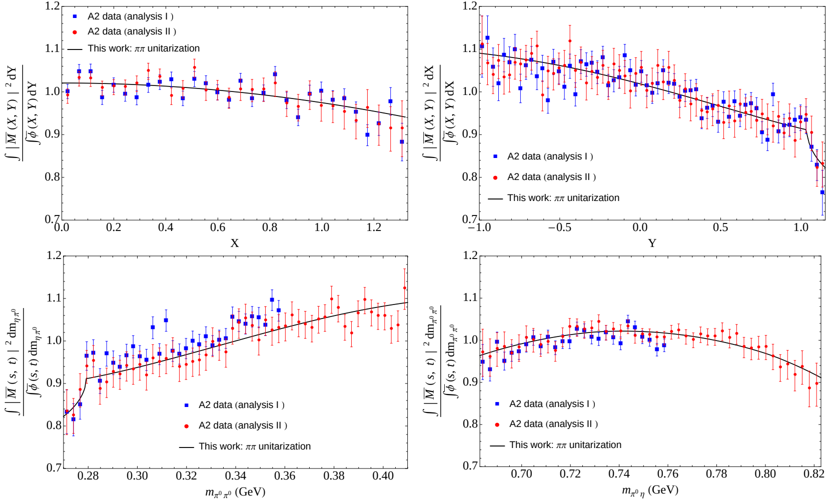
$$P_0(\cos\theta_s) = 1, \quad P_2(\cos\theta_s) = \frac{1}{2}\left[-1+3(\cos\theta_s)^2\right]$$

Fits to experimental data

- Fit 1: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d = c_m$

$$\text{Fit param.}: M_S = 982(15)\,\text{MeV}\,,\, c_{d,m} = 28.2(1.2)\,\text{MeV}\,,\, a_{\pi\pi}(\mu) = 0.71\,,\, \chi^2_{\text{dof}} = 1.13$$

$$\text{Dalitz parameters}: \quad a = -0.080\,, \quad b = -0.035\,, \quad d = -0.040\,.$$



Fits to experimental data: $\pi\pi$ FSI

- Fit 2: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_m = 53.2$ MeV

$$\text{Fit parameters}: \quad M_S = 967\,\text{MeV}\,, \quad c_d = 26.5\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 1.13$$
$$\text{Dalitz parameters}: \quad a = -0.067\,, \quad b = -0.034\,, \quad d = -0.031\,.$$

- Fit 3: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ taking c_m from $4c_d c_m = f^2$

$$\text{Fit parameters}: \quad M_S = 1018\,\text{MeV}\,, \quad c_d = 32.4\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 1.13$$
$$\text{Dalitz parameters}: \quad a = -0.079\,, \quad b = -0.035\,, \quad d = -0.048\,.$$

- We also have tried a fit letting all couplings to float i.e. $c_m, \tilde{c}_m, c_d$ and $\tilde{c}_d$, but in this case the fit becomes unstable since there are too many parameters to fit.

Elastic $\pi\eta$ final state interactions

- N/D representation accounting for $\pi\eta$ FSI

$$m^{IJ}_{\eta'\to\eta\pi\pi}(t,u) = [1+N^{IJ}_{\pi\eta}(t)g_{\pi\eta}(t)]^{-1}R^{IJ}_{\eta'\to\eta\pi\pi}(t) + [1+N^{IJ}_{\pi\eta}(u)g_{\pi\eta}(u)]^{-1}R^{IJ}_{\eta'\to\eta\pi\pi}(u)\,,$$

$$\begin{aligned} N^{IJ}_{\pi\eta}(t) &= t^{IJ}_{\pi\eta}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}\,, \\ R^{IJ}_{\eta'\to\eta\pi\pi}(t) &= m^{IJ}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}\,, \end{aligned}$$

- Avoid double counting of $m^{IJ}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}$ terms

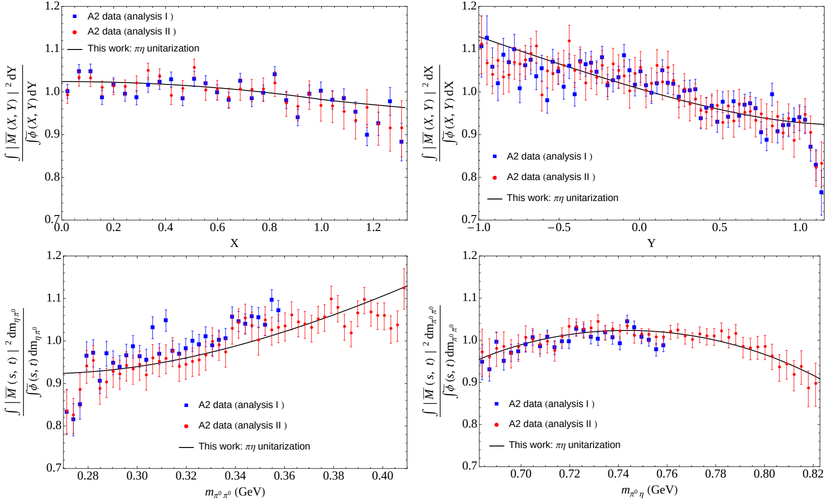
$$\begin{aligned} &\mathcal{M}^{I=1}(t,u,\cos\theta_t,\cos\theta_u) = \mathcal{M}(s,t,u)^{(2)+\mathrm{Res}+\mathrm{Loop}} \\ &+32\pi P_0(\cos\theta_t)\frac{m^{10}(t)}{1+16\pi g_{\pi\eta}(t)t^{10}_{\pi\eta}(t)} + 32\pi P_0(\cos\theta_u)\frac{m^{10}(u)}{1+16\pi g_{\pi\eta}(u)t^{02}_{\pi\eta}(u)} \\ &-32\pi m^{10}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}+\Lambda} - 32\pi m^{10}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}\,, \end{aligned}$$

Fits to experimental data: $\pi\eta$ FSI

- Fit 1: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d = c_m$

$$\text{Fit param.}: M_S = 980(8)\,\text{MeV}\,,\ c_{d,m} = 30.3\,\text{MeV}\,,\ a_{\pi\eta}(\mu) = 0.75\,,\ \chi^2_{\text{dof}} = 1.23$$

$$\text{Dalitz parameters}:\quad a = -0.093\,,\quad b = 0.004\,,\quad d = -0.042\,.$$



Masses and couplings

Source	M_{S_8}	M_{S_1}	M_{a_0}	c_d	c_m
$a_0 \to \eta\pi$ (Guo et.al. '09)	980	$= M_{S_8}$	$= M_{S_8}$	26	80
res. saturation (Ecker et.al. '88)	983	$= M_{S_8}$	$= M_{S_8}$	32	42
$K\pi$ scattering (Jamin et.al. '00)					
	1400	$= M_{S_8}$	$= M_{S_8}$	30	43
	1190	$= M_{S_8}$	$= M_{S_8}$	45.4	$= c_d$
	1260	$= M_{S_8}$	$= M_{S_8}$	24.8	76.7
	1360	$= M_{S_8}$	$= M_{S_8}$	13	85
$PP \to PP$ $(P = \pi, K, \eta)$					
Guo et.al '11	1370^{+132}_{-57}	1063^{+53}_{-31}	$= M_{S_8}$	$15.6^{+4.2}_{-3.4}$	$31.5^{+19.5}_{-22.5}$
Guo et.al '12	1397^{+73}_{-61}	1100^{+30}_{-63}	$= M_{S_8}$	$19.8^{+2.0}_{-5.2}$	$41.9^{+3.9}_{-9.2}$
Ledwig et.al. '14	$1279(9)$	$808.9(4)$	$= M_{S_8}$	$39.8(1)$	$41.1(1)$
This work					
Resonances+loops	$972(6)$	$= M_{S_8}$	$= M_{S_8}$	$29.9(4)$	$= c_d$
Resonances+loops	953	$= M_{S_8}$	$= M_{S_8}$	27.8	53.2
$\pi\pi$ final state interactions	$982(15)$	$= M_{S_8}$	$= M_{S_8}$	$28.2(1.2)$	$= c_d$
	1018	$= M_{S_8}$	$= M_{S_8}$	32.4	$4c_m c_d = f^2$
$\pi\eta$ final state interactions	$980(8)$	$= M_{S_8}$	$= M_{S_8}$	$30.3(8)$	$= c_d$

Summary

- $\eta' \to \eta\pi\pi$ analyzed within $U(3)$ ChPT at one-loop with resonances
- We have illustrated a method (N/D) to resumme two-particle FSI
- Dalitz plot parameters:
  - Y-variable is linear in s: Importance of $\pi\pi$ FSI
  - X-variable appear in the form $\cos\theta_s = X f(Y)$: Importance of the D-wave
  - $\pi\eta$ FSI effects are small

Experiment	a[Y]	b[Y^2]	c[X]	d[X^2]
GAMS4π (c=0) '09	$-0.067(16)(4)$	$-0.064(29)(5)$	0	$-0.067(20)(3)$
VES '07	$-0.127(16)(8)$	$-0.106(28)(14)$	$0.015(11)(14)$	$-0.082(17)(8)$
BESIII '11	$-0.047(11)(3)$	$-0.069(19)(9)$	$0.019(11)(3)$	$-0.073(12)(3)$
A2'17	$-0.074(8)(6)$	$-0.063(14)(5)$	—	$-0.050(9)(5)$
BESIII'17	$-0.087(9)(6)$	$-0.073(14)(5)$	0	$-0.074(9)(4)$
BESIII'17	$-0.056(4)(3)$	$-0.049(6)(6)$	$2.7(2.4)(1.8)\cdot 10^{-3}$	$-0.063(4)(4)$
Previous Estimates				
Borasoy et.al.'05	$-0.127(9)$	$-0.049(36)$	0	$0.011(21)$
Borasoy et.al.'05	$-0.116(11)$	$-0.042(34)$	0	$0.010(19)$
Escribano et.al.'10	$-0.098(48)$	$-0.050(1)$	0	$-0.092(8)$
Escribano et.al.'10	$-0.098(48)$	$-0.033(1)$	0	$-0.072(1)$
Fariborz et.al.'14	-0.024	0.0001	0	-0.029
This talk				
Resonances	$-0.096(9)$	$0.002(1)$	0	$-0.037(6)$
Resonances+loops	-0.095	0.005	0	-0.037
$\pi\pi$ FSI	-0.080	-0.035	0	-0.040
$\pi\eta$ FSI	-0.093	0.004	0	-0.042

Summary

- $\eta' \to \eta\pi\pi$ analyzed within $U(3)$ ChPT at one-loop with resonances
- We have illustrated a method (N/D) to resumme two-particle FSI
- Dalitz plot parameters:
  - Y-variable is linear in s: Importance of $\pi\pi$ FSI
  - X-variable appear in the form $\cos\theta_s = X f(Y)$: Importance of the D-wave
  - $\pi\eta$ FSI effects are small
- In progress: coupled channels effects
- Improvements:
  - Constrains from $I = 0$, $J = 0$ $\pi\pi \to \pi\pi$ phase shift
  - Constrains from $\pi\eta$ distribution

Tensor Resonance contributions

Ecker and Zauner, EPJC 52, 315 (2007)

$$\mathcal{L}_T = -\frac{1}{2}\langle T_{\mu\nu} D_T^{\mu\nu,\rho\sigma} T_{\rho\sigma}\rangle + g_T \langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle + \beta\langle T^\mu_\mu u_\nu u^\nu\rangle$$

$$g_T = 28\,\mathrm{MeV}\,, \quad \beta = -g_T\,, \quad M_T = 1300\,\mathrm{MeV}$$