Quark chiral condensate from overlap quark propagators[1]

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Outline

1. Introduction
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   - Condensate Σ from OPE
2. Numerical results
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   - Condensate from scalar form factor $\mathcal{S}(p^2)$
3. Summary

Quark chiral condensate $\Sigma = -\langle\overline{\psi}\psi\rangle$

- One of the two low energy constants of chiral perturbation theory at leading order. [f_π]
- An order parameter for chiral symmetry breaking in QCD.
- An important input parameter in QCD sum rules.
- Many ways to determining $\langle\overline{\psi}\psi\rangle$ on the lattice,
  - Bank-Casher relation [L. Giustia & M. Lüscher, JHEP2009] $\lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\sum_{k=1}^{\infty}\langle\delta(\lambda-\lambda_k)\rangle = -\frac{\langle\overline{\psi}\psi\rangle}{\pi}$
  - Chiral extrapolation of PS meson masses and decay constant.
  - PS current 2-pt, $\frac{\delta_{ab}}{N}\langle\overline{\psi}\psi\rangle = -Z_p m_q \sum_x \langle P^a(x)P^b(0)\rangle$.
  - Distribution of low-lying eigenvalues of the Dirac operators.
  - Gradient flow with quark field.

Lattice QCD at a glance

- Calcluation from first principle:

$$S\left[U_\mu, \bar{\psi}, \psi\right] = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \bar{\psi}(\not{D} + m)\psi \right\} \equiv S_G + S_F$$

- Time-space discretization and Wick rotation,

$$t \rightarrow i\tau, -(t^2 - \vec{x}^2 \rightarrow (\tau^2 + \vec{x}^2), e^{iS_M} \rightarrow e^{-S_E}$$

- Operator vacuum expectation

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[\psi, \overline{\psi}] \mathcal{D}[U] \mathcal{O}[\psi, \overline{\psi}, U] e^{-(S_G[U] + S_F[\psi, \overline{\psi}, U])}}{\int \mathcal{D}[\psi, \overline{\psi}] \mathcal{D}[U] e^{-(S_G[U] + S_F[\psi, \overline{\psi}, U])}}$$

- MC to generating $U_\mu(x)$ with weight e^{-S}

$$Z = \sum_s e^{-S[s]}, \langle \mathcal{O} \rangle = \frac{1}{Z} \sum_s e^{-S[s]} \mathcal{O}[s].$$

Extracting $\langle\overline{\psi}\psi\rangle$ from lattice data

- Quark propagator in momentum space $S(p)$ can be written as
$$S(p) = \mathcal{S}(p^2)/p^2 + \mathcal{V}(p^2)\not{p}/p^2.$$
- $\mathcal{S}(p^2)$ and $\mathcal{V}(p^2)$ have the following OPE forms
$$\mathcal{S}(p^2) = \mathcal{S}_0(p^2)m_q + \frac{C_{m_q^3}(p^2)}{p^2}m_q^3 + \frac{C_{mA^2}(p^2)}{p^2}m_q\langle A^2\rangle + \frac{C_{\overline{\psi}\psi}(p^2)}{p^2}\langle\overline{\psi}\psi\rangle$$
$$\mathcal{V}(p^2) = \mathcal{V}_0(p^2) + \frac{C_{m^2}(p^2)}{p^2}m_q^2 + \frac{C_{A^2}(p^2)}{p^2}\langle A^2\rangle$$
- Operators up to dimension three are included,
$$A^2 \equiv A_\mu^a A^{a\mu}, m, m^2, m^3, mA^2, \overline{\psi}\psi$$
- $\mathcal{S}_0(p^2)$ [Chetyrkin&Retey, 2000] and $C_{[?]}$ [Chetyrkin&Maier, 2010)] are known up to 3-loop.
- Fitting $\mathcal{S}(p^2)/\mathcal{V}(p^2)$ and $\mathcal{S}(p^2)$ from $S(p)$ in Landau guage to extract $\langle\overline{\psi}\psi\rangle$.
$$\frac{\mathcal{S}(p^2)}{p^2} = \frac{1}{12}\mathrm{Tr}[S(p)], \frac{\mathcal{V}(p^2)}{p^2} = \frac{1}{12p^2}\mathrm{Tr}[\not{p}S(p)], \Lambda_{QCD}^2 \ll p^2 \ll (\pi/a)^2$$

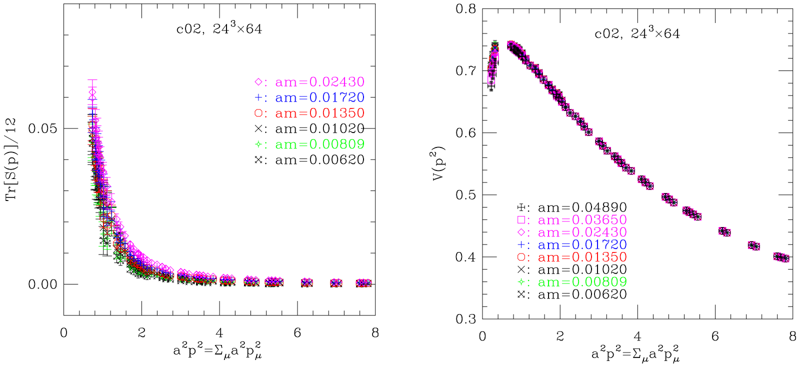
Lattice setup

Table 1: Parameters of configurations with 2+1 flavor domain wall fermions [Aoki et al., PRD 2011]

label	am_{sea}	size	N_{conf}
c005	0.005/0.04	$24^3 \times 64$	92 (92×8)
c01	0.01/0.04	$24^3 \times 64$	88
c02	0.02/0.04	$24^3 \times 64$	99 (99×8)

- Lattice spacing $1/a = 1.75(4)$ GeV from Yang et al., PRD2015.
- Overlap valence quark mass varies from 0.0062 to 0.0489.
- Corresponding pion mass varies from about 220 Mev to 600 Mev.
- Renormalization for ψ using RI/MOM proccedure.

Valence quark mass dependence



- $\mathcal{S}(p^2)$ has a visible valence quark mass dependence, $\mathcal{V}(p^2)$ not.
- Contributions from m_q^2 or higher order terms quite small for $\mathcal{V}$, and $\mathcal{S}$ [Seeing later]

$\langle\overline{\psi}\psi\rangle$ from $\mathcal{S}(p^2)/\mathcal{V}(p^2)$

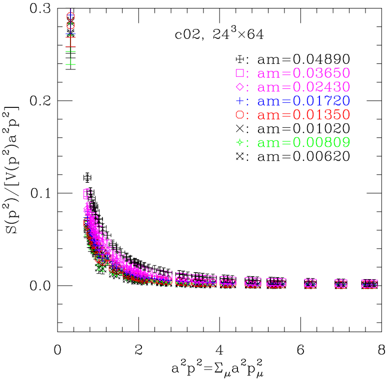
- $\mathcal{S}(p^2)$ and $\mathcal{V}(p^2)$ may heavily suffer from artifacts of $\mathcal{O}(a^2g^2)$, while $\mathcal{R}(a^2p^2) \equiv \mathcal{S}(a^2p^2)/\mathcal{V}(a^2p^2)$ not. [F. Burger, PRD2013]
- Condensensate extracted from $\mathcal{R}(a^2p^2)$ as final result and difference with that from $\mathcal{S}(a^2p^2)$ as systematic error.
- Contribution from $\langle A^2\rangle$ is ignored compared to our fitting range$[\approx 0.15]$ and because of $|C_{A^2}|^2 \approx 0.3$.
- The ratio can be expanded as

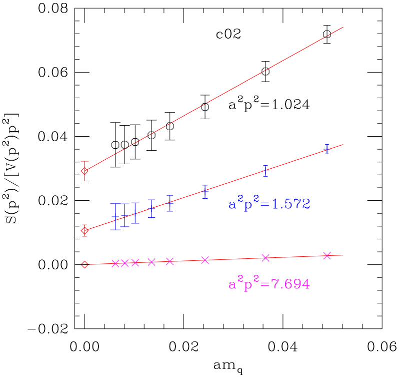
$$\mathcal{R}(a^2p^2) = \frac{\mathcal{S}(a^2p^2}{\mathcal{V}(a^2p^2)} = \frac{\mathcal{S}_0(a^2p^2)}{\mathcal{V}_0(a^2p^2}m_q + \frac{C_{\overline{\psi}\psi}(\mu,p^2)}{(a^2p^2)^2V_0}a^3\langle\overline{\psi}\psi\rangle$$

- In the chiral limit with discretization error

$$\lim_{m_q\to 0}\frac{\mathcal{R}(a^2p^2)}{a} = \frac{C_{\overline{\psi}\psi}(\mu,p^2)}{(a^2p^2)^2V_0}a^3\langle\overline{\psi}\psi\rangle + Ba^2p^2$$

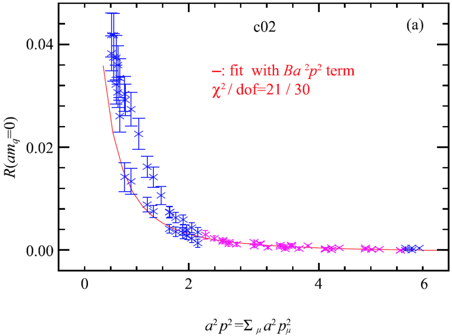
$\langle\bar{\psi}\psi\rangle$ from $\mathcal{S}(p^2)/\mathcal{V}(p^2)$





- $\mathcal{R}(a^2p^2)$ depends on valence quark mass linearly as expected.

$\langle\overline{\psi}\psi\rangle$ from $\mathcal{S}(p^2)/\mathcal{V}(p^2)$: error estimation



- $\Lambda_{QCD}^2 \ll p^2 \ll (\pi/a)^2$
- $B(a^2p^2)$ term is important!. $\chi^2/\text{dof}: 50/31 \to 21/31$
- Fitting range sensitive, vary that to obtain uncertainty.

- Truncation errors from the perturbative expansion of $C_?$ considered
- Errors from lattice spacing considered.

$\langle \bar{\psi}\psi \rangle$ from $\mathcal{S}(p^2)/\mathcal{V}(p^2)$

Table 2: $\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2\text{ GeV})$ on three ensembles

ID	$a^2p^2 \in$	χ^2/dof	$-\Sigma^{1/3}$/MeV
c005	[1.4,3.1]	0.92	-306(11)(7)(13)
c01	[1.8,3.8]	1.01	-296(39)(7)(11)
c02	[2.2,5.3]	0.71	-303(36)(7)(9)

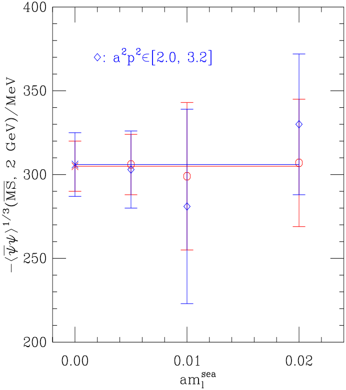
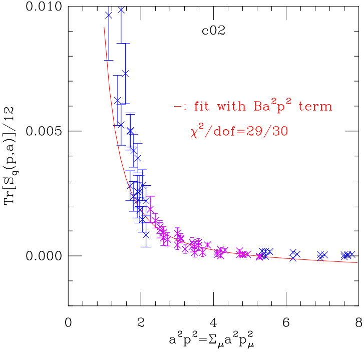


Table 3: $\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2\text{ GeV})$ on three ensembles with same fitting range $a^2p^2 \in [2.0, 3.2]$

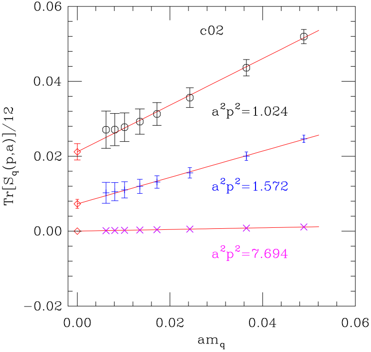
ID	χ^2/dof	$-\Sigma^{1/3}$/MeV
c005	0.97	-303(39)(8)(12)
c01	0.83	-278(53)(6)(10)
c02	0.85	-322(18)(7)(12)

$$\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2\text{ GeV}) = (-304(15)\text{ GeV})^3$$

$\langle\overline{\psi}\psi\rangle$ from scalar form factor $\mathcal{S}(p^2)$



$\mathcal{S}(a^2p^2)$ chiral extrapolation



Fitting of chiral limit results

From OPE formula of $\mathcal{O}(a^2p^2)$ in the chiral limit

$$\frac{\mathcal{S}(a^2p^2)}{a^2p^2} = \frac{1}{12}\mathrm{Tr}[S(ap)] = \frac{\mathcal{O}_R(a^2p^2)}{Z_q a^2p^2} = \frac{C_{\overline{\psi}\psi}(\mu,p^2)}{Z_q(a^2p^2)^2} a^3\langle\overline{\psi}\psi\rangle(\mu) + Ba^2p^2.$$

$\langle\overline{\psi}\psi\rangle$ from scalar form factor $\mathcal{S}(p^2)$

Table 4: $\langle\overline{\psi}\psi\rangle^{\overline{\mathrm{MS}}}$(2 GeV) extracted from $\mathcal{S}(a^2p^2)$ on three ensembles

ID	$a^2p^2 \in$	χ^2/dof	$-\Sigma^{1/3}$/MeV
c005	[1.4,3.1]	1.03	-288(10)(7)(11)
c01	[1.8,3.8]	1.02	-278(35)(6)(9)
c02	[2.2,5.3]	0.76	-288(10)(7)(11)

- A noticeable deviation from previous result.
- $\mathcal{O}(a^2g^2)$ effect may be not neglected and should be removed from finally result

- This work

$$\Sigma^{1/3} = 304(15)(20)\ \text{MeV}$$

- FLAG [S.Aoki et al, 1607.00299]:

$$\Sigma^{1/3} = 274(3)\ \text{MeV}$$

Summary

Result and conclusion

- Calculate quark chiral condensate by fitting lattice data to OPE formula of quark propagator in Landau gauge.
- Two methods to extract $\langle\overline{\psi}\psi\rangle$: the ratio $\mathcal{O}(p^2)/\mathcal{V}(p^2)$ as final result and only $\mathcal{S}(p^2)$ to estimate the lattice artifacts $\mathcal{O}(a^2g^2)$.
- Our result: $\langle\overline{\psi}\psi\rangle^{\overline{\mathsf{MS}}}(2\ \mathsf{GeV}) = (-304(15)(20)\ \mathsf{MeV})^3$.

Problems and Outlook

- $\mathcal{O}(a^2g^2)$ effect should be calculated from lattice perturbation theory and removed from lattice quark propagators.
- More configurations to decrease statistical errors.
- Calculation at more lattice spacings are needed for continuum extrapolation $a \to 0$.

Thank you!

$\langle\overline{\psi}\psi\rangle$ from chiral symmetry of overlap fermions

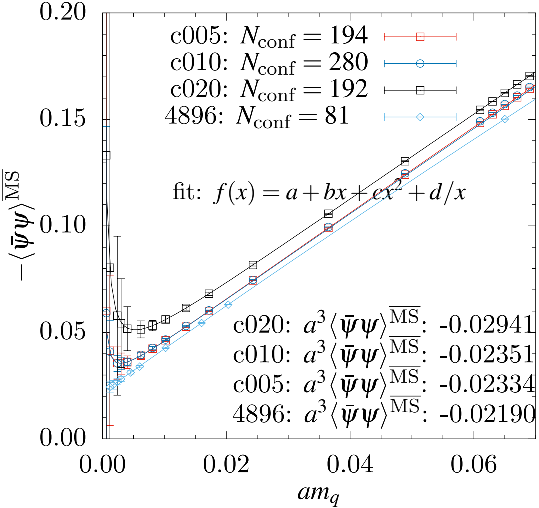


Figure 1: $\langle\overline{\psi}\psi\rangle^{\overline{\rm MS}}(2\ \text{GeV}) = (-276(2)(2)\ \text{MeV})^3$ for Ensemble 4896