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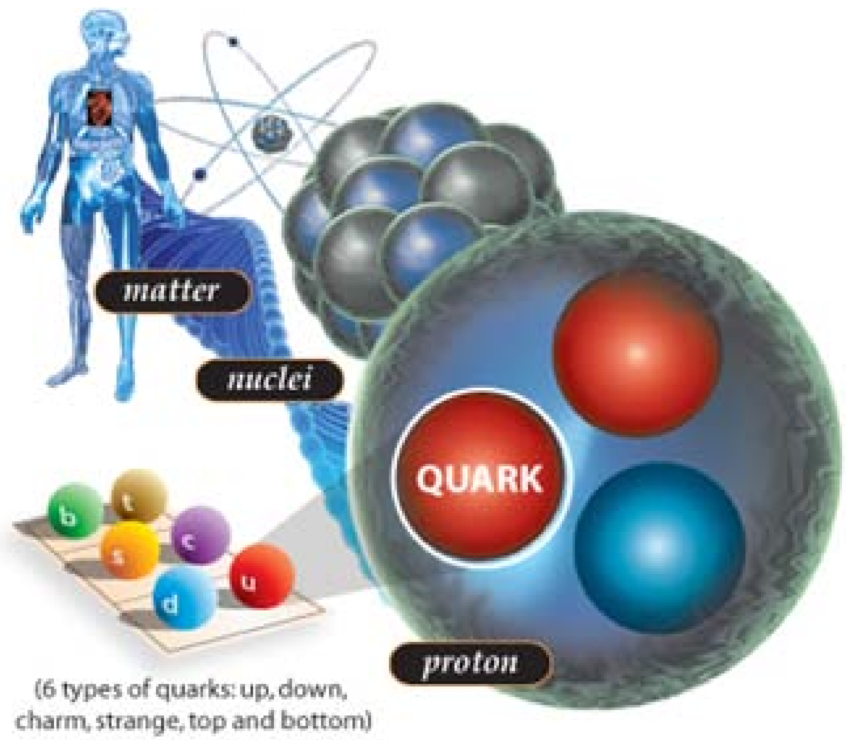
从QCD推导强子手征有效拉氏量

王　青　　2017手征有效场论研讨会　　2017年10月14日



清華大學 Tsinghua University
广西师范大学 GUANGXI NORMAL UNIVERSITY

从QCD推导介子的手征有效拉氏量

清华大学物理系　　王 青

2016年10月29日　　广西师范大学物理科学与技术学院

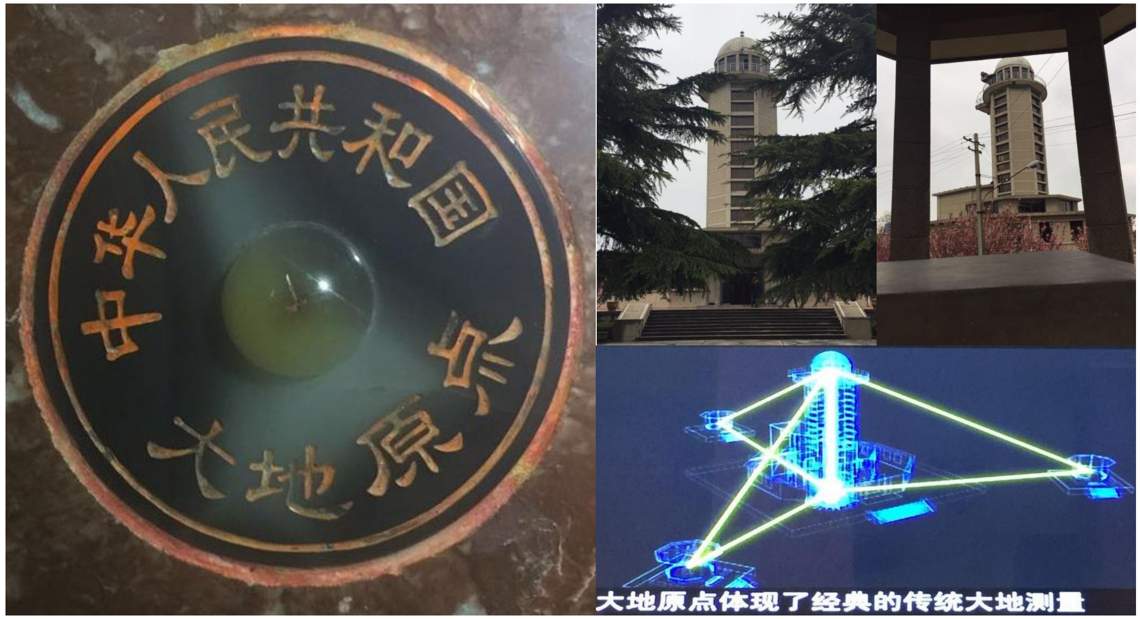
Wang Qing　　P. 1

清華大學 Tsinghua University

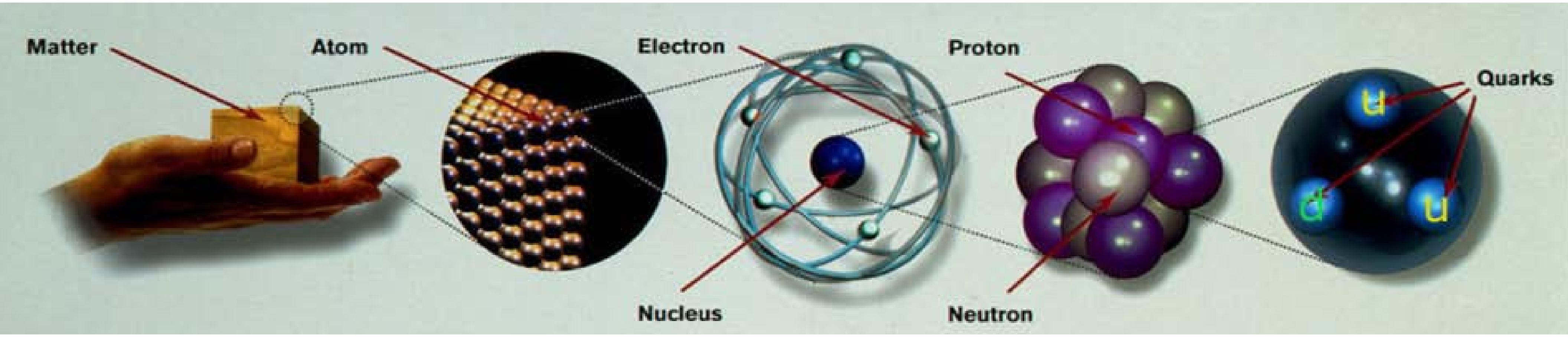
小 结

- 低能强子有效理论的第一原理推导取得了重大进展
- 赝标介子的推导被推广到了所有低能介子,重子的工作也在进行
- 有效拉氏量和束缚态场方程被自然地嵌到路径积分的体系中
- 它直接反映如何处理强作用低能的非微扰效应
- 目前主要处理夸克部分手征对称性,胶子部分用格林函数参数化色禁闭

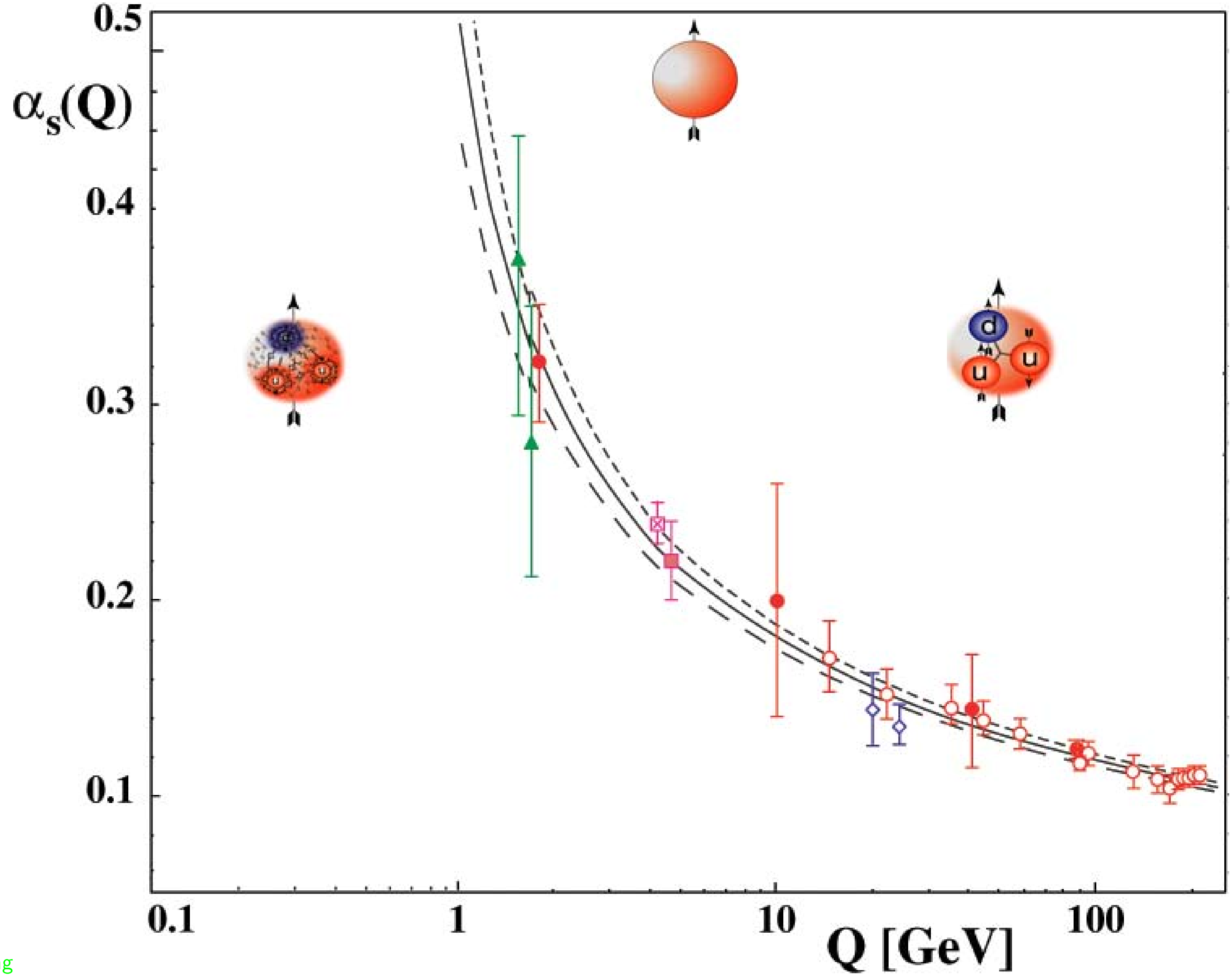
去年讲过，为啥还要再讲？



- 感谢人民群众的要求 去年介子的工作还没有发表；桂林太靠南，参会的人不完全一样
- 去年着重讲矢量介子 作为例子
- 今年针对更一般的强子 介子：赝标、矢量、标量、轴矢量；重子
- 特别着重工作所揭示的一些问题和一些新的 重要 思考 学生拖沓



- 在物质的每个层次都存在相应的物理理论 固体，分子、原子，原子核，夸克
- 更深层次的物理理论更加基本 还原论
- 用深层次的理论解释浅层次的物理虽然基本，但跨尺度难度较大
- 本报告汇报 强子 原子核与夸克之间的层次和 夸克胶子 已知的物质最基本的层次理论的关系


0.5
0.4
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$\alpha_s(Q)$
d
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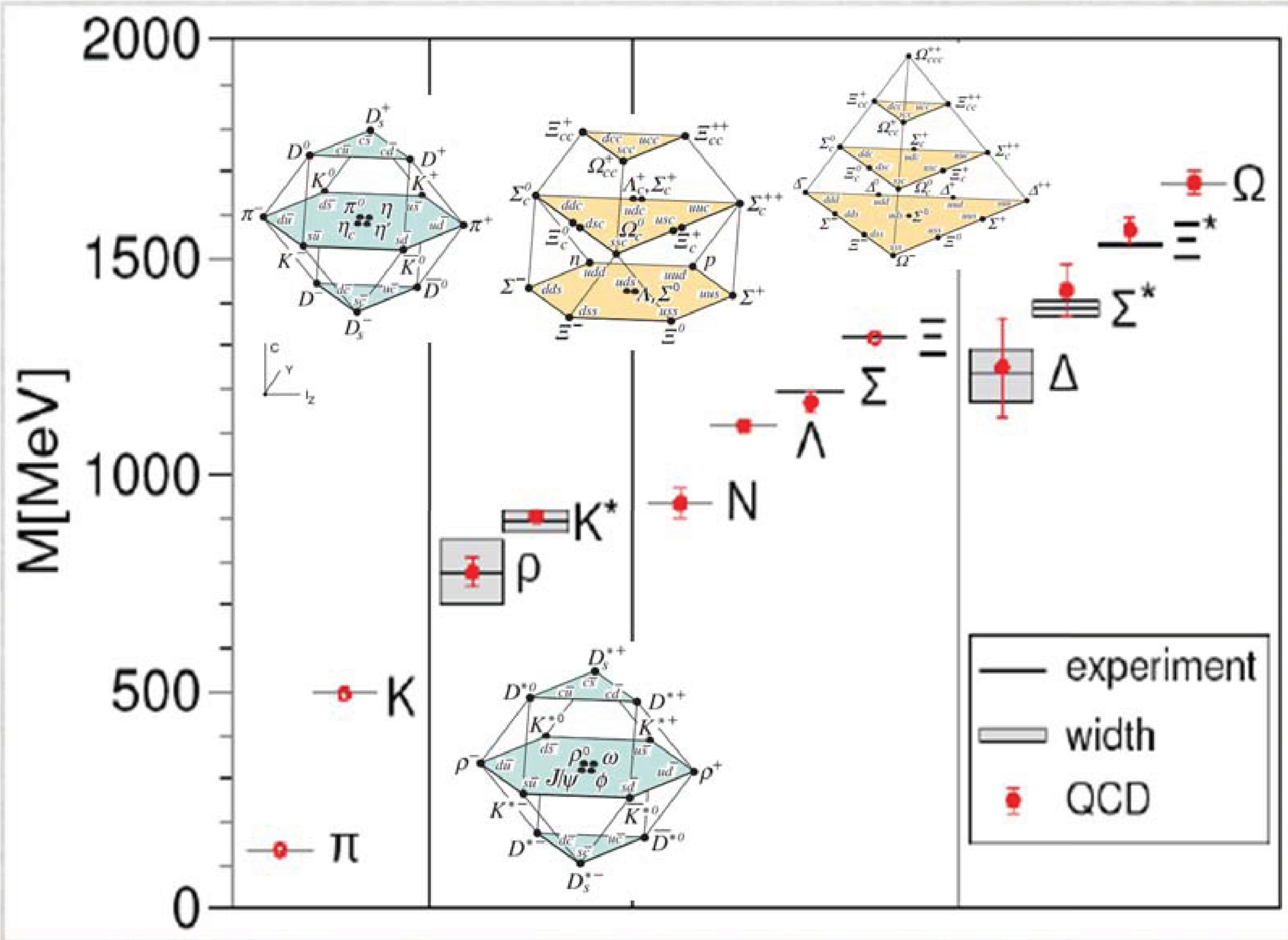
u
0.1
1
10
100
Q [GeV]


2000
1500
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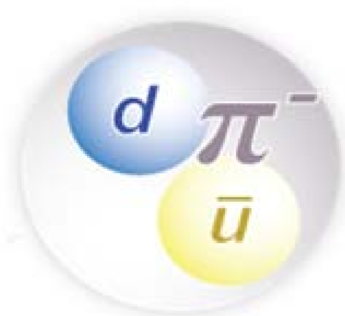
500
0
M[MeV]
π
K
ρ
K*
N
Λ
Σ
Ξ
Δ
Σ*
Ξ*
Ω
experiment
width
QCD

强子层次上物理

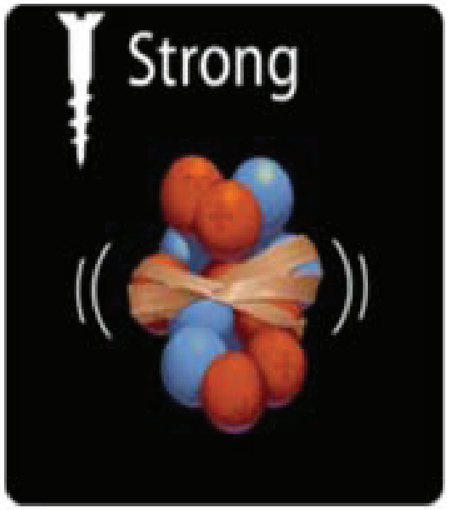
- 存在唯像的手征有效理论$\mathbf{S}_{\rm eff}[\phi,\psi]$：膺标介子，矢量介子，标量介子，重子……
- 有效理论包含所有可能的项，在数幂意义上可描写所有物理过程！
- 每个强子都有运动方程，它可通过有效理论的拉氏量求极值得到：



$$\frac{\delta \mathbf{S}_{\rm eff}[\phi,\psi]}{\delta\phi(\mathbf{x})}=\mathbf{0} \quad\Rightarrow\quad (\partial^2+\mathbf{m}_\phi^2)\phi \;+\text{相互作用项}=\mathbf{0} \qquad \text{玻色子}$$

$$\frac{\delta \mathbf{S}_{\rm eff}[\phi,\psi]}{\delta\psi(\mathbf{x})}=\mathbf{0} \quad\Rightarrow\quad (\mathbf{i}\partial\!\!\!/+\mathbf{m}_\psi)\psi \;+\text{相互作用项}=\mathbf{0} \qquad \text{费米子}$$

- 有效理论的参数在强子层次是自由参数，到高阶数目巨大丧失理论预言性。
- 需要知道有效理论参数的数值：
  - 讨论和描写强子物理
  - 检验基本理论的正确性



- **QCD action:**
  matter (fermions); gauge (gluons+ghosts)

$$S_{\rm QCD} = \int d^4x\,(\mathcal{L}_{\rm I} + \mathcal{L}_{\rm GF+FPG})$$

夸克胶子层次

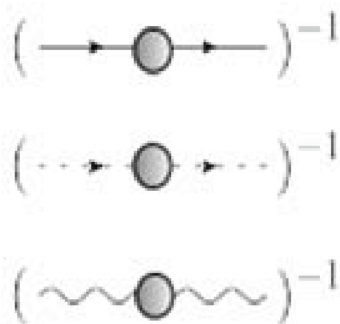
$$\mathcal{L}_{\rm I} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \bar{\psi}^i_{\rm f}(i\gamma^\mu D_\mu - m)_{ij}\psi^j_{\rm f}$$

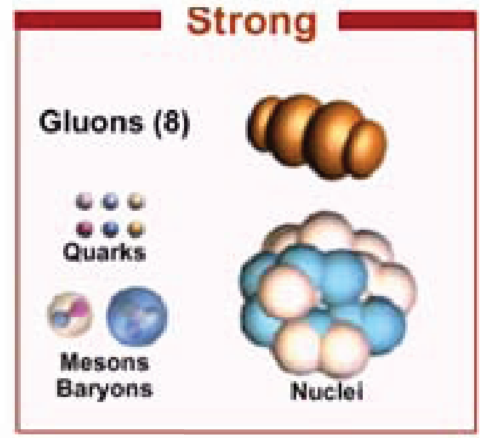
$$\mathcal{L}_{\rm GF+FPG} = s(\bar{c}^a\mathcal{F}^a - \xi/2\bar{c}^a b^a)$$

- **Gauge invariant part**
- **Gauge fixing + FP ghost:** BRST exact, does not appear in the spectrum

- **Properties are encoded in Green's functions:**
  Schwinger-Dyson (SD) equations are their quantum eom

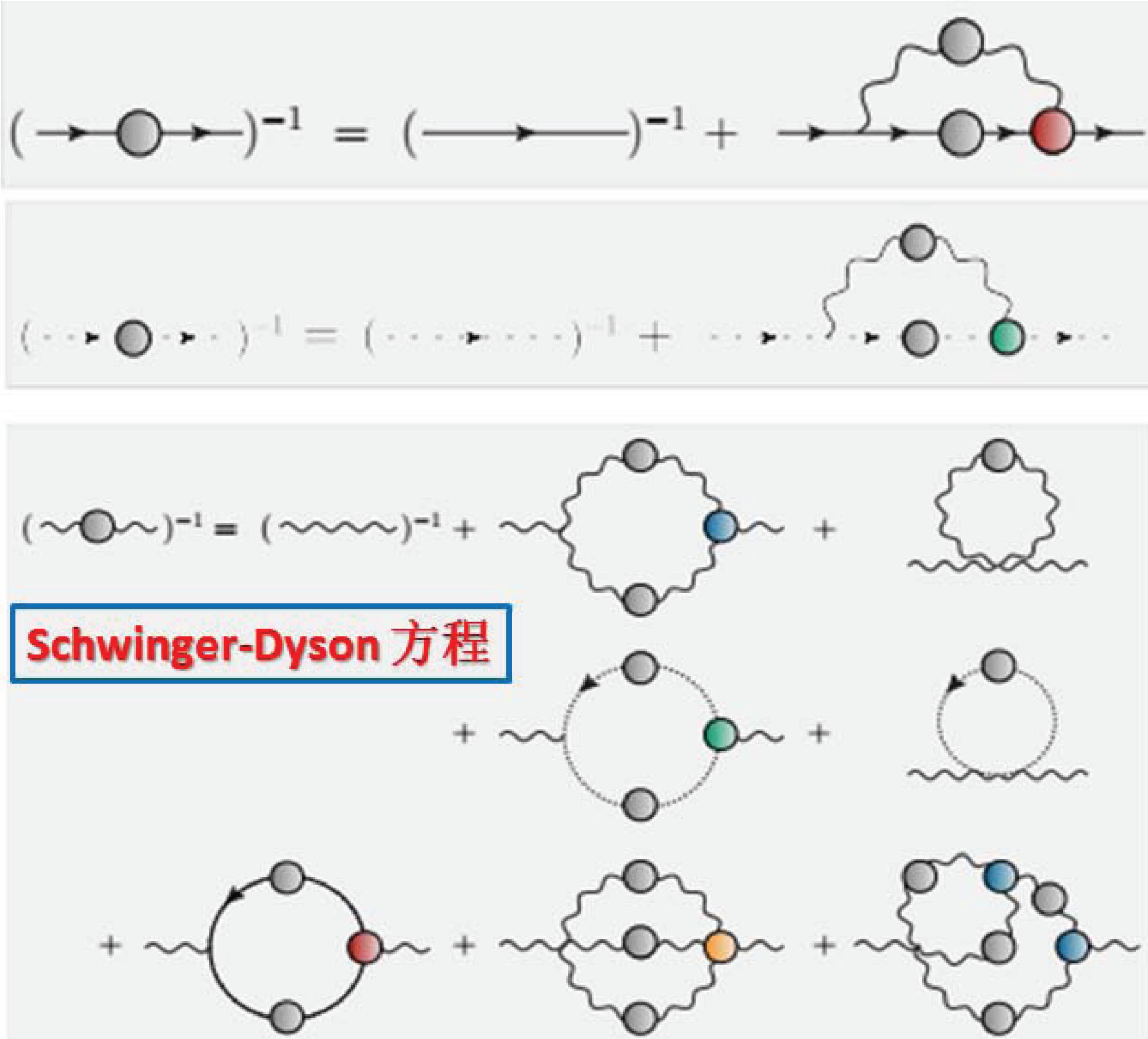
  - **Quark propagator:**
  - **Ghost propagator:**
  - **Gluon propagator:**



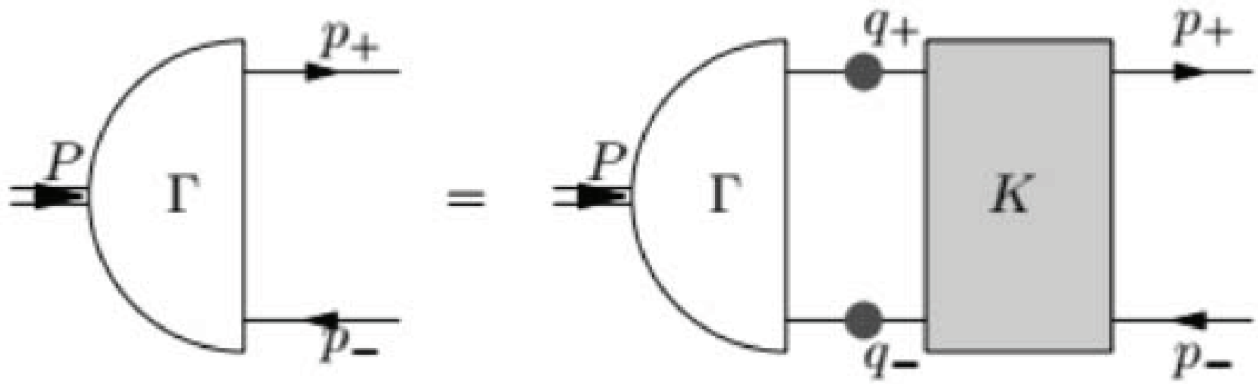


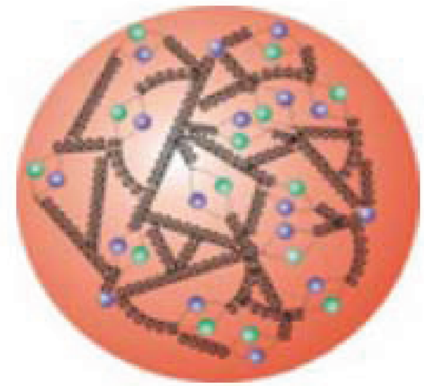
- **Nonperturbative, covariant, IR/UV, light/heavy quarks; but:**
  infinite system of coupled integral equations

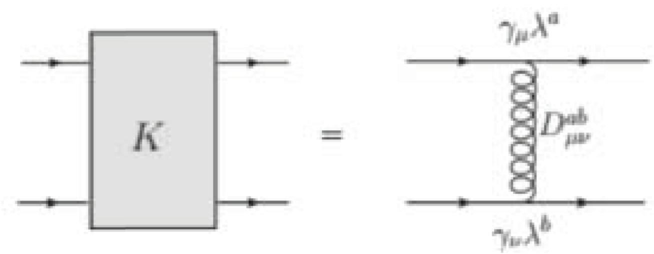
  - **Truncations:** possibly gauge invariant
  - **Ansätze/input:** pQCD, lattice, hadron properties


Schwinger-Dyson 方程

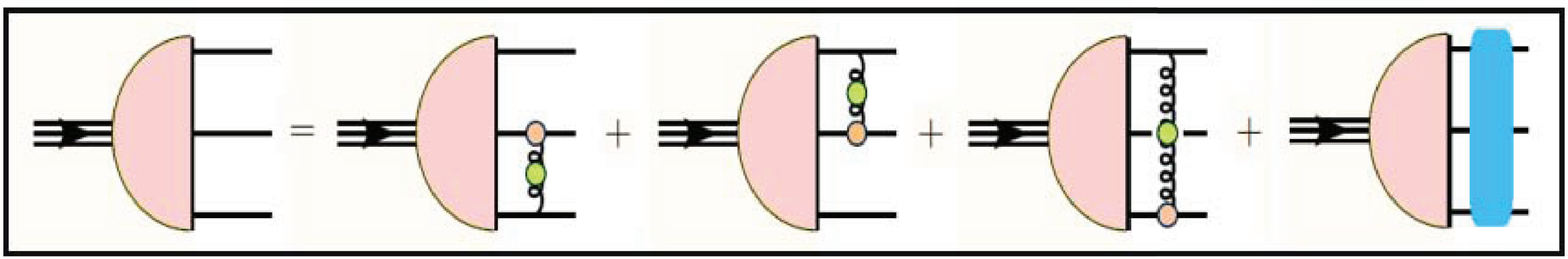
Bethe-Salpeter方程







Faddeev方程



PHYSICAL REVIEW C, VOLUME 60, 055214

Bethe-Salpeter study of vector meson masses and decay constants

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(Received 27 May 1999; published 21 October 1999)

The masses and decay constants of the light vector mesons ρ/ω, ϕ, and $K^{\star}$ are studied within a ladder-rainbow truncation of the coupled Dyson-Schwinger and Bethe-Salpeter equations of QCD with a model two-point gluon function. The approach is consistent with quark and gluon confinement, reproduces the correct one-loop renormalization group behavior of QCD, generates dynamical chiral symmetry breaking, and preserves the relevant Ward identities. The one phenomenological parameter and two current quark masses are fixed by requiring that the calculated f_π, m_π, and m_K are correct. The resulting f_K is within 3% of the experimental value. For the vector mesons, all eight transverse covariants are included and the dominant ones are identified; the complete angle dependence of the amplitudes is also retained. The calculated values for the masses m_ρ, m_ϕ, and $m_{K^\star}$ are within 5%, while the decay constants f_ρ, f_ϕ, and $f_{K^\star}$ for electromagnetic and leptonic decays are within 10% of the experimental values. [S0556-2813(99)04511-2]

PACS number(s): 14.40.Cs, 24.85.+p, 11.10.St, 12.38.Lg

I. INTRODUCTION

A realistic description of vector mesons at the quark-

example, the axial Ward identity dictates that the chiral limit Bethe-Salpeter (BS) amplitude for a pseudoscalar $\bar{q}q$ bound state in the dominant γ_5 channel is given by $B_0(p^2)/f_P$

and $m(\mu)$ depend on the quark flavor, although we have not indicated this explicitly. However, in our analysis we assume, and employ, a flavor independent renormalization scheme and hence all the renormalization constants are flavor-independent.

A. Meson Bethe-Salpeter equation

The renormalized, homogeneous BSE for a bound state of a quark of flavor a and an antiquark of flavor b having total momentum P is given by

$$\Gamma_M^{ab}(p;P)=\int^{\Lambda}\frac{d^4q}{(2\pi)^4}K(p,q;P) \times S^a(q+\eta P)\Gamma_M^{ab}(q;P)S^b(q-\bar{\eta}P), \tag{4}$$

where $\eta+\bar{\eta}=1$ describes momentum sharing, $\Gamma_M^{ab}(p;P)$ is the BS amplitude, and M specifies the meson type: pseudoscalar, vector, axial-vector, or scalar. In this paper we consider the pseudoscalar and vector amplitudes only. The kernel K operates in the direct product space of color and Dirac

B. Ladder-rainbow truncation

We use a ladder truncation for the BSE

$$K_{tu}^{rs}(p,q;P)\to -\mathcal{G}[(p-q)^2]D_{\mu\nu}^{\rm free}(p-q)\left(\frac{\lambda^a}{2}\gamma_\mu\right)^{ru} \otimes\left(\frac{\lambda^a}{2}\gamma_\nu\right)^{ts}, \tag{7}$$

which is consistent with a rainbow truncation for the quark DSE

$$Z_1\int_q^{\Lambda}g^2D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma_\nu^a(q,p) \to\int_q^{\Lambda}\mathcal{G}[(p-q)^2]D_{\mu\nu}^{\rm free}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu . \tag{8}$$

Here $D_{\mu\nu}^{\rm free}(k)$ is the perturbative gluon propagator in Landau gauge. The model is completely specified once a form is chosen for the "effective coupling" $\mathcal{G}(k^2)$.

TABLE II. Comparison of the results for the vector mesons for the three different parameter sets for the effective interaction, using all eight BS amplitudes (top), and using the five leading BS amplitudes only (bottom).

	ρ		$K^\star$		ϕ	
	m_ρ	f_ρ	m_{K*}	f_{K*}	m_ϕ	f_ϕ
Experiment	0.770	0.216	0.892	0.225	1.020	0.237
All amplitudes F_1-F_8						
$\omega=0.3$ GeV, $D=1.20$ GeV^2	0.747	0.197	0.956	0.246	1.088	0.255
$\omega=0.4$ GeV, $D=0.93$ GeV^2	0.742	0.207	0.936	0.241	1.072	0.259
$\omega=0.5$ GeV, $D=0.79$ GeV^2	0.74	0.215	0.94	0.25	1.08	0.266
Amplitudes $F_1 \ldots F_5$ only						
Maris–Roberts Ref. [10]	0.71		0.95		1.1	
$\omega=0.3$ GeV, $D=1.20$ GeV^2	0.737	0.192	0.942	0.235	1.080	0.247
$\omega=0.4$ GeV, $D=0.93$ GeV^2	0.729	0.199	0.919	0.229	1.062	0.250
$\omega=0.5$ GeV, $D=0.79$ GeV^2	0.731	0.207	0.926	0.237	1.072	0.259

PHYSICAL REVIEW C VOLUME 56, NUMBER 6 DECEMBER 1997

π- and K-meson Bethe-Salpeter amplitudes

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(Received 18 August 1997)

Independent of assumptions about the form of the quark-quark scattering kernel K, we derive the explicit relation between the flavor-nonsinglet pseudoscalar-meson Bethe-Salpeter amplitude Γ_H and the dressed-quark propagator in the chiral limit. In addition to a term proportional to γ_5, Γ_H necessarily contains qualitatively and quantitatively important terms proportional to $\gamma_5 \gamma \cdot P$ and $\gamma_5 \gamma \cdot k k \cdot P$, where P is the total momentum of the bound state. The axial-vector vertex contains a bound state pole described by Γ_H, whose residue is the leptonic decay constant for the bound state. The pseudoscalar vertex also contains such a bound state pole and, in the chiral limit, the residue of this pole is related to the vacuum quark condensate. The axial-vector Ward-Takahashi identity relates these pole residues, with the Gell-Mann–Oakes–Renner relation a corollary of this identity. The dominant ultraviolet asymptotic behavior of the scalar functions in the meson Bethe-Salpeter amplitude is fully determined by the behavior of the chiral limit quark mass function, and is characteristic of the QCD renormalization group. The rainbow-ladder *Ansatz* for K, with a simple model for the dressed-quark-quark interaction, is used to illustrate and elucidate these general results. The model preserves the one-loop renormalization group structure of QCD. The numerical studies also provide a means of exploring procedures for solving the Bethe-Salpeter equation without a three-dimensional reduction. [S0556-2813(97)04112-5]

PACS number(s): 14.40.Aq, 24.85.+p, 11.10.St, 12.38.Lg

PHYSICAL REVIEW D **80,** 114010 (2009)

Survey of $J = 0, 1$ mesons in a Bethe-Salpeter approach

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(Received 22 September 2009; published 10 December 2009)

The Bethe-Salpeter equation is used to comprehensively study mesons with $J = 0, 1$ and equal-mass constituents for quark masses from the chiral limit to the b-quark mass. The survey contains masses of the ground states in all corresponding J^{PC} channels including those with "exotic" quantum numbers. The emphasis is put on each particular state's sensitivity to the low- and intermediate-momentum, i.e., long-range part of the strong interaction.

DOI: 10.1103/PhysRevD.80.114010 PACS numbers: 14.40.−n, 11.10.St, 12.38.Lg

I. INTRODUCTION

Mesons offer a prime target for studies of various approaches to quantum chromodynamics (QCD), which is widely accepted as the quantum field theory of the strong interaction. While in terms of the number of constitutents their appearance is simple at first glance, mesons provide a broad range of phenomena and challenges to both theory and experiment. On the theoretical side, the key challenge

tion as in corresponding meson studies (for recent advances, see [26–30], and references therein).

In principle, one would aim at a complete, self-consistent solution of all equations, which is equivalent to a solution of the underlying theory. While this spirit can be held up in investigations of certain aspects of the theory (see, e.g. [31,32], and references therein), numerical studies of hadronic observables require a truncation of the

✦ Some properties of mesons in DSE-BSE

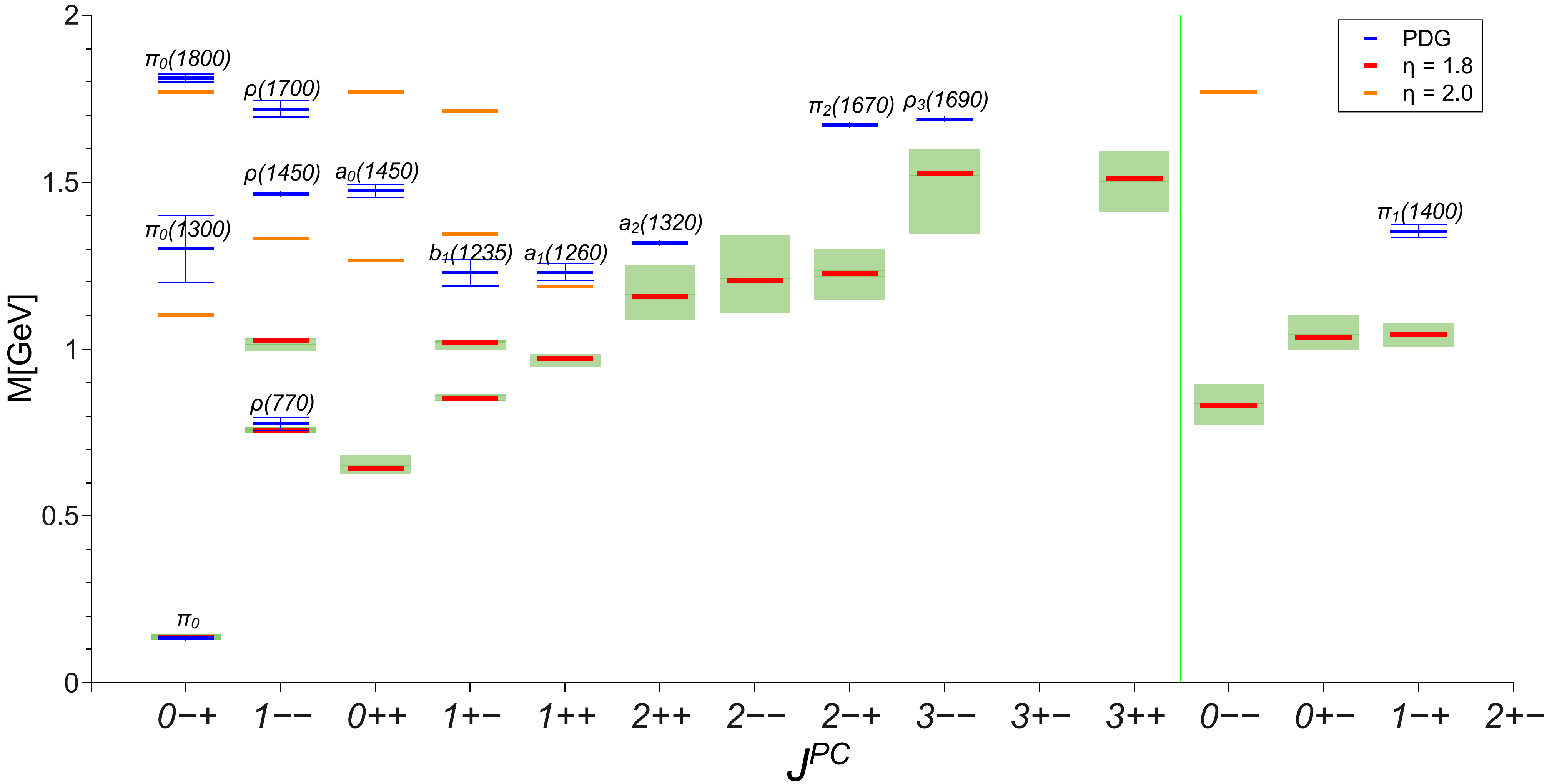
Solving the 4-dimenssional covariant B-S equation with the kernel being fixed by the solution of DS equation and flavor symmetry breaking, we obtain

	Expt. (GeV)	Calc. (GeV)	Th/Ex-1 (%)
"ρ^0"	0.7755	0.7704	-0.66
$\rho^\pm$	0.7755	0.7755	0
"ω"	0.7827	0.7806	-0.27
$K^{*\pm}$	0.8917	0.8915	-0.02
K^{*0}	0.8960	0.8969	0.10
ϕ	1.0195	1.0195	0
D^{*0}	2.0067	1.8321	-8.7
$D^{*\pm}$	2.0100	1.8387	-8.5
$D_s^{*\pm}$	2.1120	1.9871	-5.9
J/ψ	3.0969	3.0969	0
$B^{*\pm}$		4.8543	
B^{*0}		4.8613	
B_s^{*0}		5.0191	
$B_c^{*\pm}$		6.2047	
Υ	9.4603	9.4603	0

	Expt. (GeV)	Calc. (GeV)	Th/Ex-1 (%)
π^0	0.13498	0.13460	-0.3
$\pi^\pm$	0.13957	0.13499	-3.3
$K^\pm$	0.49368	0.41703	-15.5
K^0	0.49765	0.42662	-14.3
η	0.54751	0.45499	-16.9
η'	0.95778	0.91960	-4.0
D^0	1.8645	1.6195	-13.1
$D^\pm$	1.8693	1.6270	-13.0
$D_s^\pm$	1.9682	1.7938	-8.9
η_c	2.9804	3.0171	1.2
$B^\pm$	5.2790	4.7747	-9.6
B^0	5.2794	4.7819	-9.4
B_s^0	5.3675	4.9430	-7.9
$B_c^\pm$	6.286	6.1505	-2.2
η_b	9.300	9.4438	1.5

(L. Chang, Y. X. Liu, C. D. Roberts, et al., Phys. Rev. C 76, 045203 (2007))

Christian S. Fischer et al.: Mass spectra and Regge trajectories of light mesons in the Bethe-Salpeter approach



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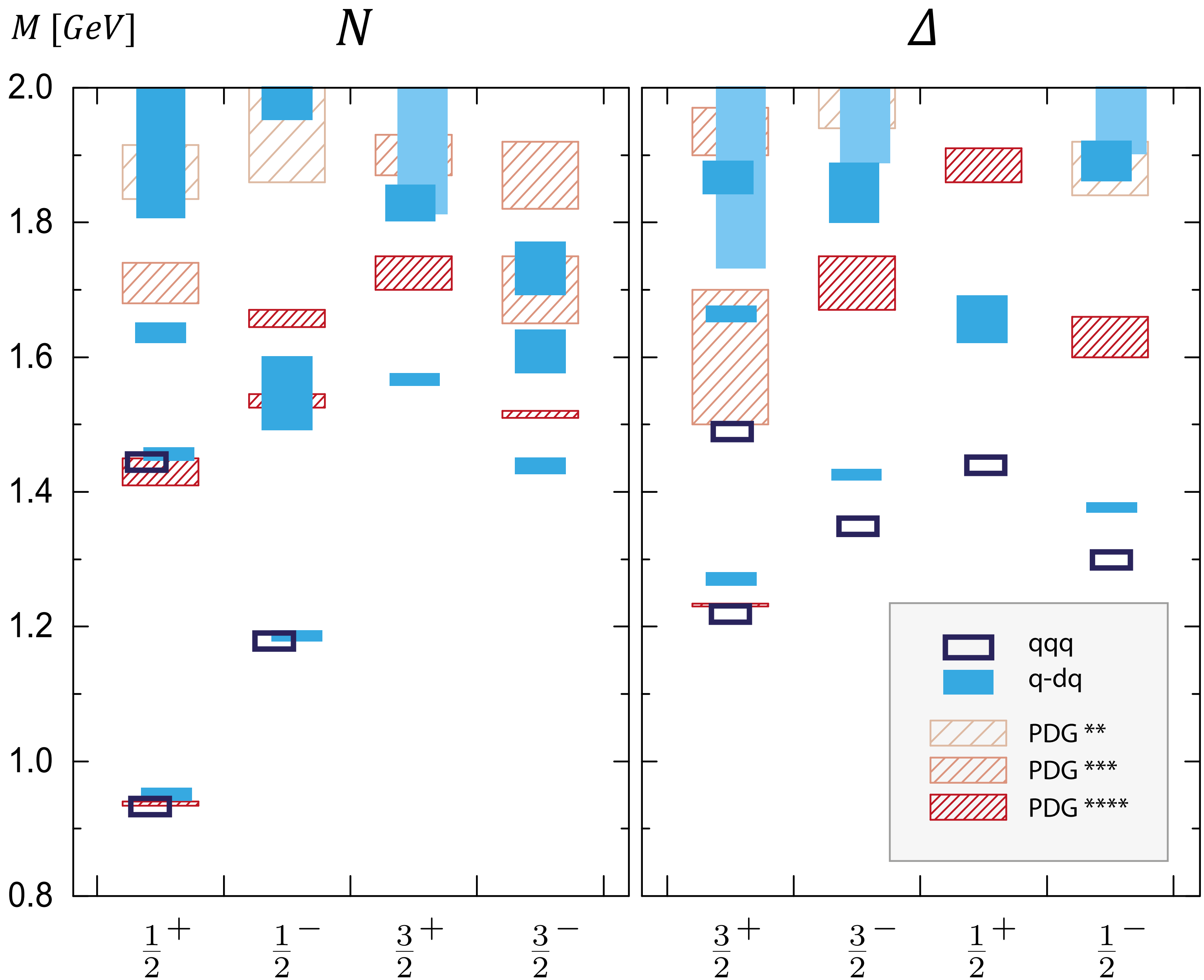
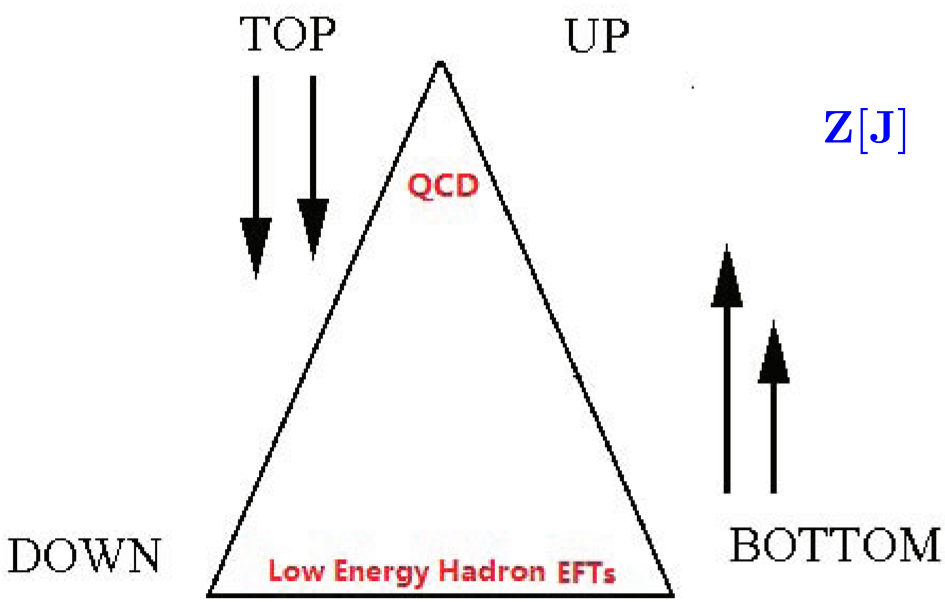


G.Eichmann, C.S.Fisher, H.Sanchis-Alepuz,PRD 94, 094033(2016)

DSE/BSE/Faddeev landscape

	I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)	III) DSE (RL)	IV) DSE (bRL)
N, Δ masses	✓	✓	✓	✓	✓
N, Δ em. FFs	✓	✓	✓	✓	
$N \to \Delta\gamma$	✓	✓	✓	✓	
Roper,...	✓	✓	✓	✓	
$N \to N^*\gamma$	✓	✓			
$N^*(1535), \ldots$	✓	✓	✓	✓	
$N \to N^*\gamma$					
Σ, Ξ, Ω	✓	✓	✓	✓	
excited strange	✓		✓	✓	
Σ, Ξ, Ω em. FFs			✓	✓	
	Cloet, Thomas, Roberts, Segovia et al.	Oettel, Alkofer, Roberts, Bloch, Segovia et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF	Sanchis-Alepuz, Williams, CF

QCD First Principle Calculation



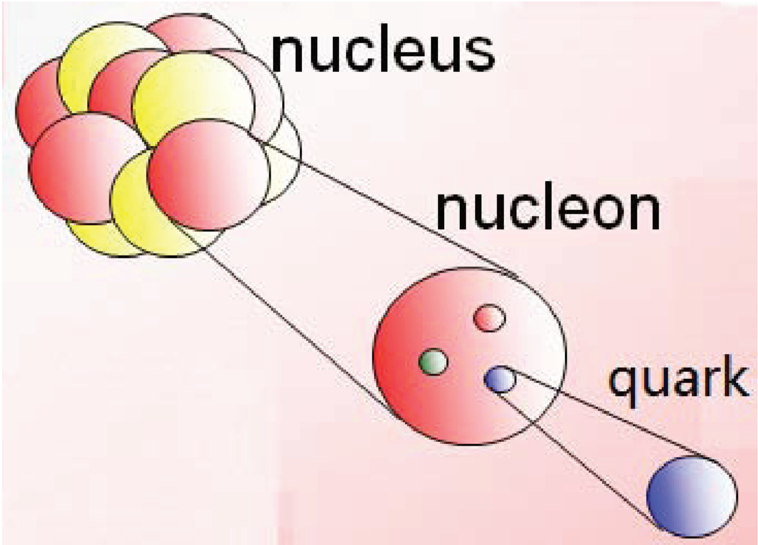
$$\mathbf{Z[J]} \equiv \underbrace{\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}q\mathcal{D}\overline{q}\; e^{i\int d^4x[\mathcal{L}_{\rm QCD}(q,\Psi,G)+F(q,G,J)]}}_{\mathbf{G,q,\Psi}\text{: 量子色动力学中的胶子、轻夸克、重夸克场}}$$

$$= \underbrace{\int \mathcal{D}\phi\; e^{iS_{\rm eff}[\phi,J]}}_{\phi\text{: 有效理论中的各种场}}$$

$$\underbrace{\mathbf{e}^{\mathbf{iS}_{\rm eff}[\phi,\mathbf{J}]} = \int \mathcal{D}\mathbf{G}\mathcal{D}\mathbf{\Psi}\mathcal{D}\overline{\mathbf{\Psi}}\mathcal{D}\mathbf{q}\mathcal{D}\overline{\mathbf{q}}\;\delta[\phi-\mathbf{H}(\mathbf{q},\mathbf{G})]\;\mathbf{e}^{\mathbf{i}\int \mathbf{d}^4\mathbf{x}[\mathcal{L}_{\rm QCD}(\mathbf{q},\mathbf{\Psi},\mathbf{G})+\mathbf{F}(\mathbf{q},\mathbf{G},\mathbf{J})]}}_{\text{强子层次有效作用量 的 夸克、胶子层次 的 路径积分表示}}$$

Low Energy Hadron EFTs & Models

我们研究第一原理推导的历史



- 走向梦中的重子EFT,Skyrmion,Faddeev的必经之路
- 很早就做了，多年一直没做完成的 事
- 以往只做成功了赝标介子PRD61,54011(2000)
- 马上就推广到了矢量介子CTP34,519(2000)； η' CTP34,683(2000)
- 发现理论中出现了任意的量纲参数，ρ介子质量似乎是任意的
- 而在路径积分中赝标介子转动角的引进和其它介子复合场是完全不一样的

赝标介子的 零质量 由SCSB决定，而作为物质场的其它介子如ρ的 非零质量 与手征对称性无关，由 很难求解的 束缚态方程 决定!

- 感觉推导有问题，尝试多年，试图与国内外合作，都没成功
- 去年找到症结，原推导没问题，对结果的理解不够深入 现在仍是这样 ！

PHYSICAL REVIEW D, VOLUME 61, 054011

Derivation of the effective chiral Lagrangian for pseudoscalar mesons from QCD

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We formally derive the chiral Lagrangian for low lying pseudoscalar mesons from the first principles of QCD considering the contributions from the normal part of the theory without taking an approximation. The derivation is based on the standard generating functional of QCD in the path integral formalism. The gluon-field integration is formally carried out by expressing the result in terms of the physical Green's functions of the gluon. To integrate over the quark field, we introduce a bilocal auxiliary field $\Phi(x,y)$ representing the mesons. We then develop a consistent way of extracting the local pseudoscalar degree of freedom $U(x)$ in $\Phi(x,y)$ and integrating out the rest degrees of freedom such that the complete pseudoscalar degree of freedom resides in $U(x)$. With certain techniques, we work out the explicit $U(x)$ dependence of the effective action up to the p^4 terms in the momentum expansion, which leads to the desired chiral Lagrangian in which all the coefficients contributed from the normal part of the theory are expressed in terms of certain quark Green's functions in QCD. Together with the exsisting QCD formulas for the anomaly contributions, the present results lead to the complete effective chiral Lagrangian for pseudoscalar mesons. The final result can be regarded as the fundamental QCD definition of the coefficients in the chiral Lagrangian. The relation between the present QCD definition of the p^2-order coefficient F_0^2 and the well-known appoximate result given by Pagels and Stokar is discussed.

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Derivation of Effective Chiral Lagrangian Involving PSGB and Vector Bosons from QCD*

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Abstract *The effective chiral Lagrangian for a matter field content consisting of pseudo-scalar Goldstone bosons and vector bosons (with hidden symmetry) is derived from the underlying QCD theory. No approximations are made. All the free parameters of the effective chiral Lagrangian are expressed in terms of QCD-based Green's functions. These may be regarded as the QCD definitions of these Lagrangian coefficients.*

PACS numbers: 11.30.Rd, 12.38.Aw, 12.38.Lg, 12.39.Fe
Key words: effective chiral Lagrangian, vector boson, QCD

I. Introduction

Due to its nonperturbative nature, studying low energy hadron physics from the perspective of QCD has long been a difficult undertaking. Thus, in place of QCD, research into this low energy realm is often based on more empirical effective chiral Lagrangian (ECL)

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Derivation of Effective Chiral Lagrangian for the Whole Nonet Pseudo-Scalar Goldstone Bosons from QCD*

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Abstract *The effective chiral Lagrangian derived from underlying QCD for pseudo-scalar Goldstone bosons has been generalized to involve the whole nonet pseudo-Goldstone bosons, no approximation is made in the derivation. The formulation offers general QCD definitions for the coefficients in effective chiral Lagrangian.*

PACS numbers: 11.30.Rd, 12.38.Aw, 12.38.Lg, 12.39.Fe
Key words: chiral Lagrangian, nonet Goldstone bosons, QCD

I. Introduction

At low energies, the effective interaction among pseudo-scalar Goldstone bosons (PSGB) induced from fundamental QCD can be described by an effective chiral Lagrangian (ECL).[1,2] This Lagrangian depends on a number of coupling coefficients which are not determined by symmetry requirements and must be taken as experimental inputs at level of phenomenology.

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第一原理推导的几个步骤主要以矢量介子为例

- 积掉胶子场得到 非局域的多夸克理论
- 积进双局域介子场得到 双局域介子场理论 为了能开展实际的计算
- 积进赝标介子场得到 赝标介子有效理论
- 积进矢量介子场得到 矢量介子有效理论
- 利用$1/N_c$展开得到介子的有效拉氏量和相应的BS方程

  大N_c极限下介子是稳定粒子
- 利用abelian近似化简BS方程，求解介子谱
- 积进其它介子场得到 介子有效理论

Tsinghua University

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Derivation of the effective chiral Lagrangian for pseudoscalar, scalar, vector, and axial-vector mesons from QCD

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A previous formal derivation of the effective chiral Lagrangian for low-lying pseudoscalar mesons from first-principles QCD without approximations [Q. Wang, Y.-P. Kuang, X.-L. Wang, and M. Xiao, Phys. Rev. D **61**, 054011 (2000)] is generalized to further include scalar, vector, and axial-vector mesons. In the large N_c limit and with an Abelian approximation, we show that the properties of the newly added mesons in our formalism are determined by the corresponding underlying fundamental homogeneous Bethe-Salpeter equation in the ladder approximation, which yields the equations of motion for the scalar, vector, and axial-vector meson fields at the level of an effective chiral Lagrangian. The masses appearing in the equations of motion of the meson fields are those determined by the corresponding Bethe-Salpeter equation.

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$$\mathbf{Z[J]} = \int \mathcal{D}\xi_{\mathbf{R}}\mathcal{D}\xi_{\mathbf{L}}\mathcal{D}\phi\ \delta(\xi_{\mathbf{R}}^{\dagger}\xi_{\mathbf{R}} - \mathbf{1})\delta(\xi_{\mathbf{L}}^{\dagger}\xi_{\mathbf{L}} - \mathbf{1})\delta(\det\xi_{\mathbf{R}} - \det\xi_{\mathbf{L}})\ \mathbf{e}^{\mathbf{i}\mathbf{S}_{\mathrm{eff}}[\xi_{\mathbf{R}},\xi_{\mathbf{L}},\phi,\mathbf{J},\boldsymbol{\Xi}_{\mathbf{c}},\tilde{\phi}_{\mathbf{c}},\boldsymbol{\Phi}_{\mathbf{c}},\boldsymbol{\Pi}_{\mathbf{c}}]}$$

$$\mathbf{e}^{\mathbf{i}\mathbf{S}_{\mathrm{eff}}[\xi_{\mathbf{R}},\ \xi_{\mathbf{L}},\ \phi,\ \mathbf{J},\ \boldsymbol{\Xi}_{\mathbf{c}},\ \tilde{\phi}_{\mathbf{c}},\ \boldsymbol{\Phi}_{\mathbf{c}},\ \boldsymbol{\Pi}_{\mathbf{c}}]} = \int \mathcal{D}\boldsymbol{\Xi}\mathcal{D}\tilde{\phi}\mathcal{D}\boldsymbol{\Phi}\mathcal{D}\boldsymbol{\Pi}\ \mathbf{e}^{\cdots\cdots}$$

⇐ 给出低能展开的有效拉氏量的表达式

$$\cdots\cdots = \mathbf{i}\mathbf{N}_{\mathbf{c}}\Big\{\int \mathbf{d}^4\mathbf{x}\ \mathrm{tr}_{\mathbf{f}}\Big[\boldsymbol{\Xi}(\mathbf{x})\Big(\mathbf{e}^{-\frac{\mathbf{i}\tilde{\theta}(\mathbf{x})}{\mathbf{N}_{\mathbf{f}}}}\xi_{\mathbf{R}}(\mathbf{x})\mathrm{tr}_{\mathbf{l}}[\mathbf{P}_{\mathbf{R}}\boldsymbol{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\xi_{\mathbf{L}}^{\dagger}(\mathbf{x}) - \mathbf{e}^{\frac{\mathbf{i}\tilde{\theta}(\mathbf{x})}{\mathbf{N}_{\mathbf{f}}}}\xi_{\mathbf{L}}(\mathbf{x})\mathrm{tr}_{\mathbf{l}}[\mathbf{P}_{\mathbf{L}}\boldsymbol{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\xi_{\mathbf{R}}^{\dagger}(\mathbf{x})\Big)\Big]$$

$$+\int \mathbf{d}^4\mathbf{x}\Big[\phi^{(\mathbf{a}\xi)(\mathbf{b}\zeta)}(\mathbf{x}) + \frac{\mathbf{1}}{\mu^4}\{[\mathbf{e}^{-\frac{\mathbf{i}\theta}{2\mathbf{N}_{\mathbf{f}}}}\xi_{\mathbf{L}}\mathbf{P}_{\mathbf{R}} + \mathbf{e}^{\mathbf{i}\frac{\mathbf{i}\theta}{2\mathbf{N}_{\mathbf{f}}}}\xi_{\mathbf{R}}\mathbf{P}_{\mathbf{L}}]\boldsymbol{\Pi}(\mathbf{x},\mathbf{x})[\mathbf{e}^{-\frac{\mathbf{i}\theta}{2\mathbf{N}_{\mathbf{f}}}}\xi_{\mathbf{R}}^{\dagger}\mathbf{P}_{\mathbf{R}} + \mathbf{e}^{\frac{\mathbf{i}\theta}{2\mathbf{N}_{\mathbf{f}}}}\xi_{\mathbf{L}}^{\dagger}\mathbf{P}_{\mathbf{L}}]\}^{(\mathbf{a}\xi')(\mathbf{b}\zeta')}\mathbf{P}^{\xi'\zeta',\xi\zeta}\Big]\tilde{\phi}^{(\mathbf{b}\zeta)(\mathbf{a}\xi)}(\mathbf{x})$$

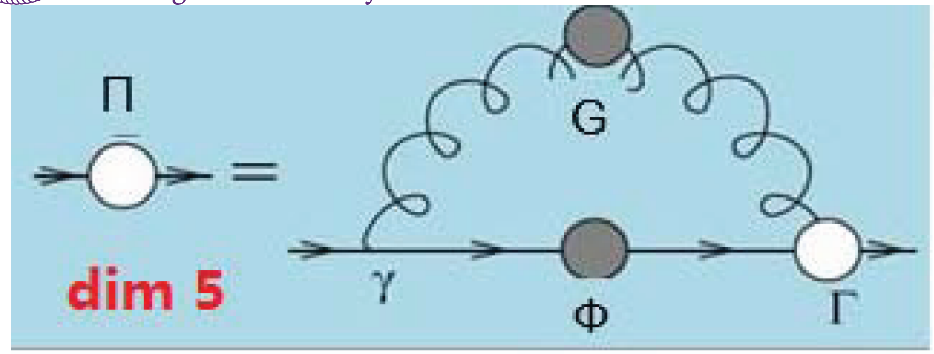
$$+\frac{\mathbf{1}}{\mathbf{N}_{\mathbf{c}}}\boldsymbol{\Gamma}_{\mathbf{I}}[\boldsymbol{\Phi}] - \mathbf{i}\mathrm{Tr}\ln(\mathbf{i}\not\partial + \mathbf{J} + \boldsymbol{\Pi}) + \int \mathbf{d}^4\mathbf{x}\mathbf{d}^4\mathbf{y}\ \boldsymbol{\Pi}^{\sigma\rho}(\mathbf{x},\mathbf{y})\boldsymbol{\Phi}^{\sigma\rho}(\mathbf{x},\mathbf{y}) + \mathbf{i}\sum_{\mathbf{n}=\mathbf{2}}^{\infty}\int \mathbf{d}^4\mathbf{x}_{\mathbf{1}}\mathbf{d}^4\mathbf{x}_{\mathbf{1}}'\cdots\mathbf{d}^4\mathbf{x}_{\mathbf{n}}\mathbf{d}^4\mathbf{x}_{\mathbf{n}}'\frac{(-\mathbf{i})^{\mathbf{n}}(\mathbf{g}^{\mathbf{2}})^{\mathbf{n}-\mathbf{1}}}{\mathbf{n}!}$$

$$\times\bar{\mathbf{G}}^{\sigma_{\mathbf{1}}\cdots\sigma_{\mathbf{n}}}_{\rho_{\mathbf{1}}\cdots\rho_{\mathbf{n}}}(\mathbf{x}_{\mathbf{1}},\mathbf{x}_{\mathbf{1}}',\cdots,\mathbf{x}_{\mathbf{n}},\mathbf{x}_{\mathbf{n}}')\boldsymbol{\Phi}^{\sigma_{\mathbf{1}}\rho_{\mathbf{1}}}(\mathbf{x}_{\mathbf{1}},\mathbf{x}_{\mathbf{1}}')\cdots\boldsymbol{\Phi}^{\sigma_{\mathbf{n}}\rho_{\mathbf{n}}}(\mathbf{x}_{\mathbf{n}},\mathbf{x}_{\mathbf{n}}')\Big\}$$

⇑ 除了必要的重整化常数之外，存在 有限的大Nc极限 ！

$$\mathbf{e}^{-\mathbf{i}\boldsymbol{\Gamma}_{\mathbf{I}}[\boldsymbol{\Phi}]} = \boldsymbol{\Pi}_{\mathbf{x}}\Big[\{\det[\mathrm{tr}_{\mathbf{l}}\mathbf{P}_{\mathbf{R}}\boldsymbol{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\det[\mathrm{tr}_{\mathbf{l}}\mathbf{P}_{\mathbf{L}}\boldsymbol{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\}^{\frac{\mathbf{1}}{\mathbf{2}}}\int \mathcal{D}\sigma\delta[(\mathrm{tr}_{\mathbf{l}}\mathbf{P}_{\mathbf{R}}\boldsymbol{\Phi}^{\mathbf{T}})(\mathrm{tr}_{\mathbf{l}}\mathbf{P}_{\mathbf{L}}\boldsymbol{\Phi}^{\mathbf{T}}) - \sigma^{\dagger}\sigma]\delta(\sigma - \sigma^{\dagger})\Big]$$

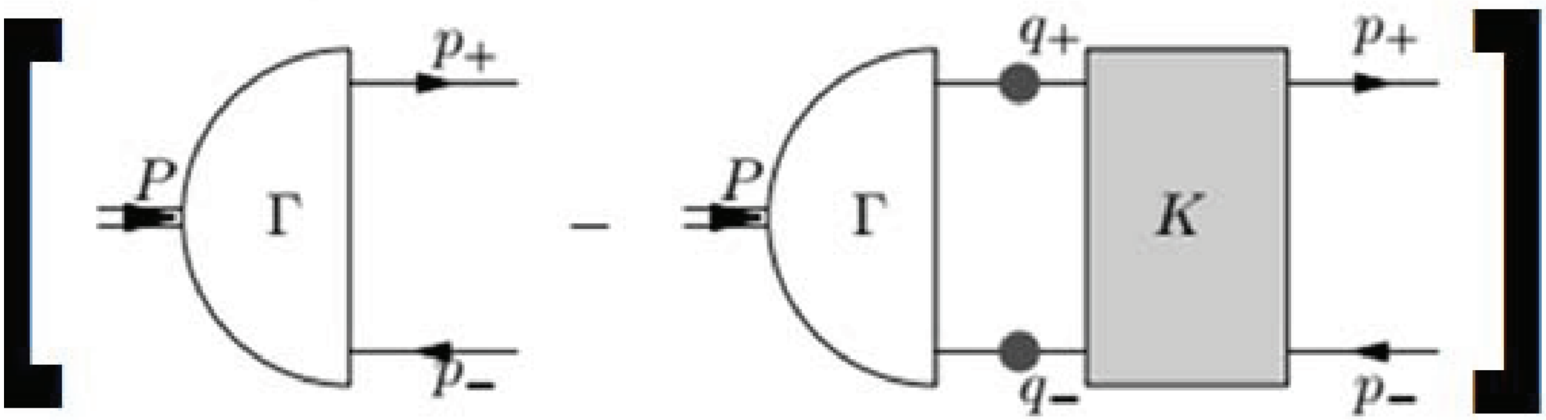
$$\mathbf{P}^{\xi'\zeta',\xi\zeta} = \frac{\mathbf{1}}{\mathbf{4}}[\delta^{\zeta'\xi'}\delta^{\xi\zeta} + (\gamma_{\mu})^{\zeta'\xi'}(\gamma^{\mu})^{\xi\zeta} - (\gamma_{\mu}\gamma_{\mathbf{5}})^{\zeta'\xi'}(\gamma^{\mu}\gamma_{\mathbf{5}})^{\xi\zeta}]$$



Extract out effective meson field from coincidence limit of quark self energy Π

$$\int \mathcal{D}\phi\,\delta\Big[\phi^{(a\xi)(b\zeta)}(x)+\frac{1}{\mu^4}\{[e^{-\frac{i\theta(x)}{2N_f}}\xi_L(x)P_R+e^{\frac{i\theta(x)}{2N_f}}\xi_R(x)P_L]\times\Pi(x,x)[e^{-\frac{i\theta(x)}{2N_f}}\xi_R^\dagger(x)P_R+e^{\frac{i\theta(x)}{2N_f}}\xi_L^\dagger(x)P_L]\}^{(a\xi')(b\zeta')}p^{\xi'\zeta'\xi\zeta}\Big],$$

free dimentional parameter $\times \dfrac{\partial S_{\mathrm{eff}}}{\partial\phi(\mathbf{x})}\delta(x-y)+$



$\times\,\phi \overset{\text{大Nc}}{=} 0$

特别可证S_{eff}中的介子质量就是BSE解出的质量！ 对重子有类似的结果，正在整理文章

小 结

- QCD低能非微扰效应强子物理 一直是强作用的硬骨头问题
- 我们开展了从QCD到强子有效理论的严格推导
- 夸克胶子层次的束缚态BS、Faddeev方程 和强子层次的强子介子、重子场方程 同居同一方程
- 手征有效拉氏量中的 强子质量 自动由BS/Faddeev方程的解给出
- 如何更深地理解和开发这个结果仍在积极地进一步讨论！

为什么对还没做积分的强子场能有场方程？

- 在 $\mathbf{S}_{\mathrm{eff}}[\xi_{\mathbf{R}}, \xi_{\mathbf{L}}, \phi, \mathbf{J}, \mathbf{\Xi}_{\mathbf{c}}, \tilde{\phi}_{\mathbf{c}}, \mathbf{\Phi}_{\mathbf{c}}, \mathbf{\Pi}_{\mathbf{c}}]$ 中，$\xi_{\mathbf{R}}, \xi_{\mathbf{L}}, \phi, \mathbf{J}$ 是独立的自由变量

- $\xi_{\mathbf{R}}, \xi_{\mathbf{L}}, \phi$ 只有在做了路径积分后才会有相应的运动方程

- 怎么可能在还没做 ϕ 的路径积分之前就出现了它的场方程？

- <u>大Nc极限</u>： 虽然强子场路径积分还没做，但做了就是此结果 不矛盾 ！