Recent Progress in Applying Lattice QCD to Kaon Physics

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Outline

“Standard” observables in Kaon physics

- $f_{K^\pm}/f_{\pi^\pm}$, $f_+(0)$, $\tau \to s$ inclusive decay and $|V_{us}|$
- B_K for SM and beyond

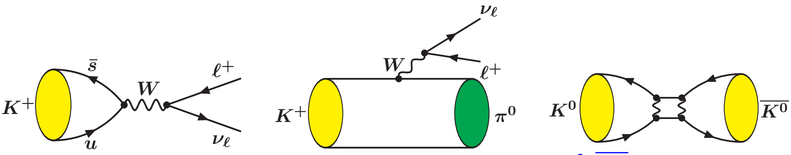
“Non-standard” observables in Kaon physics

- $K \to \pi\pi$ decays and direct CP violation
- ΔM_K and ϵ_K
- Rare Kaon decays

Lattice Kaon physics

Evaluate the hadronic matrix elements in Kaon physics

- Lattice QCD is powerful for “standard” hadronic matrix elements with



$K_{\ell 2} \to f_K$ $K_{\ell 3} \to f_+(0)$ K^0-$\overline{K^0}$ mixing $\to B_K$

  - single local operator insertion
  - only single stable hadron or vacuum in the initial/final state
  - spatial momenta carried by particles need to be small compared to $1/a$ (not a problem for Kaon physics, but essential for B decays)

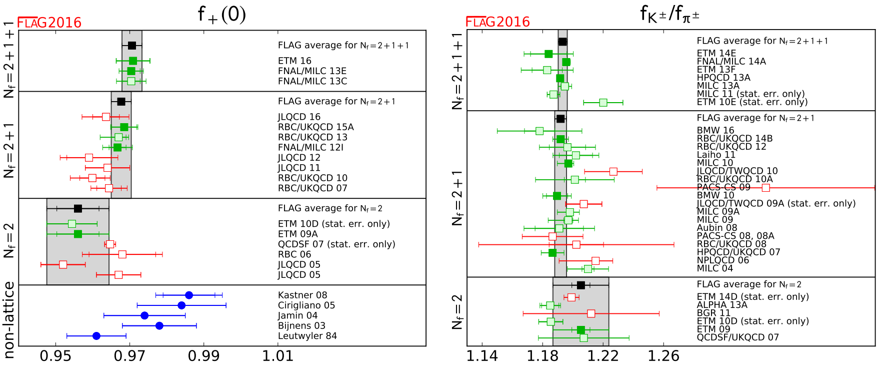
$f_{K^\pm}/f_{\pi^\pm}$, $f_+(0)$, $\tau \to s$ inclusive decay and $|V_{us}|$

“standard” quantities in Kaon physics: $f_{K^\pm}/f_{\pi^\pm}$ and $f_+(0)$

Flavor Lattice Averaging Group (FLAG) average, updated in Nov. 2016

$$f_+^{K\pi}(0) = 0.9706(27) \quad \Rightarrow \quad 0.28\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1933(29) \quad \Rightarrow \quad 0.25\% \text{ error}$$



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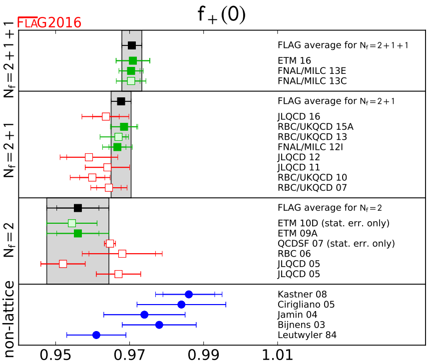

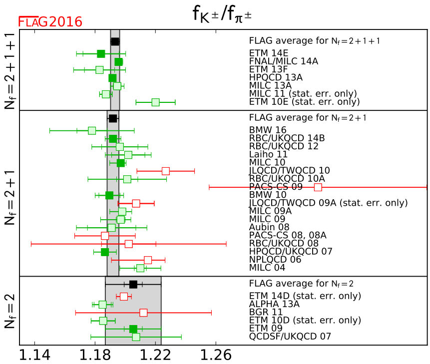

Experimental information [arXiv:1411.5252, 1509.02220]

$$K_{\ell 3} \quad \Rightarrow \quad |V_{us}| f_+(0) = 0.2165(4) \quad \Rightarrow \quad |V_{us}| = 0.2231(7)$$

$$K_{\mu 2}/\pi_{\mu 2} \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4) \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| = 0.2313(7)$$

Test the CKM unitarity

[S. Aoki et. al., FLAG report updated in Nov. 2016]

Most stringent test of CKM unitarity is given by the first row condition

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Use $|V_{us}|$ for $K_{\ell 3}$ + $|V_{us}/V_{ud}|$ for $K_{\ell 2}/\pi_{\ell 2}$ as input

$$|V_u|^2 = 0.9798(82) \quad \Rightarrow \quad 2.5\sigma \text{ deviation from } 1$$

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Most precise value of $|V_{ud}| = 0.97417(21)$ is from superallowed nuclear β decay

- Use $|V_{us}|$ for $K_{\ell 3}$ + $|V_{ud}|$ for β decay

$$|V_u|^2 = 0.9988(5) \quad \Rightarrow \quad \text{sharpen the test, still } 2.4\sigma \text{ deviation}$$

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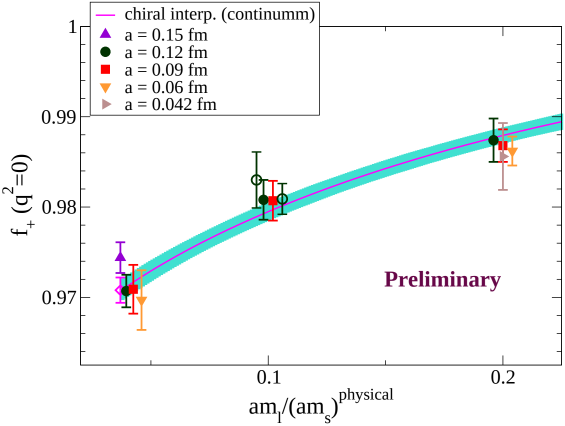
Interesting to reduce the uncertainty from $f_+(0)$ and explore the $> 2\sigma$ deviation

$f_+(0)$: recent update from Fermilab Lattice-MILC Collaboration

Use HISQ fermions on $N_f = 2+1+1$ MILC configurations [PoS LATTICE2016 286]



Plot, courtesy of E. Gámiz

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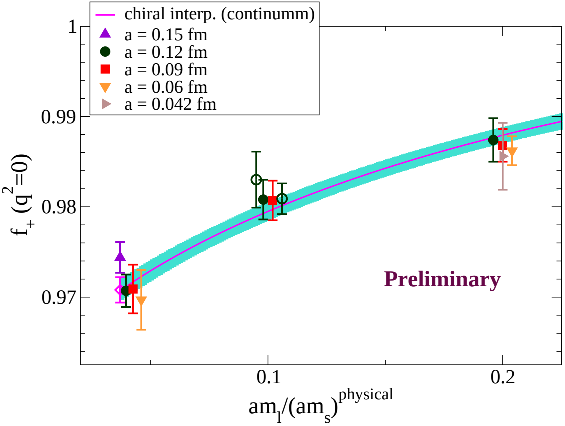

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- 4 ensembles at physical m_π
- 2 ultra-fine lattice at $a = 0.06,\ 0.042$ fm

$\Downarrow$

Stat. err reduces to $\sim 0.14\%$

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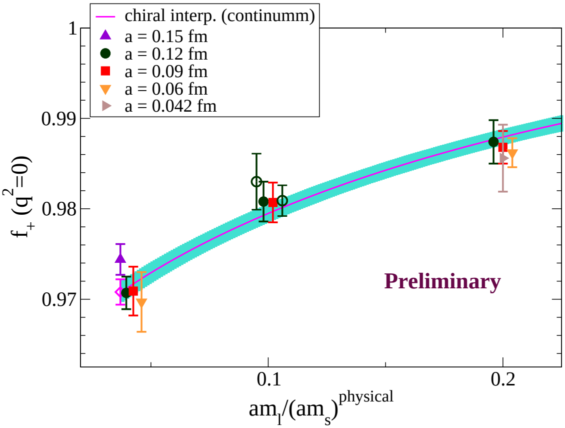

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- Use one-loop ChPT to control FV effects

  [C. Bernard et. al, JHEP 1703 (2017) 120]

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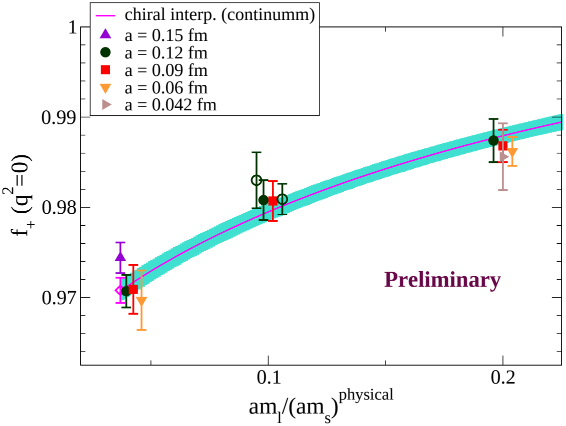

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- 4 ensembles at physical m_π
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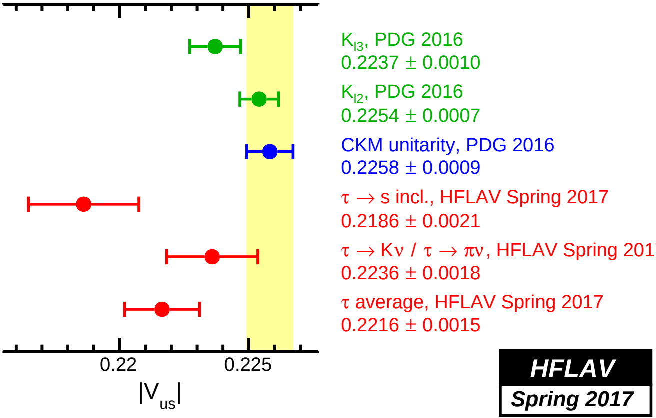
  Stat. err reduces to $\sim 0.14\%$
- Use one-loop ChPT to control FV effects

  [C. Bernard et. al, JHEP 1703 (2017) 120]

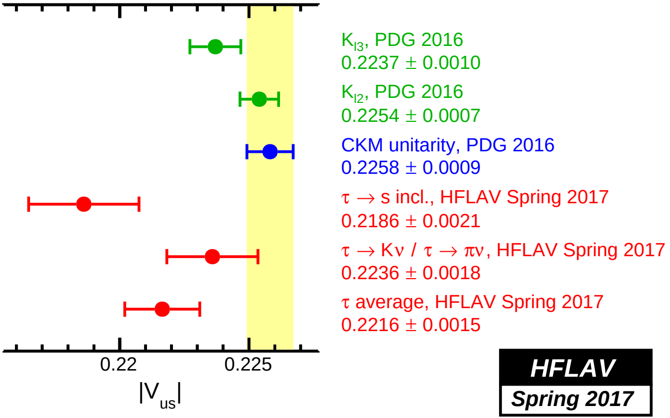
- Chiral, continuum extrapolation + discretization uncertainty + FV corrections + NNLO isospin corrections + taste-violating effects + $\cdots$

Expect to have a final error of $\sim 0.2\%$



3.2 σ deviation between $\tau \to s$ inclusive decay and CKM unitarity

$|V_{us}|$: summarized by HFAG averaging group



3.2 σ deviation between $\tau \to s$ inclusive decay and CKM unitarity

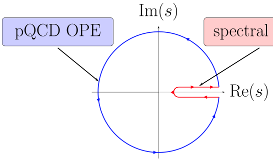
$$R = \frac{\Gamma(\tau \to \text{strange-hadrons}\,\nu_\tau)}{\Gamma(\tau \to e\,\bar{\nu}_e\,\nu_\tau)}$$

Optical theorem: Hadronic spectral func. of inclusive decay ⇔ imag. of HVP

$$\frac{dR}{ds} = \frac{12\pi|V_{us}|^2 S_{EW}}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2\left[\left(1 + 2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$\mathrm{Im}\,\Pi^{(J)}(s)$ is generically non-perturbative at small s

- Conventional approach: use dispersion relation
  [E. Braaten et. al., NPB373 (1992) 581; E. Gámiz et. al., PRL94 (2005) 011803]



$$\int_0^{s_0} ds\, W(s)\,\mathrm{Im}\,\Pi(s) = \frac{i}{2}\oint_{|s|=s_0} ds\, W(s)\Pi(s)$$

LHS given by $\frac{dR}{ds}$; RHS given by pQCD+OPE

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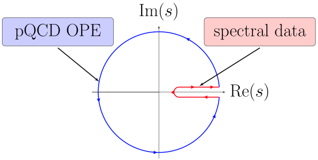

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- Study dependence on s_0 and $W(s)$ or use lattice data $\xrightarrow{\text{fit}}$ high-dim. OPE
  [R. Hudspith et. al arXiv:1702.01767]

$$|V_{us}| = \begin{cases} 0.2229(22) & \text{using BaBar } \tau \to K\pi^0\nu_\tau,\ 3.2\sigma \to 1.2\sigma \\ 0.2204(23) & \text{using HFAG } \tau \to K\pi^0\nu_\tau,\ 3.2\sigma \to 2.2\sigma \end{cases}$$

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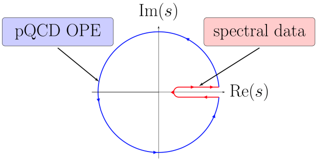

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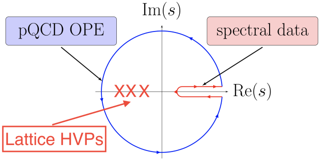
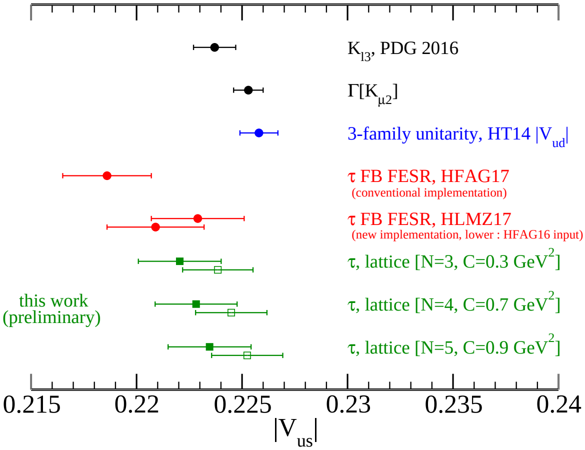
- Lattice QCD + dispersion relation [H. Ohki, Friday 17:50@Seminarios 6+7]



use $W(s) = \prod_k^{N_p} \frac{1}{s+Q_k^2}$ and let $|s| = s_0 \to \infty$

Residue at $s = -Q_k^2$ is given by Lattice HVPs

$|V_{us}|$ determined from inclusive τ decay + lattice HVPs



Plot, courtesy of T. Izubuchi & H. Ohki

$N_f = 2 + 1$ Möbius DWF

- physical m_π
- $a^{-1} = 1.73,\ 2.36$ GeV
- $V = 5\ \mathrm{fm}^3$
- Mesurements: $88 \times 48,\ 80 \times 32$

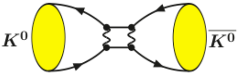
K-pole data:
$\tau \to K \nu_\tau$ (filled square)
$K_{\mu 2}$ (open square)

Choice of Q_k^2: separated by a spacing $\Delta = \frac{0.2}{N-1}$ GeV^2 and $C = \frac{Q_{min}^2 + Q_{max}^2}{2}$

- Not too large to suppress contribution from pQCD+OPE at $s > m_\tau^2$ and noisy experimental data at larger $s < m_\tau^2$
- Not too small to avoid large statistical error from lattice HVPs

B_K for SM and beyond

Short distance dominance ⇒ OPE ⇒ Wilson coeff. $C(\mu)$ × operator $Q^{\Delta S=2}(\mu)$



$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} C(\mu) Q^{\Delta S=2}(\mu)$$

- Serve as a dominant contribution to the indirect CP violation ϵ_K

$$\epsilon_K = \exp(i\phi_\epsilon)\sin(\phi_\epsilon)\left[\frac{\mathrm{Im}[\langle\overline{K^0}|\mathcal{H}_{\text{eff}}^{\Delta S=2}|K^0\rangle]}{\Delta M_K} + \frac{\mathrm{Im}[M_{0\bar{0}}^{\text{LD}}]}{\Delta M_K} + \frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]}\right]$$

- Within Standard Model, only single operator with $V - A$ structure

$$Q^{\Delta S=2} = [\bar{s}_a\gamma_\mu(1-\gamma_5)d_a][\bar{s}_b\gamma_\mu(1-\gamma_5)d_b]$$

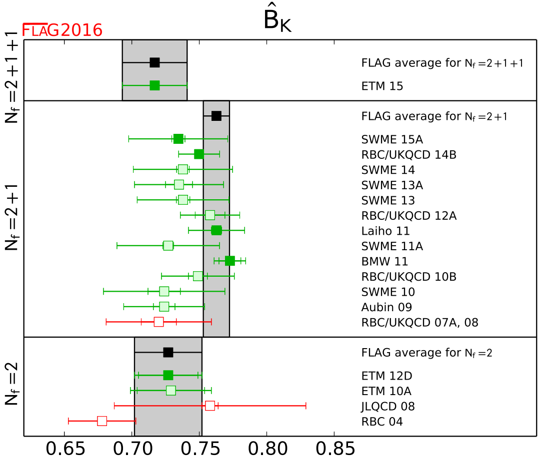
- Beyond SM, 4 other operators possible

$$Q_2^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_a][\bar{s}_b(1-\gamma_5)d_b]$$
$$Q_3^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_b][\bar{s}_b(1-\gamma_5)d_a]$$
$$Q_4^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_a][\bar{s}_b(1+\gamma_5)d_b]$$
$$Q_5^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_b][\bar{s}_b(1+\gamma_5)d_a]$$

FLAG average for Standard Model B_K

- B_K in NDR-$\overline{\mathrm{MS}}$ scheme: $B_K(\mu) = \frac{\langle \overline{K^0} | Q^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$
- Renormalization group independent B parameter $\hat{B}_K$:

$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \exp\left\{\int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0 g}\right)\right\} B_K(\mu)$$

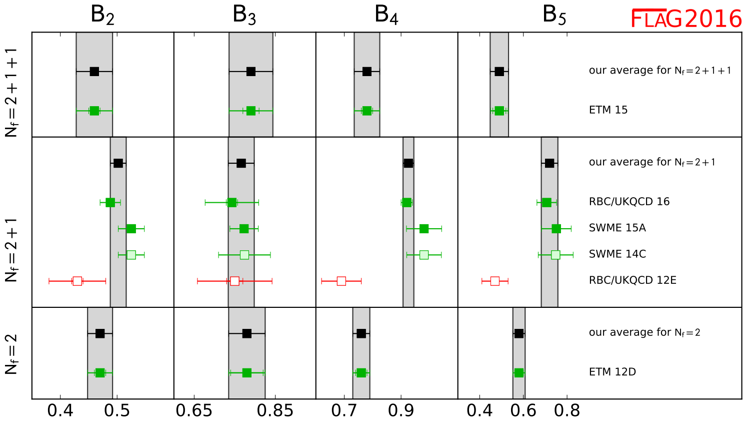


- $N_f = 2+1+1$: $\hat{B}_K = 0.717(24)$
- $N_f = 2+1$: $\hat{B}_K = 0.763(10)$
- $N_f = 2$: $\hat{B}_K = 0.727(25)$

FLAG average for BSM B_i, updated in Dec. 2016

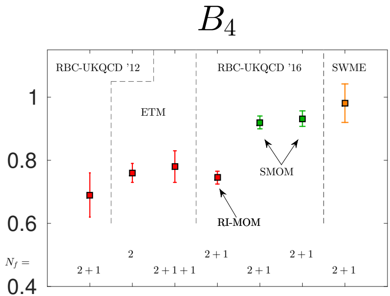
$$B_i(\mu) = \frac{\langle \overline{K^0} | Q_i(\mu) | K^0 \rangle}{N_i \langle \overline{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}, \quad \{N_2, \cdots, N_5\} = \{-5/3, 1/3, 2, 2/3\}$$

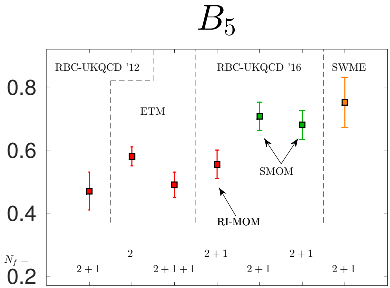
$B_i(\mu)$ at $\mu_{\overline{\rm MS}} = 3$ GeV



For $N_f = 2+1$, $B_2 = 0.502(14)$, $B_3 = 0.766(32)$, $B_4 = 0.926(19)$, $B_5 = 0.720(38)$

$N_f = 2+1$ DWF, $a = 0.08, 0.11$ fm, $m_\pi = 300$ MeV [RBC-UKQCD, JHEP11(2016)001]



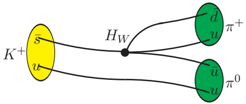


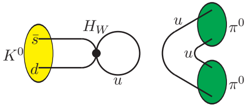
Plot, courtesy of N. Garron

- Use both RI/MOM and SMOM ⇒ the former is significantly smaller
- Use two RI/SMOM schemes, $(\not{q}, \not{q})$ and (γ_μ, γ_μ) ⇒ consistent results
- RI/(S)MOM result compatible with previous RI/(S)MOM calculation

Study suggests RI/MOM suffers from large IR artifacts ⇒ discrepancy

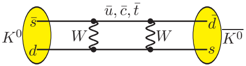
- $K \to \pi\pi$ decays and direct CP violation



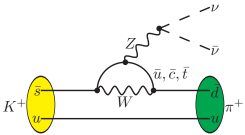


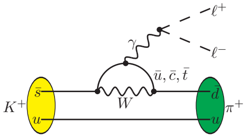
Final state involves $\pi\pi$ (multi-hadron system)

- Long-distance contributions to flavor changing processes
  - ΔM_K and ϵ_K



  - Rare kaon decays: $K \to \pi\nu\bar{\nu}$ and $K \to \pi\ell^+\ell^-$





Hadronic matrix element for bilocal operators

$$\int d^4x \, \langle f | T[Q_1(x) Q_2(0)] | i \rangle$$

$K \to \pi\pi$ decays and direct CP violation

CP violation is first observed in neutral Kaon decays

- CP eigenstates
  - Under CP transform: $CP|K^0\rangle = -|\overline{K^0}\rangle$
  - Define CP eigenstates: $K^0_\pm = (K^0 \mp \overline{K^0})/\sqrt{2}$
- Weak eigenstates
  - $K_S \to 2\pi$ $(CP = +)$
  - $K_L \to 3\pi$ $(CP = -)$
- Neglecting CP violation, we have $K_S = K^0_+$ and $K_L = K^0_-$

1964, BNL discovered $K_L \to 2\pi \Rightarrow CP$ violation $\Rightarrow$ Nobel prize (1980)

Direct and indirect CP violation

- $K_{L/S}$ are not CP eigenstates

$$|K_{L/S}\rangle = \frac{1}{\sqrt{1+\bar{\epsilon}^2}}\left(|K^0_{\mp}\rangle + \bar{\epsilon}|K^0_{\pm}\rangle\right)$$

- $K_L \to 2\pi$ $(CP = +)$
  - $K^0_+ \to 2\pi$ (indirect CP violation, ϵ or ϵ_K)
  - $K^0_- \to 2\pi$ (direct CP violation, ϵ')

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- Experimental measurement

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \eta_{+-} \equiv \epsilon + \epsilon'$$

$$\frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \eta_{00} \equiv \epsilon - 2\epsilon'$$

- Using $|\eta_{+-}|$ and $|\eta_{00}|$ as input, PDG quotes

$$|\epsilon| \approx \frac{1}{3}\left(2|\eta_{+-}| + |\eta_{00}|\right) = 2.228(11)\times 10^{-3}, \quad \mathrm{Re}[\epsilon'/\epsilon] \approx \frac{1}{3}\left(1 - \frac{|\eta_{00}|}{|\eta_{+-}|}\right) = 1.66(23)\times 10^{-3}$$

ϵ' is 1000 times smaller than the indirect CP violation ϵ

Thus direct CP violation ϵ' is very sensitive to New Physics

$K \to \pi\pi$ decays and CP violation

- Theoretically, Kaon decays into the isospin $I = 2$ and 0 $\pi\pi$ states

$$\Delta I = 3/2 \text{ transition:} \quad \langle \pi\pi(I=2)|H_W|K^0\rangle = A_2 e^{i\delta_2}$$
$$\Delta I = 1/2 \text{ transition:} \quad \langle \pi\pi(I=0)|H_W|K^0\rangle = A_0 e^{i\delta_0}$$

- If CP symmetry were protected $\Rightarrow$ A_2 and A_0 are real amplitudes

- ϵ and ϵ' depend on the $K \to \pi\pi(I)$ amplitudes A_I

$$\epsilon = \bar{\epsilon} + i\left(\frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]}\right)$$
$$\epsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}}\frac{\mathrm{Re}[A_2]}{\mathrm{Re}[A_0]}\left(\frac{\mathrm{Im}[A_2]}{\mathrm{Re}[A_2]} - \frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]}\right)$$

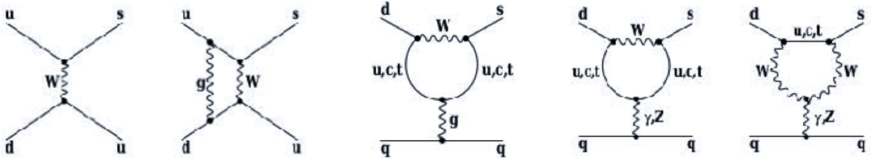
The target for lattice QCD is to calculate both amplitude A_2 and A_0

Weak Hamiltonian for $K \to \pi\pi$

Weak Hamiltonian is given by local four-quark operator

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}, \qquad \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = 1.543 + 0.635i$
- $z_i(\mu)$ and $y_i(\mu)$ are perturbative Wilson coefficients
- Q_i are local four-quark operator



Current-current operator	QCD penguin	Electro-weak penguin
Q_1, Q_2	$Q_3 - Q_6$	$Q_7 - Q_{10}$
dominate $\mathrm{Re}[A_0]$, $\mathrm{Re}[A_2]$	Q_6 dominate $\mathrm{Im}[A_0]$	Q_7, Q_8 dominate $\mathrm{Im}[A_2]$

Recent results for $K \to \pi\pi(I = 2)$

Results for A_2 [RBC-UKQCD, PRD91 (2015) 074502]

- Use two ensembles (both at $m_\pi = 135$ MeV) for continuum extrapolation

$$48^3 \times 96, \quad a = 0.11 \text{ fm}, \quad L = 5.4 \text{ fm}, \quad N_{\rm conf} = 76$$
$$64^3 \times 128, \quad a = 0.084 \text{ fm}, \quad L = 5.4 \text{ fm}, \quad N_{\rm conf} = 40$$

- After continuum extrapolation:

$$\mathrm{Re}[A_2] = 1.50(4)_{\rm stat}(14)_{\rm syst} \times 10^{-8} \text{ GeV}$$
$$\mathrm{Im}[A_2] = -6.99(20)_{\rm stat}(84)_{\rm syst} \times 10^{-13} \text{ GeV}$$

- Experimental measurement:

$$\mathrm{Re}[A_2] = 1.479(3) \times 10^{-8} \text{ GeV}$$
$$\mathrm{Im}[A_2] \text{ is unknown}$$

- Scattering phase at $E_{\pi\pi} = M_K$

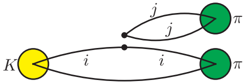
$$\delta_2 = -11.6(2.5)(1.2)^\circ$$

consistent with phenomenological analysis [Schenk, NPB363 (1991) 97]

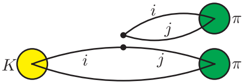
Resolve the puzzle of $\Delta I = 1/2$ rule

$\Delta I = 1/2$ rule: $A_0 = 22.5 \times A_2$ $\Rightarrow$ a > 50 year puzzle

- Wilson coefficient only contributes a factor of ~ 2
- $\mathrm{Re}[A_2]$ and $\mathrm{Re}[A_0]$ are dominated by diagrams C_1 and C_2



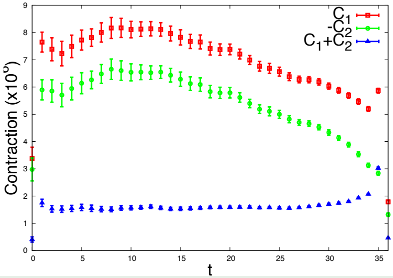
C_1: color diagonal



C_2: color mixed

Color counting in LO PT $\Rightarrow$ $C_2 = C_1/3$; Non-PT effects $\Rightarrow$ $C_2 \approx -0.7 C_1$

- $\mathrm{Re}[A_2] \propto C_1 + C_2$, while $\mathrm{Re}[A_0] \propto 2C_1 - C_2$ $\Rightarrow$ another factor of ~ 10
  - Such cancellation is first observed in an earlier calculation [RBC-UKQCD, PRL110 (2013) 152001]
  - It is further confirmed in the latest calculation of A_2 [RBC-UKQCD, PRD91 (2015) 074502]



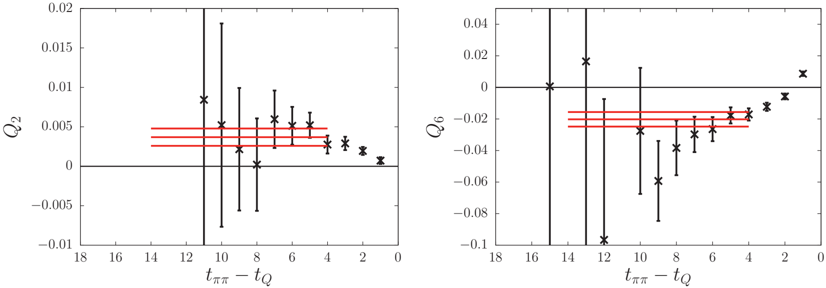
Puzzle of $\Delta I = 1/2$ rule is resolved from first principles

Results for A_0 [RBC-UKQCD, PRL115 (2015) 212001]

- Use a $32^3 \times 64$ ensemble, $N_{\rm conf} = 216$, $a = 0.14$ fm, $L = 4.53$ fm

  $M_\pi = 143.1(2.0)$ MeV, $M_K = 490(2.2)$ MeV, $E_{\pi\pi} = 498(11)$ MeV

- G-boundary condition is used: non-trivial to tune the volume $\Rightarrow M_K = E_{\pi\pi}$
- The largest contributions to $\mathrm{Re}[A_0]$ and $\mathrm{Im}[A_0]$ come from Q_2 (current-current) and Q_6 (QCD penguin) operator



- Scattering phase at $E_{\pi\pi} = M_K$: $\delta_0 = 23.8(4.9)(1.2)^\circ$
  - somewhat smaller than phenomenological expectation $\delta_0 = 38.0(1.3)^\circ$

[Courtesy of G. Colangelo]

Results for $\mathrm{Re}[A_0]$, $\mathrm{Im}[A_0]$ and $\mathrm{Re}[\epsilon'/\epsilon]$

[RBC-UKQCD, PRL115 (2015) 212001]

- Determine the $K \to \pi\pi(I = 0)$ amplitude A_0
  - Lattice results

$$\mathrm{Re}[A_0] = 4.66(1.00)_{\mathrm{stat}}(1.26)_{\mathrm{syst}} \times 10^{-7}\ \text{GeV}$$
$$\mathrm{Im}[A_0] = -1.90(1.23)_{\mathrm{stat}}(1.08)_{\mathrm{syst}} \times 10^{-11}\ \text{GeV}$$

  - Experimental measurement

$$\mathrm{Re}[A_0] = 3.3201(18) \times 10^{-7}\ \text{GeV}$$
$$\mathrm{Im}[A_0]\ \text{is unknown}$$

- Determine the direct CP violation $\mathrm{Re}[\epsilon'/\epsilon]$

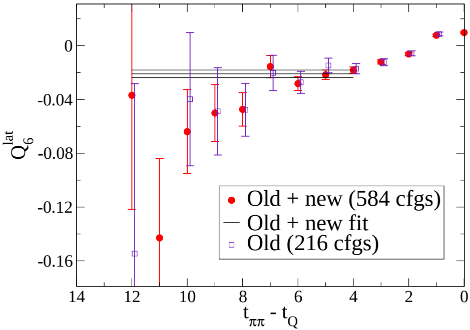
$$\mathrm{Re}[\epsilon'/\epsilon] = 0.14(52)_{\mathrm{stat}}(46)_{\mathrm{syst}} \times 10^{-3} \quad \text{Lattice}$$
$$\mathrm{Re}[\epsilon'/\epsilon] = 1.66(23) \times 10^{-3} \quad \text{Experiment}$$

2.1 σ deviation $\Rightarrow$ require more accurate lattice results

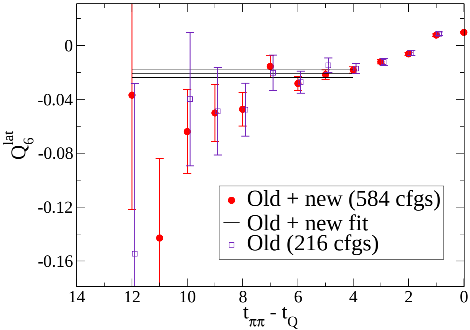
Improve both statistics and systematics

Efforts for statistics improvement

- Statistics increased: 216 → 584
- Aim to reduce stat. error by a factor of 2 within the next year



Plot, courtesy of C. Kelly



Plot, courtesy of C. Kelly

Efforts for statistics improvement

- Statistics increased: 216 → 584
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Efforts for systematic improvement

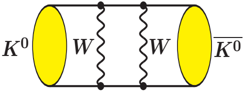
- Add the σ field to study $\sigma \to \pi\pi$ in the $I = 0$ channel
- Include EM in $K \to \pi\pi$
  - $\Delta I = 1/2$ rule may make the $O(\alpha_e)$ EM effect on A_2 20 times larger
- Calculate Wilson coefficients non-perturbatively
  - currently use unphysically light W-boson around 2 GeV

Long-distance contributions to flavor changing processes

ΔM_K and ϵ_K

K^0-$\overline{K^0}$ mixing

Weak interaction causes the mixing between K^0-$\overline{K^0}$



- Time evolution of the K^0-$\overline{K^0}$ mixing system

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left[\begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix}\right]\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- 2×2 M and Γ matrices are calculated to 2^{nd}-order in H_W

$$M_{ij} = M_K \delta_{ij} + \langle i|H_W|j\rangle + \mathcal{P}\sum\!\!\!\!\!\!\int_\alpha \frac{\langle i|H_W|\alpha\rangle\langle\alpha|H_W|j\rangle}{M_K - E_\alpha}$$

$$\Gamma_{ij} = 2\pi \sum\!\!\!\!\!\!\int_\alpha \langle i|H_W|\alpha\rangle\langle\alpha|H_W|j\rangle\,\delta(E_\alpha - M_K)$$

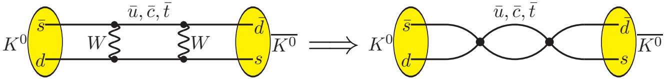
- ΔM_K and ϵ_K are related to $\mathrm{Re}[M_{0\bar{0}}]$ and $\mathrm{Im}[M_{0\bar{0}}]$, respectively

$$\Delta M_K = M_{K_L} - M_{K_S} = 2\mathrm{Re}[M_{0\bar{0}}]$$

$$\epsilon_K = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left[\frac{\mathrm{Im}[M_{0\bar{0}}]}{\Delta M_K} + \frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]}\right], \quad \phi_\epsilon = \arctan\frac{-2\Delta M_K}{\Delta\Gamma_K} \approx 45^\circ$$

Long-distance contribution to ΔM_K and ϵ_K

- $\Delta M_K \Rightarrow \mathrm{Re}[M_{0\bar{0}}] \Rightarrow$ *CP* conserving part of K^0-$\overline{K^0}$ mixing



Dominant contribution from charm-charm loop:

$$\lambda_c^2 \frac{m_c^2}{M_W^2} >> \lambda_t^2 \frac{m_t^2}{M_W^2}, \quad \text{where } \lambda_q = V_{qd} V_{qs}^*, \text{ for } q = u, c, t$$

$\Rightarrow$ historically led to the predication of the mass scale of charm quark

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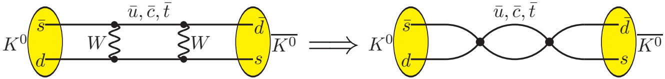

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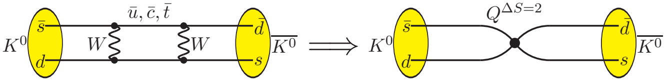
- $\epsilon_K \Rightarrow \mathrm{Im}[M_{0\bar{0}}] \Rightarrow$ *CP* violating part of K^0-$\overline{K^0}$ mixing

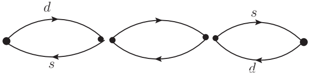


Top-top, top-charm and charm-charm loops compete in size
$\Rightarrow$ important top-top loop, thus ϵ_K is sensitive to λ_t (determined from V_{cb})

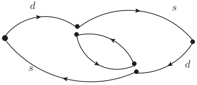
Use $32^3 \times 64$ ensemble: $a^{-1} = 1.38$ GeV, $m_\pi = 170$ MeV, $m_c = 750$ MeV

[Preliminary results from Z. Bai, for RBC-UKQCD]

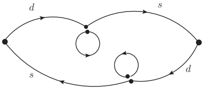
- Results based on 120 configurations



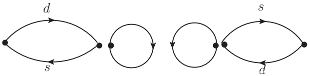
Type 1



Type 2



Type 3



Type 4

	ΔM_K $[10^{-12}$ MeV$]$
Type 1-4	3.85(46)
Type 1-2	4.49(16)
η	0
π	0.39(15)
$\pi\pi_{I=0}$	−0.06(2)
$\pi\pi_{I=2}$	$-6.25(11) \times 10^{-4}$
FV	0.024(11)
Exp	3.483(6)

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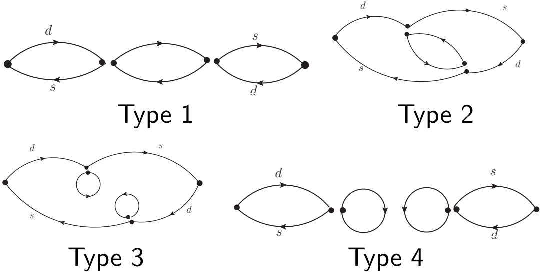

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Status for ΔM_K

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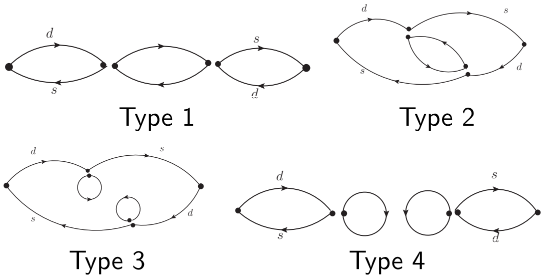

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New project: $64^3 \times 128$, $a^{-1} = 2.36$ GeV, $m_c = 1.2$ GeV, $m_\pi = 136$ MeV

- Based on 59 configurations: $\Delta M_K = 5.5(1.7) \times 10^{-12}$ MeV

Status for ϵ_K

SM predication vs Exp measurement [summarized by W. Lee @ Kaon 2016]

$$\left|\epsilon_K^{\mathrm{SM}}\right| = 1.69(17) \times 10^{-3} \quad \text{using Exclusive } V_{cb} \text{ (Lattice QCD)}$$
$$\left|\epsilon_K^{\mathrm{SM}}\right| = 2.10(21) \times 10^{-3} \quad \text{using Inclusive } V_{cb} \text{ (QCD sum rule)}$$
$$\left|\epsilon_K^{\mathrm{Exp}}\right| = 2.228(11) \times 10^{-3}$$

- 3.2 σ deviation between SM (exclusive V_{cb}) and experiment
- SM uncertainty is ~ 10%, dominated by V_{cb}
- Also important to determine LD contribution to ϵ_K (a few %)

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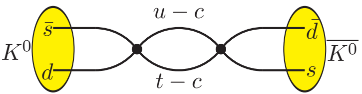
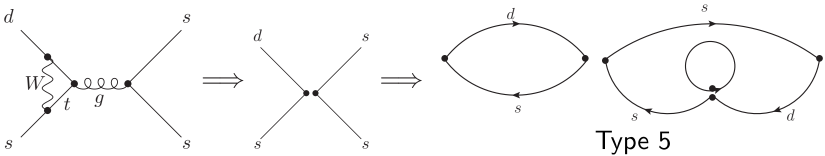


GIM subtraction of charm: $\lambda_u \times (u - c)$ and $\lambda_t \times (t - c)$

- Three terms: $\underbrace{\lambda_u^2}_{\text{irrelevant for } \epsilon_K}$ $\underbrace{\lambda_t^2}_{\text{SD dominated}}$ $\underbrace{\lambda_u \lambda_t}_{\text{need LQCD}}$

$\lambda_u \lambda_t$ contribution to ϵ_K

- In the $\lambda_u \lambda_t$ contribution, the top quark field has been integrated out
  - leaving QCD penguin operator and extra Type 5 quark contraction



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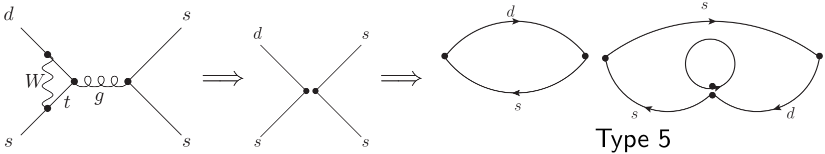
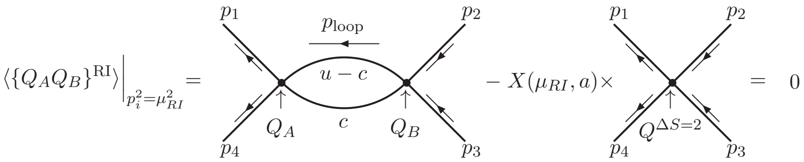


- Without top quark in the lattice QCD calculation, logarithmic divergence



  - Define the bilocal operator in the RI/SMOM scheme
  - Subtract $X(\mu_{RI}, a) Q^{\Delta S=2}$ to remove the lattice cutoff effects

Lattice results for ϵ_K

[from Z. Bai, for RBC-UKQCD]

- Use $24^3 \times 64$ lattice with DWF + Iwasaki gauge action

  $a^{-1} = 1.78$ GeV, $m_\pi = 340$ MeV, $m_K = 590$ MeV, $m_c = 970$ MeV

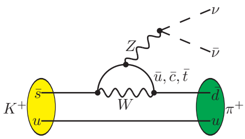
- All Type 1-5 diagrams are evaluated
- Preliminary results based on 200 configurations

μ_{RI}	$\text{Im}\, M_{\bar{0}0}^{ut,RI}$	$\text{Im}\, M_{\bar{0}0}^{ut,RI\to\overline{MS}}$	$\text{Im}\, M_{\bar{0}0}^{ut,ld\ corr}$	$\epsilon_K^{ut,ld\ corr}$
1.54 GeV	−0.75(39)	0.28	−0.46(39)	$0.091(76) \times 10^{-3}$
1.92 GeV	−0.91(39)	0.38	−0.53(39)	$0.104(76) \times 10^{-3}$
2.11 GeV	−0.99(39)	0.43	−0.55(39)	$0.108(76) \times 10^{-3}$
2.31 GeV	−1.05(39)	0.49	−0.57(39)	$0.111(77) \times 10^{-3}$
2.56 GeV	−1.12(39)	0.55	−0.57(39)	$0.111(77) \times 10^{-3}$

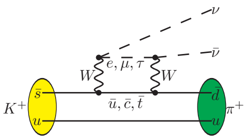
Experimental value for $|\epsilon_K| = 2.228(11) \times 10^{-3}$

- LD correction to ϵ_K is about 5% at unphysical kinematics

Rare Kaon decays



Z-exchange



W-W

$K^+ \to \pi^+ \nu\bar{\nu}$: largest contribution from top quark loop, thus theoretically clean

$$\mathcal{H}_{eff} \sim \frac{G_F}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\rm EM}}{2\pi \sin^2\theta_W} \lambda_t X_t(x_t)}_{\mathcal{N} \sim 2\times 10^{-5}} \cdot (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

Probe the new physics at scales of $\mathcal{N}^{-\frac{1}{2}} M_W = O(10\ \text{TeV})$

Past experimental measurement is 2 times larger than SM prediction

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \qquad \text{[BNL E949, '08]}$$

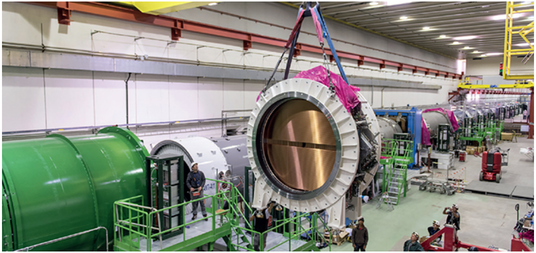
$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = 9.11 \pm 0.72 \times 10^{-11} \qquad \text{[Buras et. al., '15]}$$

but still consistent with $> 60\%$ exp. error

New experiments

New generation of experiment: NA62 at CERN

- aims at observation of $O(100)$ events [2014-2018]
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})$



Latest results reported at FPCP 2017

- Detector installation completed in 09.2016
- 5% of 2016 data $\Rightarrow$ no event yet
- Full 2016 data $\Rightarrow$ $O(1)$ events

2^{nd}-order weak interaction and bilocal matrix element

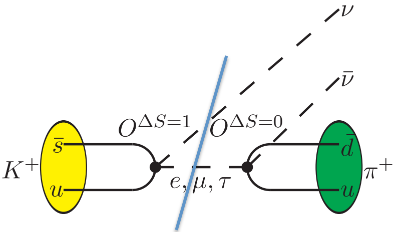
Hadronic matrix element for the 2^{nd}-order weak interaction

$$\int_{-T}^{T} dt\, \langle \pi^+ \nu \bar{\nu} | T\left[Q_A(t) Q_B(0) \right] | K^+ \rangle$$
$$= \sum_n \left\{ \frac{\langle \pi^+ \nu \bar{\nu} | Q_A | n \rangle \langle n | Q_B | K^+ \rangle}{M_K - E_n} + \frac{\langle \pi^+ \nu \bar{\nu} | Q_B | n \rangle \langle n | Q_A | K^+ \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n) T} \right)$$

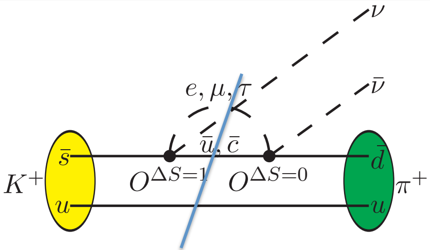
- For $E_n > M_K$, the exponential terms exponentially vanish at large T
- For $E_n < M_K$, the exponentially growing terms must be removed
- $\sum_n$: principal part of the integral replaced by finite-volume summation
  - possible large finite volume correction when $E_n \to M_K$

[Christ, XF, Martinelli, Sachrajda, PRD 91 (2015) 114510]

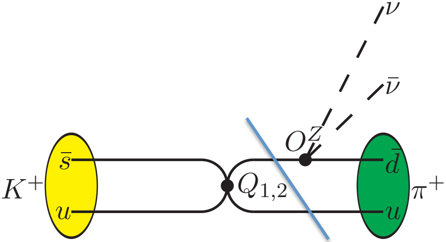
Low lying intermediate states



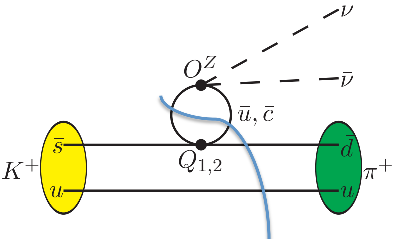
$K^+ \to \ell^+ \nu \quad \& \quad \ell^+ \to \pi^+ \bar{\nu}$



$K^+ \to \pi^0 \ell^+ \nu \quad \& \quad \pi^0 \ell^+ \to \pi^+ \bar{\nu}$



$K^+ \xrightarrow{H_W} \pi^+ \quad \& \quad \pi^+ \xrightarrow{V_\mu} \pi^+$



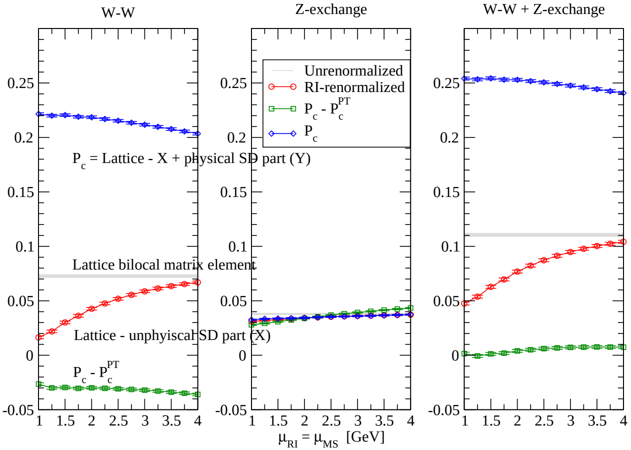
$K^+ \xrightarrow{H_W} \pi^+ \pi^0 \quad \& \quad \pi^+ \pi^0 \xrightarrow{A_\mu} \pi^+$

Lattice results

First results @ $m_\pi = 420$ MeV, $m_c = 860$ MeV

[Bai, Christ, XF, Lawson, Portelli, Schrajda, PRL 118 (2017) 252001]

$$P_c = 0.2529(\pm 13)_{\rm stat}(\pm 32)_{\rm scale}(-45)_{\rm FV}$$



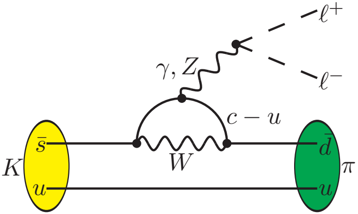
Lattice QCD is now capable of first-principles calculation of rare kaon decay

- The remaining task is to control various systematic effects

$K \to \pi \ell^+ \ell^-$: CP conserving chanel

CP conserving decay: $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$

- Involve both γ- and Z-exchange diagram, but γ-exchange is much larger



- Unlike Z-exchange, the γ-exchange diagram is LD dominated
  - By power counting, loop integral is quadratically UV divergent
  - EM gauge invariance reduces divergence to logarithmic
  - $c - u$ GIM cancellation further reduces log divergence to be UV finite

Focus on γ-exchange

- Hadronic part of decay amplitude is described by a form factor

$$T^{\mu}_{+,S}(p_K, p_\pi) = \int d^4x\, e^{iqx} \langle \pi(p_\pi) | T\{J^{\mu}_{em}(x) \mathcal{H}^{\Delta S=1}(0)\} | K^+/K_S(p_K) \rangle$$
$$= \frac{G_F M_K^2}{(4\pi)^2} V_{+,S}(z) \left[z(p_K + p_\pi)^\mu - (1 - r_\pi^2) q^\mu \right]$$

with $q = p_K - p_\pi$, $z = q^2/M_K^2$, $r_\pi = M_\pi/M_K$

The target for lattice QCD is to calculate the form factor $V_{+,S}(z)$

- Lattice calculation strategy (I): [RBC-UKQCD, PRD92 (2015) 094512]
  - Use conserved vector current to protect the EM gauge invariance
  - Use charm as an active quark flavor to maintain GIM cancellation

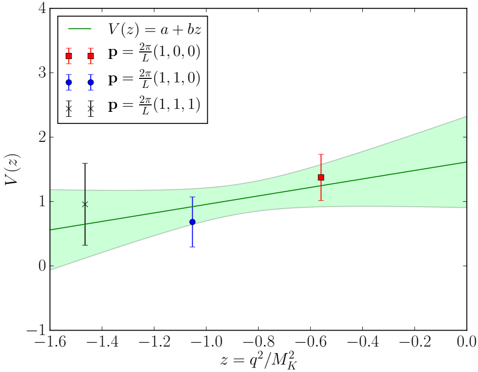
First exploratory calculation on $K^+ \to \pi^+ \ell^+ \ell^-$

Use $24^3 \times 64$ ensemble, $N_{\rm conf} = 128$
[RBC-UKQCD, PRD94 (2016) 114516]

$a^{-1} = 1.78$ GeV, $m_\pi = 430$ MeV
$m_K = 625$ MeV, $m_c = 530$ MeV

Momentum dependence of $V_+(z)$

$$V_+(z) = a_+ + b_+ z$$
$$\Rightarrow \quad a_+ = 1.6(7), \; b_+ = 0.7(8)$$



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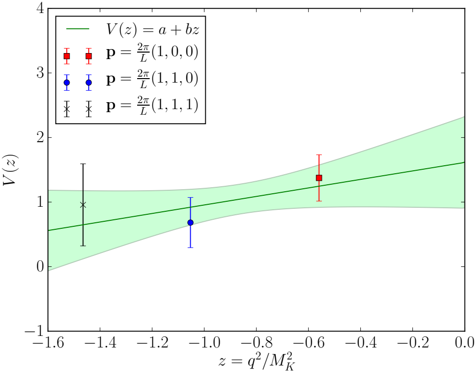


$K^+ \to \pi^+ e^+ e^-$ data + phenomenological analysis: $a_+ = -0.58(2)$, $b_+ = -0.78(7)$
[Cirigliano, et. al., Rev. Mod. Phys. 84 (2012) 399]

$$V_j(z) = a_j + b_j z + \underbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z - z_0)}{G_F M_K^2 r_\pi^4}}_{K\to\pi\pi\pi} \underbrace{\left[1 + \frac{z}{r_V^2}\right]}_{F_V(z)} \underbrace{\left[\phi(z/r_\pi^2) + \frac{1}{6}\right]}_{\text{loop}}, \quad j = +, S$$

- Experimental data only provide $\frac{d\Gamma}{dz} \Rightarrow$ square of form factor $|V_+(z)|^2$
- Need phenomenological knowledge to determine the sign for a_+, b_+

Conclusion

- For "standard" quantities such as f_K/f_π, $f_+(0)$ and B_K

	N_f	FLAG average	Frac. Err.
f_K/f_π	$2+1+1$	1.1933(29)	0.25%
$f_+(0)$	$2+1+1$	0.9706(27)	0.28%
$\hat{B}_K$	$2+1$	0.7625(97)	1.27%

  lattice QCD calculations play important role in precision flavor physics

- It's time to go beyond "standard"
  - $K \to \pi\pi$ and ϵ'
  - ΔM_K and ϵ_K
  - rare kaon decays: $K \to \pi\nu\bar{\nu}$ and $K \to \pi\ell^+\ell^-$

- Lattice QCD is now capable of first-principles calculation of the above "beyond-standard" quantities

- Realistic calculation of some of these quantities may require the next generation of super-computers

Backup slides

$K_L \to \pi^0 \ell^+ \ell^-$ decay: CP violating channel

$K_L \to \pi^0 \ell^+ \ell^-$ decay contains important CPV information

- Indirect CPV: $K_L \xrightarrow{\epsilon} K_+^0 \to \pi^0 \gamma^* \to \pi^0 \ell^+ \ell^-$
- Direct + indirect CPV contribution to branching ratio

[Cirigliano et. al., Rev. Mod. Phys. 84 (2012) 399]

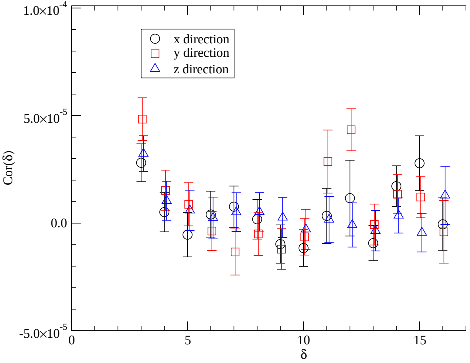
$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{CPV} = 10^{-12} \times \left[15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right)^2 \right]$$

  - $\mathrm{Im}\,\lambda_t$-term from direct CPV, $\lambda_t \approx 1.35 \times 10^{-4}$
  - $|a_S|$-term from indirect CPV, $a_S = V_S(0)$
  - $\pm$ arises due to the unknown sign of a_S

Even a determination of the sign of a_S from lattice is desirable

Duplicated RNG seeds used in quark forces $\Rightarrow$ unphysical correlation

- Such correlation is observed in plaquettes separated by 12 in y-direction
- Its size is only $\sim 5 \times 10^{-5}$
- Unlikely affect A_2, A_0 strongly, whose errors are ~ 1000 times larger



Average plaquette

- Correct ensemble 0.512239(3)(7)
- Incorrect ensemble 0.512239(6)

$K \to \pi\pi$: Uncertainty budget

Systematic error breakdown for Re A_2 and Im A_2

[RBC-UKQCD, PRD91 (2015) 074502]

Systematic errors	Re A_2	Im A_2
NPR (nonperturbative)	0.1%	0.1%
NPR (perturbative)	2.9%	7.0%
Finite-volume corrections	2.4%	2.6%
Unphysical kinematics	4.5%	1.1%
Wilson coefficients	6.8%	10%
Derivative of the phase shift	1.1%	1.1%
Total	9%	12%

Systematic error for individual operator contributions to Re(A_0), Im(A_0)

[RBC-UKQCD, PRL115 (2015) 212001]

Description	Error	Description	Error
Finite lattice spacing	12%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			27%

ΔM_K and ϵ_K: Removal of the exponentially growing terms

- Determine the hadronic matrix element for all low-lying intermediate states

$$\frac{\langle \overline{K^0}|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K - E_n}\left(1 - e^{(M_K - E_n)T}\right)$$

- Change of weak operator $H_W \to H_W + c_s\bar{s}d + c_p\bar{s}\gamma_5 d$ does not affect the physical amplitude
  - Apply the chiral Ward identity

$$\begin{aligned}\partial_\mu \bar{s}\gamma_\mu d &= (m_s - m_d)\bar{s}d \\ \partial_\mu \bar{s}\gamma_\mu\gamma_5 d &= (m_s + m_d)\bar{s}\gamma_5 d\end{aligned}$$

  - K^0-$\overline{K^0}$ transition amplitude is given by

$$\int d^4x\, \langle \overline{K^0}|T[H_W(x)H_W(0)]|K^0\rangle$$

$\partial_\mu \bar{s}\gamma_\mu d$ and $\partial_\mu \bar{s}\gamma_\mu\gamma_5 d$ do not contribute to the $\int d^4x$ integral

- Choose appropriate c_s and c_p, e.g.

$$\langle 0|H_W + c_p\bar{s}\gamma_5 d|K^0\rangle = 0$$
$$\langle \eta|H_W + c_s\bar{s}d|K^0\rangle = 0$$

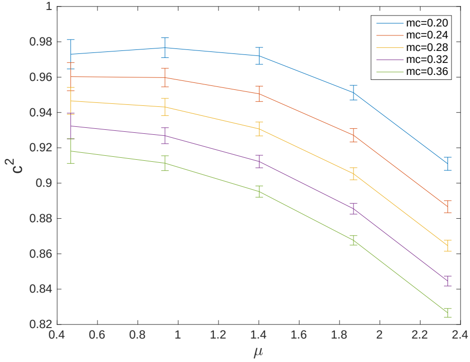
ΔM_K: Lattice artifacts with physical charm quark mass

Naive estimate of lattice artifacts $\sim (m_c^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})a)^2 = 25\%$ with $a^{-1} = 2.36$ GeV

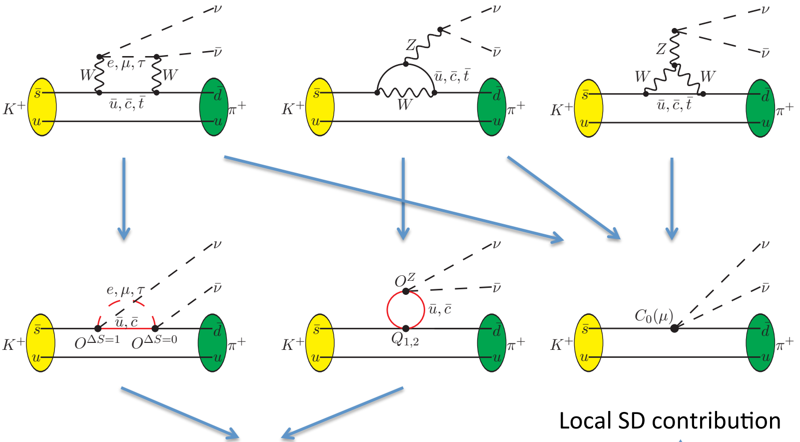
D meson dispersion relation

$$c^2 = \frac{E^2 - m^2}{p^2}, \quad \mu^2 = p^2$$

- The physical charm quark mass is related to bare mass $m_c a = 0.32$
- c^2 value deviate from 1 by $\sim 10\%$



$K^+ \to \pi^+\nu\bar{\nu}$: OPE to separate SD and LD parts



Local SD contribution

Bilocal LD contribution

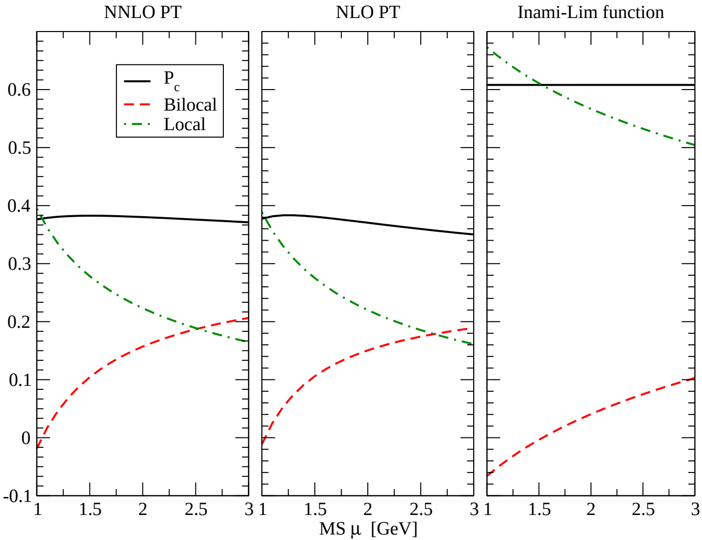
Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu\bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

$K^+ \to \pi^+ \nu\bar{\nu}$: Bilocal contribution vs local contribution

Bilocal $C_A^{\overline{\rm MS}}(\mu) C_B^{\overline{\rm MS}}(\mu) r_{AB}^{\overline{\rm MS}}(\mu)$ vs Local $C_0^{\overline{\rm MS}}(\mu)$

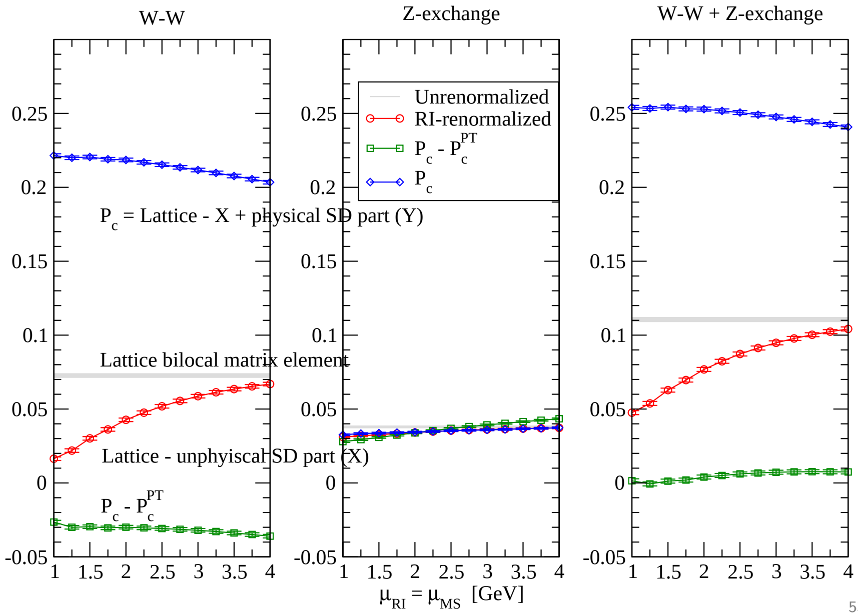
[Buras, Gorbahn, Haisch, Nierste, '06]



At $\mu = 2.5$ GeV, 50% charm quark contribution from bilocal term

$K^+ \to \pi^+\nu\bar{\nu}$: Lattice results

Use m_π = 420 MeV, m_c = 860 MeV [RBC-UKQCD, arXiv:1701.02858]



$K \to \pi \ell^+ \ell^-$ in 3-flavor theory

From 4-flavor to 3-flavor theory

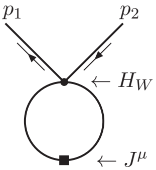
$$C^{N_f=4}(\mu_c)\underbrace{\langle H_W^{N_f=4}(\mu_c) J^\mu \rangle}_{\text{UV finite}} = C^{N_f=3}(\mu_c)\underbrace{\langle H_W^{N_f=3}(\mu_c) J^\mu \rangle}_{\text{log divergent}} + \sum_i \underbrace{C_i(\mu_c)\langle Q_i^\mu(\mu_c)\rangle}_{\text{counter term}}$$

- The local counter term is mainly given by the penguin operator

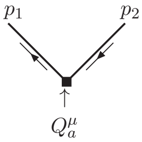
$$Q_a^\mu = (\delta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)\bar{s}\gamma_\nu(1-\gamma_5)d$$

Use NPR to convert bare lattice bilocal operator to RI/SMOM scheme

$\langle\{H_W J^\mu\}^{\mathrm{RI}}\rangle\Big|_{p_i^2=\mu_{RI}^2} =$



$- \quad X(\mu_{RI}, a) \quad \times$



$= \quad 0$

Use PT to convert RI/SMOM bilocal operator to $\overline{\mathrm{MS}}$ scheme

Lattice calculation strategy (II)

Important to have a physical point simulation, however

- physical m_π requires large lattice volume to control FV effects
- physical m_c requires ultra-fine lattice spacing

$\Rightarrow$ very high demanding on computer resources

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- Explore dispersion relation and unphysical poles for Möbius DWF

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- Explore dispersion relation and unphysical poles for Möbius DWF

Another solution is to integrate out charm quark $\Rightarrow$ strategy (II)

- Perturbatively treat the charm quark contribution
- Lattice calculation uses physical pion mass + rather coarse lattice
- No GIM cancellation, thus log divergence exists for lattice calculation

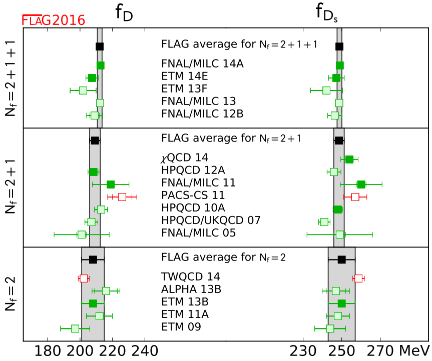
“standard” quantities in charm physics: f_D and f_{D_s}

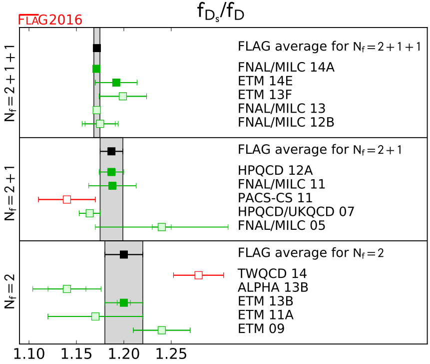
Flavor Lattice Averaging Group (FLAG) average, updated in Nov. 2016

$$f_D = 212.15(1.45)\ \text{MeV} \quad \Rightarrow \quad 0.68\%\ \text{error}$$

$$f_{D_s} = 248.83(1.27)\ \text{MeV} \quad \Rightarrow \quad 0.51\%\ \text{error}$$

$$f_{D_s}/f_D = 1.1716(32) \quad \Rightarrow \quad 0.27\%\ \text{error}$$





Experimental determination of f_D and f_{D_s} [quoted by PDG 2015 update]

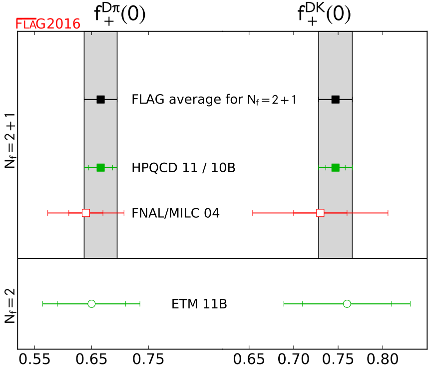
$$f_D = 203.7(4.8)\ \text{MeV} \quad \Rightarrow \quad 2.4\%\ \text{error}$$

$$f_{D_s} = 257.8(4.1)\ \text{MeV} \quad \Rightarrow \quad 1.6\%\ \text{error}$$

Charm physics: $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$

Flavor Lattice Averaging Group (FLAG) average, updated in Nov. 2016

$$f_+^{D\pi}(0) = 0.666(29) \quad \Rightarrow \quad 4.4\% \text{ error}$$
$$f_+^{DK}(0) = 0.747(19) \quad \Rightarrow \quad 2.5\% \text{ error}$$



Experimental averages from HFAG 2014

$$f_+^{D\pi}(0)|V_{cd}| = 0.1425(19) \text{ MeV} \quad \Rightarrow \quad 1.3\% \text{ error}$$
$$f_+^{DK}(0)|V_{cs}| = 0.728(5) \text{ MeV} \quad \Rightarrow \quad 0.69\% \text{ error}$$