手 征 有 效 场 论 研 讨 会

陕西-西安，2017年10月13日 – 17日

Relativistic chiral nucleon-nucleon interaction

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OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and perspectives

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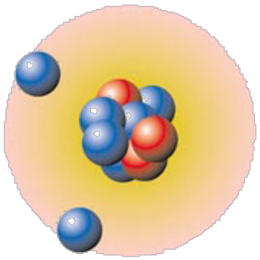
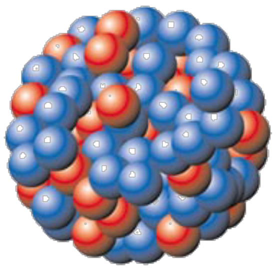
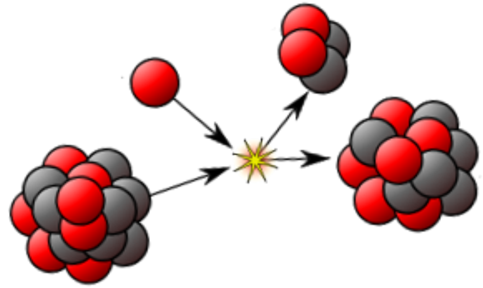
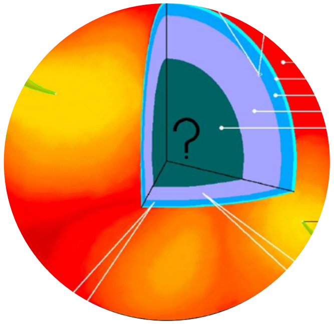
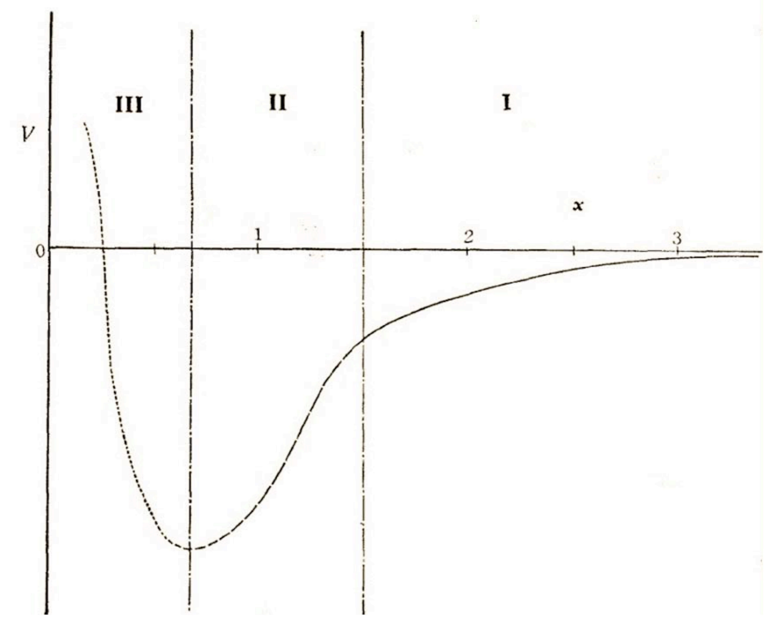
Basic for all nuclear physics

☐ Precise understanding of the nuclear force

☐ Complexity of the nuclear force (vs. electromagnetic force)

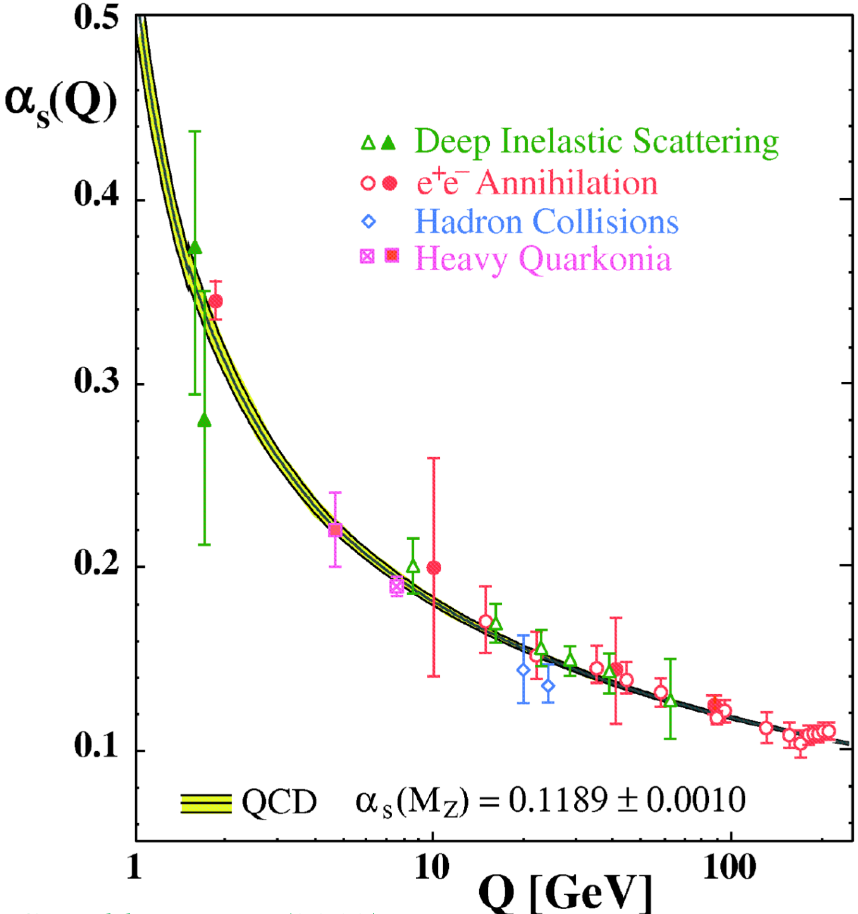
- Finite range
- Intermediate-range **attraction**
- Short-range **repulsion**-“hard core”
- Spin-dependent **non-central** force
  - Tensor interaction
  - Spin-orbit interaction
- Charge independent (approximate)



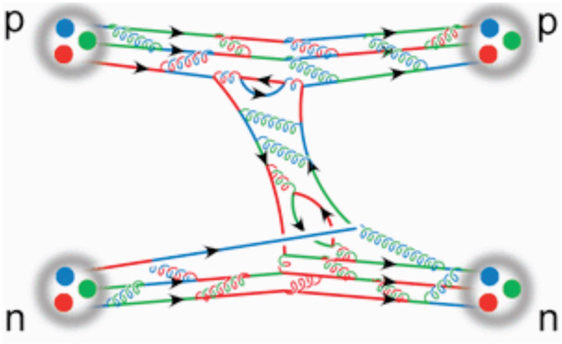
M. Taketani, Suppl.PTP3(1956)1

Nuclear force (NF) from QCD

- **Residual quark-gluon strong interaction**
- **Understood from QCD**



S.Bethke, PPNP(2013)



At low-energy region

- Running coupling constant $\alpha_s \geq 1$
- **Nonperturbative QCD -- unsolvable**

⇨
- **Phenomenological models**
- **Lattice QCD simulation**
- **Chiral effective field theory**

NF from phenomenological models

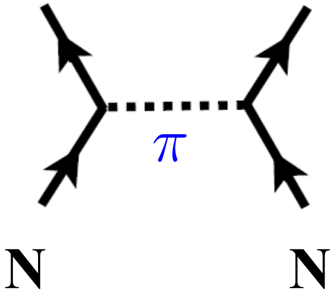
- **Phenomenological analysis**
  - Operator structures (allowed by symmetries)

$$
\begin{aligned}
V_{NN} = & \; V_0(r) + V_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_\tau(r)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + V_{\sigma\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + V_{LS}(r)\boldsymbol{L}\cdot\boldsymbol{S} + V_{LS\tau}(r)(\boldsymbol{L}\cdot\boldsymbol{S})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + V_T(r)S_{12} + V_{T\tau}(r)S_{12}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& + V_Q(r)Q_{12} + V_{Q\tau}(r)Q_{12}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& + V_{PP}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{p}) + V_{PP\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{p})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + \ldots\ldots
\end{aligned}
$$

Gammel-Thaler (1957)
Hamada-Johnston (1962)
Reid 68, Argonne V14
Reid 93, Argonne V18

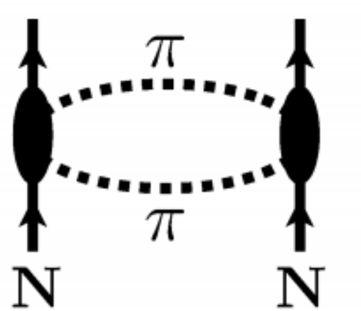
- **Meson "theory"**

OPE



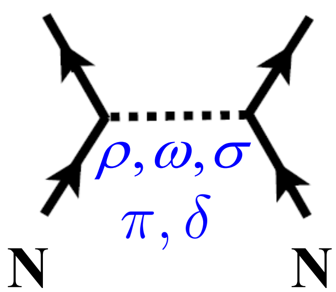
1935

TPE, MPE



1950's

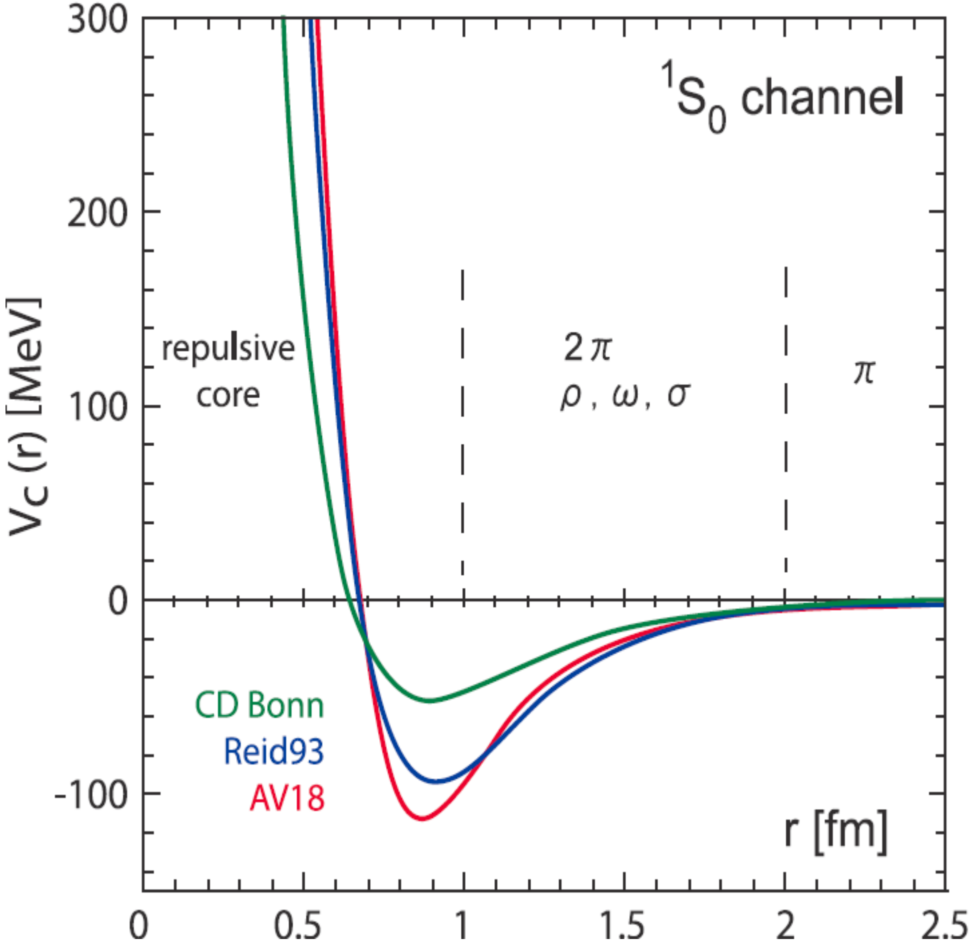
OBE



1960's,1970's

Partovi-Lomon (1970)
Stony Brook (1975)
Paris potential (1980)
Bonn (1987),
CD-Bonn(2001)

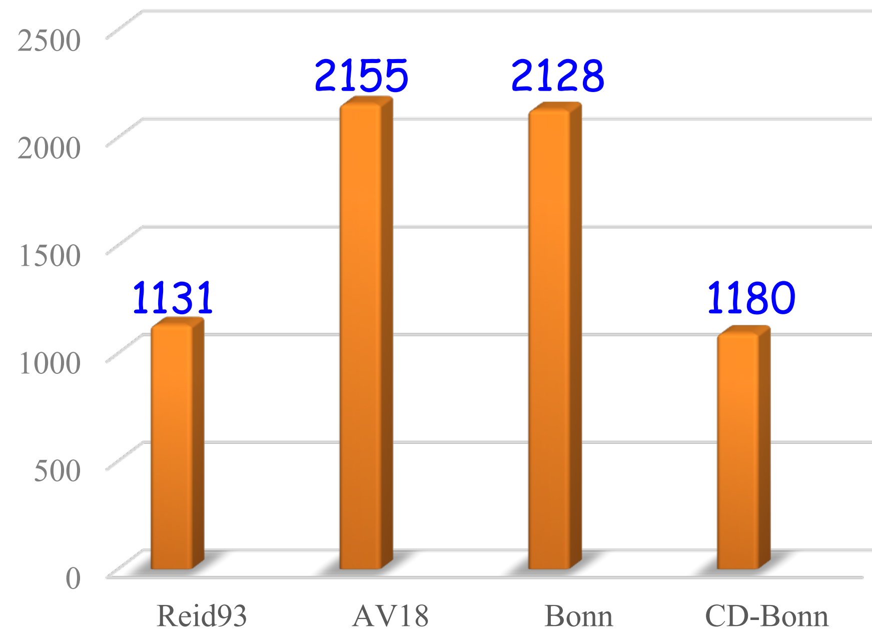
NF from phenomenological models



N. Ishii et al. PRL(2007)

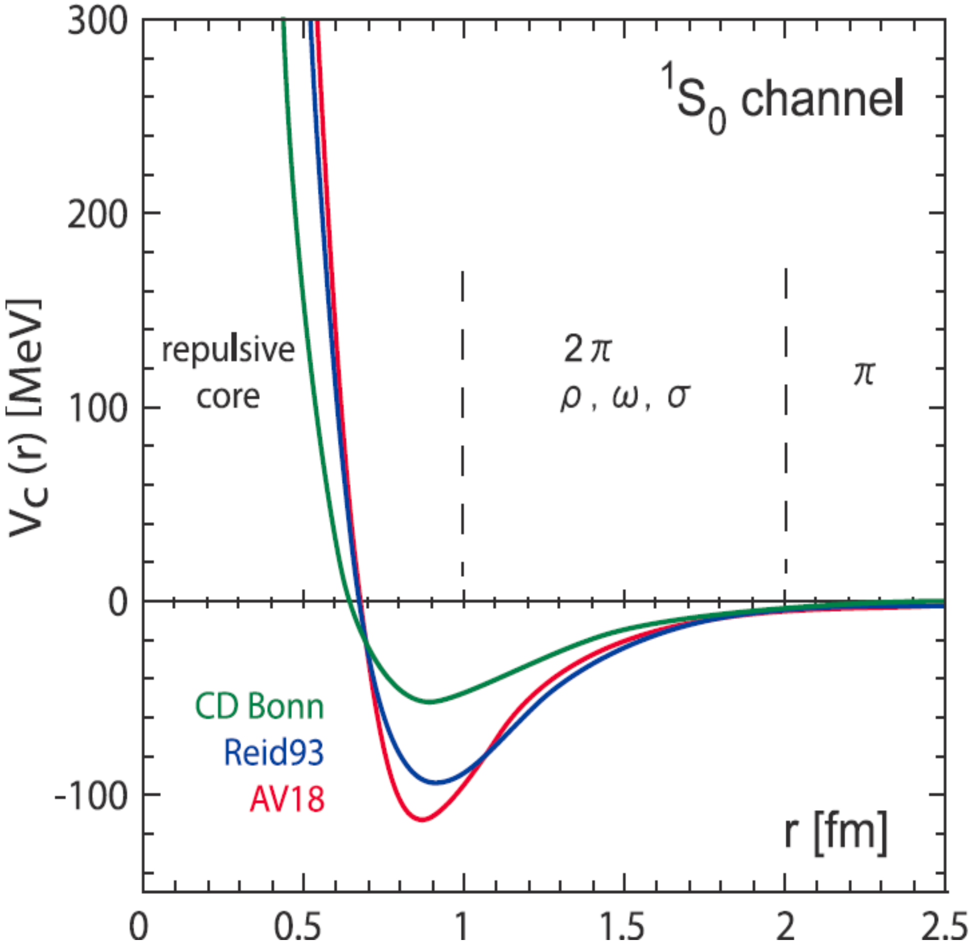
Citations from Inspire-hep

Oct. 12, 2017

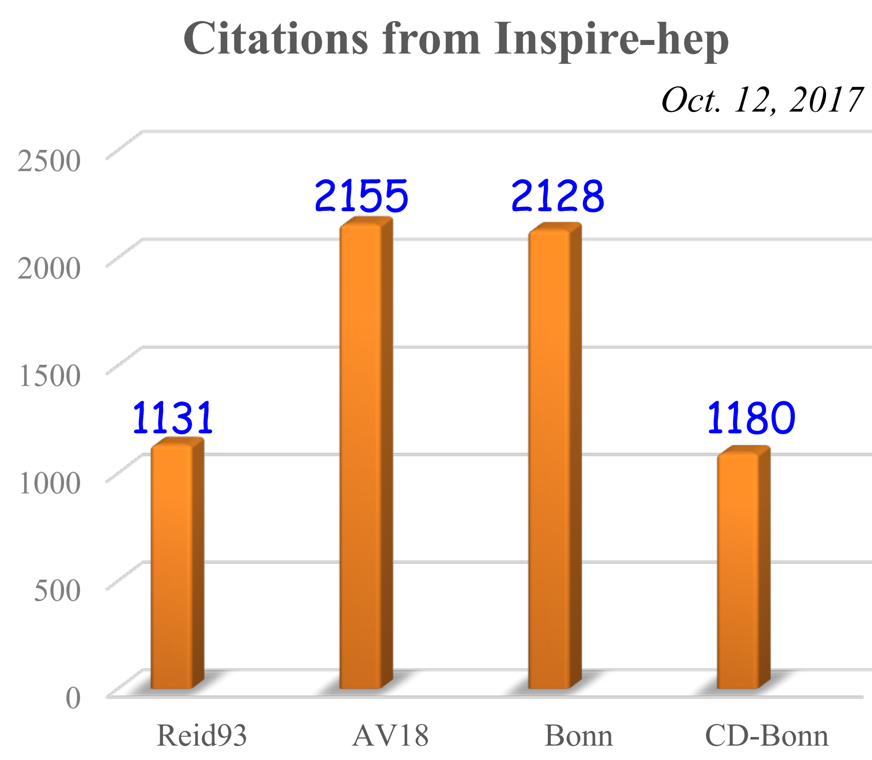


extensively applied to the nuclear physics

NF from phenomenological models



N. Ishii et al. PRL(2007)

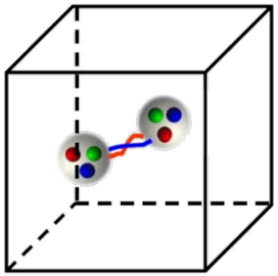


extensively applied to the nuclear physics

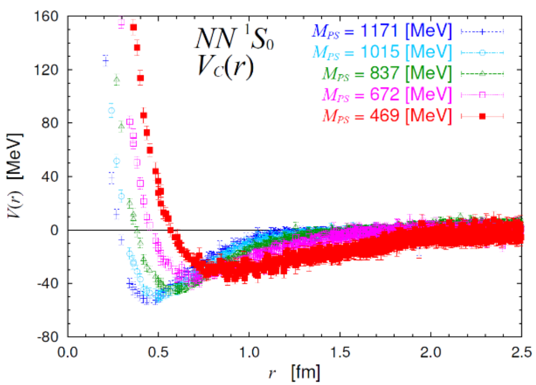
But, these potentials are not constructed from the fundamental level.

NF from Lattice QCD

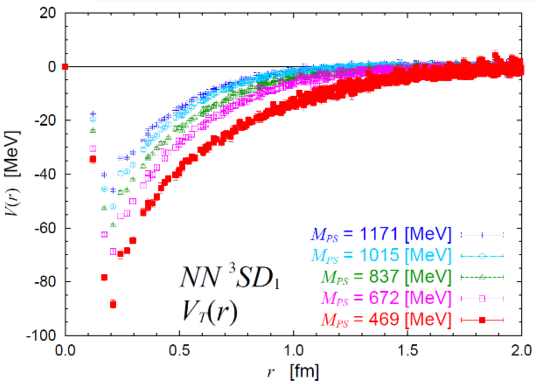
- Lattice QCD: numerical method of QCD *Wilson, PRD1974*
  - Discretized Euclidean space-time
  - Monte Carlo method



- Extract the nuclear force
  - **HAL QCD** coll. *T. Hatsuda, S. Aoki, et al.*
  - **NPLQCD** coll. *S. R. Beane, M. J. Savage, et al.*
    - **CalLat coll. / T. Yamazaki et al.**



The bulk properties of nuclear force can be produced from first principle



Input m_π=469 MeV is still larger than its physical value ~140 MeV

- ✓ **Repulsive core**
- ✓ **Attractive pocket**
- ✓ **Tensor force**

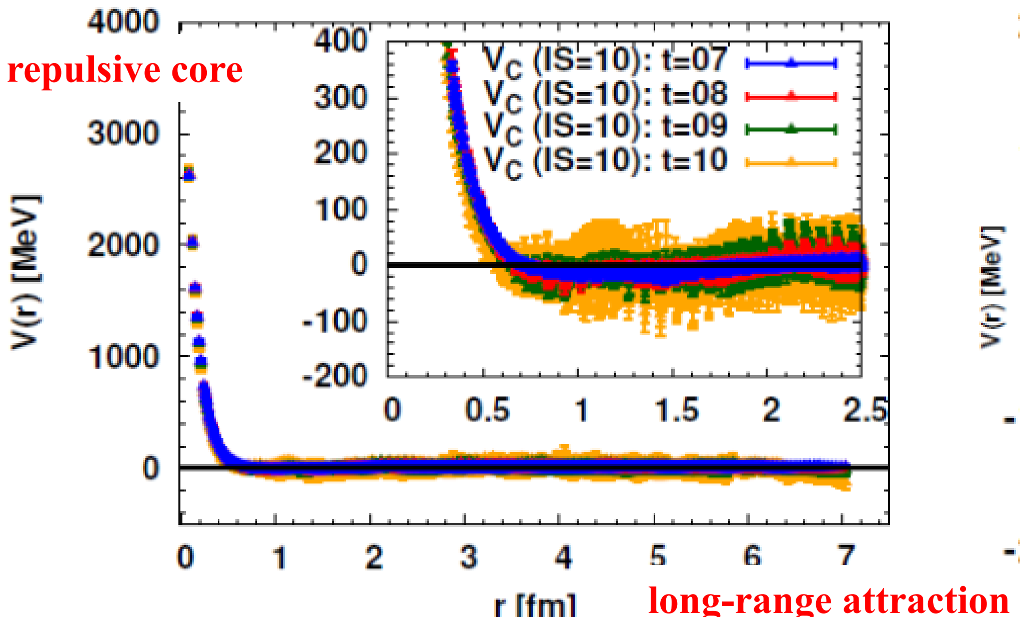
HAL QCD PRL(2007), arXiv: 1511.04871

Preliminary results at physical point

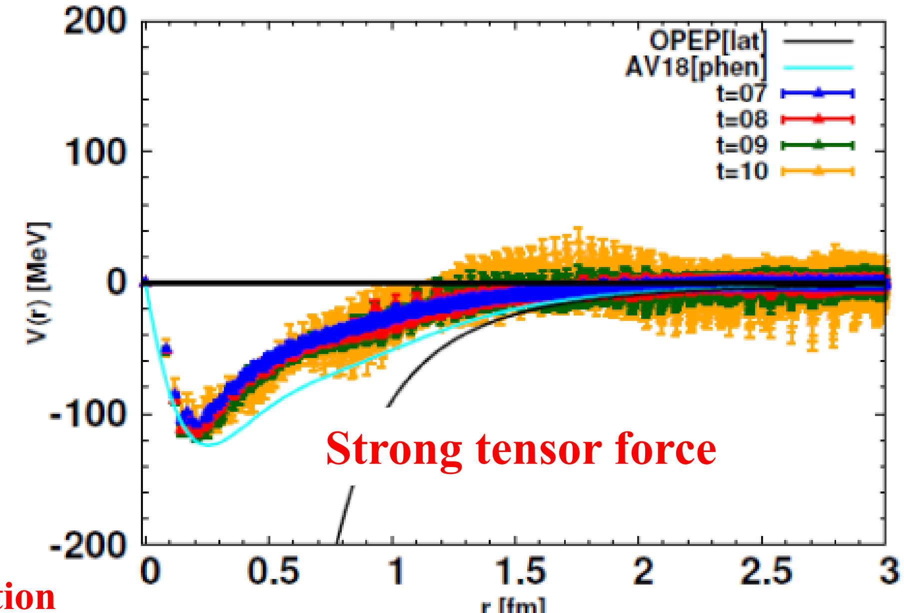
- Lattice set-up
  - Pion mass: m_{π} ~= **145 MeV**
  - Lattice box size: L ~= 8 fm
  - Lattice spacing: $1/a$ ~= 2.3 GeV
- Central/Tensor forces for NN

T. Doi, Lattice2016

1S0: center force



3S1-3D1: tensor force



NF from Chiral EFT

- Chiral effective field theory *S. Weinberg,Phys.A1979*
  - Effective field theory (EFT) of **low-energy QCD**
  - **Model independent** to study the nuclear force *S. Weinberg,PLB1990*
- Main advantages of chiral nuclear force
  - **Self-consistently include** many-body forces

$$V = V_{2N} + V_{3N} + \cdots + V_{iN} + \cdots$$

  - **Systematically improve** NF order by order

$$V_{iN} = V_{iN}^{\mathrm{LO}} + V_{iN}^{\mathrm{NLO}} + V_{iN}^{\mathrm{NNLO}} + \cdots$$

  - **Systematically estimate** theoretical uncertainties

$$|V_{iN}^{\mathrm{LO}}| > |V_{iN}^{\mathrm{NLO}}| > |V_{iN}^{\mathrm{NNLO}}| > \cdots$$

Current status of chiral NF

Nonrelativistic (NR) chiral NF

- NN interaction
  - up to NLO *U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997*
  - up to NNLO *U. van Kolck et al.,PRC1994; E. Epelbaum, et al.,NPA2000*
  - up to **N^3LO** *R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005*
  - up to **N^4LO** *E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015*
  - up to **N^5LO** (dominant terms) *D.R. Entem, et al., PRC2015*
- 3N interaction
  - up to NNLO *U. van Kolck, PRC1994*
  - up to N^3LO *S. Ishikwas, et al, PRC2007; V. Bernard et al, PRC2007*
  - up to **N^4LO** *H. Krebs, et al., PRC2012-13*
- 4N interaction
  - up to N^3LO *E. Epelbaum, PLB 2006, EPJA 2007*

P. F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339
E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

Chiral NN potential is of high precision

	Phenomenological forces			NR Chiral nuclear force				
	Reid93	**AV18**	**CD-Bonn**	LO	NLO	NNLO	**N^{3}LO**	**N^{4}LO**
No. of para.	**50**	**40**	**38**	2+2	9+2	9+2	**24+2**	**24+3**
χ^2/datum ***np data 0-290 MeV***	**1.03**	**1.04**	**1.02**	94	36.7	5.28	**1.27**	**1.10**

D.Entem, et al., PRC96(2017)024004

Chiral force has been extensively applied in the study of nuclear structure and reactions within the non-relativistic few-/many-body theories.

E. Epelbaum, et al., PRL 106(2011) 192501, PRL109(2012)252501, PRL112(2014)102501; S. Elhatisari, et al., Nature 528 (2015) 111, arXiv:1702.05177; G. Hagen, et al., PRL109(2012)032502; H. Hergert, et al., PRL110(2013)24501; G.R. Jansen, et al., PRL113(2014)102501; S.K.Bogner, et al., PRL113(2014)142501; J.E. Lynn, et al., PRL113(2014)192501; V. Lapoux, et al., PRL117(2016)052501……

Limitations of current chiral NF

- **Not "renormalization group invariance"**
  - Dependent on the UV cutoff
  - Impact on multi-nucleon system
- **Based on heavy baryon ChEFT**
  - **Cannot be used directly in relativistic nuclear structure studies**

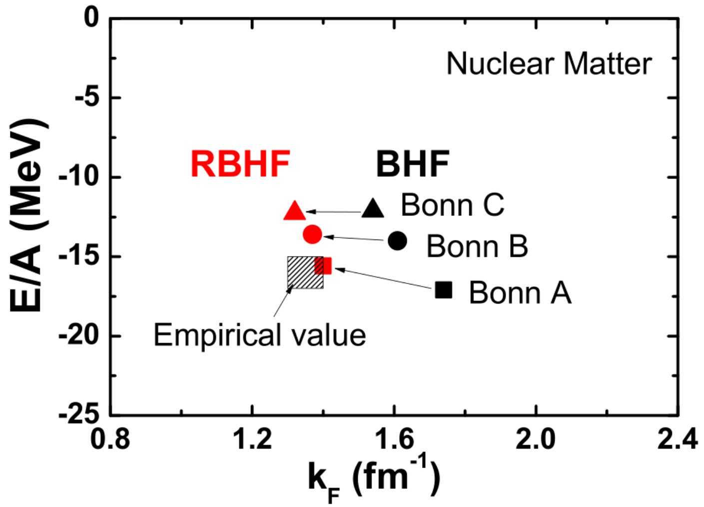
⇨ **Relativistic nuclear force based on covariant ChEFT?**

Relativistic effects are important

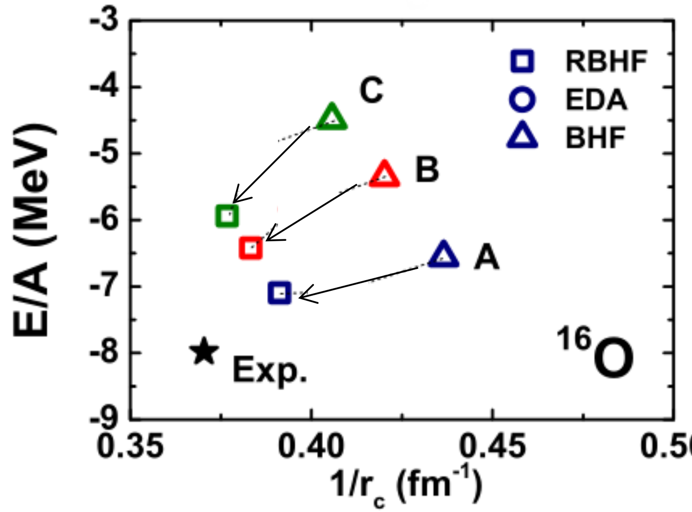
- The success of **covariant density functional theory (CDFT)** in the nuclear structure studies.

P. Ring, PPNP (1996), D.Vretenar et al.,Phys.Rept.(2005), J. Meng, PPNP(2006), Phys.Rept.(2015), IRNP(2016)

  - Relativistic Brueckner-Hartree-Fock theory in nuclear matter and finite nuclei **(input: relativistic Bonn)**



R. Brockmann & R. Machleidt, PRC(1990)



S.H. Shen, et al., CPL(2016), PRC(2017)

Relativistic nuclear force based on ChEFT is needed

Relativistic effects are important

- The success of **covariant density functional theory (CDFT)** in the nuclear structure studies.

  P. Ring, PPNP (1996), D.Vretenar et al.,Phys.Rept.(2005), J. Meng, PPNP(2006), Phys.Rept.(2015), IRNP(2016)

- Covariant ChEFT with ***extended-on-mass-shell*** scheme

  J.Gegelia,PRD(1999), Fuchs,PRD(2003)

  - Maintains all the symmetry and analyticity
  - Successfully applied to the **one-nucleon(baryon)** sector
    - Baryon mass, magnetic moments, π-N scattering …

      V. Pascalutsa,PLB2004; L.S.Geng,PRL2008; XLR,JHEP2012; Y.H.Chen,PRD(2013), ...
    - Shows a faster convergence than the NR ChEFT case

Relativistic chiral force has relatively fast convergence?

In this work

We extend covariant ChEFT to the nucleon-nucleon sector and construct a relativistic nuclear force up to next-to-leading order

- Construct the kernel potential in **covariant power counting**
  - Employ the Lorentz invariant chiral Lagrangains
  - Retain the complete form of Dirac spinor
  - Use naïve dimensional analysis to determine the chiral dimension
- Employ the 3D-reduced **Bethe-Salpeter** equation, such as **Kadyshevsky** equation, to resum the potential.

OUTLINE

- Introduction
- Theoretical framework
  - NN potential concepts
  - Relativistic chiral force up to NLO
- Results and discussion
- Summary and perspectives

NN potential concept

- Often-thought as nonrelativistic quantity
  - Appear in the **Schrödinger** equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(t,\boldsymbol{r}) + V(\boldsymbol{r})\Psi(t,\boldsymbol{r}) = i\hbar\frac{\partial}{\partial t}\Psi(t,\boldsymbol{r}).$$

  - (or) Appear in the **Lippmann-Schwinger** equation

$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d\boldsymbol{k}}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{k}) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k},\boldsymbol{p}).$$

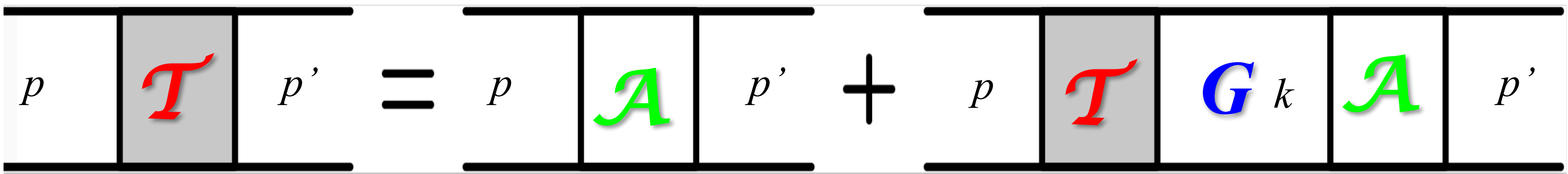
- Generalize the definition of potential
  - An interaction quantity appearing in a three-dimensional scattering equation can be referred as a **NN potential**.

⇨ **Relativistic potential**

M.H. Partovi, E.L. Lomon, PRD2 (1970) 1999
K. Erkelenz, Phys.Rept. 13C(1974) 191

Bethe-Salpeter equation

- For the relativistic nucleon-nucleon scattering



$W = \sqrt{s}/2$

Bethe-Salpeter equation with an operator form:

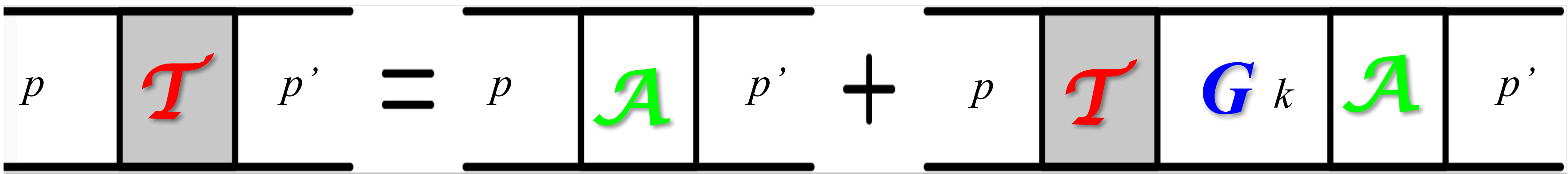
$$\mathcal{T}(p', p|W) = \mathcal{A}(p', p|W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p', p|W) G(k|W) T(k, p|W),$$

- $\mathcal{T}$: Invariant scattering amplitude
- $\mathcal{A}$: **Interaction kernel** (sum all the irreducible diagrams)
- G: Two-nucleon's Green function

$$G(k|W) = i \frac{1}{[\gamma^\mu (W+k)_\mu - m_N + i\epsilon]^{(1)} \, [\gamma^\mu (W-k)_\mu - m_N + i\epsilon]^{(2)}},$$

Bethe-Salpeter equation

- For the relativistic nucleon-nucleon scattering



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- $\mathcal{T}$: Invariant scattering amplitude
- $\mathcal{A}$: **Interaction kernel** (sum all the irreducible diagrams)
- G: Two-nucleon's Green function

It is hard to solve the BS equation, one always perform the 3-dimensional reduction.

Reduction of BS equation

- Introduce a three dimensional Green function ***g***
  - Maintain the same elastic unitarity of ***G*** at physical region
  - We choose the Kadyshevsky propagator *V. Kadyshevsky, NPB (1968).*

$$g = 2\pi \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\boldsymbol{k})\Lambda_+^{(2)}(-\boldsymbol{k})}{\sqrt{s} - 2E_k + i\epsilon} \delta[k_0 - (E_k - \frac{\sqrt{s}}{2})].$$

- To replace ***G*** with ***g***, one can introduce the effective interaction kernel $\mathcal{V}$

$$\mathcal{T} = \mathcal{A} + \mathcal{A}G\mathcal{T}. \quad \begin{cases} \mathcal{T} = \mathcal{V} + \mathcal{V}\, g\, \mathcal{T}. \\ \mathcal{V} = \mathcal{A} + \mathcal{A}\,(G - g)\,\mathcal{V}. \end{cases}$$

Reduction of BS equation

- BS equation reduces to the **Kadyshevsky equation**:

$$\mathcal{T} = \mathcal{V} + \mathcal{V}\, g\, \mathcal{T}$$
$$= \mathcal{V} + \int \frac{d\boldsymbol{k}}{(2\pi)^3} \int \frac{dk_0}{2\pi}\, \mathcal{V} \times 2\pi \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\boldsymbol{k})\Lambda_+^{(2)}(-\boldsymbol{k})}{\sqrt{s} - 2E_k + i\epsilon} \delta[k_0 - (E_k - \frac{\sqrt{s}}{2})] \times \mathcal{T}$$
$$= \mathcal{V} + \int \frac{d\boldsymbol{k}}{(2\pi)^3}\, \mathcal{V}\, \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\boldsymbol{k})\Lambda_+^{(2)}(-\boldsymbol{k})}{\sqrt{s} - 2E_k + i\epsilon}\, \mathcal{T}, \quad \text{with } k_0 = E_k - \frac{\sqrt{s}}{2}.$$

- Sandwiched by Dirac spinors:

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3}\, V(\boldsymbol{p}', \boldsymbol{k})\, \frac{m_N^2}{2E_k^2} \frac{1}{E_p - E_k + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p}),$$

V. Kadyshevsky, NPB (1968).

- Relativistic potential definition:

$$V(\boldsymbol{p}', \boldsymbol{p}) = \bar{u}(\boldsymbol{p}', s_1)\bar{u}(-\boldsymbol{p}', s_2) \times \mathcal{V}(p_0' = E_{p'} - \sqrt{s}/2, \boldsymbol{p}'; p_0 = E_p - \sqrt{s}/2, \boldsymbol{p}|W) \times u(\boldsymbol{p}, s_1)u(\boldsymbol{p}', s_2).$$

Calculate potential in ChEFT

- To obtain the potential

$$V(\boldsymbol{p}', \boldsymbol{p}) = \bar{u}_1 \bar{u}_2 \, \mathcal{V}(p, p') \, u_1 u_2 .$$

- Solve the iterated equation perturbatively

$$\mathcal{V} = \mathcal{A} + \mathcal{A}(G - g)\mathcal{V}.$$

$$\begin{aligned} \mathcal{V}^{(0)} &= \mathcal{A}^{(0)}, \\ \mathcal{V}^{(2)} &= \mathcal{A}^{(2)} + \mathcal{A}^{(0)}(G - g)\mathcal{A}^{(0)} \end{aligned}$$

Large cancellation, neglected

K. Erkelenz, ZPA1973, Phys.Rept.1974
R. Machleit, Phys.Rept.1987

- Interaction kernel, $\mathcal{A}$, can be calculated by using covariant ChEFT order by order.

Interaction kernel in covariant ChEFT

- Perturbative expansion

$$\mathcal{A} = \sum_i C[g_i(\mu)] \left(\frac{Q}{\Lambda_\chi}\right)^{n_\chi}$$

- Expansion parameters

$(Q/\Lambda_\chi)^{n_\chi}$ light --- $Q \sim p, m_\pi,$ heavy --- $\Lambda_\chi \sim 1$ GeV

- Chiral dimension n_χ (naïve dimensional analysis)

$$n_\chi = 4L - 2N_\pi - N_n + \sum_k kV_k$$

- **We have the power counting to collect the effective Lagrangians and corresponding diagrams.**

Interaction kernel up to NLO

- Covariant chiral Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

  - LO contact Lagrangian

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2}\left[C_S(\bar\Psi\Psi)(\bar\Psi\Psi) + C_A(\bar\Psi\gamma_5\Psi)(\bar\Psi\gamma_5\Psi) + C_V(\bar\Psi\gamma_\mu\Psi)(\bar\Psi\gamma^\mu\Psi) + C_{AV}(\bar\Psi\gamma_5\gamma_\mu\Psi)(\bar\Psi\gamma_5\gamma^\mu\Psi) + C_T(\bar\Psi\sigma_{\mu\nu}\Psi)(\bar\Psi\sigma^{\mu\nu}\Psi).\right]$$

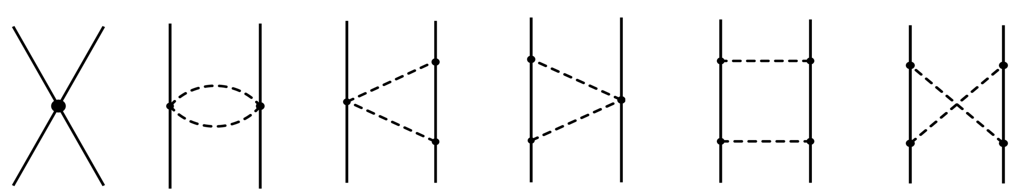
*H. Polinder, J. Haidenbauer, U.-G. Meißner,NP**A779**, 244 (2006)*

  - NLO contact Lagrangian --- **to be constructed**

- Feynman diagrams

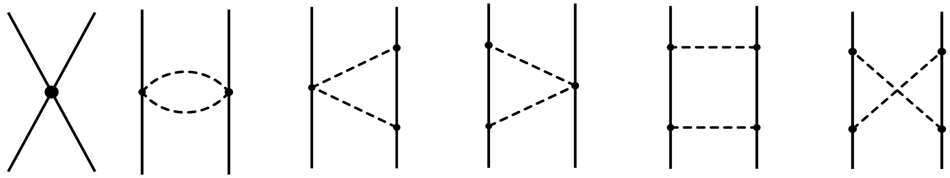
$(Q/\Lambda_\chi)^0$

$(Q/\Lambda_\chi)^2$



Relativistic chiral NF up to NLO

$V_{\mathrm{LO}} = \bar{u}_1 \bar{u}_2$ () $u_1 u_2$

$V_{\mathrm{NLO}} = \bar{u}_1 \bar{u}_2$ () $u_1 u_2$

Scattering equation and Phase shifts

- Perform the partial wave projection, one can obtain the Kadyshevesky equation in |LSJ> basis

$$T^{SJ}_{L',L}(\boldsymbol{p}',\boldsymbol{p}) = V^{SJ}_{L',L}(\boldsymbol{p}',\boldsymbol{p}) + \sum_{L''}\int_0^{+\infty}\frac{\boldsymbol{k}^2 dk}{(2\pi)^3}V^{SJ}_{L',L}(\boldsymbol{p}',\boldsymbol{k})\frac{M_N^2}{2(\boldsymbol{k}^2+M_N^2)}\frac{1}{\sqrt{\boldsymbol{p}^2+M_N^2}-\sqrt{\boldsymbol{k}^2+M_N^2}+i\epsilon}T^{SJ}_{L'',L}(\boldsymbol{k},\boldsymbol{p}).$$

V. Kadyshevsky, NPB (1968).

- Cutoff renormalization for scattering equation
  - Potential regularized by an **exponential regulator function**

$$V(\boldsymbol{p}',\boldsymbol{p}) \to V(\boldsymbol{p}',\boldsymbol{p})\exp[-(|\boldsymbol{p}'|/\Lambda)^{2n}-(|\boldsymbol{p}|/\Lambda)^{2n}]. \quad n=2$$

E.Epelbaum et al., NPA(2000)

- On-shell ***S*** matrix and phase shift ***δ***

$$S^{SJ}_{L'L} = \delta_{L'L} - \frac{i}{8\pi^2}\frac{M_N^2|\boldsymbol{p}|}{E_p}T^{SJ}_{L'L}.$$

$$S = \exp(2i\boldsymbol{\delta})$$

For couple channel: Stapp parameterization

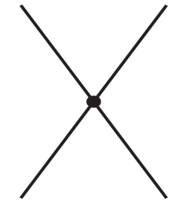
Results and discussion for LO potential

XLR, K.-W. Li, L.-S. Geng, B. Long, P. Ring, J. Meng, accepted by Chinese Physics C, arXiv: 1611.08475

XLR, K.-W. Li, L.-S. Geng, J. Meng, *et al.*, in preparation

Relativistic chiral potential at LO

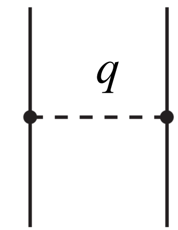
- Contact potential (momentum space):



$$V_{\rm CTP} = C_S(\bar{u}_2 u_2)(\bar{u}_1 u_1) + C_A(\bar{u}_2\gamma_5 u_2)(\bar{u}_1\gamma_5 u_1) + C_V(\bar{u}_2\gamma_\mu u_2)(\bar{u}_1\gamma^\mu u_1) + C_{AV}(\bar{u}_2\gamma_\mu\gamma_5 u_2)(\bar{u}_1\gamma^\mu\gamma_5 u_1) + C_T(\bar{u}_2\sigma_{\mu\nu} u_2)(\bar{u}_1\sigma_{\mu\nu} u_1).$$

- One-pion-exchange potential (momentum space):

$$V_{\rm OPEP} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{(\bar{u}_1\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_2\gamma^\nu\gamma_5 q_\nu u_2)}{(E_{p'}-E_p)^2-\boldsymbol{q}^2-m_\pi^2}.$$



Retardation effect included

- In the static limit (m_N → infinity), the NR results can be recovered

$$V^{\rm NonRel.} = \underbrace{(C_S+C_V)}_{C_S^{\rm HB}} - \underbrace{(C_{AV}-2C_T)}_{C_T^{\rm HB}}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}+m_\pi^2+i\epsilon} + \mathcal{O}(\frac{1}{M_N}).$$

S. Weinberg, PLB1990

Relativistic potential in LSJ basis

$\langle p'|V_{\rm LO}|p\rangle$ → $\langle L'SJ|V_{\rm LO}|LSJ\rangle$

rotation invariant

conservation of total spin

All partial waves with J = 0, 1

$$
\begin{aligned}
V_{1S0} &= \xi_N\left[\boldsymbol{C_{1S0}}\left(1+R_p^2R_{p'}^2\right)+\boldsymbol{\hat{C}_{1S0}}\left(R_p^2+R_{p'}^2\right)\right],\\
V_{3P0} &= -2\xi_N\boldsymbol{C_{3P0}}R_pR_{p'},\\
V_{1P1} &= -\frac{2\xi_N}{3}\boldsymbol{C_{1P1}}R_pR_{p'},\\
V_{3P1} &= -\frac{4\xi_N}{3}\boldsymbol{C_{3P1}}R_pR_{p'},\\
V_{3S1} &= \frac{\xi_N}{9}\left[\boldsymbol{C_{3S1}}\left(9+R_p^2R_{p'}^2\right)+\boldsymbol{\hat{C}_{3S1}}\left(R_p^2+R_{p'}^2\right)\right],\\
V_{3D1} &= \frac{8\xi_N}{9}\boldsymbol{C_{3S1}}R_p^2R_{p'}^2,\\
V_{3S1-3D1} &= \frac{2\sqrt{2}\xi_N}{9}\left(\boldsymbol{C_{3S1}}R_p^2R_{p'}^2+\boldsymbol{\hat{C}_{3S1}}R_p^2\right),\\
V_{3D1-3S1} &= \frac{2\sqrt{2}\xi_N}{9}\left(\boldsymbol{C_{3S1}}R_p^2R_{p'}^2+\boldsymbol{\hat{C}_{3S1}}R_{p'}^2\right).
\end{aligned}
$$

$$
\begin{aligned}
C_{1S0} &= (C_S+C_V+3C_{AV}-6C_T),\\
\hat{C}_{1S0} &= (3C_V+C_A+C_{AV}+6C_T).
\end{aligned}
$$

$$C_{3P0} = (C_S-4C_V+C_A-4C_{AV}).$$

$$C_{1P1} = (C_S+C_A).$$

$$C_{3P1}=(C_S-2C_V-C_A+2C_{AV}+4C_T).$$

$$
\begin{aligned}
C_{3S1} &= (C_S+C_V-C_{AV}+2C_T),\\
\hat{C}_{3S1} &= 3(C_V-C_A-C_{AV}+2C_T).
\end{aligned}
$$

7 combinations, only 5 independent.

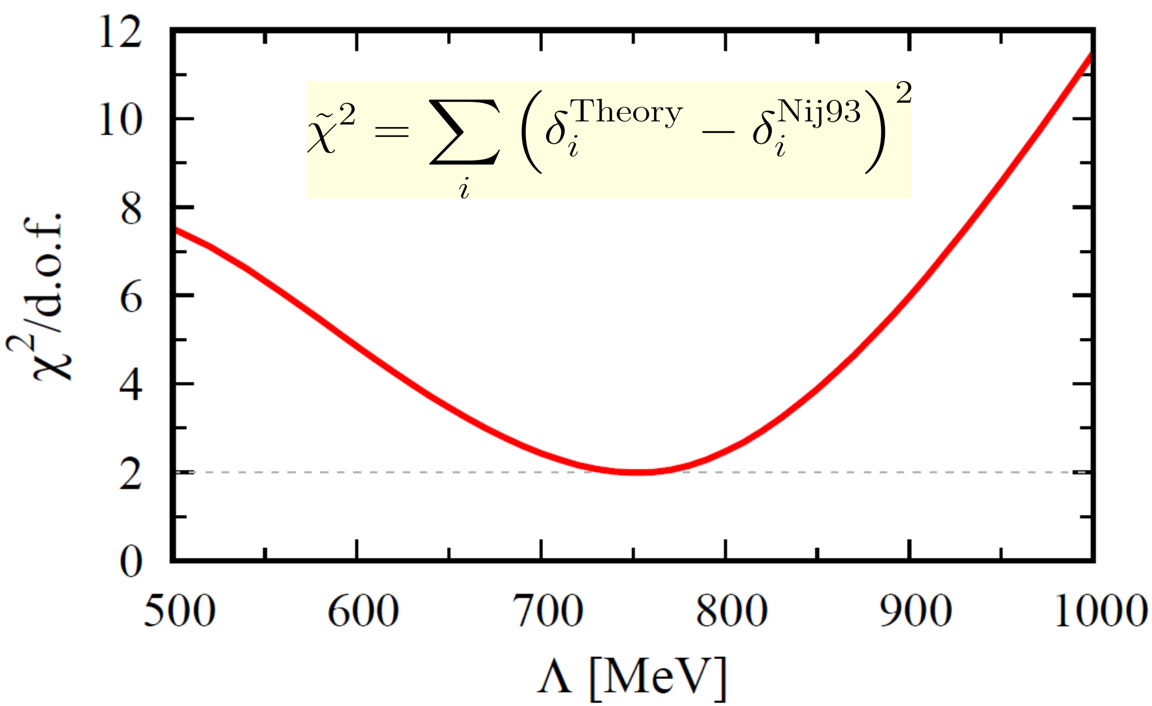
$\xi_N=4\pi N_p^2N_{p'}^2$, $R_p=|\vec{p}|/\epsilon_p$, and $R_{p'}=|\vec{p}\,'|/\epsilon_{p'}$.

Numerical details

- 5 LECs $C_{S,A,V,AV,T}$ are determined by fitting
  - **NPWA**: ***p-n*** scattering phase shifts of Nijmegen 93

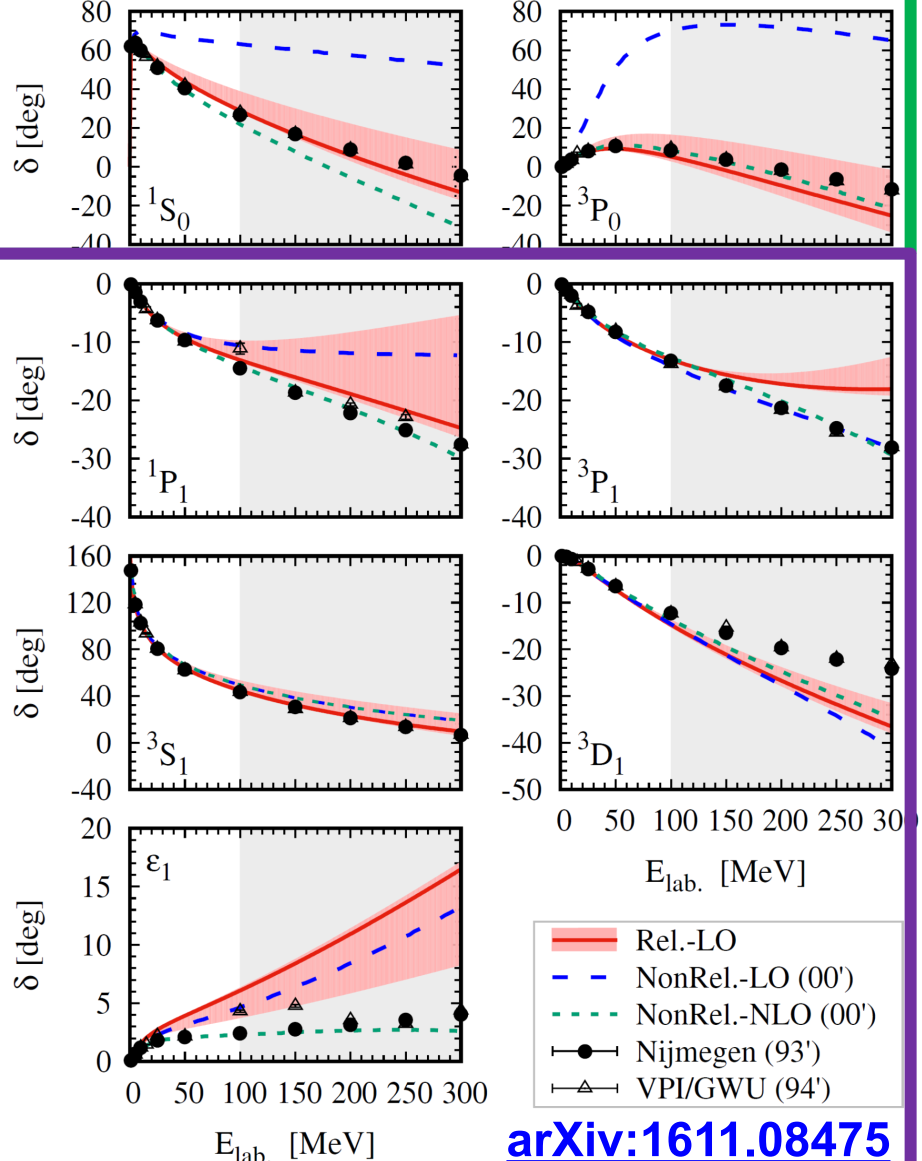
    V. Stoks et al., PRC48(1993)792
  - **7** partial waves: ***J=0, 1*** $^1S_0,\ ^3P_0,\ ^1P_1,\ ^3P_1,\ ^3D_1,\ ^3S_1,\ \epsilon_1$
  - **42** data points: 6 data points for each partial wave

    $(E_{\rm lab} = 1,\ 5,\ 10,\ 25,\ 50,\ 100\ \text{MeV})$



LECs	Values [10^4 GeV^{-2}]
C_S	-0.125
C_A	0.040
C_V	0.248
C_{AV}	0.221
C_T	0.059

Description of J=0, 1 partial waves

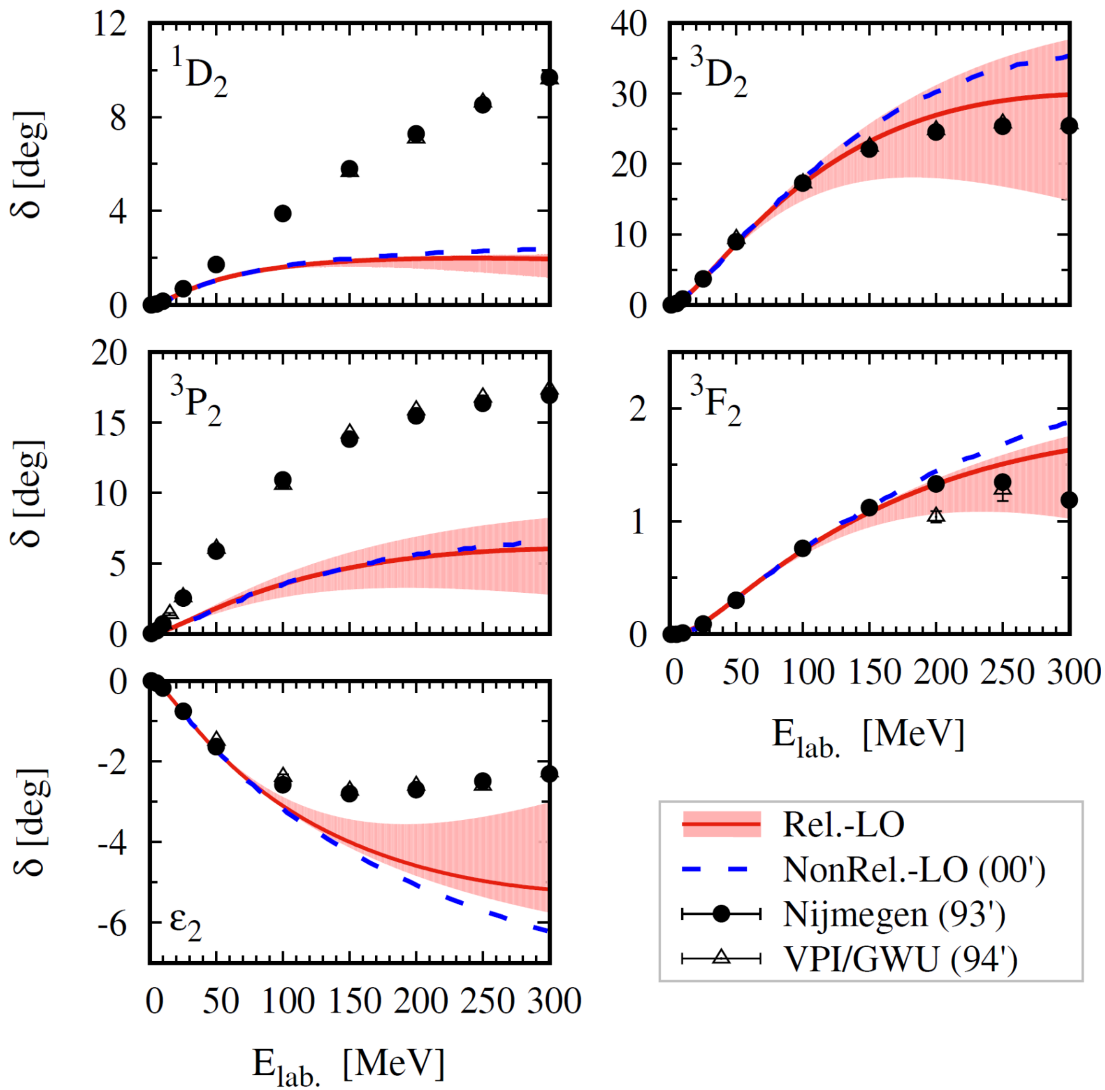


arXiv:1611.08475

- Red variation bands: **cutoff 500~1000 MeV**
- **Improve description** of $^1S_0$, $^3P_0$ phase shifts
- **Quantitatively similar** to the nonrelativistic case for J=1 partial waves

Higher partial waves

Only OPEP contributes



- The relativistic results are almost **the same** as the non-relativistic case.
- Relativistic correction of OPEP is small !

1S0 wave phenomena

- Interesting phenomena of 1S0 wave
  - Large variance of phase shift from 60 to -10 (zero point: $k_0=340.5$ MeV)
  - Virtual bound state at very low-energy region (pole postition: $-i10$ MeV)
  - Significantly large scattering length ($a=-23.7$ fm)

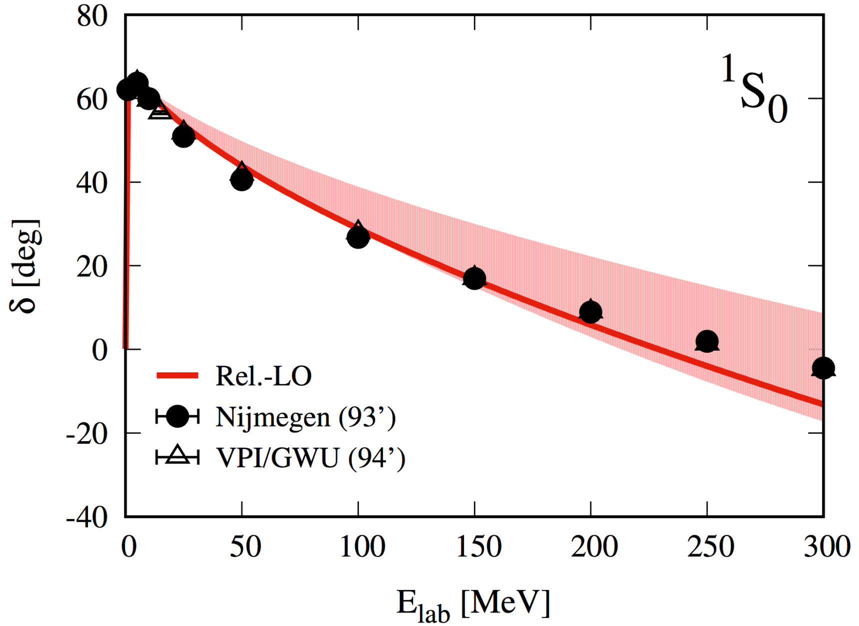
These typical energy scales are smaller than chiral symmetry breaking scale (~1GeV)

⇨ The 1S0 phenomena **should be roughly reproduced simultaneously** at the **lowest order** of chiral nuclear force

Bira van Kolck, et al., 1704.08524.

1S0 in relativistic chiral force (LO)

- A good description of 1S0 phase shift:



- Predicted results: (**reproduced simultaneously**)

	Nijmegen PWA	Global-Fit
Λ [MeV]	–	750^{500}_{1000}
scattering length a [fm]	-23.7	$-20.3^{-19.8}_{-16.2}$
effective range r [fm]	2.70	$2.45^{2.41}_{2.24}$
virtual pole position $i\gamma$ [MeV]	$-i10$	$-i9.2^{-i9.4}_{-i11.4}$

XLR, L.-S. Geng, J. Meng, et al., in preparation

Work in progress:

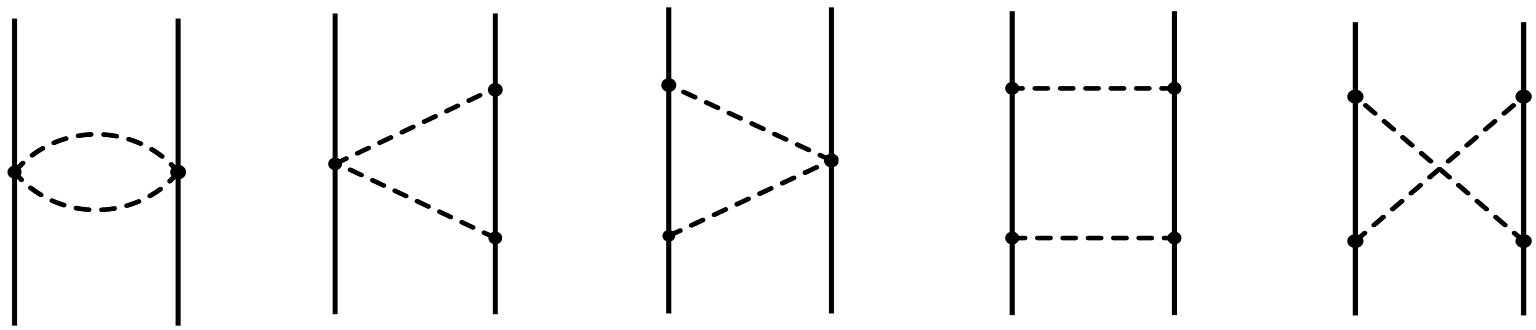
Construction NLO potential

In collaboration with:

L.-S. Geng, J. Meng, E. Epelbaum

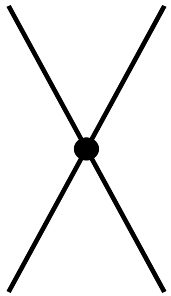
NLO corrections for chiral force

- Two pion exchange:

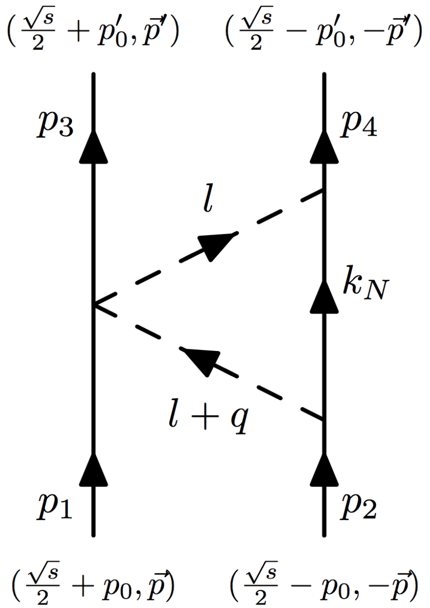


  - Except football diagram, the expresses are **very complicated** with 3-/4-point functions
  - Introduce the **power counting breaking** terms
  - Keep the four **external legs off-shell** (cannot use Dirac eq.)

- Contact potential:
  - Construct the effective Lagrangian with **two derivatives**



Take left-triangle diagram for example



□ In the momentum space

$$V = \frac{ig_A^2}{8f_\pi^4}\vec{\tau}_1\cdot\vec{\tau}_2\int\frac{d^4l}{(2\pi)^4}\frac{(\bar{u}_3\gamma^\mu(2l+q)_\mu u_1)\left(\bar{u}_4\gamma^\nu\gamma_5 l_\nu(\not{p}_4-\not{l}+M_N)\gamma^\rho\gamma_5(l+q)_\rho u_2\right)}{(l^2-m_\pi^2+i\epsilon)[(l+q)^2-m_\pi^2+i\epsilon][(p_4-l)^2-M_N^2+i\epsilon]}$$

- Perform the one loop integration in **FeynCalc** (**D-dimension**)

□ Transform to the Helicity basis

- Apply four identities related to Dirac spinor to simplify the tensor structures

$$\begin{aligned}
\not{p}_1 u_1(\vec{p},\lambda) &= \left[m_N+\gamma^0(p_{1,0}-E_p)\right]u_1(\vec{p},\lambda),\\
\not{p}_2 u_2(-\vec{p},\lambda) &= \left[m_N+\gamma^0(p_{2,0}-E_p)\right]u_2(-\vec{p},\lambda),\\
\bar{u}_3(\vec{p}',\lambda)\not{p}_3 &= \bar{u}_3(\vec{p}',\lambda)\left[m_N+\gamma^0(p_{3,0}-E_{p'})\right],\\
\bar{u}_4(-\vec{p}',\lambda)\not{p}_4 &= \bar{u}_4(-\vec{p}',\lambda)\left[m_N+\gamma^0(p_{4,0}-E_{p'})\right].
\end{aligned}$$

"(off-shell) Dirac equations"

M.J. Zuilhof et al., PRC26, 1277 (1982)

⇨ Left triangle contributions

$$\begin{aligned}
V = \frac{ig_A^2}{8F_\pi^4}\vec{\tau}_1\cdot\vec{\tau}_2 \Big[&(\bar{u}_3u_1)(\bar{u}_4u_2)\times F_{LT}^1(A_0,B_0,C_0...)+(\bar{u}_3u_1)(\bar{u}_4\gamma^0u_2)\times F_{LT}^2(A_0,B_0,C_0...)\\
&+(\bar{u}_3\gamma^0u_1)(\bar{u}_4u_2)\times F_{LT}^3(A_0,B_0,C_0...)+(\bar{u}_3\gamma^0u_1)(\bar{u}_4\gamma^0u_2)\times F_{LT}^4(A_0,B_0,C_0...)\\
&+(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma^0\gamma_\mu u_2)\times F_{LT}^5(A_0,B_0,C_0...)+(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma_\mu\gamma^0u_2)\times F_{LT}^6(A_0,B_0,C_0...)\\
&+(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma^0\gamma_\mu\gamma^0u_2)\times F_{LT}^7(A_0,B_0,C_0...)+(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma_\mu u_2)\times F_{LT}^8(A_0,B_0,C_0...)\Big]
\end{aligned}$$

Compact form of TPE contributions

- Using the aforementioned **“(off-shell) Dirac eqs.”**, one can express TPE diagrams in terms of **twenty tensor structures**

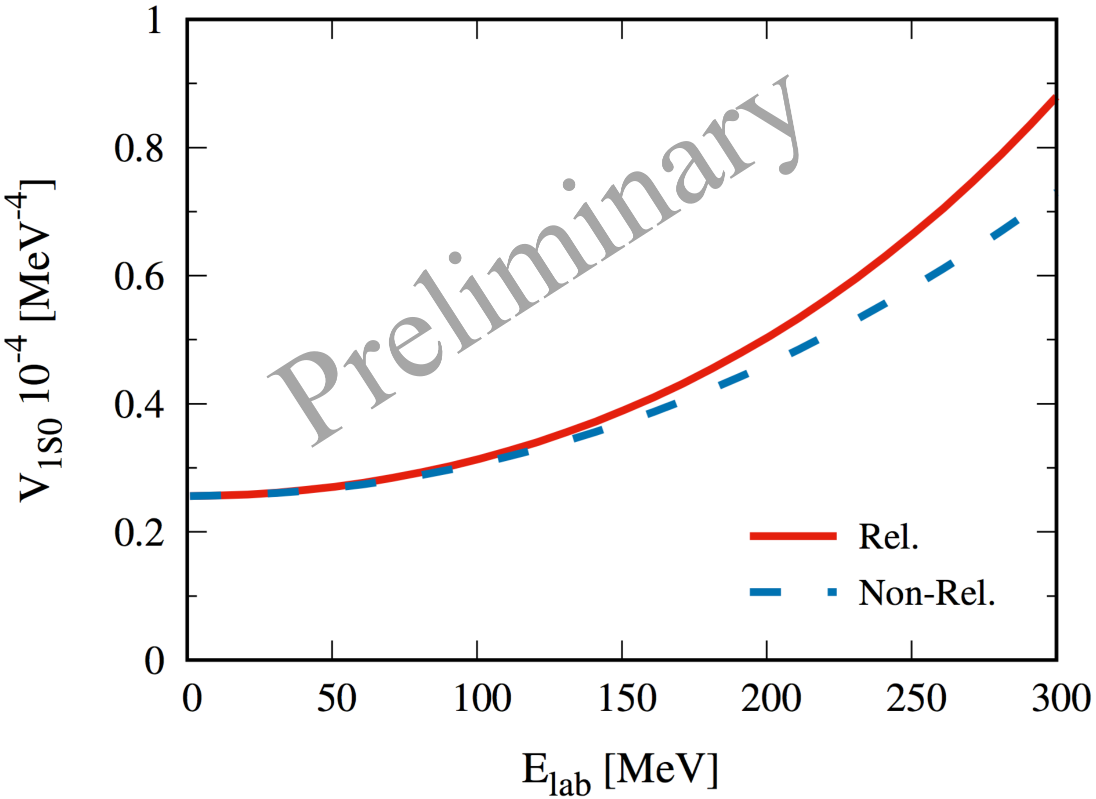
$$O_1 = 1^{(1)}1^{(2)}, \quad O_2 = 1^{(1)}\gamma_0^{(2)}, \quad O_3 = \gamma_0^{(1)}1^{(2)}, \quad O_4 = \gamma_0^{(1)}\gamma_0^{(2)}, \quad O_5 = \gamma_\mu^{(1)}\gamma^{\mu\,(2)},$$
$$O_6 = \gamma_\mu^{(1)}(\gamma_0\gamma^\mu)^{(2)}, \quad O_7 = \gamma_\mu^{(1)}(\gamma^\mu\gamma_0)^{(2)}, \quad O_8 = (\gamma_0\gamma_\mu)^{(1)}\gamma^{\mu\,(2)}, \quad O_9 = (\gamma_\mu\gamma_0)^{(1)}\gamma^{\mu\,(2)},$$
$$O_{10} = (\gamma_0\gamma_\mu)^{(1)}(\gamma_0\gamma^\mu)^{(2)}, \quad O_{11} = (\gamma_0\gamma_\mu)^{(1)}(\gamma^\mu\gamma_0)^{(2)}, \quad O_{12} = (\gamma_\mu\gamma_0)^{(1)}(\gamma_0\gamma^\mu)^{(2)}, \quad O_{13} = (\gamma_\mu\gamma_0)^{(1)}(\gamma^\mu\gamma_0)^{(2)},$$
$$O_{14} = (\gamma_\mu)^{(1)}(\gamma_0\gamma^\mu\gamma_0)^{(2)}, \quad O_{15} = (\gamma_0\gamma_\mu\gamma_0)^{(1)}(\gamma^\mu)^{(2)}, \quad O_{16} = (\gamma_0\gamma_\mu)^{(1)}(\gamma_0\gamma^\mu\gamma_0)^{(2)}, \quad O_{17} = (\gamma_\mu\gamma_0)^{(1)}(\gamma_0\gamma^\mu\gamma_0)^{(2)},$$
$$O_{18} = (\gamma_0\gamma_\mu\gamma_0)^{(1)}(\gamma_0\gamma^\mu)^{(2)}, \quad O_{19} = (\gamma_0\gamma_\mu\gamma_0)^{(1)}(\gamma^\mu\gamma_0)^{(2)}, \quad O_{20} = (\gamma_0\gamma_\mu\gamma_0)^{(1)}(\gamma_0\gamma^\mu\gamma_0)^{(2)}.$$

$$V_{\mathrm{TPE}} = \sum_n \boldsymbol{o_n}\, \boldsymbol{F_n(A_0, B_0, C_0, D_0, \ldots)}$$

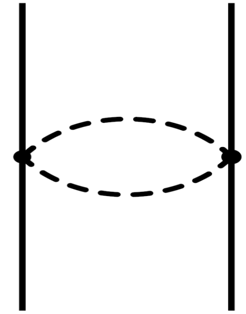
- Superposition of **PaVe functions**
- Evaluated by **LoopTools / Package-X**
- Contain **power-counting breaking (PCB) terms**

Football diagram contribution

- Partial wave potential V_{1S0} (Rel. vs. NR)



- **Relativistic correction is small**
- Only pion propagators in football diagram

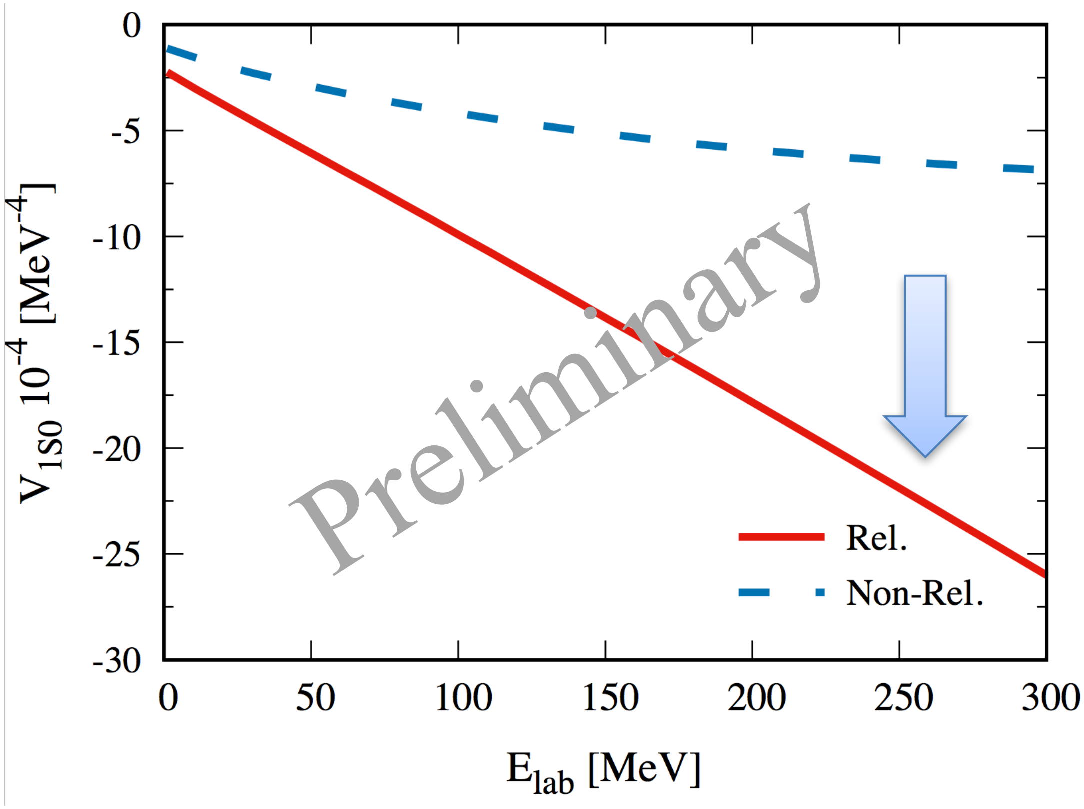


- Consistent with one-pion-exchange diagram.

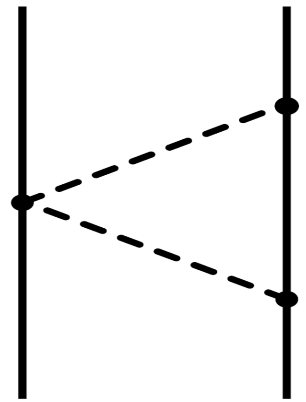
working on

(Left) triangle diagram contribution

- Partial wave potential V_{1S0} (Rel. vs. NR)



- **Relativistic correction is relatively large**
- Contains the nucleon propagator in triangle diagram



- **Box diagrams are working on!**

working on

NLO contact Lagrangian

- Chiral dimension of building blocks:
  - Clifford algebra and fields

$$1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma_{\mu\nu}\ \sim \mathcal{O}(p^0) \qquad \psi,\ \bar{\psi}\ \sim \mathcal{O}(p^0)$$

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2}\left[\boldsymbol{C_S}(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + \boldsymbol{C_A}(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) + \boldsymbol{C_V}(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + \boldsymbol{C_{AV}}(\bar{\Psi}\gamma_5\gamma_\mu\Psi)(\bar{\Psi}\gamma_5\gamma^\mu\Psi) + \boldsymbol{C_T}(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi).\right]$$

*H. Polinder, J. Haidenbauer, U.-G. Meißner, NPA**779**, 244 (2006)*

  - **Partial derivative** --- to increase the chiral order
    - acting on the whole bilinear

$$\partial^\mu\left(\bar{\psi}\psi\right) \sim \bar{u}_1 i(p_3^\mu - p_1^\mu)u_1 \sim \mathcal{O}(p^1)$$

    - acting on the inside of bilinear *(contracted pair)*

$$(\bar{\psi}\partial_\mu\psi)(\bar{\psi}\partial^\mu\psi) \sim -p_1\cdot p_2(\bar{\psi}\psi)(\bar{\psi}\psi) \sim \mathcal{O}(p^0)$$

We need subtract the mass terms: *D. Djukanovic, et al., FBS41(2007)141*

$$(\bar{\psi}\partial_\mu\psi)(\bar{\psi}\partial^\mu\psi) \sim \left[-p_1\cdot p_2 + m_N^2\right](\bar{\psi}\psi)(\bar{\psi}\psi) \sim \mathcal{O}(p^2)$$

working on

Summary

- **We performed an exploratory study to construct the relativistic nuclear force up to leading order in covariant ChEFT**
  - Relativistic chiral force can **improve the description of $^1S_0$ and $^3P_0$** phase shifts at LO
  - For the phase shifts of partial waves with high angular momenta (J>=1), the relativistic results are **quantitatively similar to** the nonrelativistic counter parts.

- **We are now working on the NLO studies**
  - Calculate the two-pion exchange potentials (**almost finished**)
  - Construct the contact Lagrangians with two derivatives
  - Expect to achieve a better description of phase shifts ⇨ **Stay tuned**

Perspectives

Chiral order	χ^2/datum (Fit: 0-100MeV)	
	Rel. chiral NF	Nonrel. chiral NF
LO	2.0~6.0	~100
NLO		2.5
NNLO		1.0

- **Our final goal: construct a high precision chiral nuclear force**
  - Study the **chiral extrapolation** of nuclear force from LQCD
  - Study the few-body systems by using the **Gaussian Expansion Method**
  - Study the nuclear structure by using the **Dirac Brueckner–Hartree–Fock theory**

Thank you very much

for your attention!

Back up slides

Hint at a more efficient formulation

V_{1S0}: $1/m_N$ expansion

$$V_{1S0} = 4\pi\left[C_{1S0} + (C_{1S0} + \hat{C}_{1S0})\left(\frac{\vec{p}^2 + \vec{p}'^2}{4M_N^2} + \cdots\right)\right]$$

$$+\frac{\pi g_A^2}{2f_\pi^2}\int_{-1}^{1}\frac{dz}{\vec{q}^2 + m_\pi^2}\left[\vec{q}^2 - \left(\frac{(\vec{p}^2 - \vec{p}'^2)^2}{4M_N^2} + \cdots\right)\right].$$

- Relativistic corrections are suppressed
- One has to be careful with **the new contact term, the momentum dependent term**, which is desired to achieve a reasonable description of the phase shifts of 1S0 channel.

J. Soto et al., PRC(2008), B. Long, PRC (2013)

Only two LECs fit:

$$V_{\rm CTP}^{\rm NonRel.} = (C_S + C_V) - (C_{AV} - 2C_T)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \mathcal{O}(\frac{1}{M_N}).$$

- Take CS and CAV as free parameters
- Best fit result:
  - chi^2/d.o.f. = **84.5**

	Relativistic Chiral NF	**Non-relativistic Chiral NF**	
Chiral order	LO	LO	NLO*
No. of LECs	**5**	2	**9**
χ^2/d.o.f.	**2.9**	**147.9**	**~2.5**

Errors and correlation matrix

TABLE I: The best fit results of five LECs appearing in the contact terms (in unit of $10^4 \mathrm{GeV}^{-2}$) with the momentum cutoff $\Lambda = 747$ MeV.

LECs	C_S	C_A	C_V	C_{AV}	C_T
Best fit	0.13515 ± 0.00307	-0.055963 ± 0.018217	-0.26857 ± 0.01151	-0.24427 ± 0.01141	-0.062538 ± 0.001319

	C_S	C_A	C_V	C_{AV}	C_T
C_S	**1.00**	0.21	-0.93	-0.58	-0.39
C_A	0.23	1.00	-0.15	0.45	0.21
C_V	**-0.93**	-0.15	1.00	0.77	0.69
C_{AV}	-0.57	0.45	**0.77**	1.00	0.89
C_T	-0.39	0.21	0.69	**0.89**	1.00

Tlab [MeV]	1	50	100	150	200	250	300
Pcm [MeV]	21.67	153.22	216.68	265.38	306.43	342.60	375.30
Vcm	0.023 **c**	0.16 **c**	0.23 **c**	0.28 **c**	0.33 **c**	0.36 **c**	0.40 **c**
E_corr(2n) [MeV]	0.25	12.5	25	37.5	50	62.5	75

$$p_{cm} = \sqrt{\frac{m_N T_{\mathrm{lab}}}{2}} \qquad V_{\mathrm{cm}} = \frac{p_{\mathrm{cm}}}{m_N} c$$

$$E_T^{\mathrm{corr}} = \frac{p_{\mathrm{cm}}^2}{2 m_N}$$

Strategies to construct NLO Lagrangian

$$\mathcal{O}^{(n)}_{\Gamma_A\Gamma_B} \sim (\overline{\psi} i \overleftrightarrow{\partial}^{\mu_1} i \overleftrightarrow{\partial}^{\mu_2} \cdots i \overleftrightarrow{\partial}^{\mu_n} \Gamma^{\alpha}_{A} \psi)(\overline{\psi} i \overleftrightarrow{\partial}_{\mu_1} i \overleftrightarrow{\partial}_{\mu_2} \cdots i \overleftrightarrow{\partial}_{\mu_n} \Gamma_{B\alpha} \psi)$$

$$\mathcal{O}^{(n)}_{\Gamma_A\Gamma_B} \sim [(p_1 + p_3) \cdot (p_2 + p_4)]^n$$

- Keep *n=1* terms   *L. Girlanda, et al., PRC81(2010)034005*
  - perform non-rel. expansion

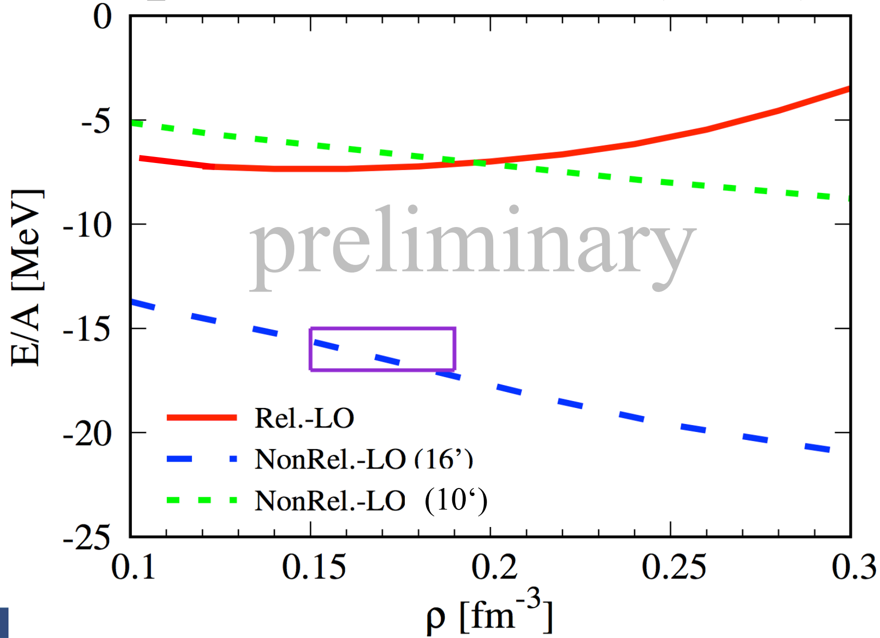
Outlook: application to nuclear matter

- Relativistic Brueckner-Hartree-Fock theory
  - Kadyshevsky equation in nuclear matter (angle average)

$$G(\boldsymbol{p}',\boldsymbol{p}|\boldsymbol{P}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{k}) \frac{{M^*}^2}{2{E^*}^2_{\boldsymbol{P}/2+\boldsymbol{k}}} \frac{\bar{Q}(\boldsymbol{k},\boldsymbol{P})}{E^*_{\boldsymbol{P}/2+\boldsymbol{p}} - E^*_{\boldsymbol{P}/2+\boldsymbol{k}}} G(\boldsymbol{k},\boldsymbol{p}|\boldsymbol{P})$$

  - **G matrix**: effective interaction in nuclear matter
  - $M^* = M_N - U_S$:effective mass; $Q(\mathbf{k},\mathbf{P})$: Pauli operator

- Equation of state (EoS) for symmetric NM



- Saturated around $\rho = 0.15\ \mathrm{fm}^{-3}$
- $E/A = -7.4$ MeV

*R. Machleidt et al., PRC**81**, 024001 (2010)*

J.N. Hu et al., arXiv:1612.05433