Chiral Lagrangians with $\Delta(1232)$ to one loop

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Outline

- Background & motivations
- Review of the spin-$\frac{3}{2}$ field and CHPT
- A method to get chiral Lagrangian with $\Delta(1232)$
- Results
- summary

Background & motivation

- $\Delta(1232)$ in the low-energy QCD

  πN scattering, nucleon magnetism moment, electromagnetic interactions of nucleons, and so on

- $(m_\Delta - m_N)/\Lambda_\chi \sim p/\Lambda_\chi \sim m_\pi/\Lambda_\chi \sim O(1/N)$

  Hemmert, Holstein, and Kambor, JPG24, 1831(1998)

  $[(m_\Delta - m_N)/\Lambda_\chi]^2 \sim m_\pi/\Lambda_\chi$ Pascalutsa and Phillips, PRC67, 055202(2003)

- Current situation (Baryon Lagrangian):

  πN up to $O(p^4)$: 1+7+13+118

  Fettes, Meißner, Mojžiš and Steininger, AP(NY)283, 273(2000)

  MB up to $O(p^4)$: 3+16+78+540

  Oller, Verbeni, and Prades, JHEP0609, 079(2006); Jiang, Chen, and Liu, PRD95, 014012(2017)

  $\pi\Delta$ $(\pi N\Delta)$ up to $O(p^2)$: 3+10 (1+5)

  Hemmert, Holstein, and Kambor, JPG24, 1831(1998)

- High order results with $\Delta(1232)$

  nucleon-Δ transition, properties of Δ, scattering processes with nucleon or Δ, etc.

Review of the free spin-3/2 fields I

- Vector-spinor representation: Ψ^μ Rarita and Schwinger, PR60, 61(1941)

  1 spin-$\frac{3}{2}$ fields+2 unphysical spin-1/2 fields

- The free spin-$\frac{3}{2}$ field Lagrangian

$$\mathscr{L}_{\rm f} = \bar{\Psi}_\mu \Lambda_A^{\mu\nu} \Psi_\nu, \quad A \neq -\frac{1}{2},$$ Moldauer and Case, PR102, 27(1956)

$$\Lambda_A^{\mu\nu} = -\Big[(i\not\partial - m_\Delta) g^{\mu\nu} + iA(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) + \frac{i}{2}(3A^2 + 2A + 1)\gamma^\mu \not\partial \gamma^\nu + m_\Delta (3A^2 + 3A + 1)\gamma^\mu \gamma^\nu\Big]$$

- EOM + 2 subsidiary conditions

$$(i\not\partial - m_\Delta)\Psi_\mu = 0, \quad \gamma^\mu \Psi_\mu = 0, \quad \partial^\mu \Psi_\mu = 0$$

The subsidiary conditions eliminate the unphysical spin-1/2 freedoms

Review of the free spin-3/2 fields II

- Lagrangian is invariant under the Point transformation

$$\Psi_\mu \to \Psi'_\mu = \Psi_\mu + \frac{1}{2}a\gamma_\mu\gamma_\nu\Psi^\nu,\ A \to A' = \frac{A-a}{1+2a},\ a \neq -\frac{1}{2}$$

A is unphysical Kamefuchi, O' Raifeartaigh, and Salam, NP28, 529(1961); Pilling, IJMPA20, 2715(2005); Krebs, Epelbaum, and Meißner, PLB683, 222(2010)

- Absorbing A into spin-$\frac{3}{2}$ field Pascalutsa, arXiv:hep-ph/9412321

$$\mathscr{L}_{\rm f} = \bar{\psi}_{A\mu}\Lambda^{\mu\nu}\psi_{A\nu}, \quad \psi^\mu_A \equiv (g^{\mu\nu} + \frac{1}{2}A\gamma^\mu\gamma^\nu)\Psi_\nu$$

$$\Lambda^{\mu\nu} = -(i\partial\!\!\!/ - m_\Delta)g^{\mu\nu} + \frac{1}{4}\gamma^\mu\gamma^\lambda(i\partial\!\!\!/ - m_\Delta)\gamma_\lambda\gamma^\nu$$

ψ^μ_A satisfies the same EOM and subsidiary conditions

Building blocks in CHPT (without Δ)

- Two-flavor QCD Lagrangian Only a^μ is traceless, $\langle a^\mu \rangle = 0$

$$\mathscr{L} = \mathscr{L}^0_{\rm QCD} + \bar{q}(\not{v} + \not{a}\gamma_5 - s + ip\gamma_5)q \,,$$

$$S = S^i\tau_i + S_s I_2,\ S = s,\ p,\ v^\mu,\ a^\mu$$

- Building blocks

$$u^\mu = i\{u^\dagger(\partial^\mu - ir^\mu)u - u(\partial^\mu - il^\mu)u^\dagger\},$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u,$$

$$h^{\mu\nu} = \nabla^\mu u^\nu + \nabla^\nu u^\mu,$$

$$f_+^{\mu\nu} = uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u,$$

$$f_-^{\mu\nu} = uF_L^{\mu\nu}u^\dagger - u^\dagger F_R^{\mu\nu}u = -\nabla^\mu u^\nu + \nabla^\nu u^\mu,$$

$r^\mu = v^\mu + a^\mu,\ l^\mu = v^\mu - a^\mu,\ \chi = 2B_0(s+ip),\ F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu],$
$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]$

- Covariant derivative

$$\nabla^\mu X \equiv \partial^\mu X + [\Gamma^\mu, X],\ \Gamma^\mu = \frac{1}{2}\{u^\dagger(\partial^\mu - ir^\mu)u + u(\partial^\mu - il^\mu)u^\dagger\}$$

CHPT with Δ

- Nucleon fields: $\psi = \begin{pmatrix} p \\ n \end{pmatrix}$

- Isovector-isospinor representation Tang and Ellis, PLB387, 9(1996)

$$\psi_1^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{3}}\Delta^0 - \Delta^{++} \\ \Delta^- - \frac{1}{\sqrt{3}}\Delta^+ \end{pmatrix}^\mu, \psi_2^\mu = -\frac{i}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{3}}\Delta^0 + \Delta^{++} \\ \Delta^- + \frac{1}{\sqrt{3}}\Delta^+ \end{pmatrix}^\mu, \psi_3^\mu = \sqrt{\frac{2}{3}} \begin{pmatrix} \Delta^+ \\ \Delta^0 \end{pmatrix}^\mu$$

ψ_i^μ: Spinor space $\times$ isospinor space $\times$ isovector space

- Subsidiary condition: $\tau^i \psi_i^\mu = 0 \quad (i = 1, 2, 3)$

- $\pi\Delta\Delta$ interactions: $\bar{\psi}_i^\mu \mathcal{O}_{A\mu\nu}^{ij} \psi_j^\nu$

  $\pi N \Delta$ interactions: $\bar{\psi} \mathcal{O}^{i\mu} \Theta_{A,n,\mu\nu}(z_n) \psi_i^\nu + \text{h.c.}$

  $\Theta_{A,n,\mu\nu}(z_n) = g_{\mu\nu} + [z_n + \frac{1}{2}(1 + 4z_n)A]\gamma_\mu\gamma_\nu$

  z_n parameters can be obtained from experiments

Building blocks in CHPT (with Δ)

- Chiral rotation (R):

$$u \to g_L u h^\dagger = h u g_R^\dagger, \quad u^2 = U, \quad K_{ij} \equiv \frac{1}{2}\langle \tau_i h \tau_j h^\dagger \rangle$$

$$X \xrightarrow{R} X' = hXh^\dagger, \Rightarrow X^i \xrightarrow{R} K^{ij} X_j, \; X_s \xrightarrow{R} X_s$$

X are building blocks in CHPT without Δ

$$\psi \xrightarrow{R} \psi' = h\psi, \quad \psi_i^\mu \xrightarrow{R} K_{ij} h \psi_j^\mu \quad (i, j = 1, 2, 3)$$

- Building blocks: X_i, X_s, ψ, $\bar{\psi}$, ψ_i^μ, $\bar{\psi}_i^\mu$, $\psi_{A,n,i}^\mu$, and $\bar{\psi}_{A,n,i}^\mu$, and their covariant derivatives

- Covariant derivatives

$$\begin{aligned}
\nabla^\mu X^i =& \partial^\mu X^i - 2i\epsilon^{ijk} X_j \Gamma_k^\mu, \\
\nabla^\mu X_s =& \partial^\mu X_s, \\
D^\mu \psi =& \partial^\mu \psi + (\Gamma_i^\mu \tau^i + \Gamma_s^\mu I_2)\psi, \\
D^\mu \psi^{i\nu} =& \partial^\mu \psi^{i\nu} - 2i\epsilon^{ijk}\Gamma_k^\mu \psi_j^\nu + \Gamma_j^\mu \tau^j \psi^{i\nu} + \Gamma_s^\mu \psi^{i\nu},
\end{aligned}$$

Properties of the building blocks I

- Expanding chiral Lagrangian by momentum
- The requirement of chiral Lagrangian: symmetry
- Chiral rotation (R)
- Parity transformation (P): $P_{X_i} = P_{X_s} = P_X$
- Charge conjugation (C): $c^{11} = -c^{22} = c^{33} = 1$

$$X \xrightarrow{C} (-1)^c X^T \Rightarrow X^i \xrightarrow{C} (-1)^c c^{ij} X_j, \; X_s \xrightarrow{C} (-1)^c X_s$$

- Hermitian transformation (h.c.): X_i, X_s and X are similar

Properties of the building blocks II

	Dim	P	C	h.c.
u^μ	1	$-u_\mu$	$(u^\mu)^T$	u^μ
$h^{\mu\nu}$	2	$-h_{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$
$\chi_\pm$	2	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$f_\pm^{\mu\nu}$	2	$\pm f_{\pm\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$
u_i^μ	1	$-u_{i,\mu}$	$c_{ij}u_j^\mu$	u_i^μ
$h_i^{\mu\nu}$	2	$-h_{i,\mu\nu}$	$c_{ij}h_j^{\mu\nu}$	$h_i^{\mu\nu}$
$\chi_{\pm,i}$	2	$\pm\chi_{\pm,i}$	$c_{ij}\chi_{\pm,j}$	$\pm\chi_{\pm,i}$
$\chi_{\pm,s}$	2	$\pm\chi_{\pm,s}$	$\chi_{\pm,s}$	$\pm\chi_{\pm,s}$
$f_{\pm,i}^{\mu\nu}$	2	$\pm f_{\pm,i\mu\nu}$	$\mp c_{ij}f_{\pm,j}^{\mu\nu}$	$f_{\pm,i}^{\mu\nu}$
$f_{+,s}^{\mu\nu}$	2	$f_{+,s\mu\nu}$	$-f_{+,s}^{\mu\nu}$	$f_{+,s}^{\mu\nu}$

Red ones are used

Properties of $\gamma, \epsilon, \varepsilon$ and τ I

- Invant monomials:

$$\pi\Delta\Delta:\ \bar{\psi}_i^{\mu} A_{\dots}^{ij}\Theta_{\dots}\psi_j^{\nu} + \text{H.c.} \quad \pi N\Delta:\ \bar{\psi} A_{\dots}^{i}\Theta_{\dots}\psi_{A,n,i}^{\mu} + \text{H.c.}$$

$\Theta_{\dots}$ contains γ, ϵ, ε, τ and D (ϵ: Isovector space, ε: Lorentz space)

- Chiral rotation (R): invariant

- Parity transformation (P): $\psi \xrightarrow{P} \gamma_0\psi,\ \psi_\mu^i \xrightarrow{P} -\gamma_0\psi^{i,\mu}$

- Charge conjugation (C): $\psi \xrightarrow{C} -i(\bar{\psi}\gamma^0\gamma^2)^T,\ \psi^{i,\mu} \xrightarrow{C} -ic^{ij}(\bar{\psi}^{j,\mu}\gamma^0\gamma^2)^T$
  $\epsilon^{ijk}X_iY_jZ_k \xrightarrow{C} -(-1)^{x+y+z}\epsilon^{ijk}X_iY_jZ_k$ Absorbing "−" into $C_{\epsilon^{ijk}}$

- Hermitian transformation (h.c.) Similar as C

- A sign difference for $D^\mu\psi_i^\nu$ because of partial integration Seeing below

Properties of $\gamma, \epsilon, \varepsilon$ and τ II

	Dim	$P_{\Delta\Delta}$	$C_{\Delta\Delta}$	h.c.$_{\Delta\Delta}$	$P_{N\Delta}$	$C_{N\Delta}$	h.c.$_{N\Delta}$
1	0	+	+	+	−	+	+
γ_5	1	−	+	−	+	+	−
γ^μ	0	+	−	+	−	−	+
$\gamma_5\gamma^\mu$	0	−	+	+	+	+	+
$\sigma^{\mu\nu}$	0	+	−	+	−	−	+
$\varepsilon^{\mu\nu\lambda\rho}$	0	−	+	+	−	+	+
ϵ^{ijk}	0	+	−	+	+	−	+
τ_i	0	+	+	+	+	+	+
$D^\mu\psi_i^\nu$	0	+	−	−	+	+	+

Linear relations I

- Subsidiary condition: $\tau^i \psi_i^\mu = 0$

$$\tau_i \tau_j = \delta_{ij} + i\epsilon_{ijk}\tau^k$$

$$\tau_i \tau_j \tau_k = \delta_{jk}\tau_i + \delta_{ij}\tau_k - \delta_{ik}\tau_j + i\epsilon_{ijk},$$

$$\epsilon^{ijk}\epsilon^{lmn} = \begin{vmatrix} \delta^{il} & \delta^{im} & \delta^{in} \\ \delta^{jl} & \delta^{jm} & \delta^{jn} \\ \delta^{kl} & \delta^{km} & \delta^{kn} \end{vmatrix}$$

$$\Rightarrow \begin{array}{l} \epsilon^{ijk}\psi_k^\mu = i\tau^i\psi^{j\mu} - i\tau^j\psi^{i\mu}, \\ \epsilon^{ijk}\tau_k\psi_j^\mu = (-i\tau^i\tau^j + i\delta^{ij})\psi_j^\mu \end{array}$$

- Schouten identity

$$\varepsilon^{\mu\nu\lambda\rho}A^\sigma - \varepsilon^{\sigma\nu\lambda\rho}A^\mu - \varepsilon^{\mu\sigma\lambda\rho}A^\nu - \varepsilon^{\mu\nu\sigma\rho}A^\lambda - \varepsilon^{\mu\nu\lambda\sigma}A^\rho = 0$$

$$\epsilon_{ijk}A_l - \epsilon_{ljk}A_i - \epsilon_{ilk}A_j - \epsilon_{ijl}A_k = 0$$

$$\Rightarrow \epsilon^{ijk}\mathcal{O}^l\psi_l^\mu = i\mathcal{O}_i\tau_j\psi_k^\mu + \mathrm{P}(i,j,k)$$

$$i\mathcal{O}_l\psi_i^\mu - i\mathcal{O}_i\psi_l^\mu - \epsilon^{ilk}\mathcal{O}_j\tau_k\psi_j^\mu - \epsilon^{ijl}\mathcal{O}_k\tau_k\psi_j^\mu = 0$$

$P(i,j,k)$ means all permutations for the indices i, j, and k and an odd permutation $P(i,j,k)$ gives a minus sign.

Linear relations II

- Partial integration

$$0 = \bar{\psi}^{i\nu}\overleftarrow{D}^{\mu}\mathcal{O}^{\cdots}\psi^{k\lambda} + \bar{\psi}^{i\nu}\nabla^{\mu}\mathcal{O}^{\cdots}\psi^{k\lambda} + \bar{\psi}^{i\nu}\mathcal{O}^{\cdots}D^{\mu}\psi^{k\lambda}$$

$$0 = \bar{\psi}\overleftarrow{D}^{\mu}\mathcal{O}^{\cdots}\psi^{j\nu} + \bar{\psi}\nabla^{\mu}\mathcal{O}^{\cdots}\psi^{j\nu} + \psi\mathcal{O}^{\cdots}D^{\mu}\psi^{j\nu}$$

$$\Rightarrow \bar{\psi}^{i\nu}\overleftarrow{D}^{\mu}\mathcal{O}^{\cdots}\psi^{k\lambda} \doteq -\bar{\psi}^{i\nu}\mathcal{O}^{\cdots}D^{\mu}\psi^{k\lambda},\ \bar{\psi}\overleftarrow{D}^{\mu}\mathcal{O}^{\cdots}\psi^{j\nu} \doteq -\psi\mathcal{O}^{\cdots}D^{\mu}\psi^{j\nu}$$

"$\doteq$" means that both sides are equal if high order terms are ignored

- Equations of motion:

π: $\nabla_{\mu}u^{\mu} = \frac{i}{2}\left(\chi_{-} - \frac{1}{N_f}\langle\chi_{-}\rangle\right)$

Δ: $\mathscr{L}_{\pi\Delta} = \bar{\psi}^{i}_{\mu}\Lambda^{\mu\nu}_{A,ij}\psi^{j}_{\nu}$

$$\Lambda^{\mu\nu}_{A,ij} = -\Big[(i\not{D} - m_{\Delta})g^{\mu\nu} + iA(\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu}) + \frac{i}{2}(3A^2 + 2A + 1)\gamma^{\mu}\not{D}\gamma^{\nu} + m_{\Delta}(3A^2 + 3A + 1)\gamma^{\mu}\gamma^{\nu}\Big]\delta_{ij} + O^{\mu\nu}_{1,ij}$$

$$\Rightarrow (i\not{D} - m_{\Delta})\psi^{\mu}_{i} \doteq 0,\ D_{\mu}\psi^{\mu}_{i} \doteq 0,\ \gamma_{\mu}\psi^{\mu}_{i} \doteq 0$$ Similar as the free field

The form of γ and ε are similar as πN chiral Lagrangian

Linear relations III

- Equations of motion (summary): Fettes, etc., AP(NY)283, 273(2000)
  - $\gamma^\mu \Longleftrightarrow -iD^\mu$, γ^μ does not exist.
  - The Lorentz indices of D's are different from that of ψ_i^μ or $\bar{\psi}_i^\mu$.
  - The indices of D's are totally different and totally symmetric

$$D_{\nu\lambda\rho\cdots} = D_\nu D_\lambda D_\rho \cdots + \text{full permutation of } D\text{'s.}$$

  - When $\epsilon^{\mu\nu\lambda\rho}$ exists, neither $\gamma_5\gamma^\mu$ nor $\sigma^{\mu\nu}$ exists
  - The Lorentz indices of $\gamma_5\gamma^\mu$ and $\sigma^{\mu\nu}$ are different from that of ψ_i^μ or $\bar{\psi}_i^\mu$ and that of covariant derivative acting on the baryon fields.
  - $\bar{\psi}(^\lambda_i)\nabla^\mu B^{\nu\cdots}\gamma_5\gamma_\rho D_\mu \psi_j^\alpha$ does not exist
  - $0 \doteq \bar{\psi}(^\lambda_i) A^{\mu\nu\lambda\rho}{}_{\mu\nu}\sigma_{\lambda\rho} D_\delta \psi_j^\delta + \mathrm{P}(\mu,\nu,\lambda,\rho,\delta)$
    P means all permutations of the subscripts behind it, and an odd permutation gives a minus sign.
  - $\psi_i^\mu \doteq \psi_{Ai}^\mu$, and both of them are useful
  - $\varepsilon^{\mu\nu\lambda\rho} D_\rho \psi_{\lambda,i} \doteq \gamma^\mu\gamma_5\psi_i^\nu - \gamma^\nu\gamma_5\psi_i^\mu$

Linear relations IV

- All possible structures of γ matrix and ε

$$
\begin{aligned}
\mathcal{O}(p^1)_{\pi N\Delta} &: 1, \\
\mathcal{O}(p^1)_{\pi\Delta\Delta}, \mathcal{O}(p^2)_{\pi N\Delta} &: D^{\mu}, \gamma_5\gamma^{\mu}, \\
\mathcal{O}(p^2)_{\pi\Delta\Delta}, \mathcal{O}(p^3)_{\pi N\Delta} &: 1, D^{\mu\nu}, \gamma_5\gamma^{\mu}D^{\nu}, \sigma^{\mu\nu}, \epsilon^{\mu\nu\lambda\rho}, \\
\mathcal{O}(p^3)_{\pi\Delta\Delta}, \mathcal{O}(p^4)_{\pi N\Delta} &: D^{\mu}, D^{\mu\nu\lambda}, \gamma_5\gamma^{\mu}, \gamma_5\gamma^{\mu}D^{\nu\lambda}, \sigma^{\mu\nu}D^{\lambda}, \\
&\quad \epsilon^{\mu\nu\lambda\rho}D_{\rho}, \epsilon^{\mu\nu\lambda\rho}D^{\sigma}, \\
\mathcal{O}(p^4)_{\pi\Delta\Delta} &: 1, D^{\mu\nu}, D^{\mu\nu\lambda\rho}, \gamma_5\gamma^{\mu}D^{\nu}, \gamma_5\gamma^{\mu}D^{\nu\lambda\rho}, \sigma^{\mu\nu}, \sigma^{\mu\nu}D^{\lambda\rho}, \\
&\quad \epsilon^{\mu\nu\lambda\rho}, \epsilon^{\mu\nu\lambda\rho}{D_{\rho}}^{\sigma}, \epsilon^{\mu\nu\lambda\rho}D^{\sigma\delta}.
\end{aligned}
$$

Linear relations V

- Covariant derivatives and Bianchi identity

$$\nabla^\mu\Gamma^{\nu\lambda}+\nabla^\nu\Gamma^{\lambda\mu}+\nabla^\lambda\Gamma^{\mu\nu}=0,\ [\nabla^\mu,\nabla^\nu]X^i=-2i\epsilon^{ijk}X_j\Gamma_k^{\mu\nu}$$

$$\nabla^\mu\Gamma_i^{\nu\lambda}+\nabla^\nu\Gamma_i^{\lambda\mu}+\nabla^\lambda\Gamma_i^{\mu\nu}=0,\ \nabla^\mu\Gamma_s^{\nu\lambda}+\nabla^\nu\Gamma_s^{\lambda\mu}+\nabla^\lambda\Gamma_s^{\mu\nu}=0$$

$\Gamma^{\mu\nu}=\frac{1}{4}[u^\mu,u^\nu]-\frac{i}{2}f_+^{\mu\nu}$

- Cayley-Hamilton relations: Implied in the Pauli matrix relations

- Contact terms

$$F_L^{\mu\nu}=\frac{1}{2}u^\dagger(f_+^{\mu\nu}+f_-^{\mu\nu})u,\quad F_R^{\mu\nu}=\frac{1}{2}u(f_+^{\mu\nu}-f_-^{\mu\nu})u^\dagger,$$

$$\chi=\frac{1}{2}u(\chi_++\chi_-)u,\quad \chi^\dagger=\frac{1}{2}u^\dagger(\chi_+-\chi_-)u^\dagger$$

Only 6 terms in the p^4 order

$$\bar\psi^{i\mu}\langle F_{L\mu}{}^{\nu}F_{L\nu}{}^{\lambda}\rangle\psi_{i\lambda}+\text{H.c.},\quad \bar\psi^{i\mu}\langle F_L{}^{\nu\lambda}F_{L\nu\lambda}\rangle\psi_{i\mu}+\text{H.c.}$$

$$\bar\psi^{i\mu}\langle F_{L\mu}{}^{\nu}F_L{}^{\lambda\rho}\rangle D_{\nu\lambda}\psi_{i\rho}+\text{H.c.},\quad \bar\psi^{i\mu}\langle F_L{}^{\nu\lambda}F_{L\nu}{}^{\rho}\rangle D_{\lambda\rho}\psi_{i\mu}+\text{H.c.}$$

$$\bar\psi^{i\mu}\langle\chi\chi^\dagger\rangle\psi_{i\mu},\quad \bar\psi^{i\mu}\det\chi\psi_{i\mu}+\text{H.c.}$$

Constructing chiral Lagrangian I

- Piling building blocks (all terms)
- C and Hermitian conjugate is invariant
- Picking out contact terms
- Removing linear depend terms
- By computer

Constructing chiral Lagrangian II

- Reduced building blocks:

$$f_{+i}^{\mu\nu} \longleftrightarrow i\Gamma_i^{\mu\nu}, \quad f_{+s}^{\mu\nu} \longleftrightarrow i\Gamma_s^{\mu\nu}$$

- Permutation: Permuting all reduced building blocks and their covariant derivatives

- Primary screening:
  Coding each operator and each script a number, choosing the smallest one with Einstein summation convention, symmetry and anti-symmetry indices and C-number positions

- Substitutions: Revealing the covariant derivatives

$$h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu, \quad f_{-,\mu\nu} = \nabla_\mu u_\nu - \nabla_\nu u_\mu, \text{ and so on}$$

Constructing chiral Lagrangian III

- classifications: classifying terms by numbers of external sources

  $D_{i,j}$: Contain $h_{\mu\nu}, f_{-,\mu\nu}$, $E_{i,k}$: Revealing ∇

$$D_{i,j} = \sum_k A_{i,jk} E_{i,k}, \quad i\text{: source scripts}$$

  All linear relations in the same classification

- linear relations: $\sum_k R_{i,jk} E_{i,k} = 0$, reduced row echelon form of $R_{i,jk}$ to search independent ones

$$R_{i,jk} \to S_{i,jk} = \begin{pmatrix} 1 & O & O & O & O & \cdots & \cdots & \cdots \\ & & 1 & O & O & \cdots & \cdots & \cdots \\ & & & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & 1 & \cdots & \cdots & \cdots \\ & & & & & O & O & O \end{pmatrix}$$

- Adding contact terms: Eliminating some terms

- Hermitian conjugation: Adding some appropriate "i"

Results

Checking

Recovering the πNN Lagrangians, with the following relations

Fettes, Meißner, Mojžiš and Steininger, AP(NY)283, 273(2000)

$$\langle XY \rangle = 2X^i Y_i + 2X_s Y_s,$$
$$\langle XYZ \rangle = 2i\epsilon^{ijk} X_i Y_j Z_k + 2X^i Y_i Z_s + 2X^i Z_i Y_s + 2Y^i Z_i X_s + 2X_s Y_s Z_s$$

$O(p^1)$ order

- Our results:

$$\mathscr{L}_{\pi\Delta\Delta}^{(1)} = -\bar{\psi}_\mu^i \big[(i\not{D} - m_\Delta)g^{\mu\nu} + iA(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + \frac{i}{2}(3A^2 + 2A + 1)\gamma^\mu \not{D}\gamma^\nu + m_\Delta(3A^2 + 3A + 1)\gamma^\mu\gamma^\nu\big]\psi_\nu^i + c_1^{(1)}\bar{\psi}^{i\mu} u^{j\nu}\tau_j\gamma_5\gamma_\nu\psi_{i\mu}$$

$$\mathscr{L}_{\pi N\Delta}^{(1)} = g_{\pi N\Delta}\bar{\psi} u_i^\mu \psi_{A,n,\mu}^i + \text{H.c.}$$

- Hemmert's: Hemmert, Holstein, and Kambor, JPG24, 1831(1998)

$$\mathscr{L}_{\pi\Delta\Delta}^{(1)} = -\bar{\psi}_\mu^i \big[(i\not{D} - m_\Delta)g^{\mu\nu} + iA(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + \frac{i}{2}(3A^2 + 2A + 1)\gamma^\mu \not{D}\gamma^\nu + m_\Delta(3A^2 + 3A + 1)\gamma^\mu\gamma^\nu + \frac{g_1}{2}\not{u}\gamma_5 + \frac{g_2}{2}(\gamma_\mu u_\nu + u_\mu\gamma_\nu)\gamma_5 + \frac{g_3}{2}\gamma_\mu\not{u}_\mu\gamma_5\gamma_\nu\big]\psi_\nu^i$$

$$\mathscr{L}_{\pi N\Delta}^{(1)} = g_{\pi N\Delta}\bar{\psi} u_i^\mu \psi_{A,n,\mu}^i + \text{H.c.}$$

- Reason: Removing redundant off-shell parameters

Krebs, Epelbaum, and Meißer, PLB683, 222(2010), Krebs, Epelbaum, and Meißner, PRC80, 028201(2009)

g_1, g_2, g_3 are not independent Wies, Gegelia, and Scherer, PRD73, 094012(2006)

$O(p^2)$ order: $\pi\Delta\Delta$

The LECs relations between ours and Hemmert's

n	$O_n^{(2)}$	$c_n^{(2)}$
1	$\bar{\psi}^{i\mu} u_{i\mu} u^{j\nu} \psi_{j\nu}$	$2a_{10}$ independent
2	$\bar{\psi}^{i\mu} u_i{}^{\nu} u^j{}_{\mu} \psi_{j\nu}$	$2a_{10}$ independent
3	$\bar{\psi}^{i\mu} u_i{}^{\nu} u^j{}_{\nu} \psi_{j\mu}$	$4a_8$
4	$\bar{\psi}^{i\mu} u^j{}_{\mu} u_j{}^{\nu} \psi_{i\nu}$	$2a_{11}$
5	$\bar{\psi}^{i\mu} u^{j\nu} u_{j\nu} \psi_{i\mu}$	a_3
6	$\bar{\psi}^{i\mu} u_i{}^{\nu} u^{j\lambda} D_{\nu\lambda} \psi_{j\mu}$	$-2a_9$
7	$\bar{\psi}^{i\mu} u^{j\nu} u_j{}^{\lambda} D_{\nu\lambda} \psi_{i\mu}$	$-a_2/2$
8	$i\bar{\psi}^{i\mu} u_i{}^{\nu} u^{j\lambda} \sigma_{\nu\lambda} \psi_{j\mu}$	a_4
9	$i\bar{\psi}^{i\mu} f_{s,+\mu}{}^{\nu} \psi_{i\nu}$	$a_6 + a_7/2$
10	$\bar{\psi}^{i\mu} f_{s,+}{}^{\nu\lambda} \sigma_{\nu\lambda} \psi_{i\mu}$	additional term
11	$i\bar{\psi}^{i\mu} f_{+}{}^{j}{}_{\mu}{}^{\nu} \tau_j \psi_{i\nu}$	a_6
12	$\bar{\psi}^{i\mu} f_{+}{}^{j\nu\lambda} \tau_j \sigma_{\nu\lambda} \psi_{i\mu}$	additional term
13	$\bar{\psi}^{i\mu} \chi_{+,s} \psi_{i\mu}$	a_1
14	$\bar{\psi}^{i\mu} \chi_{+}{}^{j} \tau_j \psi_{i\mu}$	a_5

$O(p^2)$ order: $\pi N\Delta$

- Our results:

$$\mathscr{L}^{(2)}_{\pi N\Delta} = d_1^{(2)}\bar{\psi}u^{i\mu}u^{j\nu}\tau_i\gamma_5\gamma_\mu\psi_{A,n,j\nu} + d_2^{(2)}\bar{\psi}u^{i\mu}u^{j\nu}\tau_i\gamma_5\gamma_\nu\psi_{A,n,j\mu} + d_3^{(2)}i\bar{\psi}f_+{}^{i\mu\nu}\gamma_5\gamma_\mu\psi_{A,n,i\nu} + \text{H.c.}$$

- Hemmert's Hemmert, Holstein, and Kambor, JPG24, 1831(1998)

$$\mathscr{L}^{(2)}_{\pi N\Delta} = (-\frac{1}{2}b_1 i\bar{\psi}^{\mu}_{A,1,i}f^{i}_{+\mu\nu}\gamma_5\gamma^{\nu} + b_2 i\bar{\psi}^{\mu}_{A,2,i}f^{i}_{-\mu\nu}D^{\nu} + b_3 i\bar{\psi}^{\mu}_{A,3,i}\nabla_{\mu}u^{i}_{\nu}D^{\nu} - \frac{1}{2}b_4\bar{\psi}^{\mu}_{A,4,i}u^{i}_{\mu}u^{\nu}\gamma_5\gamma_\nu - \frac{1}{2}b_5\bar{\psi}^{\mu}_{A,5,i}u_{\mu}u^{i\nu}\gamma_5\gamma_\nu)\psi + \text{H.c.}.$$

$O(p^3)$ order: $\pi\Delta\Delta$

$$\mathscr{L}^{(3)}_{\pi\Delta\Delta} = \sum_n c_n^{(3)} O_n^{(3)}$$

n	$O_n^{(3)}$	n	$O_n^{(3)}$
1	$\bar{\psi}^{i\mu} u_{i\mu} u^{j\nu} u^{k\lambda} \tau_j \gamma_5 \gamma_\nu \psi_{k\lambda}$	23	$\bar{\psi}^{i\mu} u_i{}^\nu h^j{}_\nu{}^\lambda D_\lambda \psi_{j\mu}$ + H.c.
2	$\bar{\psi}^{i\mu} u_{i\mu} u^{j\nu} u^{k\lambda} \tau_j \gamma_5 \gamma_\lambda \psi_{k\nu}$ + H.c.	24	$\bar{\psi}^{i\mu} u_i{}^\nu h^{j\lambda\rho} D_{\nu\lambda\rho} \psi_{j\mu}$ + H.c.
3	$\bar{\psi}^{i\mu} u_i{}^\nu u^j{}_\mu u^{k\lambda} \tau_k \gamma_5 \gamma_\nu \psi_{j\lambda}$ + H.c.	25	$i\bar{\psi}^{i\mu} u_i{}^\nu h^{j\lambda\rho} \sigma_{\nu\lambda} D_\rho \psi_{j\mu}$ + H.c.
4	$\bar{\psi}^{i\mu} u_i{}^\nu u^j{}_\mu u^{k\lambda} \tau_k \gamma_5 \gamma_\lambda \psi_{j\nu}$	26	$i\bar{\psi}^{i\mu} u^{j\nu} h_j{}^{\lambda\rho} \sigma_{\nu\lambda} D_\rho \psi_{i\mu}$
5	$\bar{\psi}^{i\mu} u_i{}^\nu u^j{}_\nu u^{k\lambda} \tau_j \gamma_5 \gamma_\lambda \psi_{k\mu}$ + H.c.	27	$\bar{\psi}^{i\mu} \nabla^\nu f_-{}^j{}_\nu{}^\lambda \tau_j \gamma_5 \gamma_\lambda \psi_{i\mu}$
6	$\bar{\psi}^{i\mu} u_i{}^\nu u^j{}_\nu u^{k\lambda} \tau_k \gamma_5 \gamma_\lambda \psi_{j\mu}$	28	$i\bar{\psi}^{i\mu} f_{+i\mu}{}^\nu u^{j\lambda} \gamma_5 \gamma_\nu \psi_{j\lambda}$ + H.c.
7	$\bar{\psi}^{i\mu} u^j{}_\mu u_j{}^\nu u^{k\lambda} \tau_k \gamma_5 \gamma_\nu \psi_{i\lambda}$ + H.c.	29	$i\bar{\psi}^{i\mu} f_{+i\mu}{}^\nu u^{j\lambda} \gamma_5 \gamma_\lambda \psi_{j\nu}$ + H.c.
8	$\bar{\psi}^{i\mu} u^{j\nu} u_{j\nu} u^{k\lambda} \tau_k \gamma_5 \gamma_\lambda \psi_{i\mu}$	30	$i\bar{\psi}^{i\mu} f_{+i}{}^{\nu\lambda} u^j{}_\mu \gamma_5 \gamma_\nu \psi_{j\lambda}$ + H.c.
9	$\bar{\psi}^{i\mu} u_i{}^\nu u^{j\lambda} u^{k\rho} \tau_j \gamma_5 \gamma_\nu D_{\lambda\rho} \psi_{k\mu}$ + H.c.	31	$i\bar{\psi}^{i\mu} f_{+i}{}^{\nu\lambda} u^j{}_\nu \gamma_5 \gamma_\lambda \psi_{j\mu}$ + H.c.
10	$\bar{\psi}^{i\mu} u_i{}^\nu u^{j\lambda} u^{k\rho} \tau_j \gamma_5 \gamma_\lambda D_{\nu\rho} \psi_{k\mu}$	32	$i\bar{\psi}^{i\mu} f_+{}^j{}_\mu{}^\nu u_j{}^\lambda \gamma_5 \gamma_\nu \psi_{i\lambda}$ + H.c.
11	$\bar{\psi}^{i\mu} u^{j\nu} u_j{}^\lambda u^{k\rho} \tau_k \gamma_5 \gamma_\nu D_{\lambda\rho} \psi_{i\mu}$	33	$i\bar{\psi}^{i\mu} f_+{}^j{}_\mu{}^\nu u_j{}^\lambda \gamma_5 \gamma_\lambda \psi_{i\nu}$
12	$\bar{\psi}^{i\mu} u_{i\mu} f_-{}^{j\nu\lambda} D_\nu \psi_{j\lambda}$ + H.c.	34	$i\bar{\psi}^{i\mu} f_{+i}{}^{\nu\lambda} u^{j\rho} \gamma_5 \gamma_\nu D_{\lambda\rho} \psi_{j\mu}$ + H.c.
13	$\bar{\psi}^{i\mu} u_i{}^\nu f_-{}^j{}_\mu{}^\lambda D_\nu \psi_{j\lambda}$ + H.c.	35	$i\bar{\psi}^{i\mu} f_{s,+\mu}{}^\nu u^{j\lambda} \tau_j \gamma_5 \gamma_\nu \psi_{i\lambda}$ + H.c.
14	$\bar{\psi}^{i\mu} u_i{}^\nu f_-{}^j{}_\mu{}^\lambda D_\lambda \psi_{j\nu}$ + H.c.	36	$i\bar{\psi}^{i\mu} f_{s,+\mu}{}^\nu u^{j\lambda} \tau_j \gamma_5 \gamma_\lambda \psi_{i\nu}$
15	$\bar{\psi}^{i\mu} u_i{}^\nu f_-{}^j{}_\nu{}^\lambda D_\lambda \psi_{j\mu}$ + H.c.	37	$i\bar{\psi}^{i\mu} \nabla^\nu f_{s,+\nu}{}^\lambda D_\lambda \psi_{i\mu}$
16	$\bar{\psi}^{i\mu} u^j{}_\mu f_{-j}{}^{\nu\lambda} D_\nu \psi_{i\lambda}$ + H.c.	38	$i\bar{\psi}^{i\mu} \nabla^\nu f_+{}^j{}_\nu{}^\lambda \tau_j D_\lambda \psi_{i\mu}$
17	$\bar{\psi}^{i\mu} u^{j\nu} f_{-j\mu}{}^\lambda D_\nu \psi_{i\lambda}$	39	$\bar{\psi}^{i\mu} u_i{}^\nu \chi_+{}^j \gamma_5 \gamma_\nu \psi_{j\mu}$ + H.c.
18	$i\bar{\psi}^{i\mu} u_i{}^\nu f_-{}^{j\lambda\rho} \sigma_{\nu\lambda} D_\rho \psi_{j\mu}$ + H.c.	40	$\bar{\psi}^{i\mu} u^{j\nu} \chi_{+j} \gamma_5 \gamma_\nu \psi_{i\mu}$
19	$i\bar{\psi}^{i\mu} u_i{}^\nu f_-{}^{j\lambda\rho} \sigma_{\lambda\rho} D_\nu \psi_{j\mu}$ + H.c.	41	$\bar{\psi}^{i\mu} u^{j\nu} \chi_{+,s} \tau_j \gamma_5 \gamma_\nu \psi_{i\mu}$
20	$i\bar{\psi}^{i\mu} u^{j\nu} f_{-j}{}^{\lambda\rho} \sigma_{\nu\lambda} D_\rho \psi_{i\mu}$	42	$i\bar{\psi}^{i\mu} u_i{}^\nu \chi_-{}^j D_\nu \psi_{j\mu}$ + H.c.
21	$i\bar{\psi}^{i\mu} u^{j\nu} f_{-j}{}^{\lambda\rho} \sigma_{\lambda\rho} D_\nu \psi_{i\mu}$	43	$i\bar{\psi}^{i\mu} \nabla^\nu \chi_{-,s} \gamma_5 \gamma_\nu \psi_{i\mu}$
22	$\bar{\psi}^{i\mu} u_{i\mu} h^{j\nu\lambda} D_\nu \psi_{j\lambda}$ + H.c.	44	$i\bar{\psi}^{i\mu} \nabla^\nu \chi_-{}^j \tau_j \gamma_5 \gamma_\nu \psi_{i\mu}$

$O(p^3)$ order: $\pi N \Delta$

$$\mathscr{L}_{\pi N\Delta}^{(3)} = \sum_n d_n^{(3)} (P_n^{(3)} + \text{H.c.})$$

n	$O_n^{(3)}$	n	$O_n^{(3)}$
1	$\bar{\psi} u^{i\mu} u_{i\mu} u^{j\nu} \psi_{A,n,j\nu}$	19	$\bar{\psi} \nabla^\mu f_-{}^i{}_\mu{}^\nu \psi_{A,n,i\nu}$
2	$\bar{\psi} u^{i\mu} u_i{}^\nu u^j{}_\mu \psi_{A,n,j\nu}$	20	$i\bar{\psi} \nabla^\mu f_-{}^{i\nu\lambda} \sigma_{\mu\nu} \psi_{A,n,i\lambda}$
3	$\bar{\psi} u^{i\mu} u_i{}^\nu u^{j\lambda} D_{\mu\nu} \psi_{A,n,j\lambda}$	21	$i\bar{\psi} f_{s,+}{}^{\mu\nu} u^i{}_\mu \psi_{A,n,i\nu}$
4	$\bar{\psi} u^{i\mu} u_i{}^\nu u^{j\lambda} D_{\mu\lambda} \psi_{A,n,j\nu}$	22	$i\bar{\psi} f_{s,+}{}^{\mu\nu} u^{i\lambda} D_{\mu\lambda} \psi_{A,n,i\nu}$
5	$i\bar{\psi} u^{i\mu} u_i{}^\nu u^{j\lambda} \sigma_{\mu\lambda} \psi_{A,n,j\nu}$	23	$\bar{\psi} f_{s,+}{}^{\mu\nu} u^{i\lambda} \sigma_{\mu\nu} \psi_{A,n,i\lambda}$
6	$i\epsilon^{ijk} \bar{\psi} u_i{}^\mu u_j{}^\nu u^l{}_\mu \tau_k \psi_{A,n,l\nu}$	24	$\bar{\psi} f_{s,+}{}^{\mu\nu} u^{i\lambda} \sigma_{\mu\lambda} \psi_{A,n,i\nu}$
7	$i\epsilon^{ijk} \bar{\psi} u_i{}^\mu u_j{}^\nu u^{l\lambda} \tau_k D_{\mu\lambda} \psi_{A,n,l\nu}$	25	$i\bar{\psi} f_+{}^{i\mu\nu} u^j{}_\mu \tau_i \psi_{A,n,j\nu}$
8	$\epsilon^{ijk} \bar{\psi} u_i{}^\mu u_j{}^\nu u^{l\lambda} \tau_k \sigma_{\mu\nu} \psi_{A,n,l\lambda}$	26	$i\bar{\psi} f_+{}^{i\mu\nu} u^j{}_\mu \tau_j \psi_{A,n,i\nu}$
9	$\bar{\psi} u^{i\mu} f_-{}^{j\nu\lambda} \tau_i \gamma_5 \gamma_\mu D_\nu \psi_{A,n,j\lambda}$	27	$i\bar{\psi} f_+{}^{i\mu\nu} u^{j\lambda} \tau_i D_{\mu\lambda} \psi_{A,n,j\nu}$
10	$\bar{\psi} u^{i\mu} f_-{}^{j\nu\lambda} \tau_i \gamma_5 \gamma_\nu D_\mu \psi_{A,n,j\lambda}$	28	$i\bar{\psi} f_+{}^{i\mu\nu} u^{j\lambda} \tau_j D_{\mu\lambda} \psi_{A,n,i\nu}$
11	$\bar{\psi} u^{i\mu} f_-{}^{j\nu\lambda} \tau_i \gamma_5 \gamma_\nu D_\lambda \psi_{A,n,j\mu}$	29	$\bar{\psi} f_+{}^{i\mu\nu} u^{j\lambda} \tau_i \sigma_{\mu\nu} \psi_{A,n,j\lambda}$
12	$\bar{\psi} u^{i\mu} f_-{}^{j\nu\lambda} \tau_j \gamma_5 \gamma_\mu D_\nu \psi_{A,n,i\lambda}$	30	$\bar{\psi} f_+{}^{i\mu\nu} u^{j\lambda} \tau_i \sigma_{\mu\lambda} \psi_{A,n,j\nu}$
13	$\bar{\psi} u^{i\mu} f_-{}^{j\nu\lambda} \tau_j \gamma_5 \gamma_\nu D_\mu \psi_{A,n,i\lambda}$	31	$\bar{\psi} f_+{}^{i\mu\nu} u^{j\lambda} \tau_j \sigma_{\mu\nu} \psi_{A,n,i\lambda}$
14	$\bar{\psi} u^{i\mu} f_-{}^{j\nu\lambda} \tau_j \gamma_5 \gamma_\nu D_\lambda \psi_{A,n,i\mu}$	32	$\bar{\psi} f_+{}^{i\mu\nu} u^{j\lambda} \tau_j \sigma_{\mu\lambda} \psi_{A,n,i\nu}$
15	$\bar{\psi} u^{i\mu} h^{j\nu\lambda} \tau_i \gamma_5 \gamma_\mu D_\nu \psi_{A,n,j\lambda}$	33	$\bar{\psi} u^{i\mu} \chi_{+,s} \psi_{A,n,i\mu}$
16	$\bar{\psi} u^{i\mu} h^{j\nu\lambda} \tau_i \gamma_5 \gamma_\nu D_\mu \psi_{A,n,j\lambda}$	34	$\bar{\psi} u^{i\mu} \chi_+{}^j \tau_i \psi_{A,n,j\mu}$
17	$\varepsilon^{\mu\nu\lambda\rho} \bar{\psi} u^i{}_\mu f_-{}^j{}_{\nu\lambda} \tau_i \psi_{A,n,j\rho}$	35	$\bar{\psi} u^{i\mu} \chi_+{}^j \tau_j \psi_{A,n,i\mu}$
18	$\varepsilon^{\mu\nu\lambda\rho} \bar{\psi} u^i{}_\mu f_-{}^j{}_{\nu\lambda} \tau_j \psi_{A,n,i\rho}$	36	$i\bar{\psi} \nabla^\mu \chi_-{}^i \psi_{A,n,i\mu}$

$O(p^4)$ order

$$\mathscr{L}_{\pi\Delta\Delta}^{(4)} = \sum_n c_n^{(4)} O_n^{(m)}$$

$$\mathscr{L}_{\pi N\Delta}^{(4)} = \sum_n d_n^{(4)} (P_n^{(m)} + \text{H.c.})$$

$\pi\Delta\Delta$: 385 terms, $\pi N\Delta$: 236 terms

$O_n^{(4)}$ and $P_n^{(4)}$ are in the attach file (click here)

Point transformation

$$\psi^{\mu}_{Ai} \equiv (g^{\mu\nu} + \tfrac{1}{2} A\gamma^{\mu}\gamma^{\nu})\psi_{i\nu}$$

$$\begin{aligned}\mathscr{L}_{\pi\Delta\Delta} =& -\bar{\psi}^{i}_{\mu}\big[(i\not{D} - m_{\Delta})g^{\mu\nu} + iA(\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu}) + \frac{i}{2}(3A^2 + 2A + 1)\gamma^{\mu}\not{D}\gamma^{\nu} \\ & + m_{\Delta}(3A^2 + 3A + 1)\gamma^{\mu}\gamma^{\nu}\big]\delta_{ij}\psi^{j}_{\nu} + \sum_{m,n}\bar{\psi}^{i}_{\mu}c^{(m)}_{n}(A)O^{(m)\mu\nu}_{n,ij}\psi^{j}_{\nu}\end{aligned}$$

LECs ($c^{(m)}_{n}(A)$) are dependent on A

$$\begin{aligned} =& -\bar{\psi}^{i}_{A\mu}\big[(i\not{D} - m_{\Delta})g^{\mu\nu} + iA(\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu}) + \frac{i}{2}(3A^2 + 2A + 1)\gamma^{\mu}\not{D}\gamma^{\nu} \\ & + m_{\Delta}(3A^2 + 3A + 1)\gamma^{\mu}\gamma^{\nu}\big]\delta_{ij}\psi^{j}_{A\nu} + \sum_{m,n}\bar{\psi}^{i}_{A\mu}c^{(m)}_{n}O^{(m)\mu\nu}_{n,ij}\psi^{j}_{A\nu}\end{aligned}$$

LECs ($c^{(m)}_{n}$) are independent on A

Heavy baryon projection

Without $1/m_0$ corrections

$$
\begin{aligned}
1 &\longleftrightarrow 1, \\
\gamma^\mu &\longleftrightarrow v^\mu, \\
\gamma_5\gamma^\mu &\longleftrightarrow -2S^\mu, \\
\sigma^{\mu\nu} &\longleftrightarrow 2\varepsilon^{\mu\nu\lambda\rho}v_\lambda S_\rho = -2i[S^\mu, S^\nu], \\
\psi &\longleftrightarrow N \\
\psi_i^\mu &\longleftrightarrow T_i^\mu, \\
D^\nu\psi_i^\mu &\longleftrightarrow -im_0 v^\nu T_i^\mu,
\end{aligned}
$$

Summary

- Finding a method to get chiral Lagrangian with $\Delta(1232)$
- Giving the chiral Lagrangian with $\Delta(1232)$ to p^4 order
- This method can extend to other effective Lagrangians: Decuplet ...

Thank you!

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Welcome to Guangxi University!