Pionless Effective Field Theory

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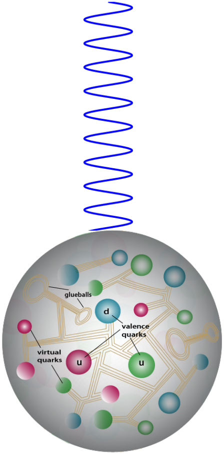
Effective Theory & Resolution Scale

- Physics at different resolution scales are described by different effective theories

QCD

- describe nucleon structures
- de Broglie wavelength: $\lambda \lessapprox 0.2$ fm

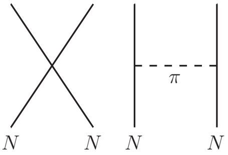


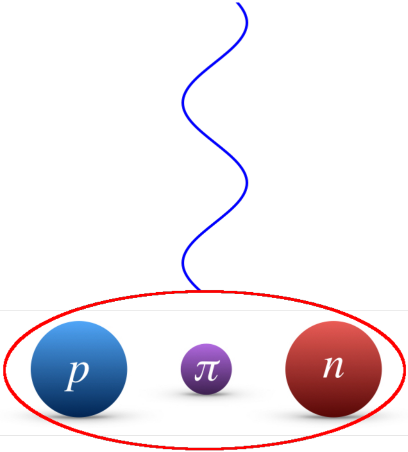
Effective Theory & Resolution Scale

- Physics at different resolution scales are described by different effective theories

chiral EFT NN interaction

- de Broglie wavelength: $\lambda \sim 1$ fm
- short range: $V_s = C_0$
- intermediate range: $V_{1\pi} \sim 1/(q^2 + m_\pi^2)$





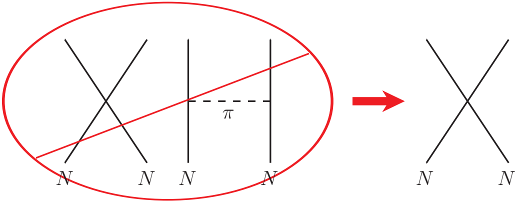
Effective Theory & Resolution Scale

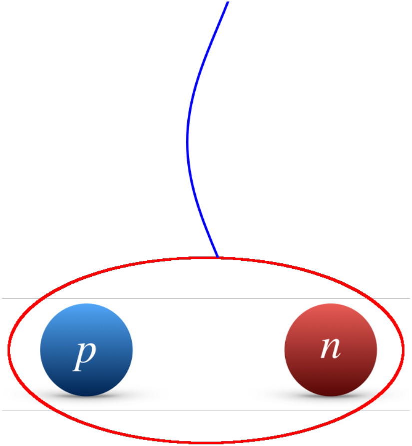
- Physics at different resolution scales are described by different effective theories

$\not\pi$EFT NN interaction

- de Broglie wavelength: $\lambda \gg 1$ fm
- NN momentum $q \ll m_\pi$

$$V_{1\pi} \sim \frac{1}{q^2 + m_\pi^2} \xrightarrow{q \ll m_\pi} \frac{1}{m_\pi^2} - \frac{q^2}{m_\pi^4} + \cdots$$





Separation of Scales & Universality

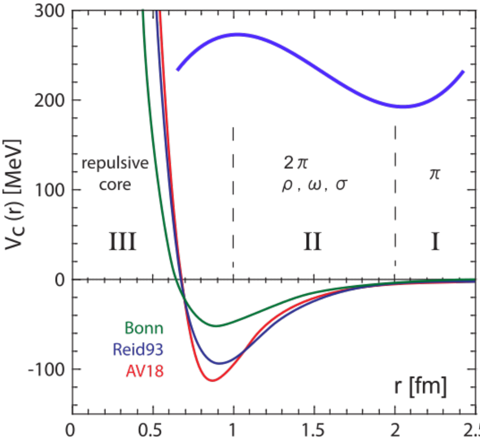
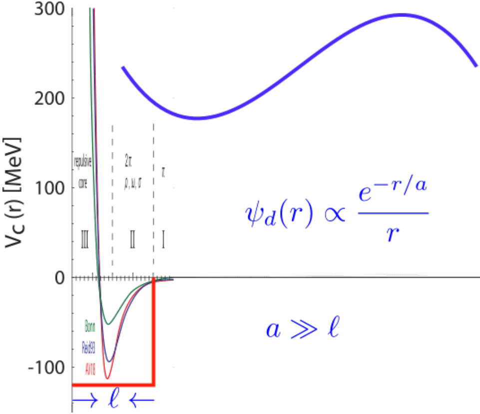


Fig: Aoki, Hatsuda, Ishii, Comp. Sci. Disc. 1 (2008) 015009

- separation of scales:
  - an observable's length scale is much larger than the interaction range $a \gg \ell$
- physics at scale a is insensitive to physics at scale ℓ

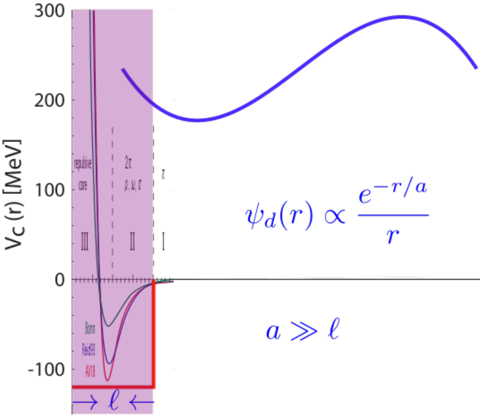


Separation of Scales & Universality

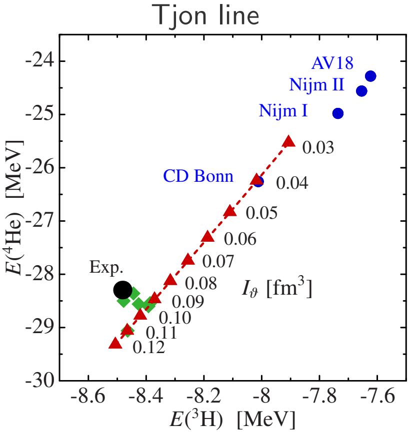
- separation of scales:
  - an observable's length scale is much larger than the interaction range $a \gg \ell$
- physics at scale a is insensitive to physics at scale ℓ
- different few-body systems share universal features
- **pionless effective field theory:**
  - represent short-range potentials with contact interactions
  - study universality in large-a physics
  - systematic expansion in ℓ/a_0

$$\mathcal{L} = \sum c_\nu (\ell/a_0)^\nu \hat{O}_\nu$$

  - implement power counting to systematically improve accuracy



- light nuclei:
  - $r_0/a_0 \sim 1/3$
  - bound states (^{2}H, ^{3}H, ^{3}He, ^{4}He)
  - elastic scattering (pd, p^3H,$\cdots$)
  - Compton scattering ($np \to d\gamma$)



see e.g., Bedaque, van Kolck,
Ann. Rev. Nucl. Part. Sci. 52 (2002) 339

Applications of Pionless Effective Field Theory

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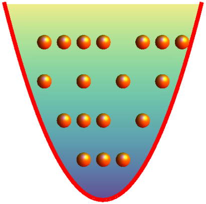
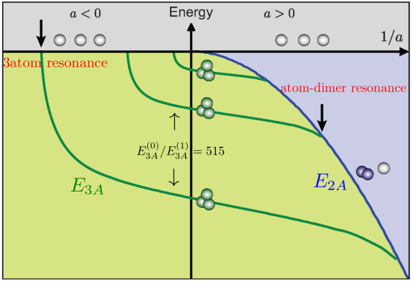
- ultracold atomic gases
  - laser cooling and trapping $\sim n$K
  - $r_0 \ll a_0$ near Feshbach resonances
  - few-body Efimov effects





see e.g., Braaten, Hammer, Phys. Rept. 428 (2006) 259 & Zhai Hui's Talk

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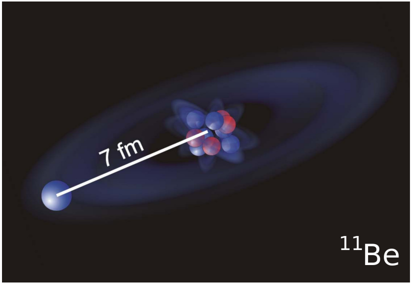
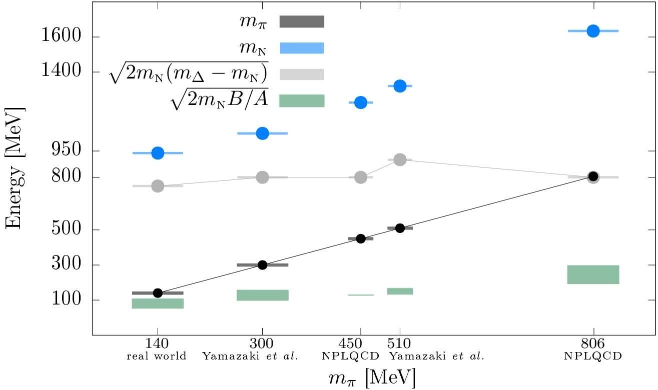
  - few-body Efimov effects
- halo nuclei
  - core + valence nucleons
  - $R_{\rm core} \ll R_{\rm halo}$
  - bound-state structure; nuclear reaction



see e.g., Hammer, CJ, Phillips, J. Phys. G 44 (2017) 103002

- Lattice simulated nuclei
  - $M_\pi \gg 140$ MeV; $B/A \ll m_\pi$



see e.g., Kirscher, Int. J. Mod. Phys. E 25 (2016) 1641001

- Effective Lagrangian with contact interactions in r_0/a_0 expansion

$$\mathscr{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C_0 \left(\psi^\dagger \psi\right)^2 - D_0 \left(\psi^\dagger \psi\right)^3 - C_2 \left[(\psi\psi)^\dagger \left(\psi \nabla^2 \psi\right) + h.c. \right] + \cdots$$

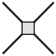
2-body contact (LO)

 $= -iC_0$

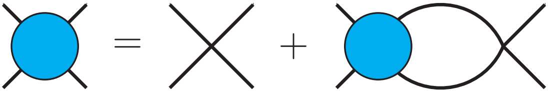
3-body contact (LO)

 $= -iD_0$

2-body contact (NLO)

 $= -iC_2$

- 2body scattering: iteratively sum up C_0 to all loop orders



- use Hubbard-Stratonovich transformation
- introduce a two-body auxiliary field (dimer/dibaryon)

$$\begin{aligned}\mathscr{L} &= \psi^\dagger\left(i\partial_0+\frac{\nabla^2}{2m}\right)\psi + d^\dagger\left[\eta\left(i\partial_0+\frac{\nabla^2}{4m}\right)+\Delta_0\right]d \\ &\quad + g\left[d^\dagger(nn)+\mathrm{h.c.}\right] + h\,(d\psi)^\dagger(d\psi)+\cdots\end{aligned}$$

- 2-body contact (LO)

$= -i\sqrt{2}g$

$g \leftarrow$ 2-body observable

- 3-body contact (LO)

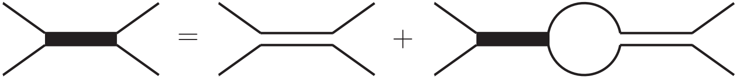
$= ih$

$h \leftarrow$ 3-body observable

Effective Field Theory for Three Identical Bosons

- non-perturbative features in EFT:
  - iteratively sum up g and h to all loop orders
  - calculate 2- and 3-body observables by solving integral equations

- 2-body Lippmann-Schwinger equation (LO, tune g)

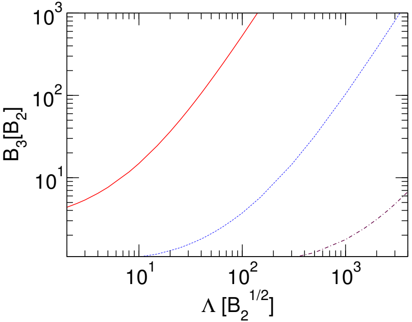


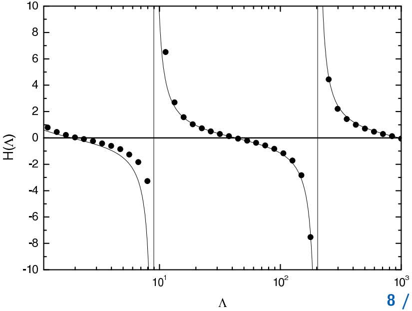
- 3-body Faddeev-STM equation (LO, tune h)

Bedaque, Hammer, van Kolck '99

LO Regularization and Renormalization

- without three-body force
  - 3-body spectrum is cutoff dependent ($\Lambda \sim 1/\ell$) Platter '09
- three-body force in renormalization
  - tune $H(\Lambda) = \Lambda^2 h/2mg^2$ to fix one 3-body observable
  - limit cycle in running coupling: $H(\Lambda)$ periodic for $\Lambda \to \Lambda(22.7)^n$ Bedaque *et al.* '00
  - discrete scaling invariance $\to$ Efimov physics Efimov '71

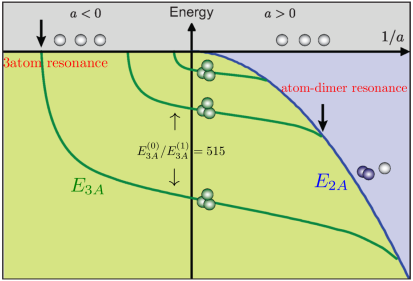




LO Prediction in Efimov Effects

- three-body systems display universal features (Efimov effects)
  - 3-body spectrum: $E_n = f(a)$
- discrete scaling symmetry
  - $E_n = E_{n-1}/\lambda^2$ in the unitary limit $a \to \infty$
  - scattering lengths at resonance/threshold: $a^\star_{(n)} = \lambda a^\star_{(n-1)}$

Discrete scaling in ultracold atoms



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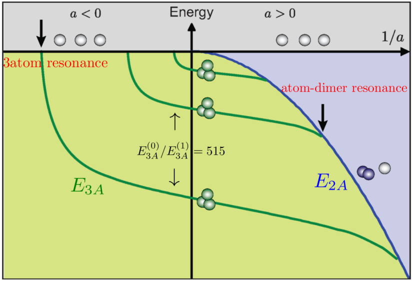

3B systems	m_1/m_2	λ
identical bosons	1	22.7
^{6}Li-Cs-Cs	0.045	4.865
^{7}Li-Cs-Cs	0.052	5.465
^{6}Li-Rb-Rb	0.069	6.835
^{7}Li-Rb-Rb	0.080	7.864
^{40}K-Rb-Rb	0.460	122.7
^{41}K-Rb-Rb	0.471	131.0

Higher Order Corrections in $\not{\pi}$EFT

- effective range corrections to 2body phase shift
  $k \cot \delta_0 = -\frac{1}{a} + \frac{r}{2} k^2 + \cdots$
- perturbative expansion of 2body propagator
  $t_{2b} =$ 
- 2body scattering t-matrix in lab frame

$$t_{2b}(E, \boldsymbol{P}) = -\frac{4\pi}{mg^2}\left(-\gamma_0 + \sqrt{\boldsymbol{P}^2/4 - mE - i\epsilon}\right)^{-1}$$
$$\times \left[1 + \frac{r_0}{2}\left(\gamma_0 + \sqrt{\boldsymbol{P}^2/4 - mE - i\epsilon}\right) + \frac{r_0^2}{4}\left(\gamma_0 + \sqrt{\boldsymbol{P}^2/4 - mE - i\epsilon}\right)^2\right]$$

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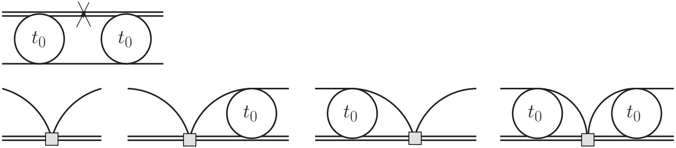
  $$\times \left[1 + \frac{r_0}{2} \left(\gamma_0 + \sqrt{\boldsymbol{P}^2/4 - mE - i\epsilon} \right) + \frac{r_0^2}{4} \left(\gamma_0 + \sqrt{\boldsymbol{P}^2/4 - mE - i\epsilon} \right)^2 \right]$$
- fixed scattering length (light nuclei, Helium molecules):
  - Hammer, Mehen '01; Vanasse, Egolf, Kerin, König, Springer '14; (NLO)
  - Bedaque, Rupak, Grießhammer, Hammer '03; Platter, Phillips '06; CJ, Phillips '13; Vanasse '13; Margaryan, Springer, Vanasse '15; Vanasse '17 (N^2LO)
- varied scattering length (ultracold atoms)
  - CJ, Platter, Phillips '09, '10; '12; CJ, Braaten, Phillips, Platter '15; Archarya, CJ, Platter '16 (NLO)

NLO Corrections in $\not{\pi}$EFT

- at next-to-leading-order $\mathcal{O}(r_0/a_0)$:
  - insert NLO piece into STM equation (1st order perturbation)



- NLO 3-body force:

  $H_1(\Lambda) = r_0\Lambda\ h_{10}(\Lambda) + r_0/a\ h_{11}(\Lambda)$
  - a **fixed:** h_{11} is absorbed (renormalise to LO 3-body parameter)
  - a **varies:** one additional 3-body parameter in renormalization

Running of Three-Body Coupling

- LO: 3BF is a log-periodic function of the cutoff: $H_0[\ln(\Lambda/\kappa_*)]$

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$$H(\Lambda) = H_0[\ln(\Lambda/\kappa_*)] + \eta\, H_0'[\ln(\Lambda/\kappa_*)]\,\ln(\Lambda/Q)\frac{r_0}{a}$$

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$$H(\Lambda) = H_0[\ln(\Lambda/\kappa_*)] + \eta\, H_0'[\ln(\Lambda/\kappa_*)]\, \ln(\Lambda/Q)\frac{r_0}{a}$$

- We treat the r_0/a term as a perturbation, and rewrite 3BF:

$$H(\Lambda) = H_0\left[\ln(\Lambda/\bar{\kappa}_*(Q,a))^{1+\eta r_0/a}\right]$$

- $\bar{\kappa}_*(Q,a) = (Q/\kappa_*)^{\eta r_0/a}\kappa_*$ is a running Efimov parameter at NLO

Limit Cycle & Renormalization-Group Improvement

- LO 3BF: RG limit cycle $\rightarrow$ discrete scaling symmetry λ^n
- Universal correlations: $a_{i,n} = \lambda^n \theta_i \kappa_*^{-1}$

CJ, Braaten, Phillips, Platter, PRA 92 (2015) 030702

Limit Cycle & Renormalization-Group Improvement

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- Universal correlations: $a_{i,n} = \lambda^n \theta_i \kappa_*^{-1}$

- NLO 3BF: RG range-modification $\to$ discrete scaling breaking
- insert running parameter $\bar{\kappa}_*(Q, a)$ into LO relation yields:

$$a_{i,n} = \lambda^n \theta_i (\lambda^n |\theta_i|)^{\eta r_0 \kappa_* / (\lambda^n \theta_i)} \kappa_*^{-1} + \tilde{J}_i r_0$$

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- expand $a_{i,n}$ up to linear-in-r_0 correction

$$a_{i,n} = \lambda^n \theta_i \kappa_*^{-1} + (J_i + n\,\eta \ln \lambda) r_0$$

- the above pattern has been verified numerically

CJ, Braaten, Phillips, Platter, PRA 92 (2015) 030702

Benchmark Range-Corrected Relations

- the three-body recombination resonance a_- in different Efimov states share a universal relation
  - LO: $(a_{-,n+1}/a_{-,n})/\lambda = 1$
  - NLO: $(a_{-,n+1}/a_{-,n})/\lambda \neq 1$ due to finite-range effects

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  - NLO: $(a_{-,n+1}/a_{-,n})/\lambda \neq 1$ due to finite-range effects
- benchmark with: Deltuva PRA 2012

  momentum-space short-range separable potential models

n	*0*	*1*	*2*	*3*	*4*
Deltuva 2012	***0.7822***	***0.9665***	***0.9976***	***0.9999***	***1.0000***
NLO	***0.7822***	***0.9665***	***0.9975***	***0.9998***	***1.0000***
RG-NLO	***0.7822***	***0.9665***	***0.9975***	***0.9998***	***1.0000***

Running Efimov Parameter in AAB Systems

- the range-corrected running Efimov parameter also exists in AAB heteronuclear atomic mixtures
- we predict universal range effects to Efimov states in AAB systems

$$a_{i,n} = \lambda^n \theta_i \kappa_*^{-1} + (J_i + n\sigma) r_{AB} + (Y_i + n\bar{\sigma}) a_{AA}$$

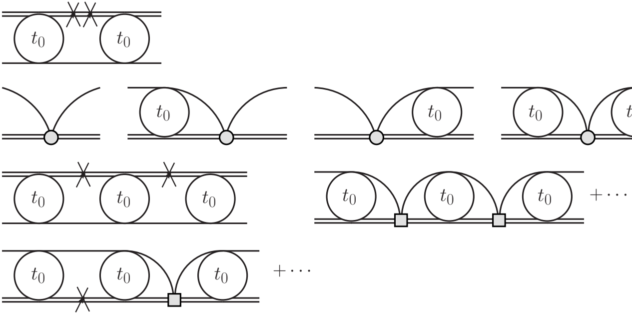
3body recombination maximum

^{6}Li-Cs-Cs	$a_-^{(0)}$	$a_-^{(1)}$	$a_-^{(2)}$	$a_-^{(3)}$
experiment†	-350	-1777	-9210	-46635
theory⋆	-267	-1677	-9210	-46635

† Ulmanis *et al.* PRA '16   ⋆ Acharya, C.J., Platter, PRA '16

N^2LO Corrections in $\not\pi$EFT

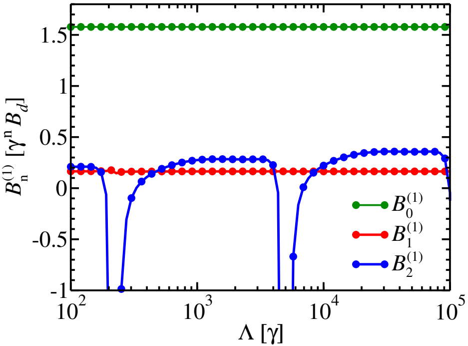
- at next-to-next-to-leading-order $\mathcal{O}(r_0^2/a_0^2)$:
  - single insertion of N^2LO piece & double insertion of NLO piece (2nd order perturbation)



- energy-dependent three-body force at N^2LO
  - $\mathcal{H}_2 = r_0^2 \Lambda^2 h_{20} + r_0^2 m E_t h_{22}$

    $\rightarrow$ one additional 3-body parameter is needed

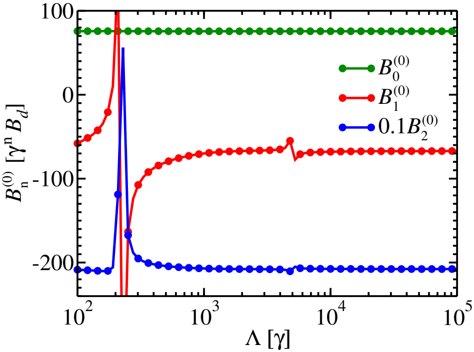
- $H_2 = r^2\Lambda^2 \, h_{20}$



$a_{ad} \rightarrow$ LO/NLO/N^{2}LO

$B_t^{(1)} = B_0^{(1)} + rB_1^{(1)} + r^2 B_2^{(1)}$

- $H_2 = r^2\Lambda^2 \, h_{20} + r^2 m E_t \, h_{22}$



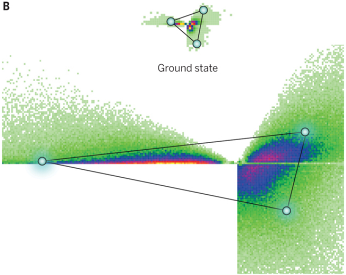
$a_{ad} \rightarrow$ LO/NLO/N^{2}LO

$B_t^{(1)} \rightarrow$ N^{2}LO

$B_t^{(0)} = B_0^{(0)} + rB_1^{(0)} + r^2 B_2^{(0)}$

Efimov states in helium trimers

The existence of an excited helium trimer, whose binding energy is much shallower than the ground state:

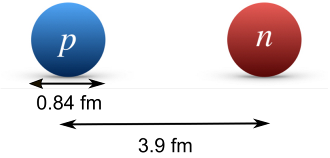


Kunitski *et al.*, Science 2015

Input		$B_t^{(1)}/B_d$	$B_t^{(0)}/B_d$	γa_{ad}
TTY potential		1.738	96.33	1.205
a_{ad}	LO	1.723	97.12	1.205
a_{ad}	NLO	1.736	89.72	1.205
a_{ad}, $B_t^{(1)}$	N^2LO	1.738	116.9	1.205
$B_t^{(1)}$	LO	1.738	99.37	1.178
$B_t^{(1)}$	NLO	1.738	89.77	1.201
$B_t^{(1)}$, a_{ad}	N^2LO	1.738	115.9	1.205

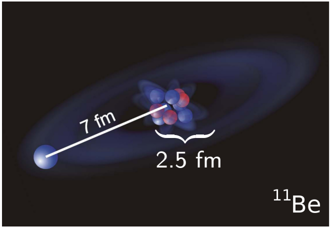
C.J., Phillips, Platter Few-Body Syst. 2013

- ^{2}H
  - simplest neutron halo



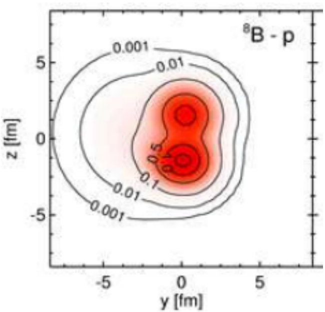
The Applications of EFT to Halo Nuclei

- ^{2}H
  - simplest neutron halo
- neutron halos with a compound nuclear core
  - ^{6}He, ^{11}Be, . . .



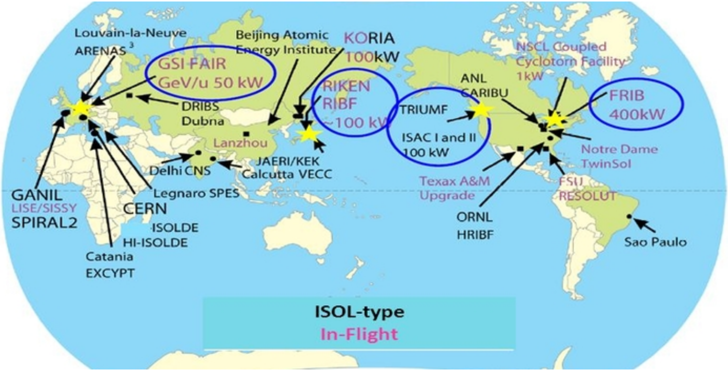
The Applications of EFT to Halo Nuclei

- ^{2}H
  - simplest neutron halo
- neutron halos with a compound nuclear core
  - ^{6}He, ^{11}Be, ...
- proton halos
  - ^{17}F* (s-wave halo)
  - ^{8}B (p-wave halo)



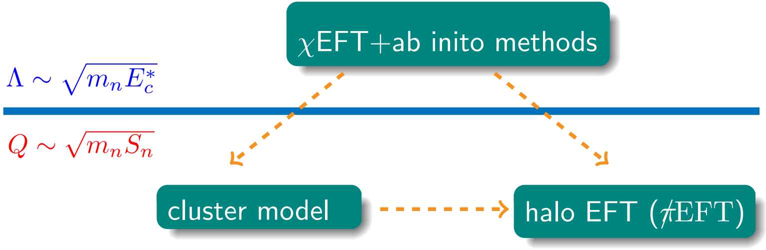
FMD calculation (T. Neff, GSI)

study neutron/proton rich nuclei beyond the nuclear dripline



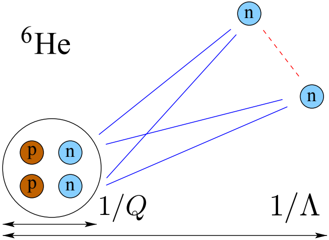
future facilities in China: BISOL & HIAF

Halo Physics Near Clustering Threshold



Halo EFT

- core + valence nucleons clustering configuration
- systematic expansion in Q/Λ
- short-range effects from underlying theory are embedded in LECs
  - e.g., anti-symmetrization of core nucleons



Halo Effective Field Theory

- Based on $\not{\pi}$EFT, we use contact interactions to describe the clustering mechanism in halo nuclei
- introduce auxiliary dimer fields for nn and nc pairs

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3 \\
\mathscr{L}_1 &= n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c \\
\mathscr{L}_2 &= s^\dagger \left[\eta_0 \left(i\partial_0 + \frac{\nabla^2}{4m_n} \right) + \Delta_0 \right] s + \sigma^\dagger \left[\eta_1 \left(i\partial_0 + \frac{\nabla^2}{2(m_n + m_c)} \right) + \Delta_1 \right] \sigma \\
&\quad + g_0 \left[s^\dagger (nn) + \text{h.c.} \right] + g_1 \left[\sigma^\dagger (nc) + \text{h.c.} \right], \\
\mathscr{L}_3 &= h \, (\sigma n)^\dagger \, (\sigma n)
\end{aligned}
$$

- 2-body contact (LO)

$= -i\sqrt{2}g$

$g \leftarrow$ 2-body observable

- 3-body contact (LO)

$= ih$

$h \leftarrow$ 3-body observable

Two Neutron Rich Halo Nuclei

- $2n$ halo with s-wave neutron-core interactions

  similarity to Efimov physics

  - ^{11}Li, ^{12}Be, ^{20}C [Canham, Hammer, EPJA '08, NPA '10]
  - ^{22}C [Yamashita et al., PLB '11; Acharya, CJ, Phillips, PLB '13]
  - ^{62}Ca [Hagen, Hagen, Hammer, Platter, PRL '13]

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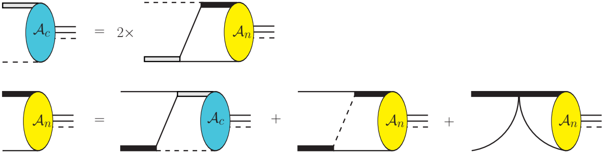
- $2n$ halo with p-wave neutron-core interactions

  universality beyond Efimov effects

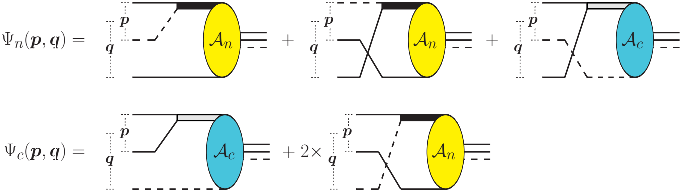
  - ^{6}He [Rotureau, van Kolck, FBS '13; CJ, Elster, Phillips, PRC '14]

Two Neutron Halos in Faddeev Formalism

- coupled-channel Faddeev equations



- three-body wave functions



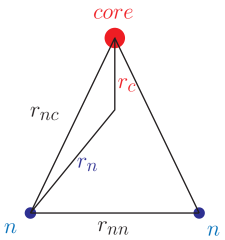
Matter and Charge Radii of 2n Halos

- two-body form factors

$$F_{2b}(k^2) = \iint d^3p d^3q \Psi_i^\dagger(\boldsymbol{p}, \boldsymbol{q}) \Psi_i(\boldsymbol{p} + \boldsymbol{k}, \boldsymbol{q})$$

- low-momentum expansion & pair distance

$$F_{2b}(k^2) = 1 - \frac{1}{6} \langle r_{2b}^2 \rangle k^2 + \cdots$$



Matter and Charge Radii of 2n Halos

- two-body form factors

$$F_{2b}(k^2) = \iint d^3p d^3q \Psi_i^\dagger(\boldsymbol{p},\boldsymbol{q})\Psi_i(\boldsymbol{p}+\boldsymbol{k},\boldsymbol{q})$$

- low-momentum expansion & pair distance

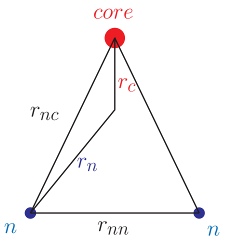
$$F_{2b}(k^2) = 1 - \frac{1}{6}\langle r_{2b}^2\rangle k^2 + \cdots$$

- matter radius (with point-like core)

$R_{\mathrm{matter}} = \sqrt{A\, r_c^2 + 2\, r_n^2}\,/\,(A+2)$

- charge radius (with point-like protons)

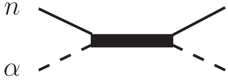
$R_{\mathrm{charge}} = r_c$



^{6}He: $2n$ Halo with P-Wave nc Interactions

- *ab intio* **calculation**
  - no-core shell model Navrátil *et al.* '01; Sääf, Forssén '14
  - NCSM-RGM/Continuum Romero *et al.* '14 '16
  - Green's function Monte Carlo Pieper *et al.* '01; '08
  - hyperspherical harmonics (EIHH) Bacca *et al.* '12
- **Halo EFT in ^{6}He ground state**
  - EFT+Gamow shell model Rotureau, van Kolck Few Body Syst. '13
  - EFT+Faddeev equation C.J., Elster, Phillips, PRC '14
  - different power-counting Ryberg, Forssén, Platter, arXiv:1701.08576

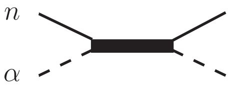
- $n - \alpha$ scattering is dominated by the $3/2^-$ resonance state



$$= \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

$n - \alpha$ P-Wave Interactions

- $n - \alpha$ scattering is dominated by the $3/2^-$ resonance state



$$= \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

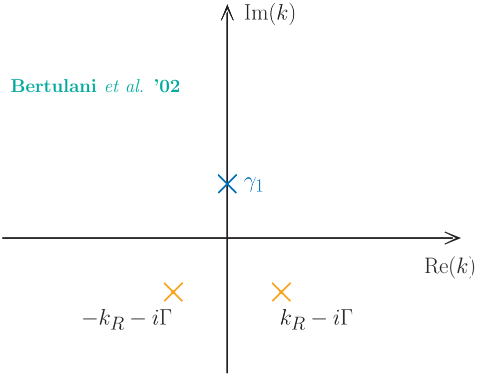
- causality $r_1 \not\to 0$ Nishida '12
- both a_1 and r_1 enter at leading order
- p-wave power counting
  - resum ik^3: $1/a_1 \sim Q^3$, $r_1 \sim Q$
    [Bertulani, Hammer, van Kolck NPA '02]
  - perturbative ik^3: $1/a_1 \sim Q^2\Lambda$, $r_1 \sim \Lambda$
    [Bedaque, Hammer, van Kolck PLB '03]

P-Wave Power Counting: ik^3 Resummation

ik^3 resummation: Bertulani, Hammer, van Kolck NPA '02

- $1/a_1 \sim Q^3 \quad r_1 \sim Q$
- two fine tunings at LO

- for $a_1 < 0$: ^{5}He $(3/2^-)$
  - a broad shallow resonance
  - a shallow bound state (spurious)



P-Wave Power Counting: Perturbative ik^3

perturbative ik^3:

Bedaque, Hammer, van Kolck PLB '03

- $1/a_1 \sim Q^2\Lambda \quad r_1 \sim \Lambda$
- one fine tuning at LO

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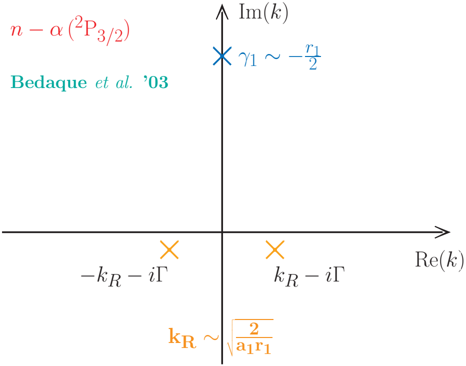

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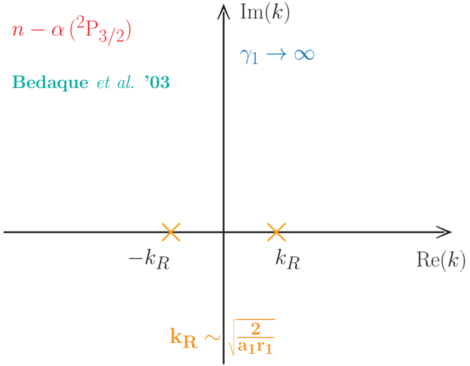

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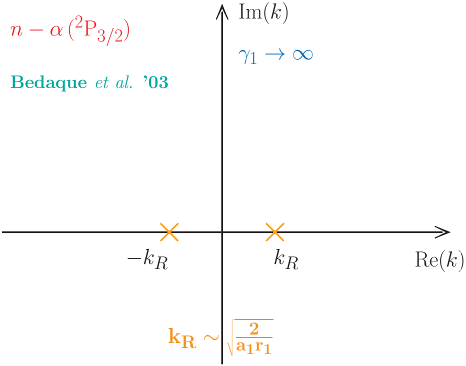
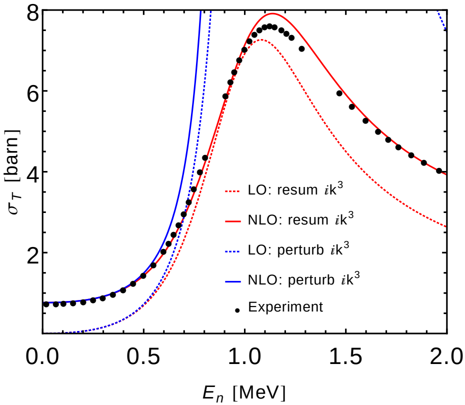
  - a narrow shallow resonance
  - a deep bound state (unphysical)
  - drop ik^3 term at LO: $\Gamma \to 0$, $\gamma_1 \to \infty$
- perturbative ik^3 is only rigorous for $|k - k_R| \gg Q^2/\Lambda$

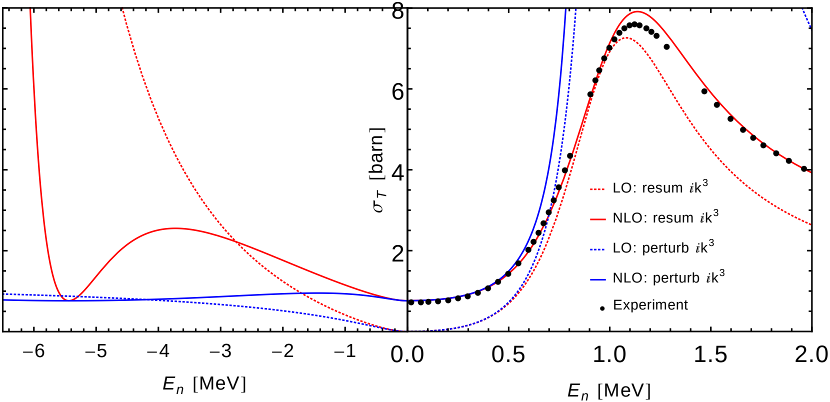


$n - \alpha$ Scattering Cross Sections



- ik^3 resummation works better when E is close to E_R

$n - \alpha$ Scattering Cross Sections



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- ik^3 resummation creates spurious bound-state pole; perturbative ik^3 does not

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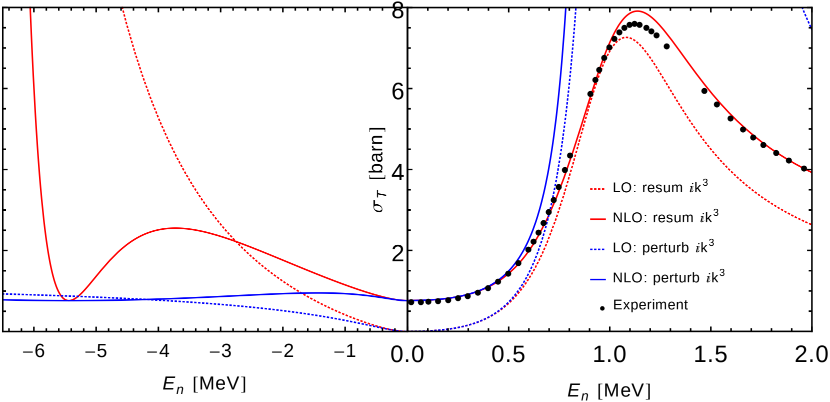
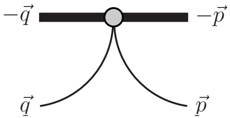


- ik^3 resummation works better when E is close to E_R
- ik^3 resummation creates spurious bound-state pole; perturbative ik^3 does not
- In ^{6}He three-body calculation, $E \to -\Lambda$ in loop integral
  - Rotureau, van Kolck, FBS '13: ik^3 resummation + spurious pole subtraction
  - C.J., Elster, Phillips, PRC '14: perturbative ik^3 (free from spurious pole)

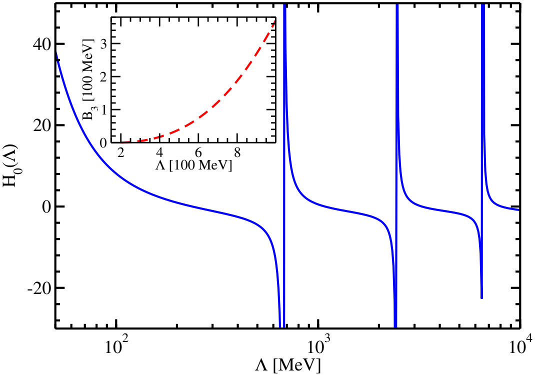
Running of P-Wave 3-Body Coupling

- p-wave 3body coupling:

  reproduce $S_{2n} = 0.973$ MeV



$$= M_n \, qp \, \frac{H(\Lambda)}{\Lambda^2}$$



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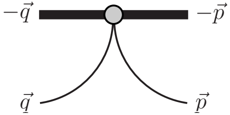
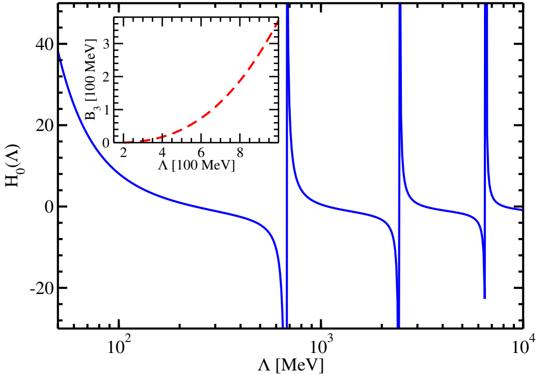
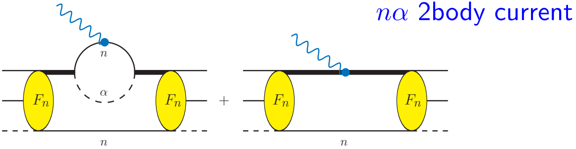


$$= M_n \, qp \, \frac{H(\Lambda)}{\Lambda^2}$$

- discrete scaling symmetry is broken due to p-wave interactions

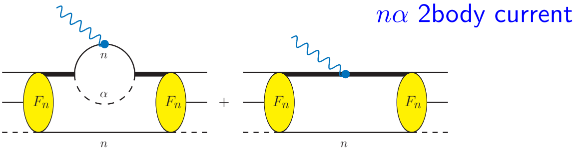


- 3-body form factor (with p-wave nc interactions)



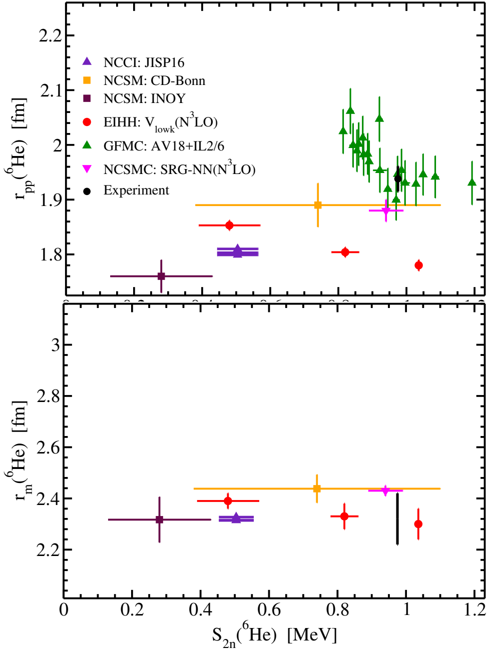
^{6}He Density Form Factors

- 3-body form factor (with p-wave nc interactions)



- The two-body current is needed for gauge invariance in p-wave channels
- The two-body current counterterm is fixed by parameter r_1 in $n\alpha$ $3/2^-$ state

Universal Correlations Among Radii & S_{2n} in ^{6}He



- He-6 charge radius
- He-6 matter radius

compare with ab initio calculations

NCCI: Caprio, Maris, Vary, PRC '14

NCSM: Caurier, Navratil, PRC '06

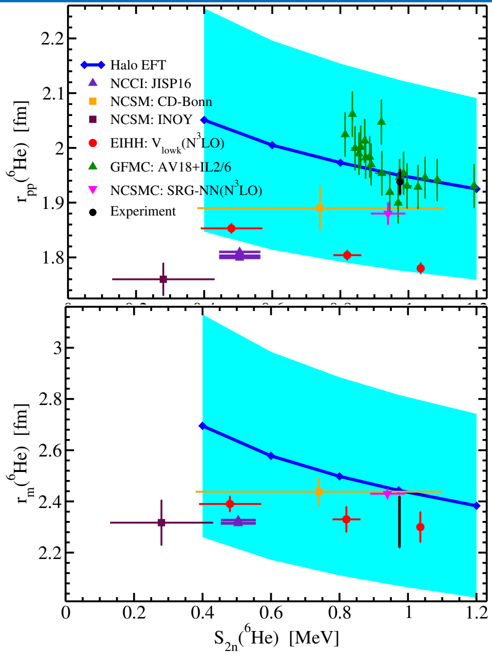
EIHH: Bacca, Barnea, Schwenk, PRC '12

GFMC: Pieper, RNC '08

NCSMC: Romero et al., PRL '16

Halo EFT: preliminary (uncertainty)

Universal Correlations Among Radii & S_{2n} in ^{6}He



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Halo EFT: preliminary (uncertainty)

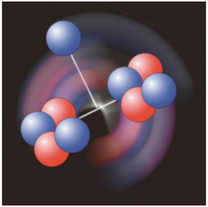
Summary

- Physics at different scales can be connected by universality in few-body systems
- $\not\pi$EFT aims to describe low-energy features in systems with short-range interactions
  - Efimov physics in ultracold atoms
  - halo nuclei beyond the nuclear dripline
- Using the systematic expansion of the Q/Λ, $\not\pi$EFT can predict few-body observables with controlled theoretical uncertainties

Combining Halo EFT with Many-Body Techniques

- ^{9}Be $\alpha - \alpha - n$
  - construct α-α and n-α interaction from halo EFT
  - combine with hyperspherical harmonics method and Lorentz integral transform
  - calculate photoabsorption cross section
  - study astrophysical reaction rate $\alpha(\alpha n,\gamma)^9$Be

with Manzata, Andreatta, Orlandini, Leidemann



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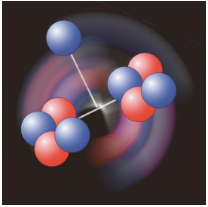
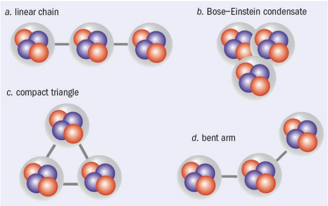


- α clustering in light/medium nuclei
  - ^{12}C $\rightarrow$ ^{16}O $\rightarrow \cdots \rightarrow$ ^{40}Ca
  - EFT potential for $\alpha\alpha$ interactions + Diffusion Monte Carlo
  - explore Efimov physics in α-clustering

  with van Kolck, Pederiva



Collaborations

D. R. Phillips; C. Elster — OHIO UNIVERSITY 1804

B. Acharya — THE UNIVERSITY of TENNESSEE KNOXVILLE

Z.-Z. Ren; M.-J. Lyu; L.-Y. Zhang — 南京大學 NANJING UNIVERSITY

G. Orlandini; W. Leidemann — TIFPA INFN Istituto Nazionale di Fisica Nucleare; UNIVERSITÀ DEGLI STUDI DI TRENTO

F. Pederiva

U. van Kolck — IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY; THE UNIVERSITY OF ARIZONA

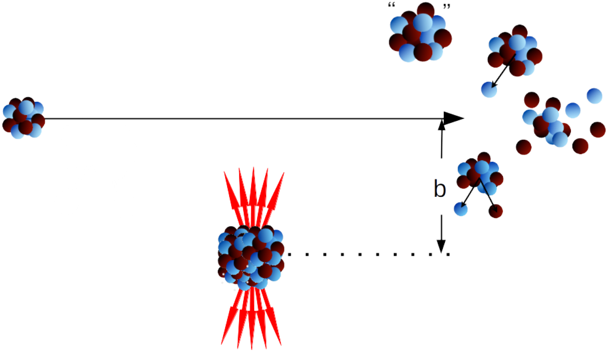
H.-W. Hammer — TECHNISCHE UNIVERSITÄT DARMSTADT

One-neutron s-wave halos

	^{2}H	^{11}Be	^{15}C	^{19}C
EXP				
S_{1n} [MeV]	2.224573(2)	0.50164(25)	1.2181(8)	0.58(9)
E_c^* [MeV]	140	3.36803(3)	6.0938(2)	1.62(2)
$\langle r_{nc}^2 \rangle^{1/2}$ [fm]	3.936(12)	6.05(23)	4.15(50)	6.6(5)
	3.95014(156)	5.7(4)	7.2±4.0	6.8(7)
		5.77(16)	4.5(5)	5.8(3)
EFT				
Q/Λ	0.33	0.39	0.45	0.6
r_0/a_0	0.32	0.32	0.43	0.33
$C_\sigma/C_{\sigma,LO}$	1.295	1.3	1.63	1.3
$\langle r_{nc}^2 \rangle^{1/2}$ [fm]	3.954	6.85	4.93	5.72

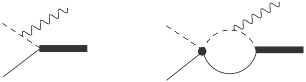
Coulomb dissociation in $1n$ halos

- Coulomb dissociation
  - breakup by colliding a halo nucleus with a high-Z nucleus
  - the halo dynamics dominates when $Q_\gamma \sim Q$

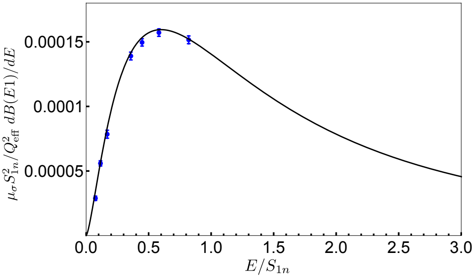


EFT on Coulomb dissociation

- **E1 transition**



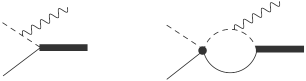
$$\frac{\mu S_{1n}^2}{Q_{eff}^2}\frac{dB(E1)}{dE} = \frac{C_\sigma^2}{C_{\sigma,LO}^2}\frac{3\alpha_{em}}{\pi^2}\frac{(E/S_{1n})^{3/2}}{(E/S_{1n}+1)^4}$$



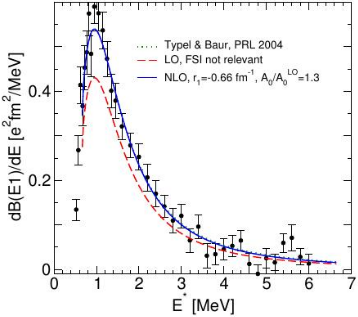
deuteron E1 strength

Hammer, CJ, Phillips, J. Phys. G 44 (2017) 103002

- **E1 transition**

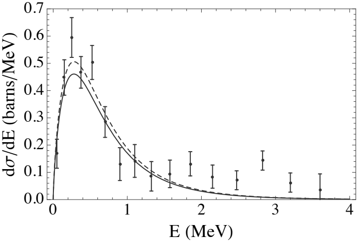


^{11}Be photo-dissociation



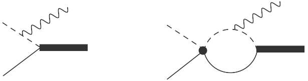
[Hammer, Phillips, NPA '11]

^{19}C photo-dissociation

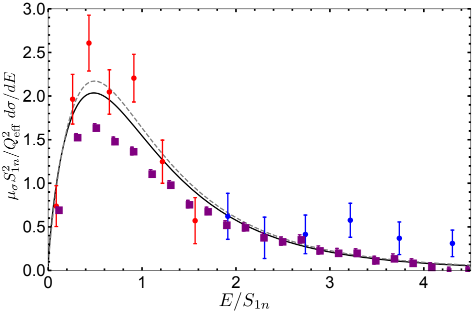


[Acharya, Phillips, NPA '13]

- **E1 transition**



$$\frac{\mu S_{1n}^2}{Q_{eff}^2}\frac{d\sigma}{dE} = \frac{16\pi^3}{9} N_{E1}(E,R)\left(\frac{\mu S_{1n}^2}{Q_{eff}^2}\frac{dB(E1)}{dE}\right)$$

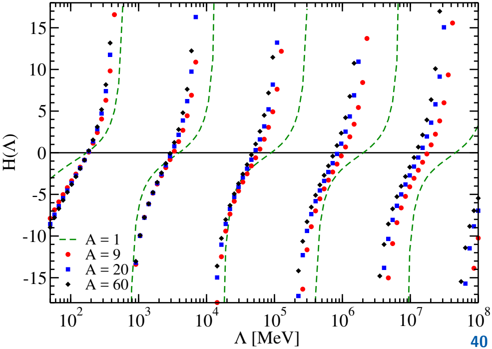


Coulomb dissociation differential cross section in ^{11}Be and ^{19}C
^{11}Be: fit to ANC in ab-initio NCSMC [Calci *et al.* PRL '16]

Hammer, CJ, Phillips, J. Phys. G 44 (2017) 103002

- **running of three-body coupling**
  - tune $H(\Lambda) = \Lambda^2 h/2mg^2$:

    reproduce one observable in a $2n$-halo



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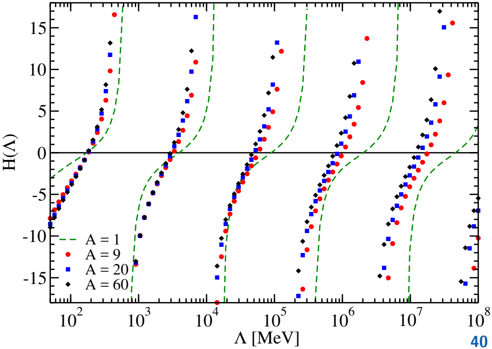
Three-body renormalization

- **running of three-body coupling**
  - tune $H(\Lambda) = \Lambda^2 h / 2mg^2$:

    reproduce one observable in a $2n$-halo
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]

Hammer, CJ, Phillips,

J. Phys. G 44 (2017) 103002



Three-body renormalization

- **running of three-body coupling**
  - tune $H(\Lambda) = \Lambda^2 h/2mg^2$:

    reproduce one observable in a $2n$-halo
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]
  - $H(\Lambda)$ appears as RG limit cycle [Mohr et al., AnnPhys '06]
  - discrete scale invariance $\to$ Efimov physics

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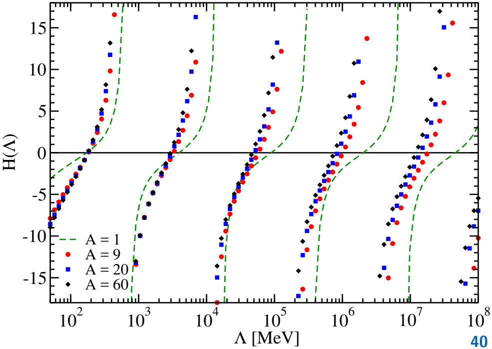
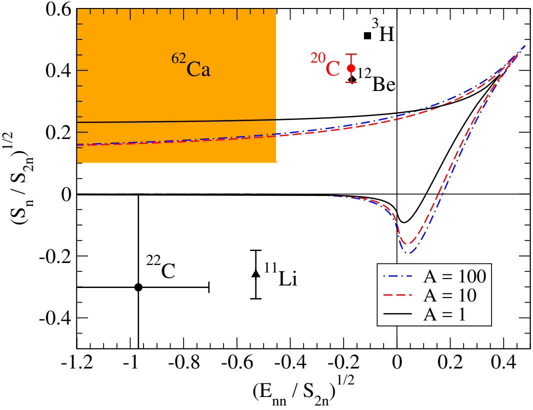


- contour constraints on ground-state energy S_{2n} if the excited-state energy $B_3^* = \max\{0, E_{nn}, S_{1n}\}$



Canham, Hammer, EPJA '08; Frederico *et al.* PPNP '12;

^{22}C: s-wave $2n$ Halo

	^{20}C	^{21}C	^{22}C
bound/unbound	bound	unbound	bound
ground state	0^+	$S_{1/2}$	0^+
binding/virtual energy [MeV]	S_{2n}: $3.50(24)$ AME2012	S_{1n}: $-0.01(47)$ AME2012 < -2.9 Mosby et al. '13	S_{2n}: $0.11(6)$ AME2012 S_{2n}: $-0.14(46)$ Gaudefroy et al. '12
matter radius r_m [fm]	$2.97^{+0.03}_{-0.05}$ Togano et al. '16		3.44(8) Togano et al. '16

Correlations in ^{22}C

use EFT universal correlations to constrain undetermined quantities

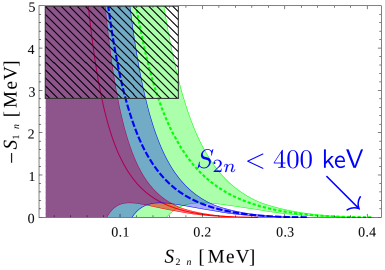
$$\langle r_m^2 \rangle_{2n-\text{halo}} = \frac{1}{m_n S_{2n}} f\left(\frac{E_{nn}}{S_{2n}}, \frac{S_{1n}}{S_{2n}}; A\right)$$

Acharya, C.J., Phillips, PLB '13

experimental input:

$\langle r_m^2 \rangle_{2n-\text{halo}} - \frac{10}{11} \langle r_m^2 \rangle_{\text{core}} = 3.81^{+0.82}_{-0.71}$ fm^2

Togano *et al.* PLB '16



Other experimental bound:

- AME2012
  $S_{2n} < 170$ keV
- Gaudefroy et al., PRL '12
  $S_{1n} < -2.9$ MeV

bands: uncertainty from NLO EFT