

Effective Potentials between Heavy Mesons at One-Loop Level and Possible Molecular States



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Lanzhou University

Peking University

My first invited talk at conferences

- Thank organizers for the opportunity

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- Met χ EFT since 2009 at the very beginning of graduate study
 - Scattering length between a Goldstone boson and a hadron
within χ EFT
 - Effective potentials between two hadrons
within χ EFT
 - Spectra of nucleon excited states
with Hamiltonian EFT
 - Corrections to axial couplings and electromagnetic moments
with χ EFT

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 - deepen understanding of SM
 - easy to learn some phenomenological models

leading order

general form

v.s.

dominate contributions

CONTENTS

1. Introduction
2. Effective potentials between heavy mesons
3. Possible molecular states

Introduction

Doubly charmed hadrons

- Discoveries of Ξ_{cc} : (ccq)

Ξ_{cc}^+ was discovered by SELEX collaboration, and

has been confirmed by LHCb group but with different mass this year

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- Interests on exotic states: ($c\bar{c}q\bar{q}$)

keeping update of X, Y, Z states from experiment

debate with different interpretations:

molecules? tetraquark? ordinary charmonium? two diquark?

Doubly charmed hadrons

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 - keeping update of X, Y, Z states from experiment
 - debate with different interpretations:
 - molecules? tetraquark? ordinary charmonium? two diquark?
- Possible states: ($cc\bar{q}\bar{q}$)
 - no mixing with ordinary charmoniums

Molecular states made of two heavy mesons

- Not unfamiliar

Deuteron is a famous molecule made of a proton and neutron

- Interactions are important

attractive interaction

- Different models and approaches

- one-boson-exchange model
- lattice study
- QCD sum rule
- diquark model
- ...

Study of Interactions within χ EFT

- EFT with respect on symmetries of QCD
- Power counting
 - systematically study, order by order, error controlled, check of SM
- Natural extension
 - 2-body force, 3-body force,...
- Wide applications

- Dealing systems with light mesons

χ EFT results can be expanded as power series of

$$m_\phi/\Lambda_\chi, q/\Lambda_\chi, \dots$$

- Power Counting Breaking (PCB) in systems with heavy hadrons involved

large masses of heavy hadrons make q^μ is never small again

expanded with the help of residual momentum \tilde{q}^μ

$$\tilde{q}^\mu = q^\mu - m(1, \vec{0}).$$

Solutions for systems with one heavy hadron

- **Heavy hadron EFT**

nonrelativistic reduction at Lagrangian level, breaking of analyticity.

Simple and still correct if not analytically extending results too far away

- **Infrared regularization**

relativistic Lagrangian, drop PCB terms at regularization

good power counting and analyticity

- **Extended on-mass-shell scheme**

relativistic Lagrangian, drop PCB terms at final results

good power counting and analyticity

Results with three different schemes will be same if

- being summarized at ALL orders, or
- the mass of heavy hadron becomes infinite.

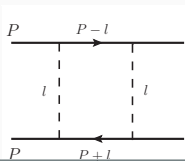
χ EFT with few hadrons involved—new trouble

The amplitude of following 2-Particle-Reducible diagram contains ¹

$$\mathcal{I} \equiv i \int d\rho \frac{i}{\rho + P^0 + i\epsilon} \frac{i}{-\rho + P^0 + i\epsilon} = \frac{\pi}{P^0 + i\epsilon} \approx \frac{\pi}{\vec{P}^2 / (2m_N) + i\epsilon}. \quad (1)$$

- naïve power counting scheme $\rightarrow \mathcal{I} \sim O(1/|\vec{P}|)$
- eq. (1) $\rightarrow \mathcal{I} \sim O(m_N/|\vec{P}|^2)$

\mathcal{I} is actually enhanced by a large factor $m_N/|\vec{P}|$.



Solid line for nucleon, dashed line for pion.

(P represents the residual momentum)

Box Diagram.

¹we have not listed the parts preserving power counting

Weinberg scheme

- not directly calculate physical observables with perturbation theory
- systematically study effective potentials first (without 2PR contribution)
- solve the dynamical equation to get the physical observables (equivalent to recover the 2PR contributions)

Effective potentials between heavy mesons

With Heavy Meson EFT, we study the systems made up of

- DD
- D^*D
- D^*D^*

Similar for $B^{(*)}B^{(*)}$ and corresponding anti-meson pair system.

We have not studied systems like $D\bar{D}$ because there exist annihilation effects and it is beyond the ability of heavy meson EFT.

- **Leading order vertex**

contact terms: $D^{(*)}D^{(*)}D^{(*)}D^{(*)}$ vertex

$D^{(*)}D^{(*)}\pi$, $D^{(*)}D^{(*)}\pi\pi$ vertex

- **Next-to-leading order vertex**

they absorb divergences, provide finite higher-order corrections

$$\begin{aligned}\mathcal{L}_{4H}^{(0)} &= D_a \text{Tr} [H\gamma_\mu \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] + D_b \text{Tr} [H\gamma_\mu \gamma_5 \bar{H}] \text{Tr} [H\gamma^\mu \gamma_5 \bar{H}] \\ &\quad + E_a \text{Tr} [H\gamma_\mu \lambda^a \bar{H}] \text{Tr} [H\gamma^\mu \lambda_a \bar{H}] + E_b \text{Tr} [H\gamma_\mu \gamma_5 \lambda^a \bar{H}] \text{Tr} [H\gamma^\mu \gamma_5 \lambda_a \bar{H}],\end{aligned}$$

$$\mathcal{L}_{H\phi}^{(1)} = -\langle (iv \cdot \partial H) \bar{H} \rangle - \langle H v \cdot \Gamma \bar{H} \rangle + g \langle H \psi \gamma_5 \bar{H} \rangle - \frac{1}{8} \Delta \langle H \sigma^{\mu\nu} \bar{H} \sigma_{\mu\nu} \rangle,$$

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$$\begin{aligned}\mathcal{L}_{4H}^{(2)} &= D_a^h \text{Tr} [H\gamma_\mu \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] \text{Tr} (\chi_+) + \dots \\ &+ D_a^d \text{Tr} [H\gamma_\mu \tilde{\chi}_+ \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] + \dots \\ &+ D_1^g \text{Tr} [(D^\mu H)\gamma_\mu \gamma_5 (D^\nu \bar{H})] \text{Tr} [H\gamma_\nu \gamma_5 \bar{H}] + \dots\end{aligned}$$

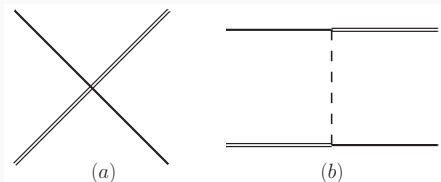
Diagrams

- **Leading order**

contact, one-pion exchange

- **Next-to-leading order**

two-pion exchange, renormalization to $D^{(*)}D^{(*)}\pi$ coupling, loop corrections to contact term, tree diagrams with NL vertice



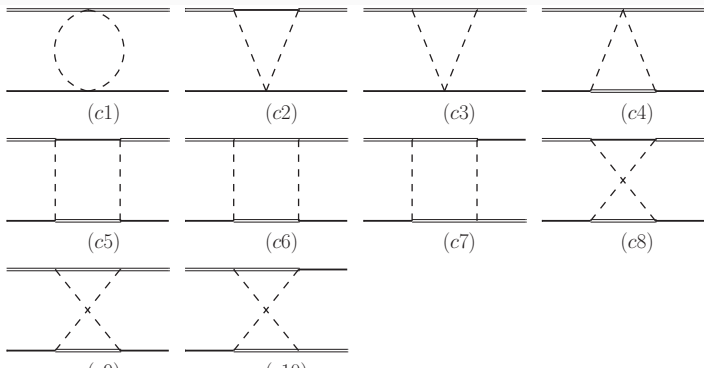
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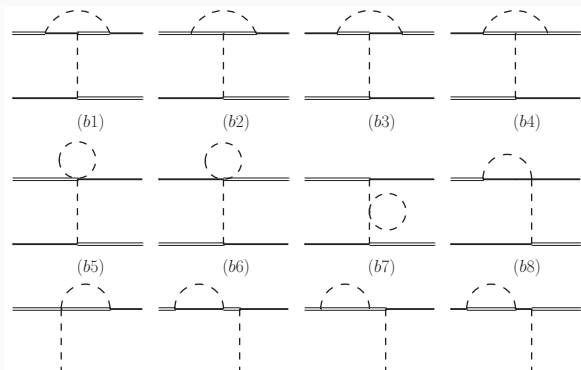
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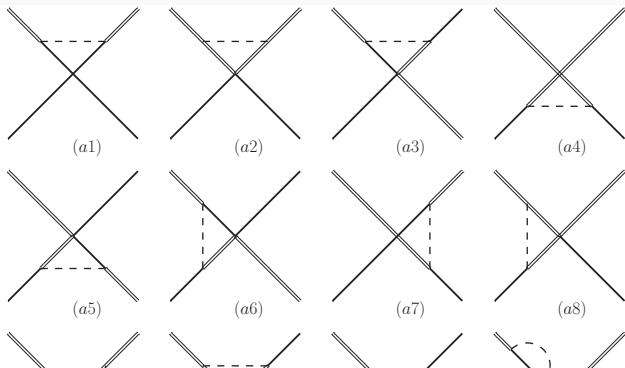
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Regularization and renormalization

We calculate diagrams with dimension regularization and modified minimal subtraction scheme.

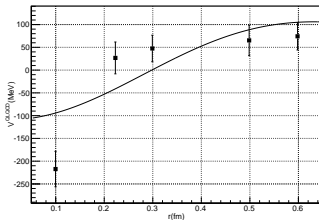
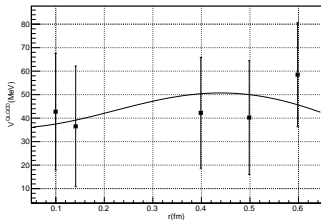
We have checked that the potentials are finite after the renormalization of the wavefunctions and vertice.

Determination of low-energy constants

- fit to experimental data
- first principle of QCD
- fit to data of Lattice QCD
- phenomenological models

Determination of low-energy constants

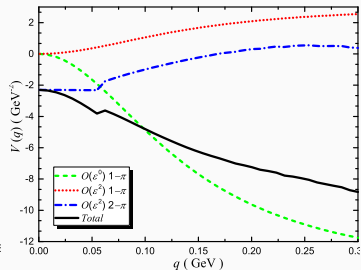
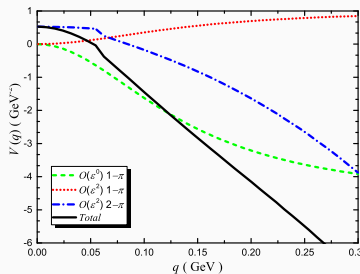
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Determination of low-energy constants

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Effective potentials in momentum space



DD^*

Possible molecular states

Search for new states

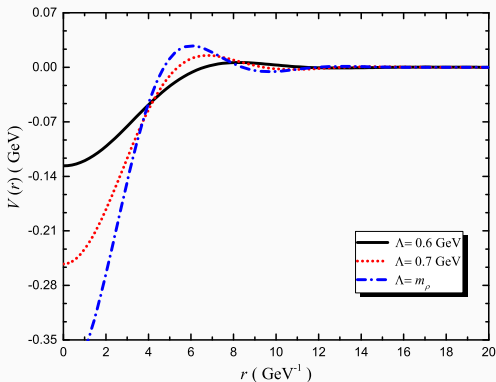
- Potentials \rightarrow partial waves, dynamical equation (momentum space)
 \rightarrow T matrices \rightarrow poles
- Potentials \rightarrow Fourier transform, dynamical equation (coordinate space)
 \rightarrow eigenvalues of bound states for different partial waves

Taking DD^* as an example

$l = 0$: bound state with around $E = 21$ MeV.

$l = 1$: no bound state.

Cutoff dependence of potentials



Binding energies are 2.5, 21.5, and 59.0 MeV as cutoff takes values of 0.6 GeV, 0.7 GeV, and m_ρ without considering cutoff dependence of couplings.

Comparison with one-boson-exchange model

Similar results as those in Phys. Rev. D 88, 114008 (2013).

ρ contribution is covered not only by the two-pion-exchange part but also by contact terms.

Summary for heavy meson potentials and possible molecules

Summary for heavy meson potentials and possible molecules

We are studying the potentials between heavy mesons.

By solving the Schrodinger equations, we found some bound states in some channels.

Study of Low-Lying Baryons with Hamiltonian Effective Field Theory



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CONTENTS

1. Introduction
2. Hamiltonian effective field theory study of the $N^*(1535)$ resonance in lattice QCD
3. Hamiltonian effective field theory study of the $N^*(1440)$ resonance in lattice QCD
4. Structure of the $\Lambda(1405)$ from Hamiltonian effective field theory
5. Summary

Introduction

Hadron Physics

Hadron physics is mainly focused on hadron scatterings, spectra, structures, interactions, etc.

- Hadron spectra are obtained from experimental Hadron scattering.
- Hadron structures and interactions \Rightarrow Hadron spectra and scattering.

Hadron physics lies in the region of low energies with a large α_s , traditional perturbation expansion in series of $(\alpha_s)^n$ cannot work here.

- constituent quark model
- effective field theory —expanded by small momenta
- lattice QCD —discretized QCD
- QCD sum rule —operator product expansion—twist
- large N_c — $1/N_c$
- ...

Low-lying Baryons

There are much more scattering data on low-lying baryons, $N^*(1440)$, $N^*(1535)$, $\Lambda(1405)$, compared to those for large-mass resonances or charmed hadrons.

Naive quark model predicts the wrong order for masses of $N^*(1440)$ and $N^*(1535)$.

IF: harmonic form for confinement potential

Then: $E = (2n_r + L + 3/2)\omega$

$N^*(1440)$: $n_r = 1, L = 0$

$N^*(1535)$: $n_r = 0, L = 1$

$\Lambda(1405)$ is lower than other members of $J^P = 1/2^-$ octet even if it contains an s quark.

Triquark or pentaquark state?

Liu, Zou, Phys. Rev. Lett. 96, 042002 (2006) ...

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

Connection between Scattering Data and Lattice QCD Data

Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

Lattice QCD Data \rightarrow Physical Data

- Lüscher Formalisms and extensions:
 - Model independent; efficient in single-channel problems
 - Spectrum \rightarrow Phaseshifts; $m_{K_L} - m_{K_S}$ etc.
- Effective Field Theory (EFT), Models, etc
 - with low-energy constants fitted by Lattice QCD data

Physical Data \rightarrow Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

Effective field theory deals with extrapolation powerfully.

Guo, Hanhart, Llanes-Estrada, Meißner, *Quark mass dependence of the pion vector form factor*, Phys.Lett.B678:90-96,2009.

Finite-volume effect can be studied by discretizing the EFT.

Molina, Doring, *Pole structure of the $N(1405)$ in a recent QCD simulation*, Phys.Rev. D94 (2016) no.5, 056010, Addendum: Phys.Rev.

D94 (2016) no.7, 079901

discretize the mass equation (in **integral form**)
(most of time, potentials are momentum independent.)

Hall, Hsu, Leinweber, Thomas, Young, *Finite-volume matrix Hamiltonian model for a $\Delta \rightarrow N\pi$ system*, Phys.Rev. D87 (2013) no.9,

094510

discretize the Hamiltonian equation (in **differential form**)

Discrete spacing effects can also be studied with EFT.

Ren, Geng, Meng, *Baryon chiral perturbation theory with Wilson fermions up to $O(a^2)$ and discretization effects of latest $nf=2+1$ LQCD*

octet baryon masses, Eur.Phys.J. C74 (2014) no.2, 2754

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

Hamiltonian Effective Field Theory

Hamiltonian Effective Field Theory (HEFT)

analyses both **experimental data at infinite volume**
and **lattice QCD results at finite volume** at the same time.

- at infinite volume
 - Lagrangian (via 2-particle irreducible diagrams) \rightarrow
 - potentials (via Betha-Salpeter Equation) \rightarrow
 - phaseshifts and inelasticities
- at finite volume
 - potentials discretized (via Hamiltonian Equation) \rightarrow spectra
 - wavefunctions: analyse the structure of the eigenstates on the lattice
- finite-volume and infinite-volume results are connected by the coupling constants etc.

We use Hamiltonian effective field theory to analyse the scatterings data at experiment and spectra of lattice QCD which are related to

- $N^*(1535)$
- $N^*(1440)$
- $\Lambda(1405)$

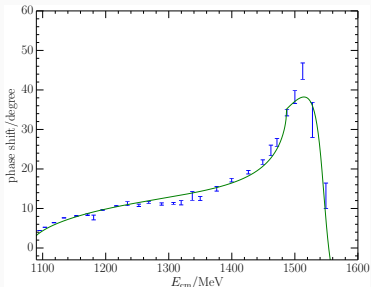
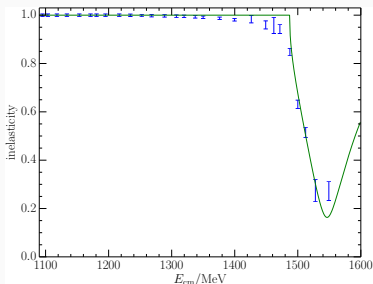
By our analyses, we try to better understand the structures of those resonances and relevant interactions.

**Hamiltonian effective field theory
study of the $N^*(1535)$ resonance
in lattice QCD**

$N^*(1535)$ with πN Scattering

$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions among the bare baryon N_0^* , πN channel, and ηN channel.
- Phase shifts and inelasticities are obtained by solving Bethe-Salpeter equation with the interactions.



πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- Pole position for $N^*(1535)$: $1531 \pm 29 - i 88 \pm 2$ MeV.
Particle Data Group (PDG): $1510 \pm 20 - i 85 \pm 40$ MeV.

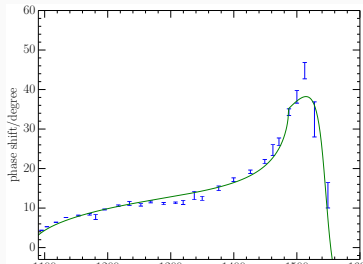
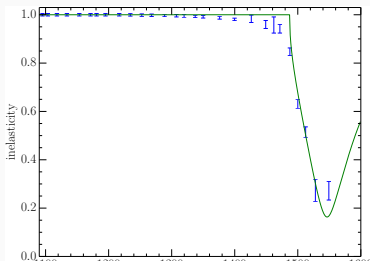
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$$G_{\pi N; N_0^*}^2(k) = \frac{3g_{\pi N; N_0^*}^2}{4\pi^2 f^2} \omega_\pi(k)$$
$$V_{\pi N; \pi N}^S(k, k') = \frac{3g_{\pi N}^S}{4\pi^2 f^2} \frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)} \frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')} \quad (1)$$

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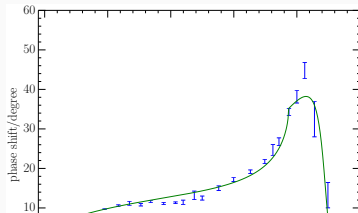
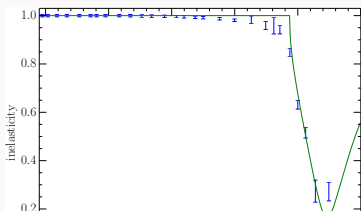


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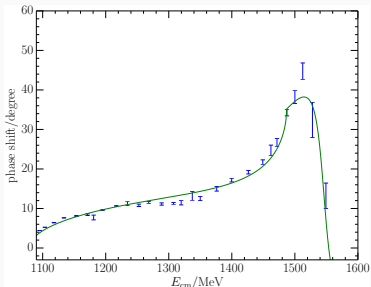
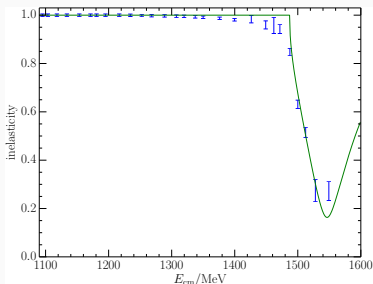
$$T_{\alpha,\beta}(k, k'; E) = V_{\alpha,\beta}(k, k') + \sum_{\gamma} \int q^2 dq$$
$$V_{\alpha,\gamma}(k, q) \frac{1}{E - \sqrt{m_{\gamma 1}^2 + q^2} - \sqrt{m_{\gamma 2}^2 + q^2} + i\epsilon} T_{\gamma,\beta}(q, k'; E)$$



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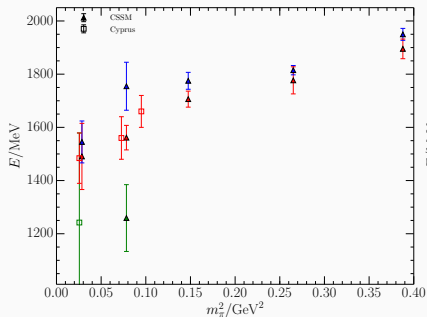
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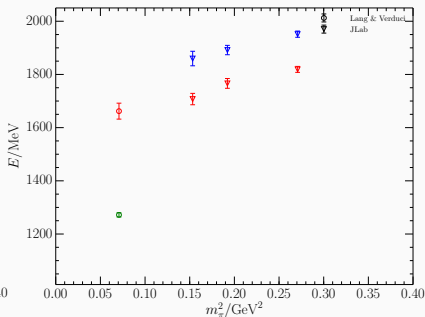
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Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes



$L \approx 3 \text{ fm}$



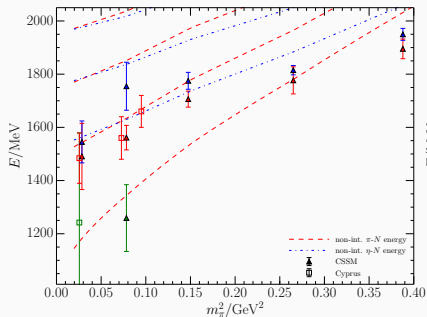
$L \approx 2 \text{ fm}$

Spectra with $I(J^P) = \frac{1}{2}(1_2^-)$ at finite volumes

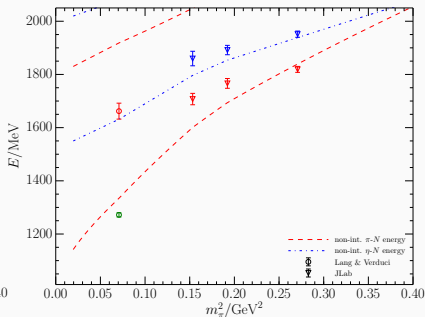
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3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



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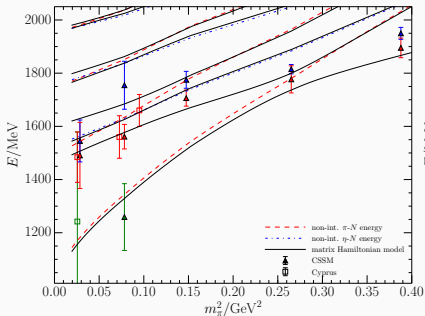


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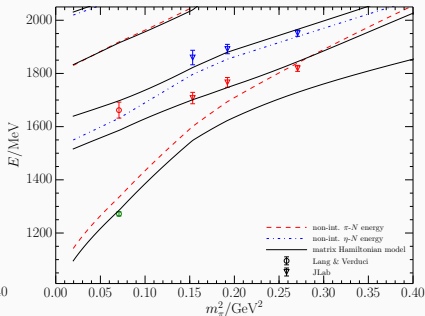
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Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes
Non-interacting energies of the two-particle channels
Eigenenergies of Hamiltonian effective field theory



$L \approx 3$ fm



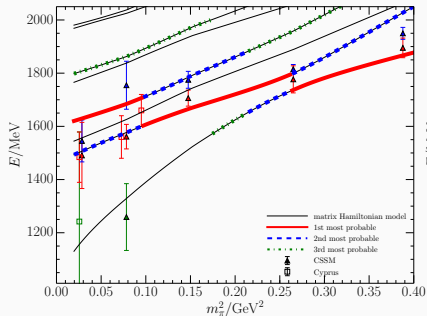
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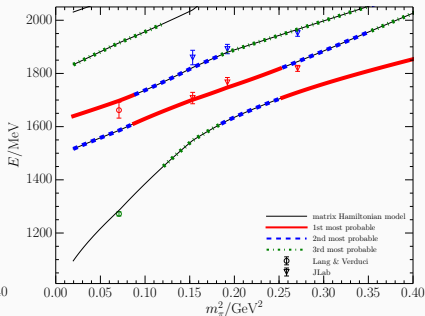
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3 sets of lattice data at different pion masses and finite volumes
Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD



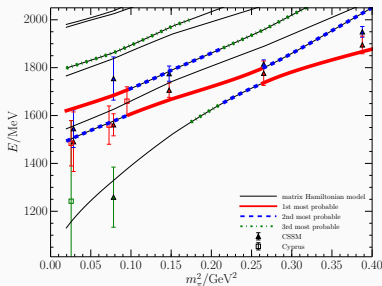
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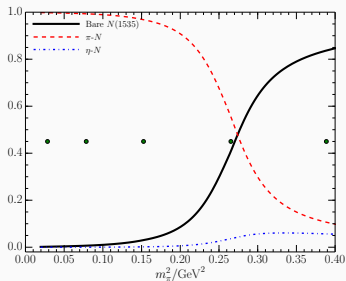
Components of Eigenstates with $L \approx 3$ fm



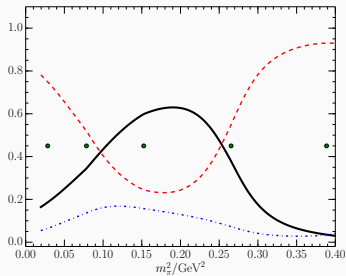
Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

- The 1st eigenstate at light quark masses is mainly πN scattering states.
- The most probable state at physical quark mass is the 4th eigenstate. It contains about 60% bare $N^*(1535)$, 20% πN and 20% ηN .

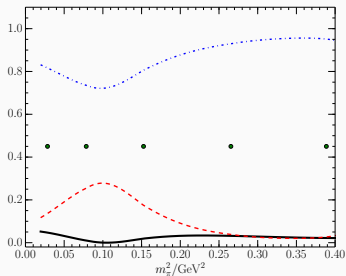
Components of Eigenstates with $L \approx 3$ fm



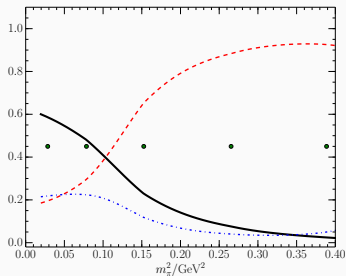
1st eigenstate



2nd eigenstate



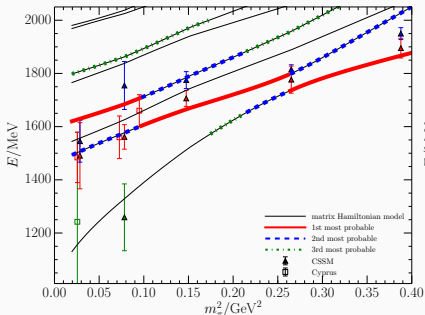
3rd eigenstate



4th eigenstate

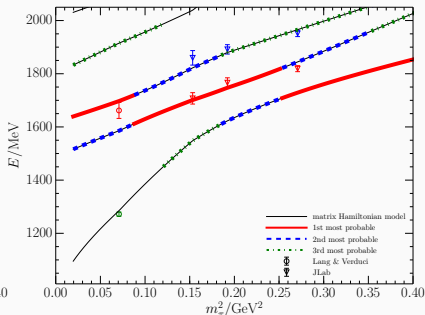
Lattice Results \rightarrow Experimental Results

- Experimental Data \rightarrow Lattice Data We have shown that.
- Lattice Data \rightarrow Experimental Data We show it here.



$L \approx 3 \text{ fm}$

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and the bare mass is fitted by LQCD data



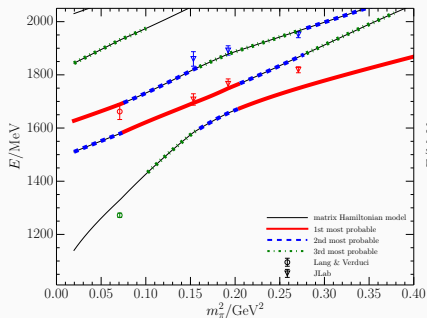
$L \approx 2 \text{ fm}$

By fitting lattice data, the pole position for $N^*(1535)$ at infinite volume is $1602 \pm 48 - i 88.6_{-2.8}^{+0.7} \text{ MeV}$.

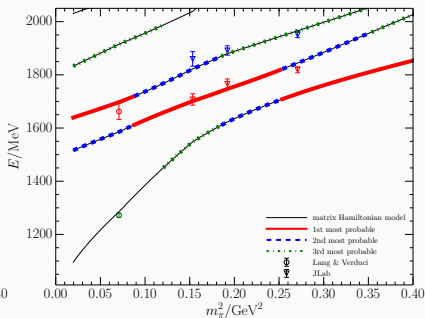
PDG: $1510 \pm 20 - i 85 \pm 40$.

Effects of Separable Potentials

fit for lattice QCD data



without separable potential



with separable potential

**Hamiltonian effective field theory
study of the $N^*(1440)$ resonance
in lattice QCD**

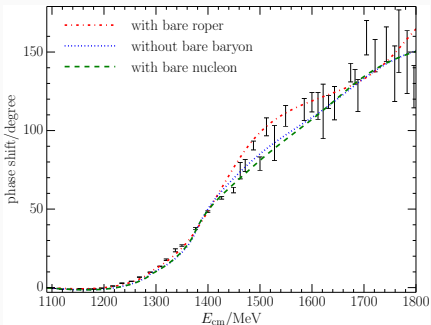
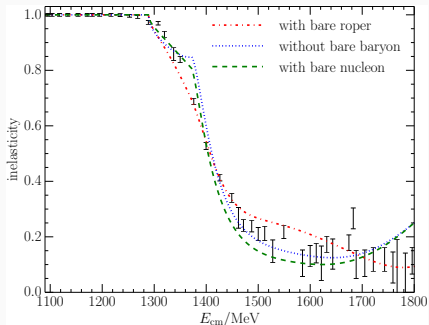
$N^*(1440)$ Resonance

- $N^*(1440)$, usually called Roper, is the excited state $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Naive quark model predicts $m_{N^*(1440)} > m_{N^*(1535)}$ if they are both dominated by 3-quark core. But contrary to experiment.

To check whether a 3-quark core largely exists in Roper, we consider models

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

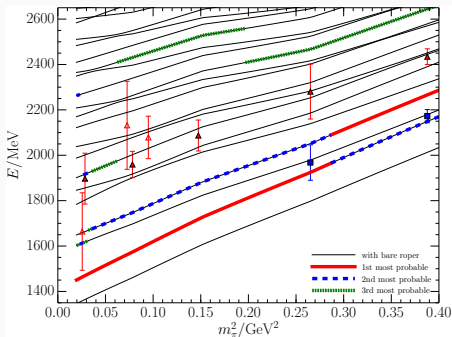
$N^*(1440)$ Resonance



πN scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

Results of the Model with a Bare Roper



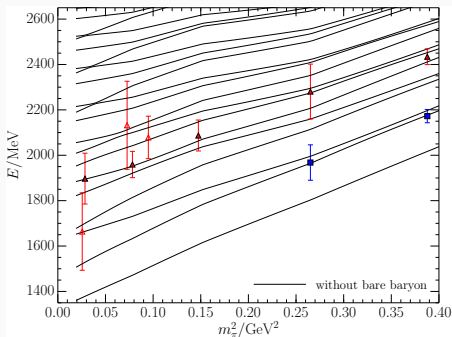
Spectrum given by the scenario with a bare Roper.

$$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$$
 and $L \approx 3$ fm.

At low pion masses, the 2nd state contains more than 20% bare Roper, so this state should be observed with a 3-quark interpolating operators on the lattice.

But it is not.

Results of the Model without Bare Baryons

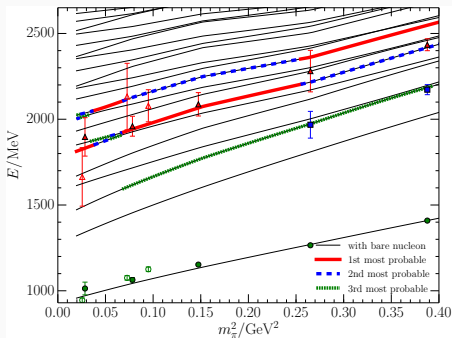


Spectrum given by the scenario without any bare baryon.

$$I(J^P) = \frac{1}{2}(\frac{1}{2}^+) \text{ and } L \approx 3 \text{ fm.}$$

- The lattice data sit on the eigenenergy spectrum of this model;
- ALTHOUGH it is hard to predict which state is easier to observe on the lattice,
- we notice that lattice QCD prefers to extract eigenstates with non-trivial mixing of scattering states.

Including the Effect of the Bare Nucleon

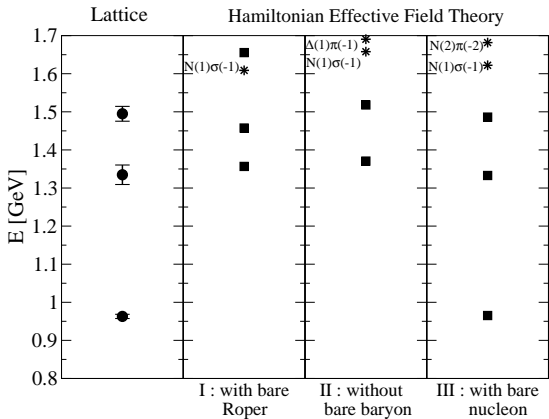


Spectrum given by the scenario with a bare nucleon.

$$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$$
 and $L \approx 3$ fm.

- The bare nucleon does not affect the spectrum very much compared to the results of the model without any bare baryons;
- We can plot the probability based on the distribution of the bare nucleon;
- It can explain both the experimental data and lattice data.

Our results are verified



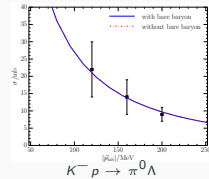
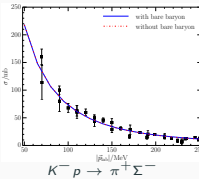
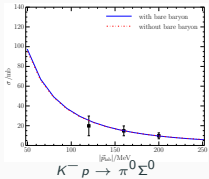
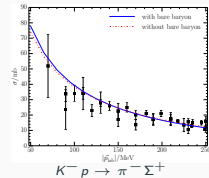
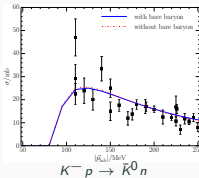
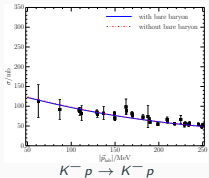
interpolating operators: $N(0)$, $N(0)\sigma(0)$, $N(p)\pi(-p)$, $\Delta(p)\pi(-p)$. from Lang, Leskovec, Padmanath, Prelovsek, PRD95 (2017) no.1, 014510.

No these two higher states with $N^{-P}(0)\pi(0)\dots$ from CMMS.

Structure of the $\Lambda(1405)$ from Hamiltonian effective field theory

$\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.
momentum-dependent, non-separable
- We can fit the cross sections of $K^- p$ well
both with and without a bare baryon.



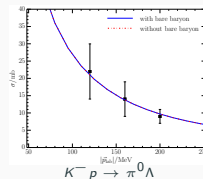
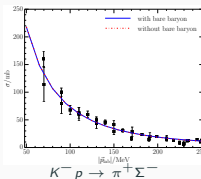
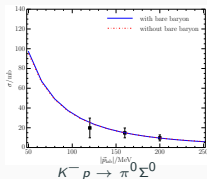
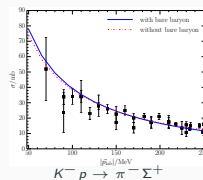
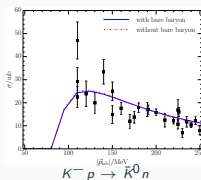
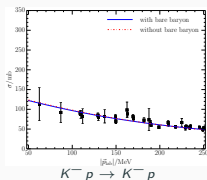
- Two-pole structure of $\Lambda(1405)$
 $1430 - i22$ MeV, $1338 - i89$ MeV

$\Lambda(1405)$ with K^-p scattering

- The well-known Weinberg-Tomozawa potentials are used.
momentum-dependent, non-separable

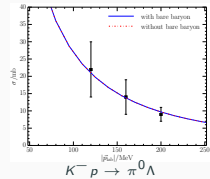
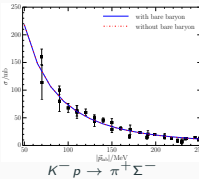
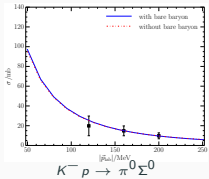
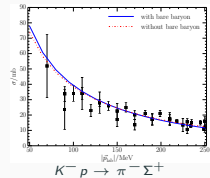
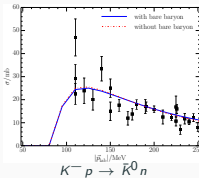
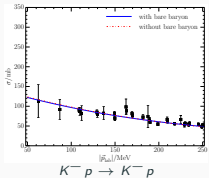
$$V_{\alpha,\beta}(k, k') = g_{\alpha,\beta} \frac{\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)} \sqrt{\omega_{\beta_M}(k')}}$$

- We can fit the cross sections of K^-p well
both with and without a bare baryon.



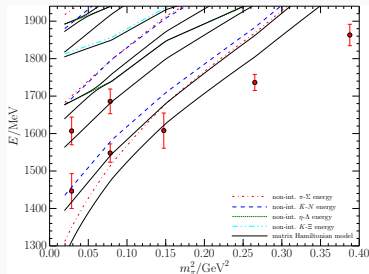
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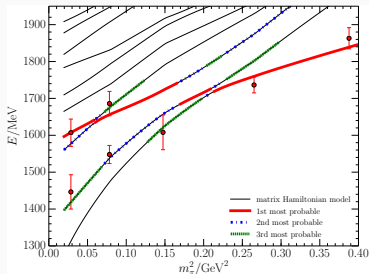


- Two-pole structure of $\Lambda(1405)$
 $1430 - i22$ MeV, $1338 - i89$ MeV

Spectrum on the Lattice



without a bare baryon



with a bare baryon

Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting the lattice QCD data at large pion masses.

The bare state introduces a new pole for $\Lambda(1670)$ at 1660-30i MeV

- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state containing very little of bare baryon at physical pion mass.

Summary

Summary

We have analysed the scattering data at experiment and the lattice spectra on the lattice relevant to $N^*(1440)$, $N^*(1535)$, and $\Lambda(1405)$ with Hamiltonian effective field theory

- $N^*(1535)$ contains a 3-quark core;
- $N^*(1440)$ should contain little of 3-quark consistent;
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state at physical quark mass, while a 3-quark core dominates at large quark masses.

Future improvement:

- couple-channel effect
- dynamical mechanism
- higher order loop effect
- ...