FCC-ee dynamic aperture with radiation from quadrupoles

A. Bogomyagkov, E. Levichev, S. Glukhov, S. Sinyatkin

Budker Institute of Nuclear Physics
Novosibirsk

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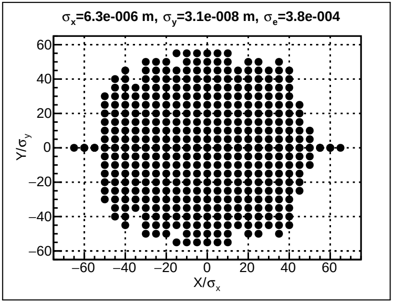
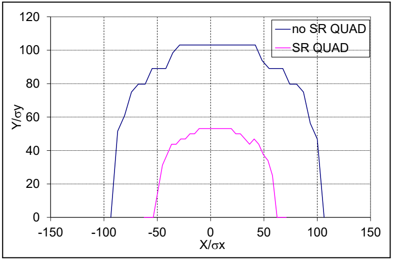
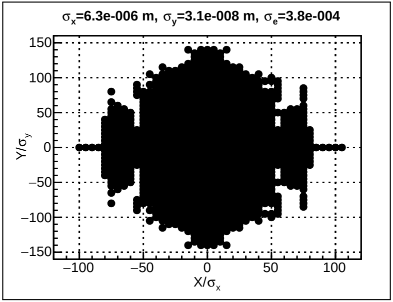
6d (SR from BEND, QUAD) and 6d tracking: *XY*

6d(SR): $R_x = 65\sigma_x$ $R_y = 55\sigma_y$





6d: $R_x = 109\sigma_x$ $R_y = 142\sigma_y$



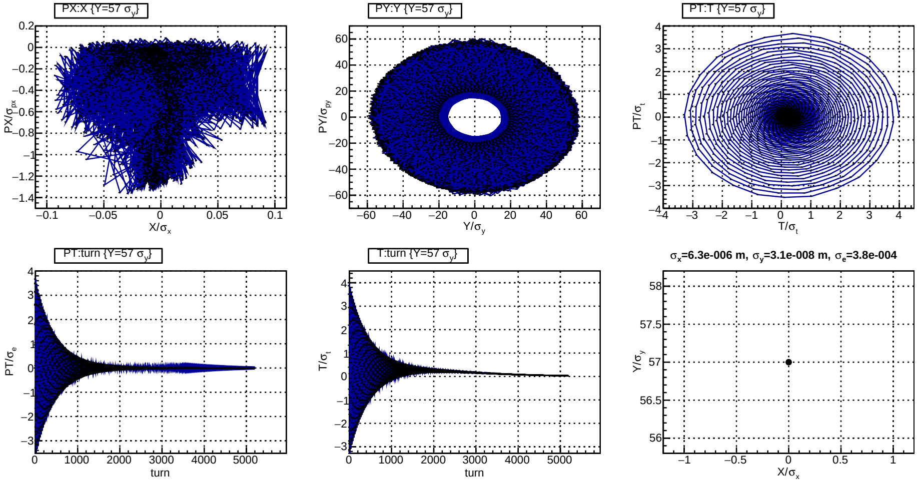
Introduction

Problem

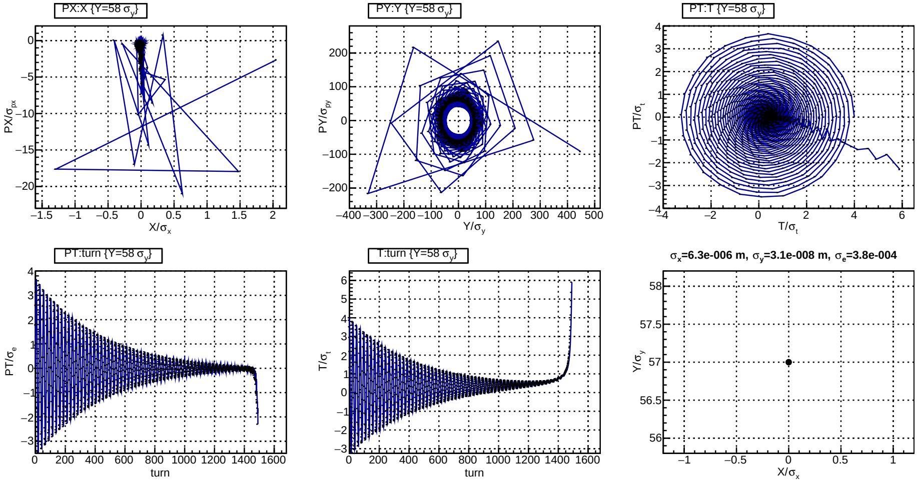
- Why vertical dynamic aperture drops from $R_y = 142\sigma_y$ to $R_y = 55\sigma_y$?
- Why horizontal dynamic aperture drops from $R_x = 109\sigma_x$ to $R_x = 65\sigma_x$?

in the FCCee_z_202_nosol_13.seq lattice at 45 GeV

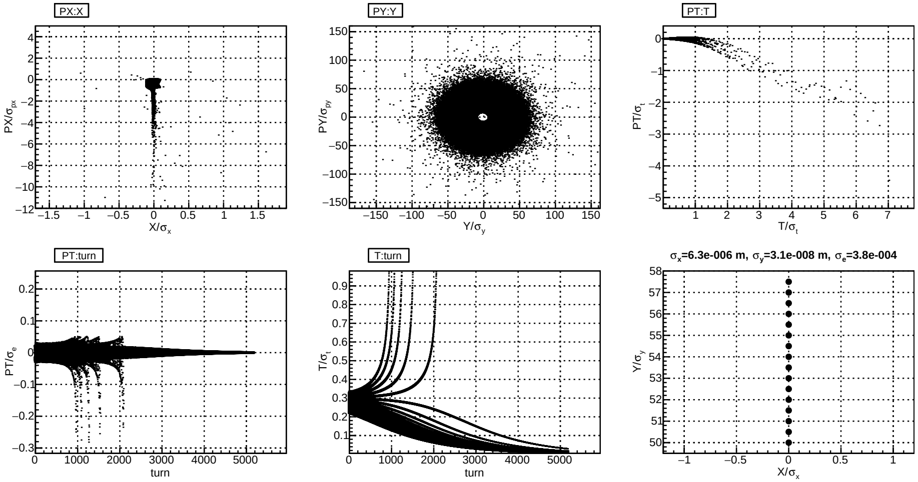
6d (SR from BEND, QUAD; last stable): $Y_0 = 57\sigma_y$



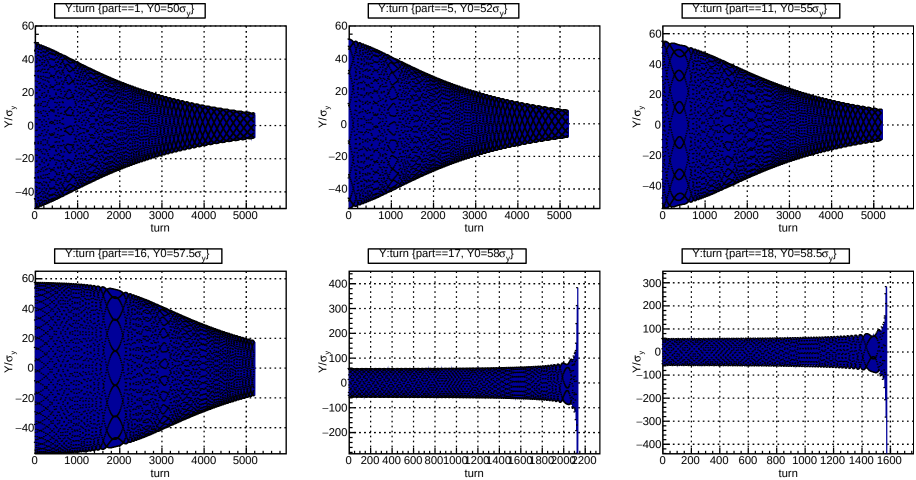
6d (SR from BEND, QUAD; first unstable): $Y_0 = 58\sigma_y$



6d (SR from BEND, QUAD; longitudinally adjusted)



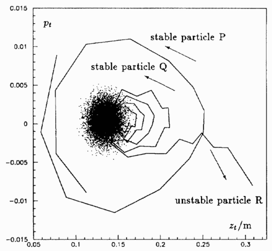
6d (SR from BEND, QUAD) damping



History

References

- John M. Jowett (SLAC), Introductory Statistical Mechanics for Electron Storage Rings, AIP Conf.Proc. 153 (1987) 864-970
- J. Jowett (CERN), Dynamic aperture for LEP: Physics and calculations, Conf.Proc. C9401174 (1994) 47-71, In *Chamonix 1994, LEP performance* 47-71
- F. Barbarin, F. C. Iselin and J. M. Jowett, Particle dynamics in LEP at very high-energy, Conf. Proc. C **940627**, 193 (1994).



Comments

- Some tracking plots are similar.
- There was no mentioning of damping turning into raising.

Parameters (radiation ON, no tappering)

- Energy: $E = 45.6$ Gev.
- Tunes: $\nu_s = 0.0413$, $\nu_y = 0.2217$, $\nu_s = 0.1366$
- Damping times [turns]: $\tau_s = 1300$, $\tau_y = 2600$, $\tau_x = 2600$
- Energy loss: $U_0 = 35.96$ MeV/turn
  - $U_d(B, arc) = 3014 \times 12.4$ keV$= U_0$,
  - $U_q(FF, 50\sigma_y) = 4 \times 0.5$ MeV, $U_q(FF, 50\sigma_x) = 4 \times 3$ MeV,
  - $U_q(QF, 50\sigma_y) = 1470 \times 2.5$ eV, $U_q(QF, 50\sigma_x) = 1470 \times 2.8$ keV,
  - $U_q(QD, 50\sigma_y) = 1468 \times 10$ eV, $U_q(QD, 50\sigma_x) = 1468 \times 1$ keV

Equations of motion: longitudinal

Exact

$$\sigma' = -K_0 x - \frac{p_x^2}{2} - \frac{p_y^2}{2}$$

$$p_t' = \left(-\frac{eV_0}{p_0 c}\right) \sin\left[\phi_s + 2\pi\frac{\sigma}{\lambda}\right] \delta(s - s_0) - \frac{C_\gamma}{2\pi}\frac{E_0^4}{p_0 c} K_0^2 (1 + 2p_t)$$
$$- \frac{C_\gamma}{2\pi}\frac{E_0^4}{p_0 c} K_1^2 (x^2 + y^2)$$

Average, $x_\beta = 0$

$$\sigma' = -\alpha p_t - \frac{J_y \langle\gamma\rangle}{2},$$

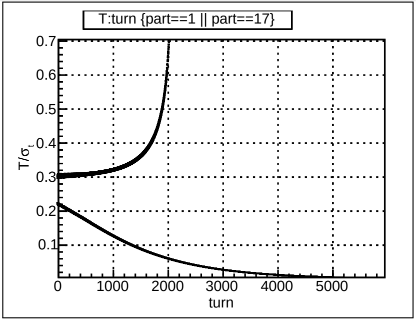
$$p_t' = \frac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_t - \frac{C_\gamma}{2\pi}\frac{E_0^4}{p_0 c \Pi}\sum_q K_1^2 L_q y_q^2, \qquad k_s^2 = \frac{(2\pi\nu_s)^2}{c^2}$$

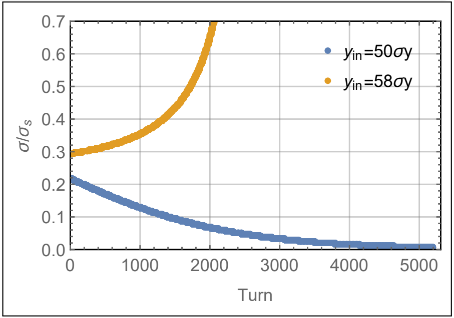
Synchronous phase

No synchrotron oscillations

$$\sigma' = 0\,,\, p_t' = 0\,, \qquad J_y = \frac{J_0 \exp\left(-2\alpha_y s\right)}{1 - \frac{\beta}{2\alpha_y}\left(1 - \exp\left(-2\alpha_y s\right)\right)}$$

$$\sigma = -\frac{\alpha_\sigma}{k_s^2} J_y \left\langle \gamma_y \right\rangle + \frac{\alpha}{k_s^2} \sum_q \frac{U_{q,y}}{\Pi p_0 c} \frac{J_y}{\varepsilon_y}\,, \quad p_t = -\frac{1}{2\alpha} J_y \left\langle \gamma_y \right\rangle$$





Equations of motion: longitudinal

Average, $x_\beta = 0$

$$\sigma' = -\alpha p_t - \frac{J_y \langle\gamma\rangle}{2}$$

$$p_t' = \frac{k_s^2}{\alpha}\sigma - 2\alpha_\sigma p_t - \sum_q \frac{U_q(\sigma_y)}{p_0 c \Pi} \frac{y_q^2}{\sigma_{q,y}^2}$$

$$y_q = \sqrt{2 J_y \beta_{q,y}} \cos(\psi_0 + \psi_q + k_y s) = A f_q + A^* f_q^*$$

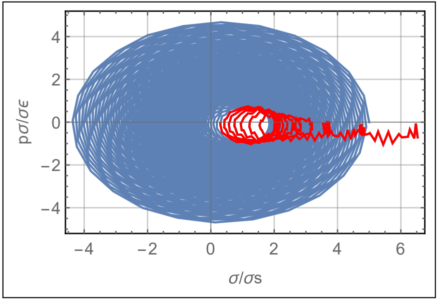
Solution

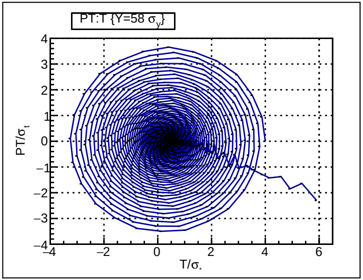
$$p_t = B e^{-\alpha_t s} \cos\left(k_s s\right) - \frac{J_y \langle\gamma\rangle}{2\alpha} - \sum_q \frac{U_{q,y}(\sigma_y)}{p_0 c} \frac{2 k_y}{\Pi(4k_y^2 - k_s^2)} \sin\left(2\psi_0 + 2\psi_q + 2k_y s\right) \frac{J_y}{\varepsilon_y}$$

$k_y = \dfrac{2\pi\{\nu_y\}}{\Pi}$, $U_{q,y}(\sigma_y)$ is radiation from quadrupole at $1\sigma_y$

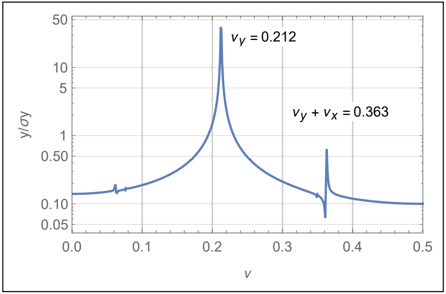
Illustration: $\nu_x = 269.14\,, \nu_y = 267.22\,, \nu_s = 0.0413$

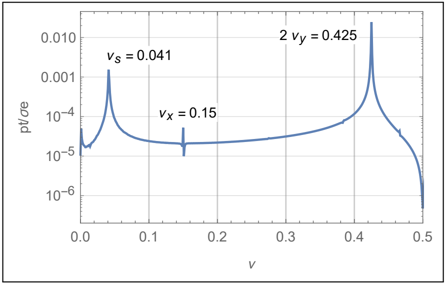
Solution of longitudinal equations





Tracking with $Y_0 = 58\sigma_y$





Equations of motion: vertical

Exact

$$y' = p_y(1 - p_t)$$

$$p_y' = K_1 y + K_2 \eta p_t y - p_y \frac{C_\gamma}{2\pi} \frac{E_0^4}{p_0 c} \left[K_0^2 + p_t \left(2K_0^2 + 2K_0 K_1 + K_0^3 \eta \right) \right] - p_y \frac{C_\gamma}{2\pi} \frac{E_0^4}{p_0 c} y^2 K_1^2$$

Map of quadrupole radiation

$$\Delta p_y = -p_{y,0} y_0^2 \frac{C_\gamma}{2\pi} \frac{E_0^4}{p_0 c} K_1^2 L_q \,, \qquad \frac{C_\gamma}{2\pi} \frac{E_0^4}{p_0 c} K_1^2 L_q = \frac{U_q(\sigma_y)}{E_0 \sigma_y^2} \approx 0.7\, m^{-2}$$

Map of quadrupole fringe

$$\Delta p_y = -p_{y,0} y_0^2 \frac{K_1}{4} \,, \qquad K_1/4 \approx 0.15\, m^{-2}$$

Vertical dynamic aperture limit

Solving: parameter variation and averaging

$$y(s) = A(s)f(s) + A(s)^* f(s)^*,$$
$$p_y(s) = A(s)f'(s) + A(s)^* f'(s)^*$$

Averaged equation

$$J_y' = -2\alpha_y J_y + \beta J_y^2\,,$$
$$\beta = \frac{k_y}{\varepsilon_y \Pi(4k_y^2 - k_s^2)} \sum_q \frac{U_{q,y}}{p_0 c} \left\langle (K_1 - K_2\eta)\beta_y \cos\left(2\psi_y - 2\psi_{q,y} - 2k_y s\right)\right\rangle$$

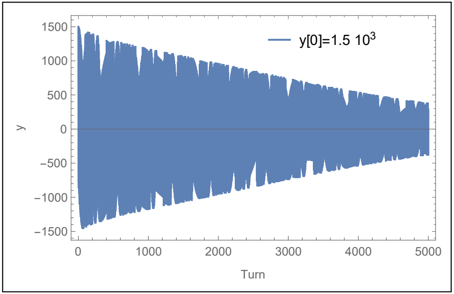
DA limit

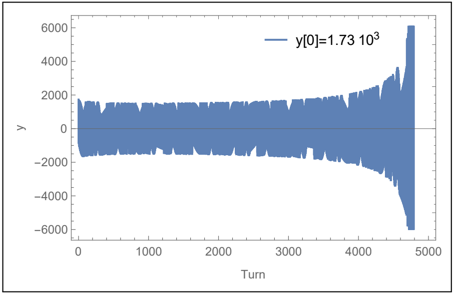
$$J_y' = 0$$
$$\frac{J_y}{\varepsilon_y} = \frac{4k_y^2 - k_s^2}{k_y} \cdot \frac{U_0}{\sum_q U_{q,y} \left\langle (K_1 - K_2\eta)\beta_y \cos\left(2\psi_y - 2\psi_{q,y} - 2k_y s\right)\right\rangle}$$

Parametric resonance and Van der Pol oscillator

Exact: $$p_y'' = K_1 p_y (1 - p_t) - p_y' \frac{C_\gamma}{2\pi} \frac{E_0^4}{p_0 c} K_0^2 - (p_y y^2)' \frac{C_\gamma}{2\pi} \frac{E_0^4}{p_0 c} K_1^2$$

Illustration: $$y'' + k_y^2 \left(1 - F_1 y^2 \cos(2 k_y s)\right) y + 2\alpha y' = 0$$





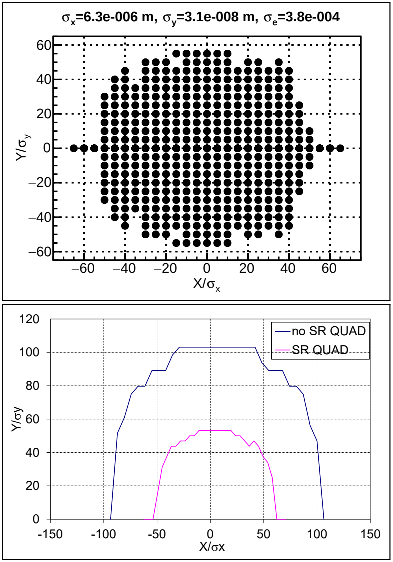
Van der Pol oscillator: $$y'' + k_y^2 y + 2\alpha y' \left(1 - F_1 y^2\right) = 0$$

Conclusion for vertical plane at 45.6 GeV

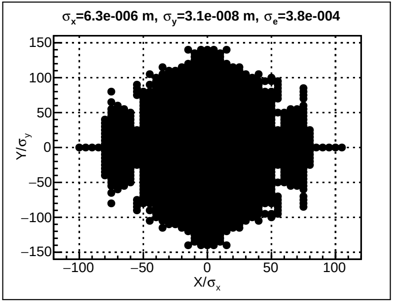
1. Observed and studied a new effect limiting dynamic aperture.
2. Radiation from FF quadrupoles modulates p_t at double betatron frequency.
3. Parametric resonance in vertical motion changes damping. It is observed in tracking and obtained by equations.
4. Estimations with some assumptions predict dynamic aperture limit $J_{y,limit} \approx 30\sigma_y$.
5. Map of radiation from FF quadrupole is similar to quadrupole fringe and kick is larger.
6. $\pi/2$ phase advance between quadrupoles will decrease p_t modulation at double betatron frequency and eliminate parametric resonance.

6d (SR from BEND, QUAD) and 6d tracking: *XY*

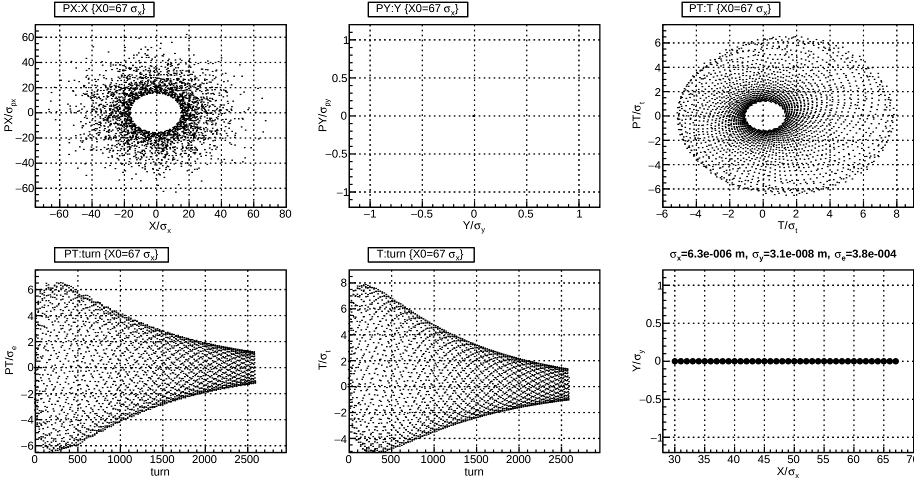
6d(SR): $R_x = 65\sigma_x$ $R_y = 55\sigma_y$



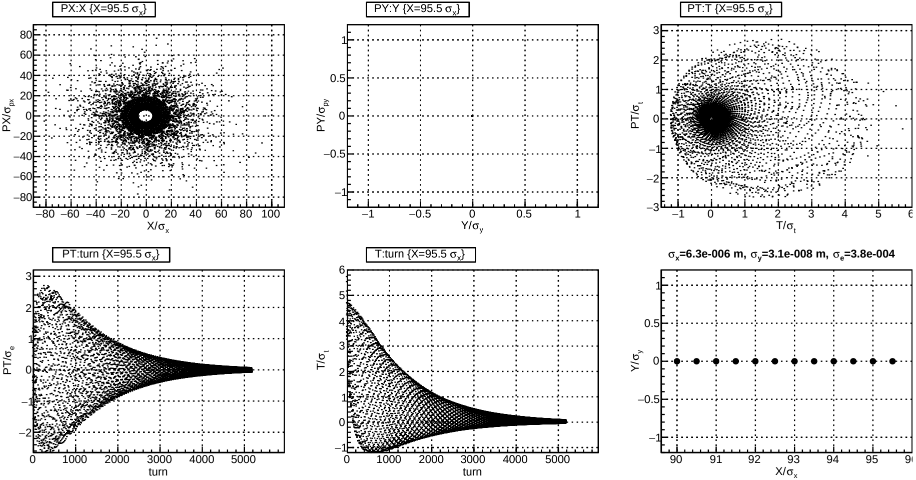
6d: $R_x = 109\sigma_x$ $R_y = 142\sigma_y$



6d (SR from BEND, QUAD; last stable): $X_0 = 67\sigma_x$

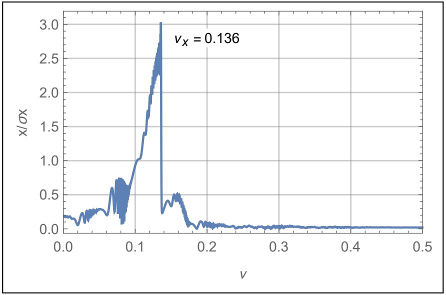


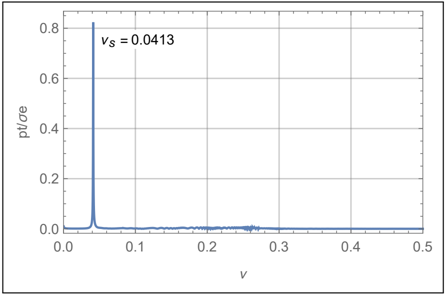
6d (SR from BEND, QUAD; longitudinally adjusted): $X_0 = 95.5\sigma_x$



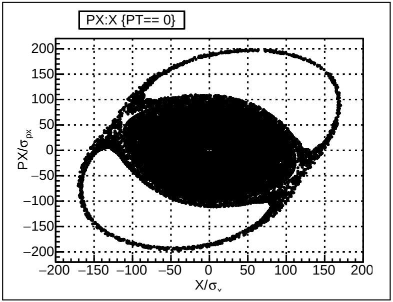
Spectra: $X_0 = 95.5\sigma_x$

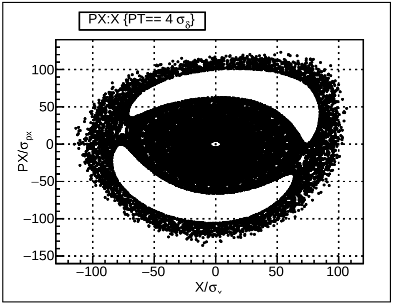
$\nu_x = 269.14$ $\nu_y = 267.22$ $\nu_s = 0.0413$

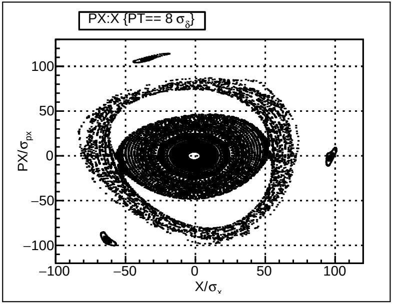


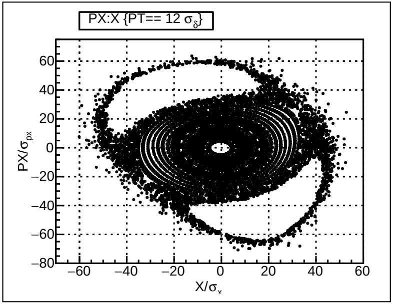


5d tracking: *PX* : *X*









Integer resonance: $PX : X$

$$\begin{pmatrix} \cos(\mu) & \beta\sin(\mu) \\ -\frac{1}{\beta}\sin(\mu) & \cos(\mu) \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix} + \begin{pmatrix} 0 \\ kx^3 \end{pmatrix} = \begin{pmatrix} x \\ p_x \end{pmatrix}$$

Solution

$$\{x, p_x\} = \{0, 0\}$$

$$\{x, p_x\} = \left\{ \frac{\sqrt{2}}{\sqrt{k\beta}} \left(\tan\frac{\mu}{2}\right)^{\frac{1}{2}}, \frac{\sqrt{2}}{\sqrt{k\beta^3}} \left(\tan\frac{\mu}{2}\right)^{\frac{3}{2}} \right\}$$

$$\{x, p_x\} = \left\{ -\frac{\sqrt{2}}{\sqrt{k\beta}} \left(\tan\frac{\mu}{2}\right)^{\frac{1}{2}}, -\frac{\sqrt{2}}{\sqrt{k\beta^3}} \left(\tan\frac{\mu}{2}\right)^{\frac{3}{2}} \right\}$$

History

J. Jowett (CERN), Dynamic aperture for LEP: Physics and calculations, Conf.Proc. C9401174 (1994) 47-71, In *Chamonix 1994, LEP performance* 47-71

”Here I shall briefly describe a new effect which I propose to call Radiative Beta-Synchrotron Coupling (RBSC). It is a non-resonant effect. A particle with a large betatron amplitude make an extra energy loss by radiating in quadrupoles. ...you can say that its ”effective stable phase angle" will change to reflect the greater energy loss. The particle will tend to oscillate about a displaced fixed point in the synchrotron phase plane. This results in a growth of the oscillation amplitude which may eventually lead the particle outside the stable region in synchrotron phase space.”

Conclusion for horizontal plane at 45.6 GeV

1. Radiation from quadrupoles shifts the synchronous phase proportional to square of the horizontal amplitude,
2. therefore increase of synchrotron oscillations amplitude.
3. Particle with energy deviation experiences different lattice, where 4d resonances limit horizontal aperture.