Analytical treatment of dynamic apertures in circular accelerators and colliders with beam-beam effects

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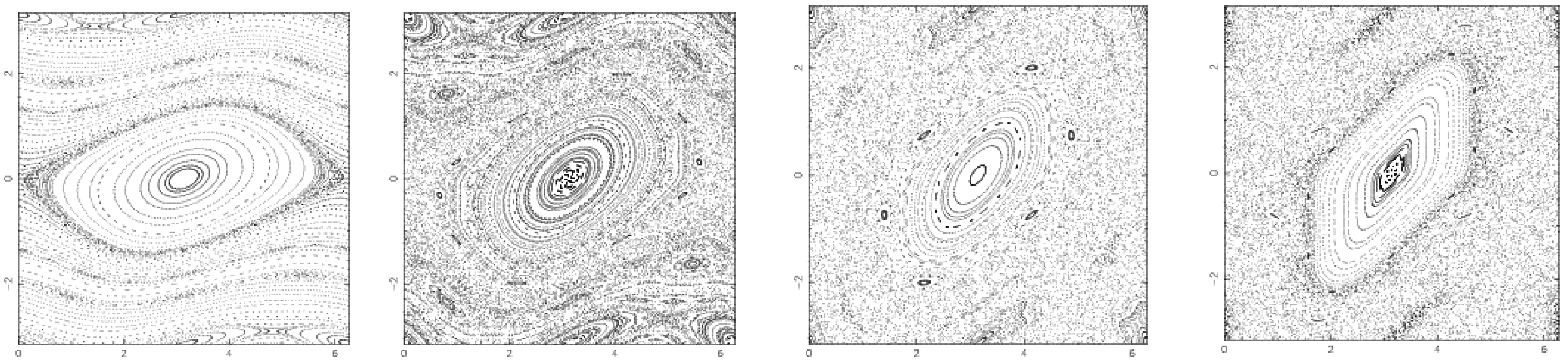
Standard Map

The progresses of nonlinear physics are the bases for understanding various long stadind beam dynamics phenomenons.

$$\bar{I} = I + K_0 \sin\theta$$

$$\bar{\theta} = \theta + \bar{I}$$

when K≥0.97164 stochastic motion starts, so called Chirikov Criterion

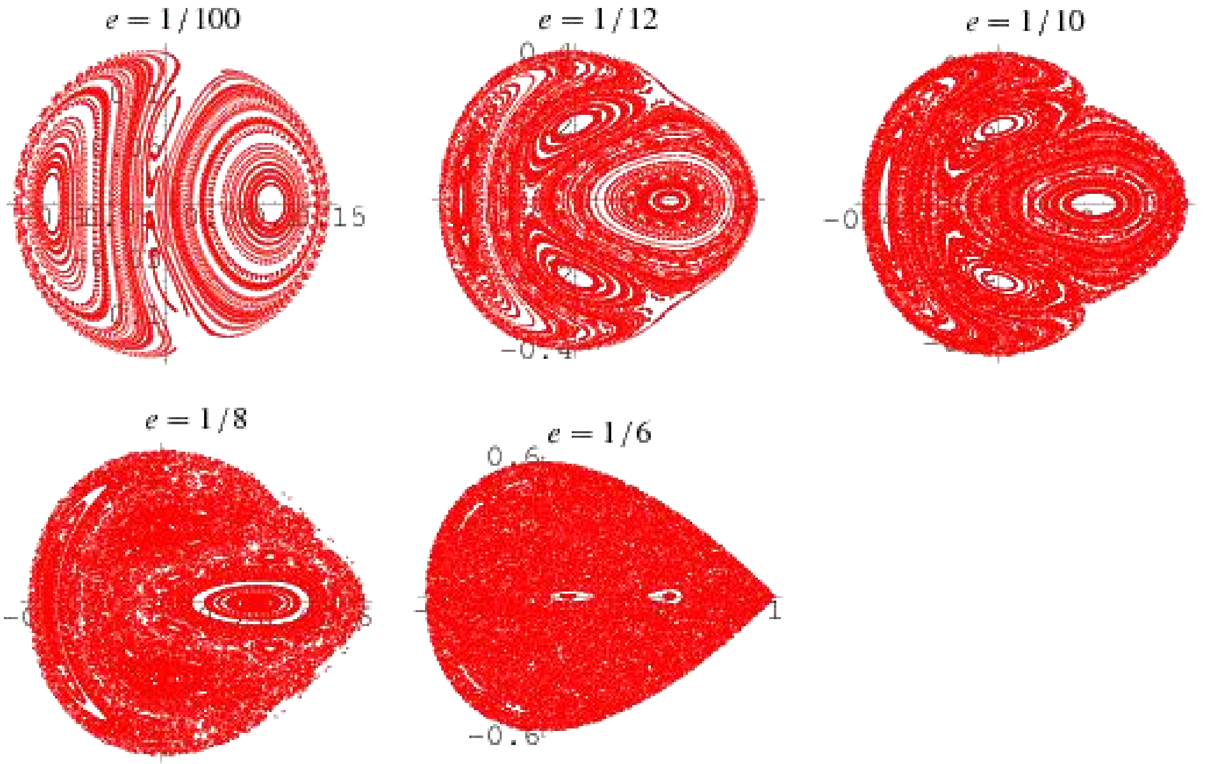


Chirikov, B. V. "A Universal Instability of Many-Dimensional Oscillator Systems." **Phys. Rep.** 52, 264-379, 1979.

*R.Z. Sagdeev, D.A. Usikov, G.M. Zaslavsky, **Nonlinear Physics, from the Pendulum to Turbulence and Chaos**, **Harwood Academic Publishers**, 1988.

Héno-Heiles Problem

$$H_{\mathrm{H\&H}} = \frac{1}{2}\left(x^2 + p_x^2 + y^2 + p_y^2 + 2y^2x - \frac{2}{3}x^3\right).$$



Hénon, M. and Heiles, C. "The Applicability of the Third Integral of Motion: Some Numerical Experiments." **Astron. J.** 69, 73-79, 1964.

Analyitcal treatment of dynamic apertures of multipoles

$$\Psi = \int_0^s \frac{ds'}{\beta_x(s')} + \phi_0$$

$$J = \frac{\varepsilon_x}{2} = \frac{1}{2\beta_x(s)}\left(x^2 + \left(\beta_x(s)x' - \frac{\beta_x' x}{2}\right)^2\right)$$

$$H(J,\Psi) = \frac{J}{\beta_x(s)}.$$

$$\Psi_1 = \Psi + \frac{2\pi\nu}{L} - \int_0^s \frac{ds'}{\beta_x(s')}$$

$$J_1 = J$$

$$H_1 = \frac{2\pi\nu}{L} J_1.$$

$$H = \frac{x^2 B_y|_{x=0,y=0}}{2\rho^2 B_0} + \frac{1}{B_0 \rho}\sum_{n=1}^{\infty} \frac{1}{n!}\frac{\partial^{n-1} B_y}{\partial x^{n-1}}\Bigg|_{x=0,y=0}(x+iy)^n - (1 + x/\rho)\left(\left(1 + \frac{\Delta P}{P_0}\right)^2 - \left(\bar{p}_x - \frac{eA_x}{P_0}\right)^2 - \left(\bar{p}_y - \frac{eA_y}{P_0}\right)^2\right)^{1/2} - \frac{e\Phi}{P_0}$$

$$x = \sqrt{2J_1\beta_x(s)}\cos\left(\Psi_1 - \frac{2\pi\nu}{L}s + \int_0^s \frac{ds'}{\beta_x(s')}\right).$$

$$\frac{dJ_1}{ds} = -\frac{\partial H_1}{\partial \Psi_1}$$

$$\frac{d\Psi_1}{ds} = \frac{\partial H_1}{\partial J_1}$$

$$\overline{J_1} = \overline{J_1}(\Psi_1, J_1)$$

$$\overline{\Psi_1} = \overline{\Psi_1}(\Psi_1, J_1)$$

$$\bar{I} = I + K_0 \sin\theta$$

$$\bar{\theta} = \theta + \bar{I}$$

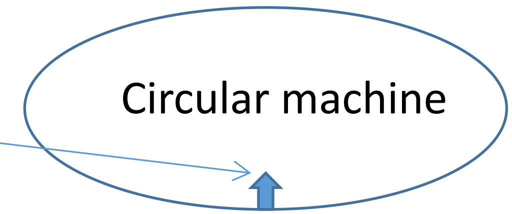
$|K_0| \le 1$ （0.97164)

Analytical DA expressions

J. Gao, “Analytical estimation of the dynamic apertures of circular accelerators”, **Nuclear Instruments and Methods in Physics Research A** 451 (2000) 545-557.

Basic theroy of dynamic aperture

$$H=\frac{p^2}{2}+\frac{K(s)}{2}x^2+\frac{1}{m!B_0\rho}\frac{\partial^{m-1}B_z}{\partial x^{m-1}}x^m L\sum_{k=-\infty}^{\infty}\delta(s-kL)$$



A nonlinear multipole

$$B_z=B_0(1+xb_1+x^2b_2+x^3b_3+\cdots+x^{m-1}b_{m-1}+\cdots)$$

For one multipole $B_z=B_0x^{m-1}b_{m-1}$ m≥3

$$A_{\mathrm{dyna},2m}=\sqrt{2\beta_x(s)}\left(\frac{1}{m\beta_x^m(s(2m))}\right)^{\frac{1}{2(m-2)}}\left(\frac{\rho}{|b_{m-1}|L}\right)^{1/(m-2)}$$

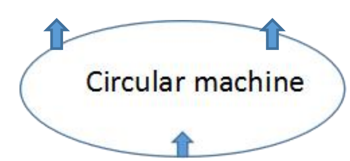
Standard Mapping
Chirikov Criterion

Relation between X and Y $A_{\mathrm{dyna},2m,y}=\sqrt{\frac{\beta_x(s(2m))}{\beta_y(s(2m))}(A^2_{\mathrm{dyna},2m,x}-x^2)}$

Hénon and Heiles problem

For more independent multipoles

$$A_{\mathrm{dyna,total}}=\frac{1}{\sqrt{\sum_i\frac{1}{A^2_{\mathrm{dyna,sext},i}}+\sum_j\frac{1}{A^2_{\mathrm{dyna,oct},j}}+\sum_k\frac{1}{A^2_{\mathrm{dyna,deca},k}}+\cdots}}$$



Many multipoles

J. Gao, "Analytical estimation of the dynamic apertures of circular accelerators", **Nuclear Instruments and Methods in Physics Research A** 451 (2000) 545-557.

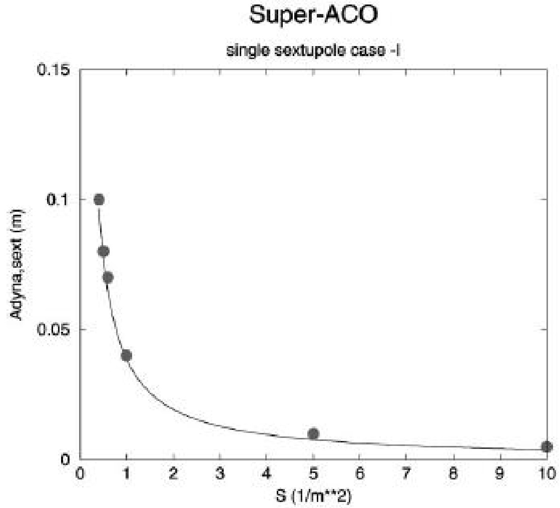


Fig. 16. The dynamic aperture of Super-ACO vs S ($S = b_2 L/\rho$) at s_1.

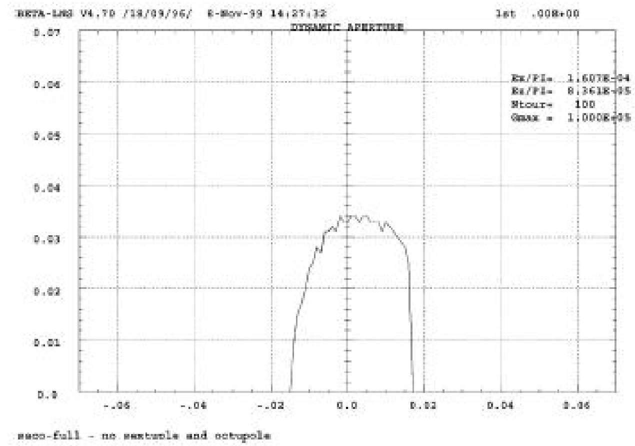


Fig. 22. The 2D dynamic aperture of Super-ACO with S = 2 located at s_2 with $\beta_x(s_2) = 15.18$ m and $\beta_y(s_2) = 4.26$ m.

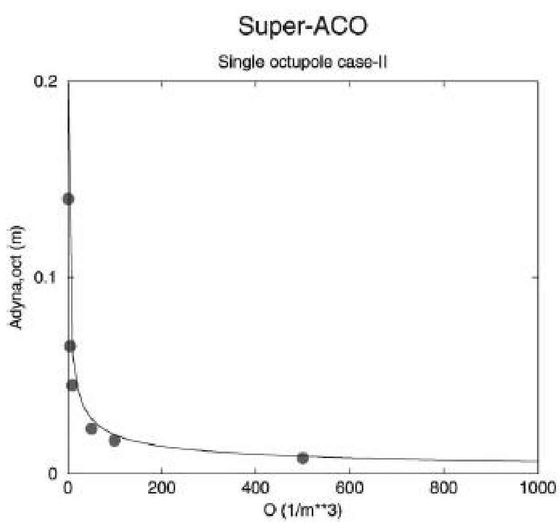


Fig. 17. The dynamic aperture of Super-ACO vs O ($O = b_3 L/\rho$) at s_2.

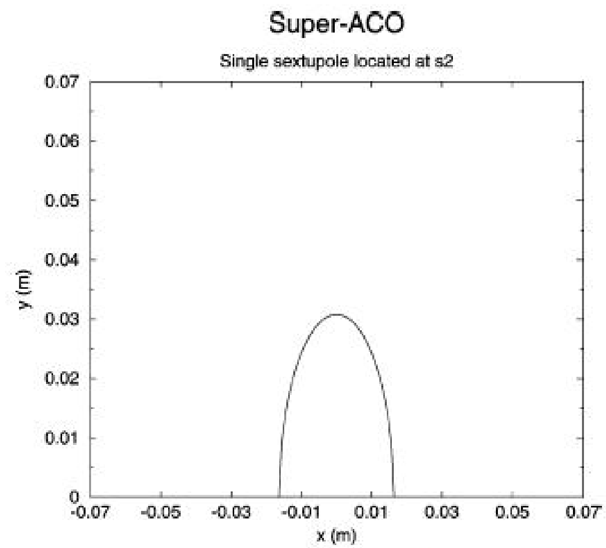


Fig. 23. The analytical estimation of the 2D dynamic aperture of Super-ACO with S = 2 located at s_2 with $\beta_x(s_2) = 15.8$ m and $\beta_y(s_2) = 4.26$ m.

Super-ACO

Table 1
Summary of parameters

Case	Multipole strength	Beta function (m)
1	$S(s_1) = 2\ (1/m^2)$	$\beta_x(s_1) = 13.6$
2	$O(s_1) = 10\ (1/m^3)$	$\beta_x(s_1) = 13.6$
3	$D(s_1) = 1000\ (1/m^4)$	$\beta_x(s_1) = 13.6$
4	$S(s_1) = 2\ (1/m^2)$, $O(s_1) = 62\ (1/m^3)$	$\beta_x(s_1) = 13.6$
5	$S(s_1) = 2\ (1/m^2)$, $O(s_2) = 62\ (1/m^3)$	$\beta_x(s_1) = 13.6$, $\beta(s_2) = 15.18$
6	$S(s_{1,2,3,4}) = 2\ (1/m^2)$	$\beta_x(s_{1,2,3,4}) = 13.6$, 15.18, 7.8, 6.8
8	$S(s_1) = 2\ (1/m^2)$	$\beta_x(s_1) = 12.42$, $\beta_x(0) = 5.1$
9	$S(s_1) = 2\ (1/m^2)$	$\beta_x(s_2) = 15.18$

Table 2
Summary of comparison results

Case	$A_{\text{dyna,analy.}}$ (m)	$A_{\text{dyna,numer.}}$ (m)
1	0.0385	0.04
2	0.055	0.054
3	0.022	0.024
4	0.0145	0.016
5	0.0138	0.0135
6	0.012	0.0135
8	0.021	0.02
9	$A_x = 0.0163$, $A_y = 0.031$	$A_x = 0.017$, $A_y = 0.034$

Super-ACO

Dynamic aperture of wigglers

A example of a sum of multipoles

$$B_x = \frac{k_x}{k_y} B_0 \sinh(k_x x) \sinh(k_y y) \cos(ks), \quad (1)$$

$$B_y = B_0 \cosh(k_x x) \cosh(k_y y) \cos(ks), \quad (2)$$

$$B_z = -\frac{k}{k_y} B_0 \cosh(k_x x) \sinh(k_y y) \sin(ks) \quad (3)$$

Wiggler fields

$$A_{N_w,y}(s) = \sqrt{\frac{3\beta(s)}{\beta_{y,m}^2}} \frac{\rho_w}{k_y\sqrt{L_w}},$$

$$A_{N_w,x}(s) = \sqrt{\frac{\beta_y(s)}{\beta_x(s)}(A_{N_w,y}(s)^2 - y^2)}.$$

$$A_{\mathrm{total},y}(s) = \frac{1}{\sqrt{1/A_y(s)^2 + \sum_{j=1}^{M} 1/A_{j,\mathrm{w},y}(s)^2}},$$

where N_w is the wiggler period number, λ_w is the wiggler period length, the wiggler length $L_w = N_w\lambda_w$, ρ_w is the radius of curvature of the wiggler peak magnetic field B_0, and $\rho_w = E_0/ecB_0$ with E_0 being the electron energy, and $\beta_{y,m}$ is the beta function value in the middle of the wiggler.

J. Gao, "Analytical estimation of dynamic apertures limited by the wigglers in storage rings", **Nuclear Instruments and Methods in Physics Research A** 516 (2004) 243–248

Super-ACO

Table 1
The dynamic apertures correspond to different ρ_w, where $A_{N_w,y,n}$ and $A_{N_w,y,a}$ correspond to numerical and analytical results, respectively

ρ_w (m)	$A_{N_w,y,n}$ (m)	$A_{N_w,y,a}$ (m)	$\beta_{y,m}$ (m)	λ_w (m)	L_w (m)
2.7	0.017	0.019	13	0.17584	3.5168
3	0.023	0.024	10.7	0.17584	3.5168
4	0.033	0.034	9.5	0.17584	3.5168

Table 2
The dynamic apertures correspond to different λ_w, where $A_{N_w,y,n}$ and $A_{N_w,y,a}$ correspond to numerical and analytical results, respectively

λ_w (m)	$A_{N_w,y,n}$ (m)	$A_{N_w,y,a}$ (m)	$\beta_{y,m}$ (m)	ρ_w (m)	L_w (m)
0.08792	0.016	0.017	9.55	4	3.5168
0.17584	0.033	0.034	9.5	4	3.5168
0.35168	0.067	0.067	9.5	4	3.5168

One wiggler case

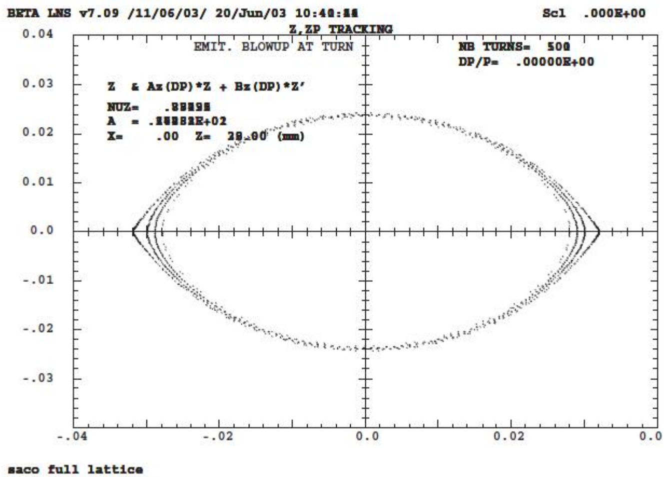


Fig. 5. The vertical phase space corresponds to the case of two wigglers.

When $\rho_w = 6$ m and $\beta_y(s) = \beta_{y,m} = 13.75$ m, one finds the vertical dynamic aperture limited by the two wigglers being 0.032 m numerically as shown in Fig. 5 and 0.03 m analytically calculated from Eqs. (19) and (23).

Two wiggler case

Nonlinear beam-beam effects-1 (e+e-)

Bsseti-Erskine formula for beam-beam induced transverse kicks

$$\delta y' + \mathrm{i}\delta x' = -\frac{N_{\mathrm{e}} r_{\mathrm{e}}}{\gamma_*} f(x, y, \sigma_x, \sigma_y)$$

$$f(x,y,\sigma_x,\sigma_y) = \sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}} \times w\left(\frac{x+\mathrm{i}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - \sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}} \times \exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\frac{\sigma_y}{\sigma_x}x+\mathrm{i}\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)$$

$$H_y = \frac{p_y^2}{2} + \frac{K_y(s)}{2}y^2 + \frac{N_{\mathrm{e}} r_{\mathrm{e}}}{\sqrt{2}\gamma_*}\left(\frac{1}{\sigma_x\sigma_y}y^2 - \frac{1}{12\sigma_x\sigma_y^3}y^4 + \frac{1}{120\sigma_x\sigma_y^5}y^6 - \frac{1}{1344\sigma_x\sigma_y^7}y^8 + \cdots\right) \times \sum_{k=-\infty}^{\infty}\delta(s-kL) \qquad \text{(FB)}, \tag{38}$$

with $p_x = \mathrm{d}x/\mathrm{d}s$ and $p_y = \mathrm{d}y/\mathrm{d}s$.

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Nonlinear beam-beam effects-2 (e+e-)

$$\tau_{bb} = \frac{\tau_y}{2}\left(\frac{\langle y^2\rangle}{y_{\max}^2}\right)\exp\left(\frac{y_{\max}^2}{\langle y^2\rangle}\right) = \frac{\tau_y}{2}\left(\frac{\sigma_y(s)^2}{A_{\mathrm{dyna},y}(s)^2}\right)\exp\left(\frac{A_{\mathrm{dyna},y}(s)^2}{\sigma_y(s)^2}\right)$$

or

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{16\gamma_*\sigma^2}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{-1}\exp\left(\frac{16\gamma_*\sigma^2}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,x} = \frac{\tau^*_x}{2}\left(\frac{6\gamma_*\sigma_x^2}{N_e r_e \beta_x(s_{\mathrm{IP}})}\right)^{-1}\exp\left(\frac{6\gamma_*\sigma_x^2}{N_e r_e \beta_x(s_{\mathrm{IP}})}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{3\sqrt{2}\gamma_*\sigma_x\sigma_y}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{-1}\exp\left(\frac{3\sqrt{2}\gamma_*\sigma_x\sigma_y}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{4}{\pi\xi^*_y}\right)^{-1}\exp\left(\frac{4}{\pi\xi^*_y}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,x} = \frac{\tau^*_x}{2}\left(\frac{3}{\pi\xi^*_x}\right)^{-1}\exp\left(\frac{3}{\pi\xi^*_x}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,y} = \frac{\tau^*_y}{2}\left(\frac{3}{\sqrt{2}\pi\xi^*_y}\right)^{-1}\exp\left(\frac{3}{\sqrt{2}\pi\xi^*_y}\right) \quad \text{(FB).}$$

More generally, one has

$$\tau^*_{bb,2m,y} = \frac{\tau^*_y}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{2/m-2}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,2m,x} = \frac{\tau^*_x}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_x}\right)^{2/m-2}\right) \quad \text{(FB)}$$

Nonlinear beam-beam effects-3 (e+e-)

$$\xi_x^* = \frac{N_e r_e \beta_{x,\mathrm{IP}}}{2\pi\gamma^* \sigma_x(\sigma_x + \sigma_y)}$$

$$\xi_y^* = \frac{N_e r_e \beta_{y,\mathrm{IP}}}{2\pi\gamma^* \sigma_y(\sigma_x + \sigma_y)}$$

Dynamic apertures limited by nonlinear beam-beam effects

$$\frac{A_{\mathrm{dyna},8,y}(s)}{\sigma_*(s)} = \left(\frac{16\gamma_*\sigma^2}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{1/2} \quad \text{(RB)} \qquad = \left(\frac{4}{\pi\xi_y^*}\right)^{1/2}$$

$$\frac{A_{\mathrm{dyna},8,x}(s)}{\sigma_{*,x}(s)} = \left(\frac{6\gamma_*\sigma_x^2}{N_e r_e \beta_x(s_{\mathrm{IP}})}\right)^{1/2} \quad \text{(FB)} \qquad = \left(\frac{3}{\pi\xi_x^*}\right)^{1/2}$$

$$\frac{A_{\mathrm{dyna},8,y}(s)}{\sigma_{*,y}(s)} = \left(\frac{3\sqrt{2}\gamma_*\sigma_x\sigma_y}{N_e r_e \beta_y(s_{\mathrm{IP}})}\right)^{1/2} \quad \text{(FB).} \quad = \left(\frac{3}{\sqrt{2}\pi\xi_y^*}\right)^{1/2}$$

Nonlinear beam-beam effects-4 (e+e-)

More generally, one has

$$\tau^*_{bb,2m,y} = \frac{\tau^*_y}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{RB}}}{4\pi\sqrt{m}\xi^*_y}\right)^{2/m-2}\right) \quad \text{(RB)}$$

$$\tau^*_{bb,2m,x} = \frac{\tau^*_x}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},x}}{\pi 2\sqrt{m}\xi^*_x}\right)^{2/m-2}\right) \quad \text{(FB)}$$

$$\tau^*_{bb,2m,y} = \frac{\tau^*_y}{2}\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},y}}{\pi\sqrt{2m}\xi^*_y}\right)^{-2/m-2}\exp\left(\left(\frac{2^{(m-2)/2}C_{m,\mathrm{FB},y}}{\pi\sqrt{2m}\xi^*_y}\right)^{2/m-2}\right) \quad \text{(FB).}$$

$$\xi^{\mathrm{RB}}_{y,\max} = \frac{4\sqrt{2}}{3}\xi^{\mathrm{FB}}_{y,\max} = 1.89\xi^{\mathrm{FB}}_{y,\max}$$

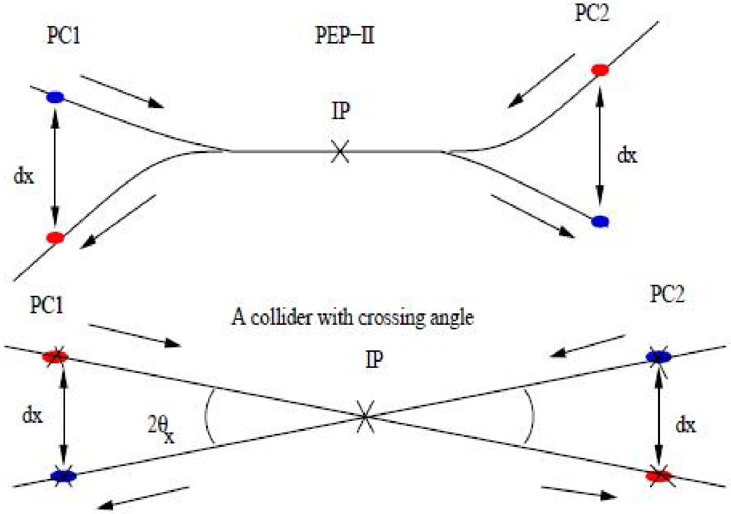
Round beam vs flat beam

and

$$\xi^{\mathrm{FB}}_{x,\max} = \sqrt{2}\xi^{\mathrm{FB}}_{y,\max}.$$

J. Gao, “Analytical estimation ofthe beam–beam interaction limited dynamic apertures and lifetimes in e+e- circular colliders”, **Nuclear Instruments and Methods in Physics Research A** 463 (2001) 50–61

Parasitic crossing beam-beam effects



$$\tau_{PC,y,RB} = \frac{\tau_y}{2}\left(\mathcal{R}_{y,PC,RB}\right)^{-1}\exp\left(\mathcal{R}_{y,PC,RB}\right)$$

$$= \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_{PC,y}}\right)^{-1}\exp\left(\frac{4}{\pi\xi_{PC,y}}\right)$$

with

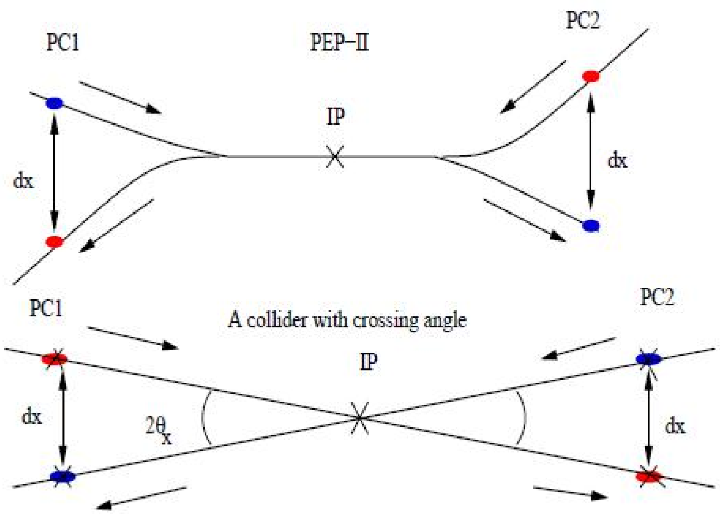
$$\xi_{PC,y} = \frac{r_e N_e \beta_{PC,x}}{2\pi\gamma_* \Sigma_{PC}^2} = \frac{r_e N_e \beta_{PC,y}}{2\pi\gamma_* d_x^2}$$

$$\Sigma_{PC}, \Sigma_{PC} = \sqrt{d_x^2 + d_y^2}$$

J. Gao, ON PARASITIC CROSSINGS AND THEIR LIMITATIONS TO E+E− STORAGE RING COLLIDERS, **Proceedings of EPAC 2004**, Lucerne, Switzerland, p. 671-673 (2004)

J. Gao, “Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings”, **Chinese Physics C** Vol. 33, No. 2, Feb., 2009, 135-144

Beam-beam effects with crossing angle



$$\mathscr{R}_{\text{syn-beta},x} = \frac{A_{\text{syn-beta},x}(s)^2}{\sigma_x(s)^2} = \frac{2}{3\pi^2}\left(\frac{1}{\xi_x^* \Phi}\right)^2$$

where $\Phi = (\sigma_z/\sigma_x)\phi$ is Piwinski angle.

J. Gao, "Analytical estimation of the effects of crossing angle on the luminosity of an e+e- circular collider", **Nuclear Instruments and Methods in Physics Research A** 481 (2002) 756–759

J. Gao, "Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings", **Chinese Physics C** Vol. 33, No. 2, Feb., 2009, 135-144

Space charge nonlinear effects

$$\left(\frac{A_{\mathrm{total},sc,y}(s)}{\sigma_y(s)}\right)^2 = \frac{3}{\sqrt{2}\pi\xi_{sc}}$$

$$\xi_{sc,y} = -\frac{r_{\mathrm{e}} N_{\mathrm{e}} \beta_{\mathrm{av},y}}{2\pi\gamma\sigma_y(\sigma_x+\sigma_y)}\left(\frac{L}{\sqrt{2\pi}\beta^2\gamma^2\sigma_z}\right)$$

J. Gao, "Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings", **Chinese Physics C** Vol. 33, No. 2, Feb., 2009, 135-144

J. Gao, Theoretical analysis of the limitation from the nonlinear space charge forces to TESLA damping ring, **TESLA 2003-12**

TESLA

TESLA COLLABORATION

Theoretical Analysis on the Limitation from the Nonlinear Space Charge Forces to TESLA Damping Ring

J. Gao

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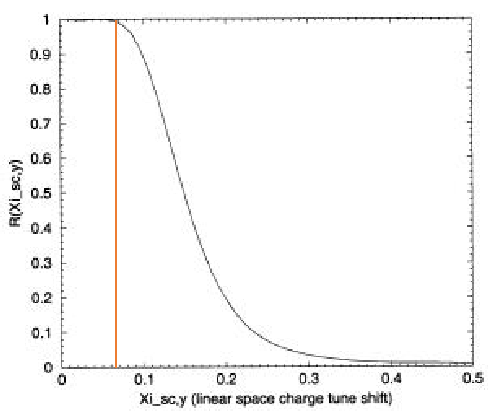

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DESY

February 2003, TESLA 2003-12

Electron cloud nonlinear effect

$$\xi'_{\mathrm{ec}}(s_0) = \frac{r_e N_{\mathrm{ec}} \beta_{+,y}(s_0)}{2\pi\gamma_+ \sigma_{+,y}(s_0)(\sigma_{+,x}(s_0) + \sigma_{+,y}(s_0))} \left(\frac{1}{2L_0}\right)$$

$$\left(\frac{A_{\mathrm{ec},y}}{\sigma_{+,y}}\right)^2 \approx \frac{3\sqrt{2}\gamma_+}{\pi r_e \beta_{av,y} \rho_{\mathrm{ec}} L}$$

$$\rho_{\mathrm{ec}} = \frac{N_{\mathrm{ec}}}{2\pi\sigma_{\mathrm{av},+,x}\sigma_{\mathrm{av},+,y}L_0}$$

J. Gao, “Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings”, **Chinese Physics C** Vol. 33, No. 2, Feb., 2009, 135-144

Combined beam-beam, space charge, electron cloud nonlinear effects

$$\mathcal{R}^2_{\mathrm{ec},y} = \left(\frac{A_{\mathrm{ec},y}}{\sigma_{+,y}}\right)^2 \approx \frac{3\sqrt{2}\gamma_+}{\pi r_{\mathrm{e}}\beta_{av,y}\rho_{\mathrm{ec}}L}, \qquad \rho_{\mathrm{ec}} = \frac{N_{\mathrm{ec}}}{2\pi\sigma_{\mathrm{av},+,x}\sigma_{\mathrm{av},+,y}L},$$

$$\mathcal{R}^2_{\mathrm{total},+,y} = \frac{1}{\dfrac{1}{\mathcal{R}^2_{bb,+,y}} + \dfrac{1}{\mathcal{R}^2_{\mathrm{ec},y}} + \dfrac{1}{\mathcal{R}^2_{sc,y}}},$$

$$\tau_{\mathrm{total},+,y} = \frac{\tau_{+,y}}{2}\left(\mathcal{R}^2_{\mathrm{total},+,y}\right)^{-1}\exp\left(\mathcal{R}^2_{\mathrm{total},+,y}\right)$$

J. Gao, “Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings”, **Chinese Physics C** Vol. 33, No. 2, Feb., 2009, 135-144

Analytical formulae for dynamic apertures with energy spread

WEPEA022 Proceedings of IPAC2013, Shanghai, China

ANALYTICAL ESTIMATIONS OF THE DYNAMIC APERTURES OF BEAMS WITH MOMENTUM DEVIATION AND APPLICATION IN FFAG*

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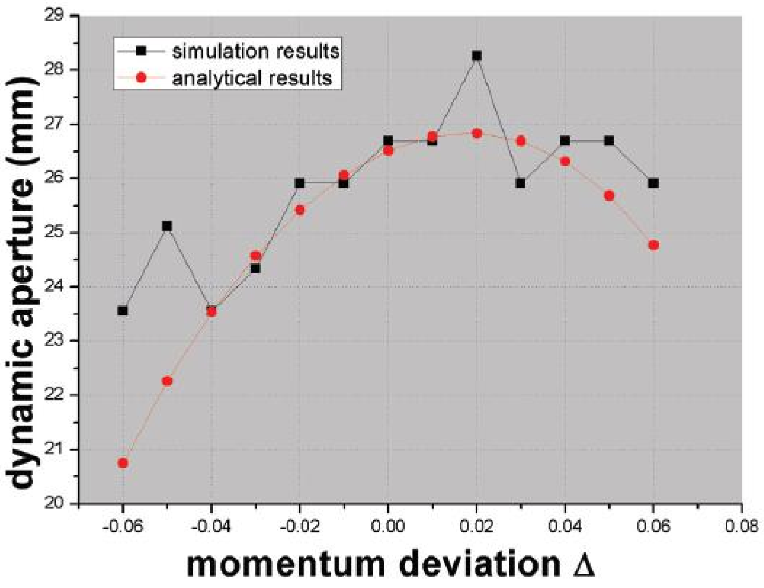
$$H = \frac{p_\beta^2}{2} - (1-\Delta)\left(K_x + \Delta SD\right)\frac{x_\beta^2}{2} + (1-\Delta)S\frac{x_\beta^3}{6}$$

$$A_{dyna,sext,\Delta} = \frac{1}{1-\Delta}\sqrt{\frac{8\tilde{\beta}_x(s)}{3(B^2+C^2)}} = \Omega \times A_{dyna,sext} \quad (16)$$

Here we call Ω the modulation factor. It is clear to tell that the dynamic aperture for off-momentum particles is modulated by both the momentum deviation and the linear lattice's characteristic.



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Hadron collider beam-beam limit formulae (pp)

$$\xi_{h,y,\max} = \frac{H_0 \gamma}{f(x_*)} \sqrt{\frac{r_h}{6\pi R N_{\rm IP}}}$$

H_0 ~2845,

$$\xi_{h,y,\max} = \frac{H_0}{2\pi f(x_*)} \sqrt{\frac{T_0}{\tau_y \gamma N_{\rm IP}}}$$

$$f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x \exp\left(-\frac{t^2}{2}\right) dt$$

$$x^2 = \frac{4 f(x)}{\pi \xi_{y,\max} N_{\rm IP}}$$

$$x_*^2 = \frac{4 f(x_*)^2}{H_0 \pi \gamma} \sqrt{\frac{6\pi R}{r_h N_{\rm IP}}}$$

f=1 corresponds electron positron colliders

Machine	N_{IP}	R (m)	γ	$\xi_{y,max}$	x^*	$f(x^*)$
$Sp\bar{p}S$	3	741	335.75	0.0026	2.187	34.8
TEVATRON	2	682	959	0.0042	2.16	32.5
HERA(p)	2	588	874	0.00426	2.155	32.1
LHC	2	2700	7460	0.005	2.1	28
SSC	2	9824	23400	0.0049	2.116	29.12

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Proceedings of the 1999 Particle Accelerator Conference, New York, 1999

SINGLE BUNCH LONGITUDINAL INSTABILITIES IN PROTON STORAGE RINGS

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$$H(I,\theta,t)^* = H(I) + \frac{1}{2}\Delta P^2 T_0 \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

$$= H(I) + \frac{(dE)^2 h^2 \eta^2}{2R_s^2 p_s^2} T_0 \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

$$= H(I) + \frac{\mathcal{U}_w^2 h^2 \eta^2 \cos^2\theta}{2R_s^2 p_s^2} T_0 \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

Microwave instability starting bunch current

$$I_{b,th} = \frac{R_s p_s}{e\sqrt{|\Omega'| T_0^2 h |\eta| \mathcal{K}_{//}^{tot}(\sigma_z)}}$$

$$|\Omega'| = \frac{1}{\pi^4 |1 - H_b/H_c|} \left(\ln \frac{32}{|1 - H_b/H_c|} \right)^3$$

where

$$\frac{H_b}{H_c} = \left(\frac{\delta E_b}{\delta E_{max}} \right)^2 = \frac{\pi h |\eta| E_s}{\beta^2 e \hat{V} G(\phi_s)} (\delta E_b)^2$$

$$G(\phi_s) = 2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s$$

J. Gao, "Single bunch longitudinal instabilities in proton storage rings", **Proceedings of PAC99**, New York, USA (1999)

Analytical estimates of halo current loss rates in space charge dominated beams

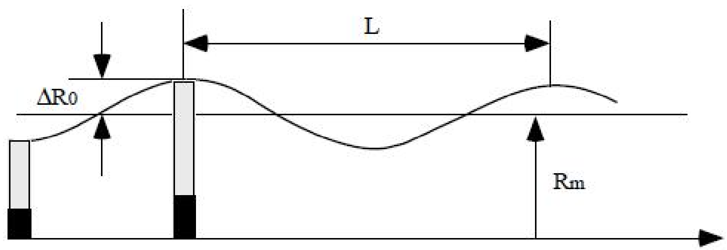

Fig. 4. Schematic illustration of halo particle loss at the mechanical boundary.

$$\frac{\Delta x_{\max}(z)}{R_0} = \frac{2R_0^3\sqrt{\beta(z)}}{\sqrt{27}LK\Delta R_0\beta(z_i)^{3/2}} \quad (27)$$

where $\beta(z)$ is the beta function of the focusing channel of the zero space charge effect. If one takes $\beta(z) = \beta(z_i) = \beta_{\mathrm{av}}$, Eq. (27) can be further simplified as:

$$\frac{\Delta x_{\max}}{R_0} = \frac{2R_0^3}{\sqrt{27}LK\Delta R_0\beta_{\mathrm{av}}} \quad (28)$$

where $\beta_{\mathrm{av}} = R_0/\sqrt{K}$.

J. Gao, "Analytical estimates of halo current loss rates in space charge dominated beams", **Nuclear Instruments and Methods in Physics Research A** 484 (2002) 27–35

THE DYNAMIC APERTURE FOR THE SYNCHROTRON RADIATION FACILITY OF TAC

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Abstract

The Turkish Accelerator Center (TAC) is a project for accelerator based fundamental and applied researches supported by Turkish State Planning Organization (TSPO). In this presentation, the dynamic aperture calculations for the TAC Synchrotron Storage Ring is made.

Analytical Method for Dynamic Aperture

The general expression of dynamic aperture in the horizontal plane (z=0) of multipole component:

$$A_{dyna,2m} = \sqrt{2\beta_x(s)}\left(\frac{1}{m\beta_x^m(s(2m))}\right)^{1/2(m-2)}\left(\frac{\rho}{|b_{m-1}|L}\right)^{1/(m-2)} \quad (4)$$

m=3 for sextupole;

$$A_{dyna,sext} = \sqrt{2J_{max,sext}\beta_x(s)} = \frac{\sqrt{2\beta_x(s)}}{\sqrt{3}\beta_x(s_1)^{3/2}}\left(\frac{\rho}{|b_2|L}\right) \quad (5)$$

At the same time the total dynamic aperture expressed as follows:

$$A_{dyna,total} = \frac{1}{\sqrt{\sum_i 1/A^2{}_{dyna,sext,i} + \sum_j 1/A^2{}_{dyna,oct,j} + \sum_k 1/A^2{}_{dyna,dec,k} + \cdots}} \quad (6)$$

Finally, six sextupole of S=1 are located at $s=s_{1,2,3,4,5,6}$ with $\beta_x(s_1)$= 3.286 m, $\beta_x(s_2)$= 1.767 m, $\beta_x(s_3)$= 11.854 m, $\beta_x(s_4)$= 13.855 m, $\beta_x(s_5)$= 2.021 m, $\beta_x(s_6)$= 3.558 m, respectively, and $\beta_x(0)$= 9.988 m, in the same time we get $A_{dyna,sext,1}$= 0.43 m, $A_{dyna,sext,2}$= 1.093 m, $A_{dyna,sext,3}$= 0.063 m, $A_{dyna,sext,4}$= 0.05 m, $A_{dyna,sext,5}$= 0.893 m, $A_{dyna,sext,6}$= 0.385 m, so the total dynamic aperture value of TAC expressed as follows:

$$A_{dyna,total} \cong \frac{1}{\sqrt{666}} = \frac{1}{25} = 0{,}0387\ m \cong 38{,}7\ mm \quad (7)$$

CONCLUSION

In this study, dynamic aperture for storage ring of the TAC is calculated. We found $A_{dyna,total}$ = 0.0387 m with analytical methods and it compared with the numerical value of 0.037 m. They are good values, but dynamic aperture can be improved by using more sextupoles (harmonic sextupoles) or octupoles to cancel the effect of the chromaticity sextupoles. Otherwise dynamic aperture should be researched and studied for top-up injection and touschek life time.

Beam Dynamics Studies and Design Optimisation of New Low Energy Antiproton Facilities

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Table I. Basic ELENA $\bar{\mathrm{p}}$ parameters.

Kinetic energy range	5.3 MeV–100 keV
Momentum range	100 MeV/c–13.7 MeV/c
Intensity	$\sim 10^7\,\bar{\mathrm{p}}$
Transverse acceptance	75 mm·mrad
Parameters at ejection:	
Number of bunches	4
$\Delta p/p$ (rms)	$\sim 0.05\%$
Bunch length (rms)	0.33 m
$\epsilon_{x,y}$ (rms)	~ 1 mm·mrad

The DA in the horizontal plane in the presence of a multipole magnet of $(2m)$th order ($m \geq 3$) can be estimated analytically [8]:

$$A_{x,2m} = \sqrt{2\beta_x(s)}\left(\frac{1}{m\beta_x^m(s(2m))}\right)^{1/(2(m-2))}\left(\frac{|k_{m-1}|L}{m-1}\right)^{-1/(m-2)}, \quad (1)$$

where $\beta_x(s)$ is the betatron function at the point where we want to calculate the DA, usually at the entrance of the lattice (injection), $\beta_x(s(2m))$ the betatron function at the position of the multipolar element, L is the effective length of the multipolar magnet, and $k_{m-1} = (1/B_0\rho)\partial^{m-1}B_y/\partial x^{m-1}$ is the normalised magnetic strength (defined according to the MAD-X [6] conventions).

If the lattice presents a certain number of independent nonlinear components, i.e. in a simple case with no special phase and amplitude relations between them, the total DA can be calculated as:

$$A_{x,\mathrm{total}} = \left(\sum_i \frac{1}{A_{x,6,i}^2} + \sum_j \frac{1}{A_{x,8,j}^2} + \sum_k \frac{1}{A_{x,10,k}^2} + \cdots\right)^{-1/2}, \quad (2)$$

where $A_{x,6}$ stands for DA of sextupoles, $A_{x,8}$ for octupoles, $A_{x,10}$ for dodecapoles, and so on.

As mentioned before, in ELENA there are just four sextupoles for chromaticity correction. For two sextupoles with $k_2 = 25.8$ m^{-3} and $\beta_x \simeq 3.6$ m, two sextupoles with $k_2 = -58.4$ m^{-3} and $\beta_x \simeq 0.54$ m, and $\beta_x(s) \simeq 2$ m at the beginning and at the end of the one-turn mapping, from Eqs. (1) and (2) we obtain $A_{x,\mathrm{total}} \approx 62$ mm. For the vertical plane, in the presence of a single sextupole at position s_i:

$$A_{y,6} = \sqrt{\frac{\beta_x(s_i)}{\beta_y(s_i)}(A_{x,6}^2 - x^2)}. \quad (3)$$

Therefore, the total vertical DA can be estimated using Eq. (3) (with $x = 0$) and Eq. (2) for y, i.e. $A_{y,\mathrm{total}} = \left(\sum_i 1/A_{y,6,i}^2\right)^{-1/2}$. Knowing that $\beta_y \simeq 3.1$ m for the four ELENA sextupoles, then $A_{y,\mathrm{total}} \approx 63$ mm.

The above results are in reasonable agreement with multi-turn particle tracking simulations. Figure 2 shows the transverse phase space for 2000 turns in the ELENA ring, using the PTC tracking module of the program MAD-X [6].

Further tracking studies of ELENA (including limiting physical apertures) for different off-momentum particles have determined a momentum deviation acceptance $|\Delta p/p_0| \lesssim 0.7\%$.

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
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Analytical estimation of the dynamic apertures of circular accelerators

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Abstract

By considering delta function sextupole, octupole, and decapole perturbations and using difference action-angle variable equations, we find some useful analytical formulae for the estimation of the dynamic apertures of circular accelerators due to single sextupole, single octupole, single decapole (single $2m$ pole in general); and their combined effects are derived based on the Chirikov criterion of the onset of stochastic motions. Comparisons with numerical simulations are made, and the agreement is quite satisfactory.

PACS: 29.20. – c

Keywords: Circular accelerators; Dynamic aperture; Stochastic motions; Nonlinear dynamics

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Analytical estimation of the beam–beam interaction limited dynamic apertures and lifetimes in e^+e^- circular colliders

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Review of some important beam physics issues in electron–positron collider designs

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Analytical treatment of the nonlinear electron cloud effect and the combined effects with beam-beam and space charge nonlinear forces in storage rings*

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Abstract In this paper we treat first some nonlinear beam dynamics problems in storage rings, such as beam dynamic apertures due to magnetic multipoles, wiggles, beam-beam effects, nonlinear space charge effect, and then nonlinear electron cloud effect combined with beam-beam and space charge effects, analytically. This analytical treatment is applied to BEPCⅡ. The corresponding analytical expressions developed in this paper are useful both in understanding the physics behind these problems and also in making practical quick hand estimations.

Key words electron cloud effect, beam-beam effect, space charge effect, storage ring, BEPCⅡ

PACS 29.20.db, 29.27.Bd

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Conlusions

1) Theoretical understanding of nonlinear motion in accelerators are very important

2) Analytical formulae of dynamic aperture in circular accelerators for multipoles have been established with applications in wigglers, beam-beam, space charge, electron cloude effects, etc

3) Aanalytical formulae have find their applications in real machine designs

4) More advanced theoretical studied are needed together with the development of advanced of nonlinear physics

5) The final nalytical treatment goal is to find the multipole settings for maximum dynamic apertures by solving equations

6) Synergies with numerical methods will not be excluded of course

Thank you for your attention