How Does Lepton Collider Indirectly Probe Neutralino Dark Matter?

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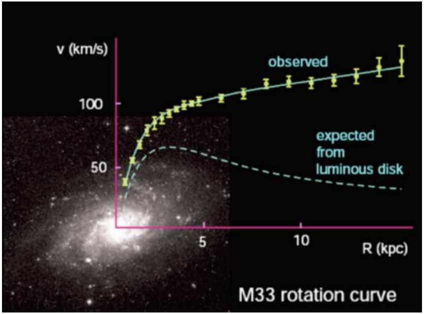
Stellar motions

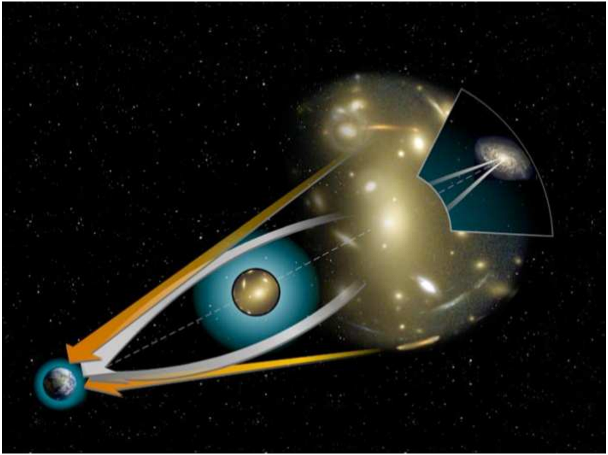
- First hint was from the observation of stars in the Milky Way by Jan Hendrik Oort in 1932.
- Similar results came out in the observation of the nearby Coma cluster of galaxies by Fritz Zwicky in 1933.
- More direct evidence is the rotation curves of galaxies (1970s, Vera Rubin).



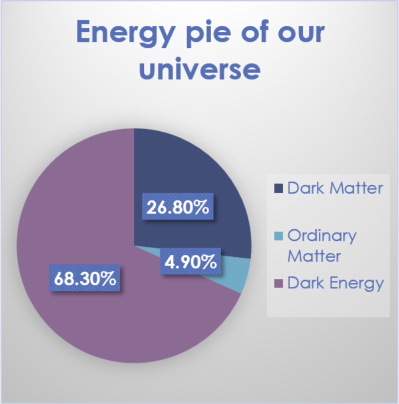
$$v(R) = \sqrt{\frac{GM(R)}{R}}$$

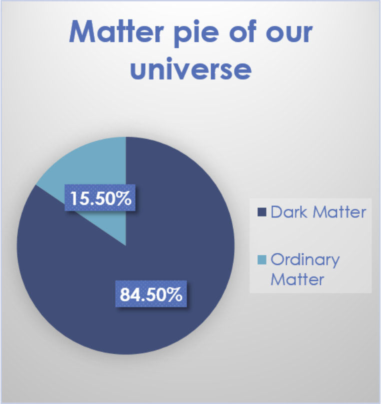
Dark matter exists

Gravitational lensing



Cosmology

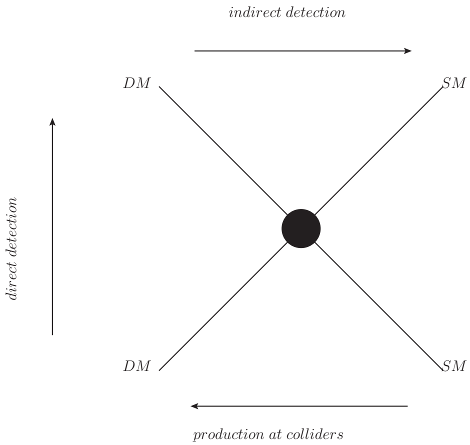




From PLANCK

Dark matter searches

- Relic density
- Direct searches (PANDAX, LUX, XENON1T, ...)
- Indirect searches
- Collider searches: directly, indirectly



SM EFT

Standard Model (SM) Effective Field Theory (EFT) is an EFT composed of Standard Model (SM) fields.

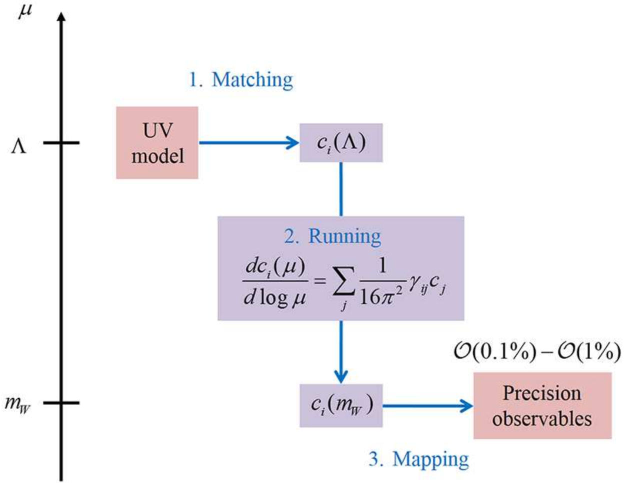
$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda^{d_i - 4}} c_i \mathscr{O}_i . \tag{1}$$

1 dimension-5 operator $\mathscr{O}^{(5)}$.
59 independent dimension-6 operators $\mathscr{O}_i^{(6)}$.

Model vs Measurements

model	<>	operators	<>	measurements
parameters in $\mathscr{L}$	<>	59 Wilson coefficients	<>	cross sections angular distributions

How does SM EFT work?



Matching

$$\mathscr{L}_{\text{UV}}[\phi, \Phi] \Longrightarrow \mathscr{L}_{\text{eff}}[\phi] \, ?$$

Feynman diagramatic mathcing

I. Write down all possible operators in $\mathscr{L}_{\text{eff}}$
II. Calculate Feynman diagrams
III. Matching to EFT operator coefficients

Functional matching, covariant derivative expansion(CDE)

I. Start from path integral, and directly derive effective operators
II. Preserve gauge invariance in intermediate steps via CDE
III. Can obtain all operators at the same time

CDE of tree- and 1-loop effective action

The effective action can be obtained by integrate out Φ

$$e^{iS_{\rm eff}[\phi]} = \int D\Phi e^{iS[\phi,\Phi]}, \qquad (2)$$

where $S[\phi,\Phi] = \int dx \mathscr{L}_{\rm UV}[\phi,\Phi]$. Expand S around Φ_c, where $\delta S[\phi,\Phi]/\delta\Phi = 0$,

$$S[\phi,\Phi_c+\eta] = S[\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}|_{\Phi_c}\eta^2 + \mathscr{O}(\eta^3).$$

Thus we have

$$\begin{aligned} S_{\rm eff}^{\rm tree} &= S[\Phi_c], \\ S_{\rm eff}^{\text{1-loop}} &= \frac{i}{2}{\rm Tr}\log\left(-\frac{\delta^2 S}{\delta\Phi^2}\bigg|_{\Phi_c}\right). \end{aligned} \qquad (3)$$

B. Henning, X. Lu and H. Murayama (HLM) [arxiv:1412.1837]

CDE of tree-level effective Lagrangian

A UV Lagrangian with heavy field Φ

$$\mathscr{L}_{\mathrm{UV}}[\Phi,\phi] \supset \left(\Phi^{\dagger}B(x)+\text{h.c.}\right)+\Phi^{\dagger}\left(-D^2-m^2-U(x)\right)\Phi+\mathscr{O}(\Phi^3). \quad (4)$$

Solve the equation of motion for Φ

$$\left(P^2-m^2-U(x)\right)\Phi=-B(x)+\mathscr{O}(\Phi^2) \quad (5)$$

we get

$$\Phi_c=-\frac{1}{P^2-m^2-U(x)}B(x), \quad (6)$$

where $P_\mu \equiv iD_\mu = i\partial_\mu + A_\mu(x)$.

CDE of tree-level effective Lagrangian

Expand Φ_c with

$$\frac{1}{A^{-1}(1-AB)} = \sum_{n=0}^{\infty}(AB)^n A, \tag{7}$$

$$\begin{aligned}
\Phi_c &= \left[1-\frac{1}{m^2}\left(P^2-U\right)\right]^{-1}\frac{1}{m^2}B \\
\Phi_c &= \frac{1}{m^2}B+\frac{1}{m^2}\left(P^2-U\right)\frac{1}{m^2}B+\frac{1}{m^2}\left(P^2-U\right)\frac{1}{m^2}\left(P^2-U\right)\frac{1}{m^2}B+\dots
\end{aligned} \tag{8}$$

Tree-level effective Lagrangian is

$$\begin{aligned}
\mathscr{L}_{\text{eff}}^{\text{tree}} &= -B^{\dagger}\frac{1}{P^2-m^2-U(x)}B+\mathscr{O}(\Phi_c^3) \\
\mathscr{L}_{\text{eff}}^{\text{tree}} &= B^{\dagger}\frac{1}{m^2}B+B^{\dagger}\frac{1}{m^2}\left(P^2-U\right)\frac{1}{m^2}B+\cdots+\mathscr{O}(\Phi_c^3)
\end{aligned} \tag{9}$$

UV Lagrangian

$$\mathscr{L}_{\mathrm{UV}}[\Phi,\phi] \supset \left(\Phi^\dagger B(x) + \text{h.c.}\right) + \Phi^\dagger\left(-D^2 - m^2 - U(x)\right)\Phi + \mathscr{O}(\Phi^3)$$

$$\Rightarrow \frac{\delta^2 S}{\delta\Phi^2} = \left(P^2 - m^2 - U(x)\right) \tag{10}$$

$$\Rightarrow S_{\text{eff}}^{\text{1-loop}} = \frac{i}{2}\text{Tr}\log\left(-P^2 + m^2 + U(x)\right) \tag{11}$$

Insert a complete set of momentum and spatial states

$$\mathrm{Tr}[f(\hat{x},\hat{q})] = \int dq\,\mathrm{tr}\langle q|f(\hat{x},\hat{q})|q\rangle \qquad \left(\int dq = \int \frac{d^4q}{(2\pi)^4}\right)$$

$$\int dx|x\rangle\langle x| = 1 \qquad \left(\int dx = \int d^4x\right)$$

we get

$$S_{\text{eff}}^{\text{1-loop}} = \frac{i}{2}\int dxdq\langle q|x\rangle\langle x|\mathrm{tr}\log(-P^2 + m^2 + U)|q\rangle. \tag{12}$$

$$S_{\text{eff}}^{\text{1-loop}} = \frac{i}{2}\int dxdq \langle q|x\rangle\langle x|\operatorname{tr}\log(-P^2+m^2+U)|q\rangle. \qquad (13)$$

Since

$$\langle x|q\rangle = e^{iq\cdot x} \qquad \langle x|\hat{\mathscr{O}}(\hat{x},\hat{p})|q\rangle = \hat{\mathscr{O}}(x,i\partial_x)e^{iq\cdot x}$$
$$\langle q|x\rangle\langle x|P_\mu|q\rangle = e^{-iq\cdot x}P_\mu e^{iq\cdot x} = iD_\mu - q_\mu = P_\mu - q_\mu$$

we obtain

$$S_{\text{eff}}^{\text{1-loop}} = \frac{i}{2}\int dxdq \operatorname{tr}\log\left(-(P_\mu - q_\mu)^2 + m^2 + U\right). \qquad (14)$$

Covariant Derivative Expansion (CDE) with:
Gaillard-Cheyette transformation[1]

$$S_{\text{eff}}^{\text{1-loop}} = \frac{i}{2}\int dxdq\,\mathrm{tr} e^{P\cdot\frac{\partial}{\partial q}} \log\left(-(P_\mu - q_\mu)^2 + m^2 + U\right) e^{-P\cdot\frac{\partial}{\partial q}} \qquad (15)$$

and Baker-Cambell-Hausdorff (BCH) formula

$$e^B A e^{-B} = \sum_{n=0}^{\infty} \frac{1}{n!} L_B^n A, \qquad L_B A = [B, A] \qquad (16)$$

[1] NPB268:669 (1986)

CDE of 1-loop effective action

The non-commutator term is n=0 term, which could be canceled as

$$\begin{aligned}
e^{P\cdot\frac{\partial}{\partial q}}(P_\mu - q_\mu)e^{-P\cdot\frac{\partial}{\partial q}} &= e^{P\cdot\frac{\partial}{\partial q}}P_\mu e^{-P\cdot\frac{\partial}{\partial q}} - e^{P\cdot\frac{\partial}{\partial q}}q_\mu e^{-P\cdot\frac{\partial}{\partial q}} \\
&= \sum_{n=0}^{\infty}\frac{1}{n!}\left(L_{P\cdot\partial/\partial q}\right)^n P_\mu - \sum_{n=0}^{\infty}\frac{1}{n!}\left(L_{P\cdot\partial/\partial q}\right)^n q_\mu \\
&= P_\mu + \sum_{n=1}^{\infty}\frac{1}{n!}\left(L_{P\cdot\partial/\partial q}\right)^n P_\mu \\
&\quad - \left\{q_\mu + P_\mu + \sum_{n=2}^{\infty}\frac{1}{n!}\left(L_{P\cdot\partial/\partial q}\right)^n q_\mu\right\} \\
&= -q_\mu + \sum_{n=1}^{\infty}\frac{n}{(n+1)!}\left(L_{P\cdot\partial/\partial q}\right)^n P_\mu \\
&= -q_\mu - \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}\left[P_{\alpha_1},\left[\ldots\left[P_{\alpha_n},\left[D_\nu,D_\mu\right]\right]\right]\right]\frac{\partial^n}{\partial q_{\alpha_1}\ldots\partial q_{\alpha_n}}\frac{\partial}{\partial q_\nu}
\end{aligned} \tag{17}$$

With

$$\widetilde{G}_{\nu\mu} = \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!} \left[P_{\alpha_1}, \left[P_{\alpha_2}, \left[\ldots \left[P_{\alpha_n}, [D_\nu, D_\mu]\right]\right]\right]\right] \frac{\partial^n}{\partial q_{\alpha_1} \partial q_{\alpha_2} \ldots \partial q_{\alpha_n}},$$

$$\widetilde{U} = e^{P\cdot\frac{\partial}{\partial q}} U e^{-P\cdot\frac{\partial}{\partial q}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left[P_{\alpha_1}, \left[P_{\alpha_2}, \left[\ldots [P_{\alpha_n}, U]\right]\right]\right] \frac{\partial^n}{\partial q_{\alpha_1} \partial q_{\alpha_2} \ldots \partial q_{\alpha_n}},$$

the 1-loop effective action becomes

$$S_{\text{eff}}^{\text{1-loop}} = i\frac{i}{2} \int dxdq \,\mathrm{tr}\log\left[-\left(q_\mu + \widetilde{G}_{\nu\mu}\frac{\partial}{\partial q_\nu}\right)^2 + m^2 + \widetilde{U}\right]. \qquad (18)$$

Expansion in mass-degenerate case

Expand in $\Delta \equiv (q^2 - m^2)^{-1}$

$$\mathscr{L}_{\text{eff}}^{\text{1-loop}} = ic_s \int dq \,\mathrm{tr} \log \left[-\left(q_\mu + \widetilde{G}_{\nu\mu} \frac{\partial}{\partial q_\nu} \right)^2 + m^2 + \widetilde{U} \right]$$
$$= -ic_s \int dq \,\mathrm{tr} \log \left\{ \Delta^{-1} \left[1 - \Delta \left(-\{q_\mu, \widetilde{G}_{\nu\mu} \partial^\nu\} - \widetilde{G}_{\sigma\mu} \widetilde{G}^\sigma_\nu \partial^\mu \partial^\nu + \widetilde{U} \right) \right] \right\} \tag{19}$$

with $c_s = i/2$ and $\partial_\mu \equiv \partial / \partial q^\mu$ for short.

Expansion in mass-degenerate case

Introduce an auxiliary integral as

$$\int dm^2 \frac{d\log(m^2+X)}{dm^2} = \int dm^2 \frac{1}{m^2+X} \tag{20}$$

The 1-loop effective Lagrangian becoms

$$\begin{aligned}
&\mathscr{L}_{\text{eff}}^{\text{1-loop}} \\
&= -ic_s \int dq \int dm^2 \mathrm{tr} \frac{1}{\left\{\Delta^{-1}\left[1-\Delta\left(-\{q_\mu,\widetilde{G}_{\nu\mu}\partial^\nu\}-\widetilde{G}_{\sigma\mu}\widetilde{G}^\sigma_\nu\partial^\mu\partial^\nu+\widetilde{U}\right)\right]\right\}} \\
&= -ic_s \sum_{n=0}^{\infty} \int dqdm^2 \mathrm{tr}\left\{\left[\Delta\left(-\{q_\mu,\widetilde{G}_{\nu\mu}\partial^\nu\}-\widetilde{G}_{\sigma\mu}\widetilde{G}^\sigma_\nu\partial^\mu\partial^\nu+\widetilde{U}\right)\right]^n\Delta\right\}.
\end{aligned} \tag{21}$$

$$
\begin{aligned}
\mathscr{L}_{\text{eff}}^{\text{1-loop}} = \frac{c_s}{(4\pi)^2}\, \text{tr}\Bigg\{ \\
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right)\right] \\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2} - 1\right)U\right] \\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2}\right)G'^2_{\mu\nu} - \frac{1}{2}\log\frac{m^2}{\mu^2}\,U^2\right] \\
&+ \frac{1}{m^2}\left[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2 - \frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} - \frac{1}{12}\left(P_\mu U\right)^2 - \frac{1}{6}U^3 - \frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{m^4}\left[\frac{1}{24}U^4 + \frac{1}{12}U\left(P_\mu U\right)^2 + \frac{1}{120}\left(P^2 U\right)^2 + \frac{1}{24}\left(U^2 G'_{\mu\nu}G'_{\mu\nu}\right)\right. \\
&\qquad \left. - \frac{1}{120}\left[(P_\mu U),(P_\nu U)\right]G'_{\mu\nu} - \frac{1}{120}\left[U[U,G'_{\mu\nu}]\right]G'_{\mu\nu}\right] \\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5 - \frac{1}{20}U^2\left(P_\mu U\right)^2 - \frac{1}{30}\left(UP_\mu U\right)^2\right] \\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned}
\tag{22}
$$

Expansion in non-degenerate case

In this case

$$M = diag(m_i) \neq M\mathbb{1}$$
$$\Rightarrow [M,U] \neq 0 \tag{23}$$

Introduce other auxiliary parameter ξ as in [arXiv: 1512.03003] [2].
Let $M = \xi diag(m_i)$ and $\Delta_\xi = 1/(q^2 - \xi M^2)$, and set $\xi = 1$ in the end.

Thus the $\mathscr{L}_{\text{eff}}^{\text{1-loop}}$ becomes

$$\mathscr{L}_{\text{eff}}^{\text{1-loop}} = -ic_s \sum_{n=0}^{\infty} \int dq d\xi \, \mathrm{tr}\Big\{ \big[\Delta_\xi \big(-\{q_\mu, \widetilde{G}_{\nu\mu}\partial^\nu\} - \widetilde{G}_{\sigma\mu}\widetilde{G}^\sigma_\nu \partial^\mu \partial^\nu + \widetilde{U} \big)\big]^n \Delta_\xi M^2 \Big\} \tag{24}$$

[2] or μ parameter as in [arXiv: 1509.05942].

After the integration, one can get the universal one-loop effective action (UOLEA).

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s\Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} \left[P_\mu, [P_\nu, U_{ij}]\right] \left[P_\mu, [P_\nu, U_{ji}]\right] + f_{12,b}^{ij} \left[P_\mu, [P_\nu, U_{ij}]\right] \left[P_\nu, [P_\mu, U_{ji}]\right] \\
& + f_{12,c}^{ij} \left[P_\mu, [P_\mu, U_{ij}]\right] \left[P_\nu, [P_\nu, U_{ji}]\right] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} [P_\mu, U_{ij}] [P_\nu, U_{jk}] G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .
\end{aligned}
$$

f_is in 1512.03003

How to deal with fermions in CDE?

Lagrangian contains Dirac fermions as

$$\mathscr{L}[\psi, \phi] = \overline{\psi}\big(i\not{D} - m - M(x)\big)\psi. \qquad (25)$$

The 1-loop effective action $S_{\text{eff}}^{\text{1-loop}} = -i\,\text{Tr}\log\big(\not{P} - m - M\big)$ could be treated with a trick as

$$S_{\text{eff}}^{\text{1-loop}} = -\frac{i}{2}\Big[\text{Tr}\log\big(-\not{P} - m - M\big) + \text{Tr}\log\big(\not{P} - m - M\big)\Big]. \qquad (26)$$

This is because the trace is invariant under changing signs of gamma matrices.

How to deal with fermions in CDE?

With $\not{P}M \equiv [\not{P}, M]$ then

$$S_{\text{eff}}^{\text{1-loop}} = -\frac{i}{2}\operatorname{Tr}\log\left(-\not{P}^2 + m^2 + 2mM + M^2 + \not{P}M\right) \tag{27}$$

We also have

$$\gamma^\mu\gamma^\nu = (\{\gamma^\mu,\gamma^\nu\} + [\gamma^\mu,\gamma^\nu])/2 = g^{\mu\nu} - i\sigma^{\mu\nu}$$
$$\Rightarrow \not{P}^2 = P^2 + \frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu] = P^2 + \frac{i}{2}\sigma\cdot G' \tag{28}$$

Then

$$S_{\text{eff}}^{\text{1-loop}} = -\frac{i}{2}\operatorname{Tr}\log\left(-P^2 + m^2 + U_{\text{ferm}}\right) \tag{29}$$

where $U_{\text{ferm}} \equiv -\frac{i}{2}\sigma^{\mu\nu}G'_{\mu\nu} + 2mM + M^2 + \not{P}M$. Then

$$\mathscr{L}_{\text{eff}}^{\text{1-loop}} = -\frac{i}{2}\int dq \operatorname{tr}\log\left[-\left(q_\mu + \widetilde{G}_{\nu\mu}\partial_\nu\right)^2 + m^2 + \widetilde{U}_{\text{ferm}}\right]. \tag{30}$$

Neutralino-Chargino sector in the MSSM

The Lagrangian with left-handed spinor field ψ_a

$$\begin{aligned}\mathscr{L}_{\text{MSSM}} \supset & i\tilde{B}^\dagger\bar{\sigma}^\mu D_\mu \tilde{B} + i\tilde{W}^\dagger\bar{\sigma}^\mu D_\mu \tilde{W} + i\tilde{H}_u^\dagger\bar{\sigma}^\mu D_\mu \tilde{H}_u + i\tilde{H}_d^\dagger\bar{\sigma}^\mu D_\mu \tilde{H}_d \\ & - \frac{1}{2}(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + c.c) \\ & - \mu(\tilde{H}_u^T \varepsilon \tilde{H}_d + c.c) \\ & - [\frac{\sqrt{2}}{2} g(H_u^\dagger \sigma^a \tilde{H}_u + H_d^\dagger \sigma^a \tilde{H}_d)\tilde{W}^a + \frac{\sqrt{2}}{2} g'(H_u^\dagger \tilde{H}_u - H_d^\dagger \tilde{H}_d)\tilde{B} + c.c], \end{aligned} \tag{31}$$

where $\tilde{H}_u^T = (\tilde{H_u^+}, \tilde{H_u^0})$, $\tilde{H}_d^T = (\tilde{H_d^0}, \tilde{H_d^-})$, $\tilde{W}^T = (\tilde{W}^1, \tilde{W}^2, \tilde{W}^3)$ and $\tilde{B}$ are higgsinos and electroweak gauginos.

The effective action

$$e^{iS_{\text{eff}}} = \int D\chi^\dagger D\chi \, \exp\left\{ i \int dx \, [i\chi^\dagger \bar{\sigma}^\mu D_\mu \chi - \frac{1}{2}(\chi^T (M + U(x))\chi + c.c)] \right\} \tag{32}$$

with $\chi^T = (\tilde{B}, \tilde{W}^T, H_u^T, \tilde{H}_d^T) = (\tilde{B}, \tilde{W}^1, \tilde{W}^2, \tilde{W}^3, \tilde{H_u^+}, \tilde{H_u^0}, \tilde{H_d^0}, \tilde{H_d^-})$, and

$$M = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 \mathbb{1}_{3\times 3} & 0 & 0 \\ 0 & 0 & 0 & \mu\varepsilon \\ 0 & 0 & \mu\varepsilon^T & 0 \end{pmatrix}, U = \begin{pmatrix} 0 & 0 & g' H^\dagger s_\beta/\sqrt{2} & -g'\tilde{H}^\dagger c_\beta/\sqrt{2} \\ 0 & 0 & gH^\dagger \vec{\sigma} s_\beta/\sqrt{2} & g\tilde{H}^\dagger \vec{\sigma} c_\beta/\sqrt{2} \\ g' H^* s_\beta/\sqrt{2} & g\vec{\sigma}^* H^* s_\beta/\sqrt{2} & 0 & 0 \\ -g'\tilde{H}^* c_\beta/\sqrt{2} & g\vec{\sigma}^* \tilde{H}^* c_\beta/\sqrt{2} & 0 & 0 \end{pmatrix}$$

$$D_\mu = \partial_\mu - i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & gW_\mu^a \lambda^a & 0 & 0 \\ 0 & 0 & gW_\mu^a \sigma^a/2 + g' B_\mu/2 & 0 \\ 0 & 0 & 0 & gW_\mu^a \sigma^a/2 - g' B_\mu/2 \end{pmatrix}$$

Transformation

With a few steps of fields transformation [3], one can obtain

$$\mathscr{L}_{\text{eff}} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - M' - U'P_L - U'^{\dagger}P_R)\psi \tag{33}$$

where

$$M' = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2\mathbb{1}_{3\times3} & 0 & 0 \\ 0 & 0 & \mu\mathbb{1}_{2\times2} & 0 \\ 0 & 0 & 0 & \mu\mathbb{1}_{2\times2} \end{pmatrix}, U' = \begin{pmatrix} 0 & 0 & g'H^{\dagger}s_{\beta}/\sqrt{2} & -g'\tilde{H}^{\dagger}c_{\beta}/\sqrt{2} \\ 0 & 0 & gH^{\dagger}\vec{\sigma}s_{\beta}/\sqrt{2} & g\tilde{H}^{\dagger}\vec{\sigma}c_{\beta}/\sqrt{2} \\ g'Hc_{\beta}/\sqrt{2} & g\vec{\sigma}Hc_{\beta}/\sqrt{2} & 0 & 0 \\ -g'\tilde{H}s_{\beta}/\sqrt{2} & g\vec{\sigma}\tilde{H}s_{\beta}/\sqrt{2} & 0 & 0 \end{pmatrix}$$

and $\psi^i = (\psi_L^i, \psi_R'^i)$.

[3] Mark Srednicki, "Quantum Field Theory", Chapter 36.

A few more tricks

Then

$$e^{2iS_{\text{eff}}} = \int D\bar{\psi}D\psi \exp\left\{\bar{\psi}(i\gamma^\mu D_\mu - M' - U'P_L - U'^\dagger P_R)\psi\right\}$$
$$= \det[i\gamma^\mu D_\mu - M' - U'P_L - U'^\dagger P_R]$$
$$\Rightarrow S_{\text{eff}} = -\frac{i}{2}\text{Log}[\det[i\gamma^\mu D_\mu - M' - U'P_L - U'^\dagger P_R]]$$
$$= -\frac{i}{2}\text{Tr Log}[i\gamma^\mu D_\mu - M' - U'P_L - U'^\dagger P_R)] \tag{34}$$

With

$$\Lambda = \frac{U'^\dagger + U'}{2}, V = \frac{U'^\dagger - U'}{2}, \tag{35}$$

one can get

$$S_{\text{eff}} = -\frac{i}{2}\text{TrLog}[\not{P} - M' - \Lambda - V\gamma_5]. \tag{36}$$

$$
\begin{aligned}
S_{\text{eff}} &= -\frac{i}{2}\text{TrLog}[\not{P} - M' - \Lambda - V\gamma_5] \\
&= -\frac{i}{4}\text{Tr}\left(\text{Log}\left[-\not{P} - M' - \Lambda + V\gamma_5\right] + \text{Log}\left[\not{P} - M' - \Lambda - V\gamma_5\right]\right) \\
&= -\frac{i}{4}\text{Tr Log}\left[-P^2 + M'^2 + U_{\text{ferm}}\right] \\
&= -\frac{i}{4}\int \text{dx} \int \text{dq tr Log}\left[-\left(q_\mu + \tilde{G}_{\nu\mu}\frac{\partial}{\partial q_\nu}\right)^2 + M'^2 + \tilde{U}_{\text{ferm}}\right] \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
U_{\text{ferm}} &= \left(\Lambda^2 - V^2 + [\not{P},\Lambda] + \{M',\Lambda\}\right) + \left(\Lambda V - V\Lambda + [\not{P},V] + [M',V]\right)\gamma_5 - \frac{i}{2}\sigma^{\mu\nu}G'_{\mu\nu} \\
\tilde{G}_{\nu\mu} &= \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}, \\
\tilde{U}_{\text{ferm}} &= \sum_{n=0}^{\infty}\frac{1}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U_{\text{ferm}}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}},
\end{aligned}
$$

$$
\mathscr{L}_{\text{eff}} = \frac{i}{4}\sum_{n=0}^{\infty}\int dqd\xi\,\text{tr}\left\{\left[\Delta_\xi\left(-\{q_\mu,\widetilde{G}_{\nu\mu}\partial^\nu\} - \widetilde{G}_{\sigma\mu}\widetilde{G}^{\sigma}_{\nu}\partial^\mu\partial^\nu + \widetilde{U}\right)\right]^n\Delta_\xi M^2\right\} \qquad (38)
$$

Operator bases

18 dimension-6 SM EFT CP-even bosonic operators.

$\mathcal{O}_{GG}$	$=$	$g_s^2 H^\dagger H G^a_{\mu\nu} G^{a,\mu\nu}$	$\mathcal{O}_H$	$=$	$\frac{1}{2}\left(\partial_\mu H^\dagger H\right)^2$
$\mathcal{O}_{WW}$	$=$	$g^2 H^\dagger H W^a_{\mu\nu} W^{a,\mu\nu}$	$\mathcal{O}_T$	$=$	$\frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
$\mathcal{O}_{BB}$	$=$	$g'^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_R$	$=$	$H^\dagger H D_\mu H^\dagger D^\mu H$
$\mathcal{O}_{WB}$	$=$	$2gg' H^\dagger t^a H W^a_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_D$	$=$	$D^2 H^\dagger D^2 H$
$\mathcal{O}_W$	$=$	$ig\left(H^\dagger t^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}$	$\mathcal{O}_6$	$=$	$(H^\dagger H)^3$
$\mathcal{O}_B$	$=$	$ig' Y_H \left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$	$\mathcal{O}_{2G}$	$=$	$-\frac{1}{2}\left(D^\mu G^a_{\mu\nu}\right)^2$
$\mathcal{O}_{3G}$	$=$	$\frac{1}{3!} g_s f^{abc} G^{a\mu}_\rho G^{b\nu}_\mu G^{c\rho}_\nu$	$\mathcal{O}_{2W}$	$=$	$-\frac{1}{2}\left(D^\mu W^a_{\mu\nu}\right)^2$
$\mathcal{O}_{3W}$	$=$	$\frac{1}{3!} g \epsilon^{abc} W^{a\mu}_\rho W^{b\nu}_\mu W^{c\rho}_\nu$	$\mathcal{O}_{2B}$	$=$	$-\frac{1}{2}\left(\partial^\mu B_{\mu\nu}\right)^2$
$\mathcal{O}_{HW}$	$=$	$2ig (D_\mu H)^\dagger t^a (D_\nu H) W^{a\mu\nu}$			
$\mathcal{O}_{HB}$	$=$	$2ig' Y_H (D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}$			

Operator bases

This is a redundant basis, in the sense that $\mathscr{O}_{HW}$ and $\mathscr{O}_{HB}$ can be switched into $\mathscr{O}_{WW}$, $\mathscr{O}_{BB}$, $\mathscr{O}_{WB}$, $\mathscr{O}_{W}$ and $\mathscr{O}_{B}$, by using the relations:

$$\begin{aligned} \mathscr{O}_{HW} &= \mathscr{O}_{W} - \frac{1}{4}(\mathscr{O}_{WW} + \mathscr{O}_{WB}); \\ \mathscr{O}_{HB} &= \mathscr{O}_{B} - \frac{1}{4}(\mathscr{O}_{BB} + \mathscr{O}_{WB}). \end{aligned} \tag{39}$$

Analytical results for coefficients

	Non-logarithmic Contributions	Logarithmic Contributions
$(4\pi)^2 c_{WW}$	$\frac{g'^2}{96\mu(\mu^2-M_1^2)^3}\Big(\mu^5+13\mu M_1^4-2\mu^3 M_1^2 +2M_1\sin 2\beta(-\mu^4+2M_1^4+5\mu^2 M_1^2)\Big) +\frac{g^2}{96\mu M_2(M_2^2-\mu^2)^3}\Big(17\mu M_2^5-10\mu^3 M_2^3+53\mu^5 M_2 -2\sin 2\beta\left(-16\mu^6+6M_2^6-25\mu^2 M_2^4+5\mu^4 M_2^2\right)\Big)$	$\frac{g'^2 M_1^4}{16(M_1^2-\mu^2)^4}\left(\mu^2+M_1^2+2\mu M_1\sin 2\beta\right)\ln\frac{M_1^2}{\mu^2} -\frac{g^2}{16(M_2^2-\mu^2)^4}\left(4\mu^4+M_2^4\right)\left(\mu^2+M_2^2+2\mu M_2\sin 2\beta\right)\ln\frac{M_2^2}{\mu^2}$
$(4\pi)^2 c_{BB}$	$\frac{g'^2}{96\mu(\mu^2-M_1^2)^3}\Big(\mu^5+13\mu M_1^4-2\mu^3 M_1^2 +2M_1\sin 2\beta\left(-\mu^4+2M_1^4+5\mu^2 M_1^2\right)\Big) +\frac{g^2}{32\mu(\mu^2-M_2^2)^3}\Big(\mu^5+13\mu M_2^4-2\mu^3 M_2^2 +2M_2\sin 2\beta\left(-\mu^4+2M_2^4+5\mu^2 M_2^2\right)\Big)$	$\frac{g'^2 M_1^4}{16(M_1^2-\mu^2)^4}\left(\mu^2+M_1^2+2\mu M_1\sin 2\beta\right)\ln\frac{M_1^2}{\mu^2} +\frac{3g^2 M_2^4}{16(M_2^2-\mu^2)^4}\left(\mu^2+M_2^2+2\mu M_2\sin 2\beta\right)\ln\frac{M_2^2}{\mu^2}$
$(4\pi)^2 c_{WB}$	$\frac{g'^2}{48\mu(\mu^2-M_1^2)^3}\Big(\mu^5+13\mu M_1^4-2\mu^3 M_1^2 +2M_1\sin 2\beta\left(-\mu^4+2M_1^4+5\mu^2 M_1^2\right)\Big) +\frac{g^2}{48\mu(\mu^2-M_2^2)^3}\Big(-25\mu^5+11\mu M_2^4+2\mu^3 M_2^2 +\sin 2\beta\left(-4M_2^5+38\mu^2 M_2^3-46\mu^4 M_2\right)\Big)$	$\frac{g'^2 M_1^4}{8(M_1^2-\mu^2)^4}\left(\mu^2+M_1^2+2\mu M_1\sin 2\beta\right)\ln\frac{M_1^2}{\mu^2} +\frac{g^2(M_2^4-2\mu^4)}{8(M_2^2-\mu^2)^4}\left(\mu^2+M_2^2+2\mu M_2\sin 2\beta\right)\ln\frac{M_2^2}{\mu^2}$
$(4\pi)^2 c_{W}$	$\frac{g'^2}{72(\mu^2-M_1^2)^4}\Big(5\mu^6+23M_1^6-17\mu^2 M_1^4-35\mu^4 M_1^2 +4\mu M_1\sin 2\beta\left(-2\mu^4+7M_1^4-11\mu^2 M_1^2\right)\Big) -\frac{g^2}{24(\mu^2-M_2^2)^4}\Big(-29\mu^6+M_2^6+41\mu^2 M_2^4+11\mu^4 M_2^2 +4\mu M_2\sin 2\beta\left(-10\mu^4+5M_2^4+11\mu^2 M_2^2\right)\Big)$	$-\frac{g'^2 M_1^3}{12(M_1^2-\mu^2)^5}\Big(M_1^5+4\mu^2 M_1^3-9\mu^4 M_1 +2\mu\sin 2\beta\left(M_1^4-3\mu^4\right)\Big)\ln\frac{M_1^2}{\mu^2} +\frac{g^2}{12(M_2^2-\mu^2)^5}\Big(-4\mu^8+M_2^8+4\mu^2 M_2^6+27\mu^4 M_2^4-16\mu^6 M_2^2 +2\mu M_2\sin 2\beta\left(-4\mu^6+M_2^6+12\mu^2 M_2^4-3\mu^4 M_2^2\right)\Big)\ln\frac{M_2^2}{\mu^2}$
$(4\pi)^2 c_{B}$	$\frac{g'^2}{72(\mu^2-M_1^2)^4}\Big(5\mu^6+23M_1^6-17\mu^2 M_1^4-35\mu^4 M_1^2 +4\mu M_1\sin 2\beta\left(-2\mu^4+7M_1^4-11\mu^2 M_1^2\right)\Big) +\frac{g^2}{24(\mu^2-M_2^2)^4}\Big(5\mu^6+23M_2^6-17\mu^2 M_2^4-35\mu^4 M_2^2 +4\mu M_2\sin 2\beta\left(-2\mu^4+7M_2^4-11\mu^2 M_2^2\right)\Big)$	$-\frac{g'^2 M_1^3}{12(M_1^2-\mu^2)^5}\Big(M_1^5+4\mu^2 M_1^3-9\mu^4 M_1 +2\mu\sin 2\beta\left(M_1^4-3\mu^4\right)\Big)\ln\frac{M_1^2}{\mu^2} -\frac{g^2 M_2^3}{4(M_2^2-\mu^2)^5}\Big(M_2^5+4\mu^2 M_2^3-9\mu^4 M_2 +2\mu\sin 2\beta\left(M_2^4-3\mu^4\right)\Big)\ln\frac{M_2^2}{\mu^2}$

Analytical results for coefficients

	Non-logarithmic Contributions	Logarithmic Contributions
$(4\pi)^2 c_{HW}$	$\frac{g'^2}{24(M_1^2-\mu^2)^4}\Big(\mu^6 - 11M_1^6 + 35\mu^2 M_1^4 - \mu^4 M_1^2 + 2\mu M_1(\mu^4 + M_1^4 + 10\mu^2 M_1^2)\sin 2\beta\Big) + \frac{g^2}{8(M_2^2-\mu^2)^4}\Big(5M_2^6 - 13M_2^4\mu^2 + 47M_2^2\mu^4 - 15\mu^6 + 2M_2\mu(M_2^4 + 10M_2^2\mu^2 + \mu^4)\sin 2\beta\Big)$	$\frac{g'^2 M_1^3}{4\left(M_1^2-\mu^2\right)^5}\left(\mu^2 + M_1^2\right)\left(M_1^3 - 3\mu^2 M_1 - 2\mu^3 \sin 2\beta\right)\ln\frac{M_1^2}{\mu^2} - \frac{g^2}{4\left(M_2^2-\mu^2\right)^5}\left(\mu^2 + M_2^2\right)\Big(-4\mu^6 + M_2^6 - 3\mu^2 M_2^4 + 12\mu^4 M_2^2 + 6\mu^3 M_2^3 \sin 2\beta\Big)\ln\frac{M_2^2}{\mu^2}$
$(4\pi)^2 c_{HB}$	$\frac{g'^2}{24(\mu^2-M_1^2)^4}\Big(\mu^6 - 11M_1^6 + 35\mu^2 M_1^4 - \mu^4 M_1^2 + 2\mu M_1 \sin 2\beta\left(\mu^4 + M_1^4 + 10\mu^2 M_1^2\right)\Big) + \frac{g^2}{8(\mu^2-M_2^2)^4}\Big(\mu^6 - 11M_2^6 + 35\mu^2 M_2^4 - \mu^4 M_2^2 + 2\mu M_2 \sin 2\beta\left(\mu^4 + M_2^4 + 10\mu^2 M_2^2\right)\Big)$	$\frac{g'^2 M_1^3}{4\left(M_1^2-\mu^2\right)^5}\left(\mu^2 + M_1^2\right)\left(M_1^3 - 3\mu^2 M_1 - 2\mu^3 \sin 2\beta\right)\ln\frac{M_1^2}{\mu^2} + \frac{3g^2 M_2^3}{4\left(M_2^2-\mu^2\right)^5}\left(\mu^2 + M_2^2\right)\left(M_2^3 - 3\mu^2 M_2 - 2\mu^3 \sin 2\beta\right)\ln\frac{M_2^2}{\mu^2}$
$(4\pi)^2 c_D$	$+\frac{g'^2}{6(M_1^2-\mu^2)^4}\Big(M_1^6 - 7M_1^4\mu^2 - 7M_1^2\mu^4 + \mu^6 - M_1\mu(M_1^4 + 10M_1^2\mu^2 + \mu^4)\sin 2\beta\Big) + \frac{g^2}{2(M_2^2-\mu^2)^4}\Big(M_2^6 - 7M_2^4\mu^2 - 7M_2^2\mu^4 + \mu^6 - M_2\mu(M_2^4 + 10M_2^2\mu^2 + \mu^4)\sin 2\beta\Big)$	$+\frac{g'^2\mu^3 M_1^3}{(M_1^2-\mu^2)^5}\Big(2M_1\mu + (M_1^2 + \mu^2)\sin 2\beta\Big)\ln\frac{M_1^2}{\mu^2} + \frac{3g^2\mu^3 M_2^3}{(M_2^2-\mu^2)^5}\Big(2M_2\mu + (M_2^2 + \mu^2)\sin 2\beta\Big)\ln\frac{M_2^2}{\mu^2}$

$\cdots$

Check our results

$h\gamma\gamma$ effective coupling

$$\mathscr{L}_{h\gamma\gamma} = c_{h\gamma\gamma} h A_{\mu\nu} A^{\mu\nu} \tag{40}$$

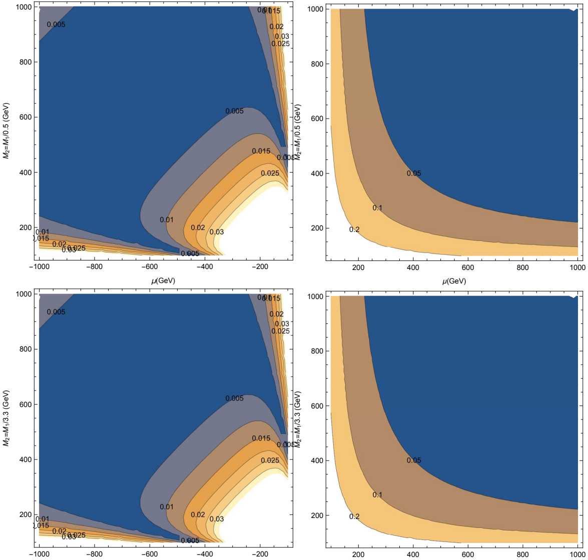
EFT result (with c_{WW}, c_{WB}, c_{WB}):

$$c_{h\gamma\gamma} = -\frac{{g_2}^4 \sin^2(\theta_w) v \sin(2\beta)}{24\sqrt{2}\pi^2 \mu M_2}. \tag{41}$$

With low energy theorem:

$$c_{h\gamma\gamma} = \frac{{g_2}^4 \sin^2(\theta_w) v \sin(2\beta)}{24\sqrt{2}\pi^2 \left({g_2}^2 v^2 \sin(2\beta) - \mu M_2\right)}. \tag{42}$$

Compared with loop calculation result, with $\Delta = c^{\mathrm{EFT}}_{h\gamma\gamma}/c^{loop}_{h\gamma\gamma} - 1$



Collider measurements

The uncertainties expected at each experiments, for the EWPT experiments (T and S), the TGC experiments (Δg_1^Z, $\Delta\kappa_\gamma$ and λ_γ) and 2 representative channels of 7 Higgs experiment channels [1] .

Observable	$10^3 T$	$10^3 S$	$10^4 \Delta g_1^Z$	$10^4 \Delta\kappa_\gamma$	$10^4 \lambda_\gamma$	$\frac{\Delta(\sigma_{Zh}\mathrm{Br}_{bb})}{\sigma_{Zh}^{\mathrm{SM}}\mathrm{Br}_{bb}^{\mathrm{SM}}}$	$\frac{\Delta(\sigma_{Zh}\mathrm{Br}_{\gamma\gamma})}{\sigma_{Zh}^{\mathrm{SM}}\mathrm{Br}_{\gamma\gamma}^{\mathrm{SM}}}$
CEPC	9	14	1.59	2.30	1.67	0.32%	9.1%

[1] J. Fan, M. Reece and L. T. Wang, JHEP **1509**, 196 (2015).

Mapping

Electroweak precision test observables

$$S = 4\pi v^2(4c_{WB} + c_W + c_B), \qquad T = v^2 c_T/\alpha \tag{43}$$

TGC Parameters

$$\delta g_1^Z = -m_Z^2 c_W, \quad \delta\kappa_\gamma = 4m_W^2 c_{WB}, \quad \lambda_\gamma = -m_W^2 c_{3W} \tag{44}$$

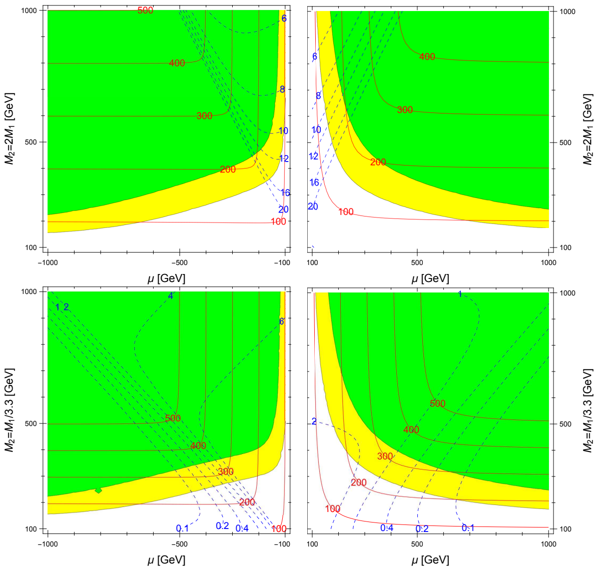
Deviations in Higgs related processes

$$\varepsilon = \frac{\sigma_{Zh}\mathrm{Br}_i}{\sigma_{Zh}^{\mathrm{SM}}\mathrm{Br}_i^{\mathrm{SM}}} - 1 \tag{45}$$

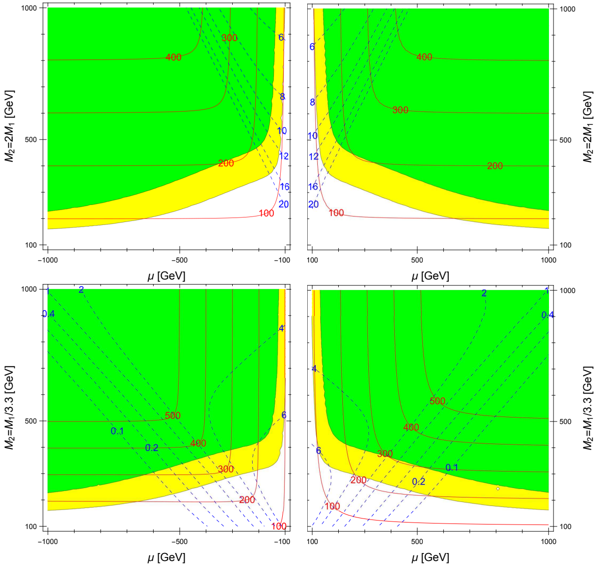
$$\begin{aligned} \mathrm{Br}_i &= \mathrm{Br}_{bb}, \mathrm{Br}_{cc}, \mathrm{Br}_{\tau\tau}, \mathrm{Br}_{\mu\mu}, \mathrm{Br}_{\gamma\gamma}, \mathrm{Br}_{WW^*}, \mathrm{Br}_{ZZ^*} \\ \varepsilon &= \varepsilon(c_{\mathrm{WW}}, \cdots) \text{ [1]} \end{aligned}$$

[1] B. Henning, X. Lu and H. Murayama, JHEP **1601**, 023 (2016).

Expected constraints from CEPC with $\tan\beta = 2$.



Expected constraints from CEPC with $\tan\beta = 20$.



Summary

- With measurements in hand, mapping to dim-6 operators, one can thus probe the model.
- Lepton collider will be useful to detect the neutralino dark matter in the MSSM.

Outlook

- Combine the sfermion-sector and neutralino-chargino sector together.
- CDE @ Composite Higgs models.