Flavor Singlet Mesons from Lattice QCD

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CRC110 meeting, Aug 29 - 31, Peking University, Beijing

Contents

Motivation

- Conventional quark model classifies $q\bar{q}$ bound states into $SU(N_f)$ flavor multiplets.
- Non-Abelian nature of QCD suggests other non-$q\bar{q}$ meson states.
- Glueballs, hybrids, tetraquarks ...
- Study lowest-lying flavor singlet scalar, pseudoscalar and tensor states.
- $U_A(1)$ anomaly relate flavor singlet pseudoscalar with topological charge density.
- For pseudoscalar channel, use conventional glueball & topological operators (CLQCD) and $q\bar{q}$ operator(C.Urbach, ETMC)

Glueball studies in Lattice QCD

Quenched approximation:

- B.Berg and A.Billoire, *Nuclear Physics* **B221** (1983):109-140
- C.Morningstar and M.Peardon, *Phys.Rev.***D56**(1997):4043-4061
- C.Morningstar and M.Peardon, *Phys.Rev.***D60**(1999)034509
- H.B.Meyer and M.J.Teper, *Phys.Lett.***B605**(2005)344-345
- Y.Chen et al, *Phys.Rev.***D73**(2006)014516
- ...

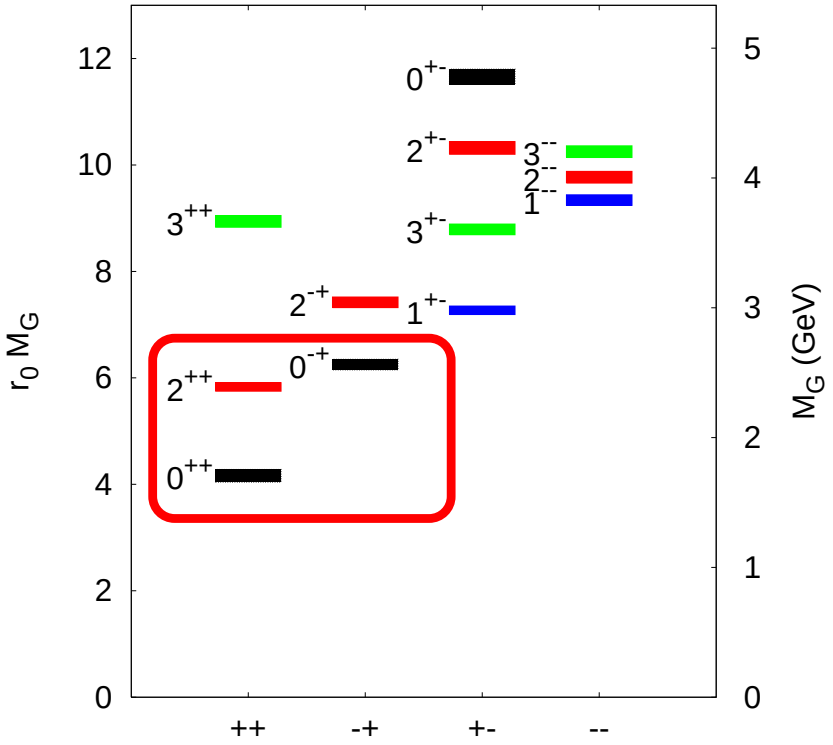
Dynamical sea quark:

- G.S.Bali et al,*Phys.Rev.***D62**(2000)054503
- A.Hart and M.Teper, *Phys.Rev.***D65**(2002)034502
- **UKQCD**, *Phys.Rev.***D82**(2010)034501
- E.Gregory et al, *JHEP* **10**(2012)170
- ...

Lowest-lying glueballs in quenched LQCD

- Lowest states with $J^{PC} = 0^{++},\ 2^{++},\ 0^{-+}$
- Masses around 1.7 GeV, 2.4 GeV and 2.6 GeV respectively.

[Y. Chen et al, *Phys. Rev.* **D 73**, 014516(2006)]



BESIII results on J/ψ radiative decay

- Radiative decay of J/ψ is a gluon rich process.
- BESIII Collaboration observed isosinglet scalar, pseudoscalar and tensor resonances in

$$J/\psi \longrightarrow \gamma X \longrightarrow \gamma\eta\eta \; and \; J/\psi \longrightarrow \gamma X \longrightarrow \gamma\phi\phi.$$

[M.Ablikim et al, *Phys.Rev.***D.87**, 092009 (2013)]

Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2 σ
$f_0(1710)$	$1759 \pm 6^{+14}_{-25}$	$172 \pm 10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0 σ
$f_0(2100)$	$2081 \pm 13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9 σ
$f_2'(1525)$	$1513 \pm 5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0 σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4 σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6 σ

[M.Ablikim et al, *Phys.Rev.***D.93**, 112011 (2016)]

Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times 10^{-4}$)	Sig.
$\eta(2225)$	2216^{+4+21}_{-5-11}	185^{+12+43}_{-14-17}	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

Lattice study on J/ψ radiatively decay into glueballs

- From experimental $J/\psi \longrightarrow \gamma X \longrightarrow \gamma\pi\pi$, $X \longrightarrow \pi\pi$ we can estimated

$$Br(J/\psi \longrightarrow \gamma f_0(1710)) = 2.5 \times 10^{-3}$$
$$Br(J/\psi \longrightarrow \gamma f_0(1500)) = 3.1 \times 10^{-4}$$

- Quenched LQCD predicted the J/ψ radiatively decay into scalar glueball with branching ratio :
  [L.C Gui et al, CLQCD Collaboration, *Phys.Rev.Lett.* **110** (2013) no.2, 021601]

$$Br(J/\psi \longrightarrow \gamma G_{0^{++}}) = 3.8(9) \times 10^{-3},$$

  which suggested the $f_0(1710)$ as scalar glueball candidate.
- J/ψ radiative decay into tensor glueball gives
  [Yi-Bo Yang et al, CLQCD Collaboration, *Phys. Rev. Lett.* **111**, 091601 (2013)]

$$Br(J/\psi \longrightarrow \gamma G_{2^{++}}) = 1.1(2) \times 10^{-2}$$

  relatively large branching ratio.

Gauge configuration details

- limited resources, GPU cluster at Hunan Normal University(24 K40 cards).
- USQCD Chroma software used.
- $N_f = 2$ anisotropic gauge configuration.
- Tadpole-improved gauge action and Clover-improved Wilson fermion action.
- Ground state 0^{++}, 2^{++}, 0^{-+} was investigated in our study. [W. Sun et al, CLQCD Collaboration, arXiv:1702.08174]

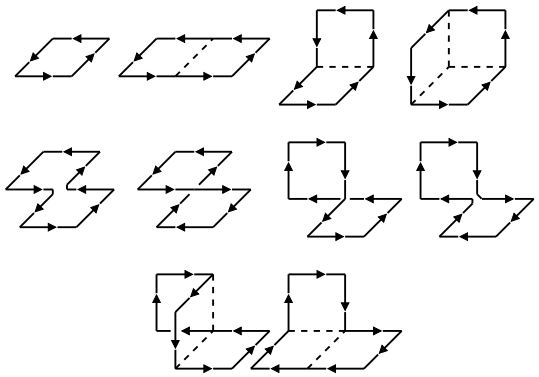
Table: Parameters of configurations

β	$L^3 \times T$	ξ	a_s	m_π	N_{conf}
2.5	$12^3 \times 128$	5	$0.114fm$	~ 650 MeV	4800
2.5	$12^3 \times 128$	5	$0.118fm$	~ 938 MeV	10400

Glueball operators construction

- Particle states denoted by J^{PC} in continuum
- SU(2)(continuum) $\xrightarrow{reduction}$ ^2O(lattice)
- Glueballs are bosons ($^2O \to O$)
- Octahedral group O has five IRs, A_1, A_2, E, T_1, T_2, denoted by R
- Subduced representation of $SU(2)$ with respect to group O is generally reducible($J \geq 2$)
- $R \not\leftrightarrow J$ ($A_1 \longrightarrow J = 0, J = 4, ...$)
- Assuming that the ground state on the lattice corresponds to the lowest spin state in continuum
- Using different spatial oriented Wilson loops to construct glueball operators with quantum number denoted by R^{PC}

R \ J	0	1	2	3	4	5
A_1	1	0	0	0	1	0
A_2	0	0	0	1	0	0
E	0	0	1	0	1	1
T_1	0	1	0	1	1	2
T_2	0	0	1	1	1	1



[C.J. Morningstar and M.J. Peardon, *Phys. Rev.* **D 60**, 034509(1999)]

Effective mass plateaus

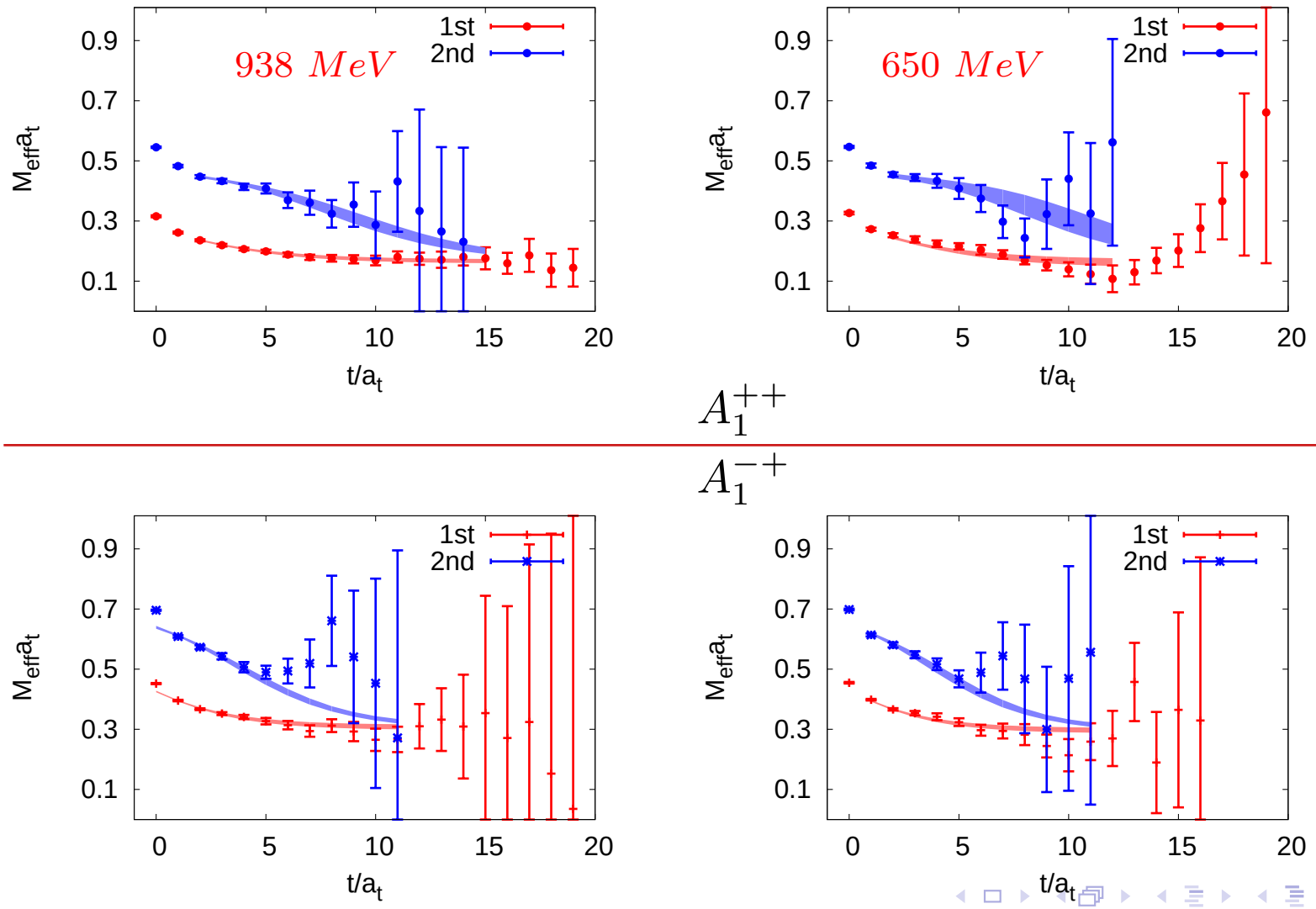
- 24 operators $\phi_\alpha^{(R^{PC})}$ for each R^{PC}
- Use the variational method to get a optimal operator $\Phi_i^{R^{PC}}$
- The optimal correlation function is

$$\tilde{C}_i^{(R^{PC})}(t) = \sum_\tau \langle 0|\Phi_i^{(R^{PC})}(t+\tau)\Phi_i^{(R^{PC})}(\tau)|0\rangle,$$

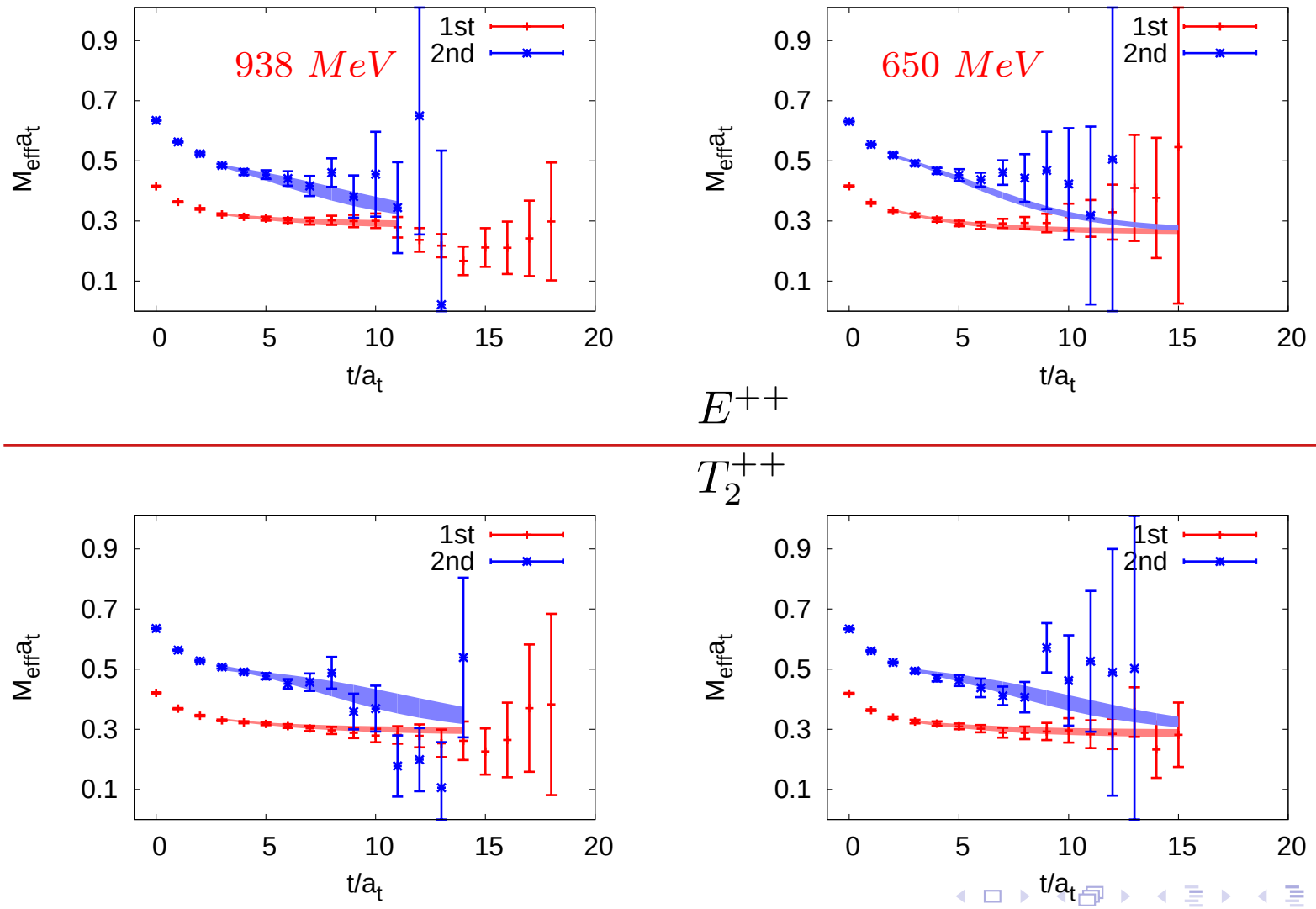
- Use two state union fit

$$\begin{aligned}\tilde{C}_1^{(R^{PC})}(t) &= W_{11}^{(R^{PC})}e^{-m_1 t} + W_{12}^{(R^{PC})}e^{-m_2 t},\\ \tilde{C}_2^{(R^{PC})}(t) &= W_{21}^{(R^{PC})}e^{-m_1 t} + W_{22}^{(R^{PC})}e^{-m_2 t},\end{aligned}$$

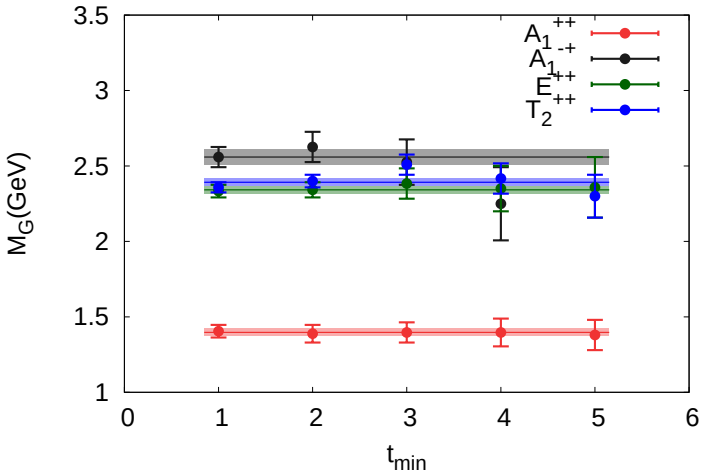
Effective mass plateaus (A_1^{++} & A_1^{-+})



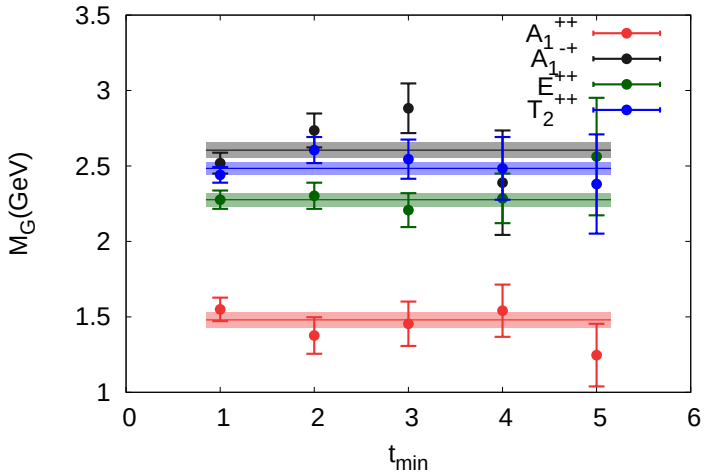
Effective mass plateaus (E^{++} & T_2^{++})



Fitted results



938 MeV



650 MeV

	m_π (MeV)	m_{0++} (MeV)	m_{2++} (MeV)	m_{0-+} (MeV)
$N_f = 2$	938	1397(25)	2367(35)	2559(50)
[this work]	650	1480(52)	2380(61)	2605(52)
$N_f = 2+1$	360	1795(60)	2620(50)	—
[E. Gregory]				
quenched	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
[C. Morningstar]				
quenched	—	1730(50)(80)	2400(25)(120)	2590(40)(130)
[Y. Chen]				

Fitted results

Table: Ground state meson spectrum

m_{PS}(MeV)	m_V(MeV)	m_S(MeV)	$m_{G^{0++}}$(MeV)
650(4)	993(16)	1362(53)	1480(52)
938(3)	1164(10)	1473(28)	1397(25)

- 0^{++} glueball mass is relatively lighter than previous results and close to the 0^{++} $q\bar{q}$ meson mass on our lattice (actually isovector a_0).
- Relatively large mass above 2 GeV for pseudoscalar glueball
- Physical isosinglet pseudoscalar around 1 GeV
- Further study on pseudoscalar state by topological charge density
- Heavy quark mass, sea quark effects?

$\eta'(\eta_2)$ *v.s.* pseudoscalar glueball

- $U(1)_A$ anomaly gives

$$\partial_\mu A^\mu(x) = 2mP(x) - N_f q(x)$$

- P(x):flavor singlet pseudoscalar density

$$P(x) = \bar{\psi}(x)\gamma_5\psi(x)$$

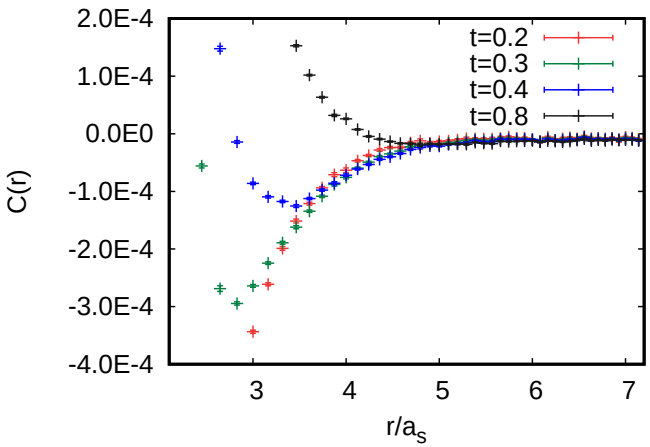
- q(x):topological charge density

$$q(x) = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} Tr F_{\mu\nu} F_{\rho\sigma}$$

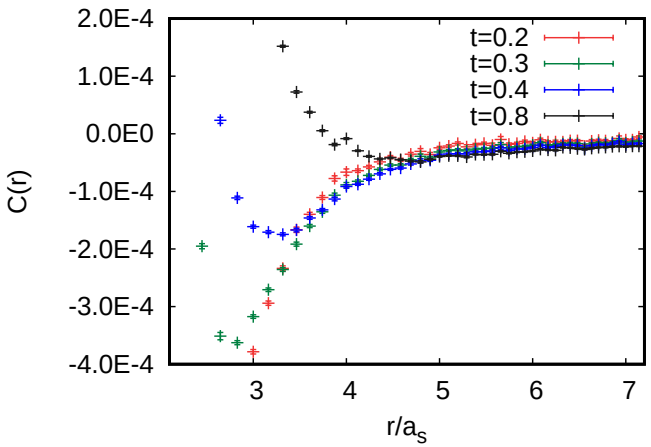
- Study correlation functions of $P(x)$(C.Urbach) and $q(x)$(this work) respectively.

correlation of $q(x)$

$$C_q(x-y) = \langle q(x)q(y)\rangle$$



(a) $m_\pi \sim 938$ MeV

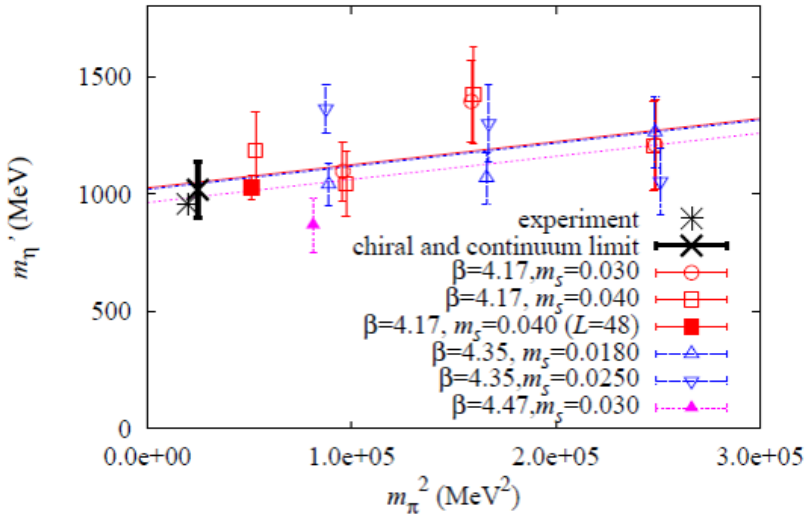


(b) $m_\pi \sim 650$ MeV

correlation of $q(x)$

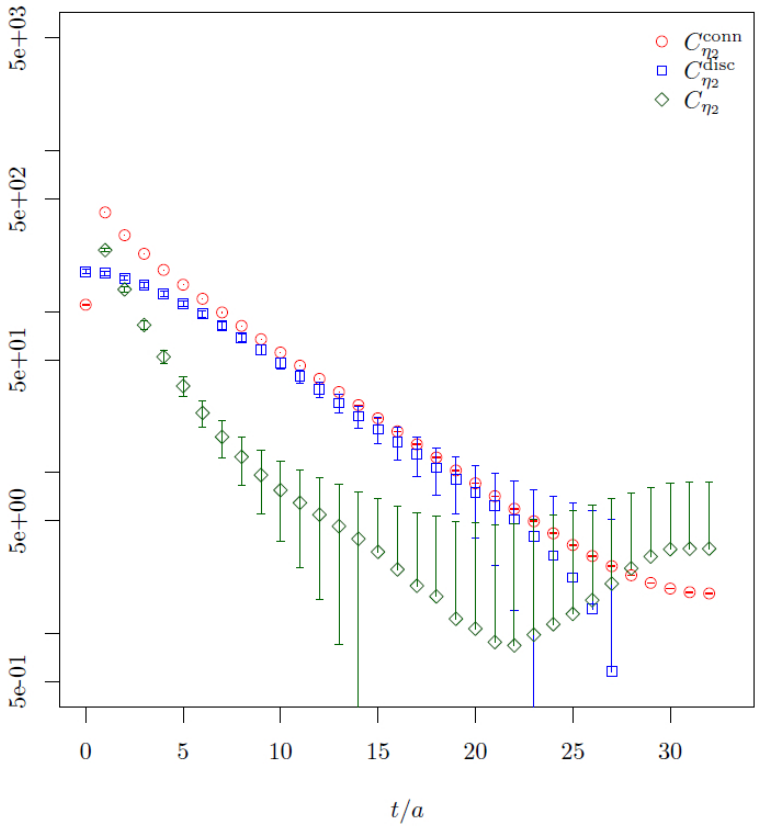
m_π	fit range(a_s)	$m_{\eta'} a_s$	$m_{\eta'}$(MeV)	χ^2/dof
938 MeV	3.74-5.92	0.856(21)	1481(36)	1.01
650 MeV	3.87-5.48	0.514(22)	890(38)	1.43

- For quenched study, the flavor singlet pseudoscalar mass agrees with pseudoscalar glueball.
  [A. Chowdhury et al., *Phys. Rev.* **D 91** (2015)074507]
- $N_f = 2 + 1$ result agrees with physical η'.
  [H. Fukaya et al, *Phy. Rev.* **D92 (R)**, 111501 (2015)]



correlation of $P(x)$ (C.Urbach, Lattice 2017)

- $N_f = 2$ Wilson twisted mass clover fermion and Iwasaki gauge action.
- lattice spacing $a \approx 0.09$fm.
- three pion mass from 130 to 350 MeV.
- two volumns per poin mass from $L/a = 24$ to $L/a = 64$.
- correlation function after Wick contraction contains connected and disconnected part
$$C_{\eta_2} = C_{\eta_2}^{conn} - C_{\eta_2}^{disc}$$
.
- stochastic Laplace-Heaviside method calculating correlation functions.



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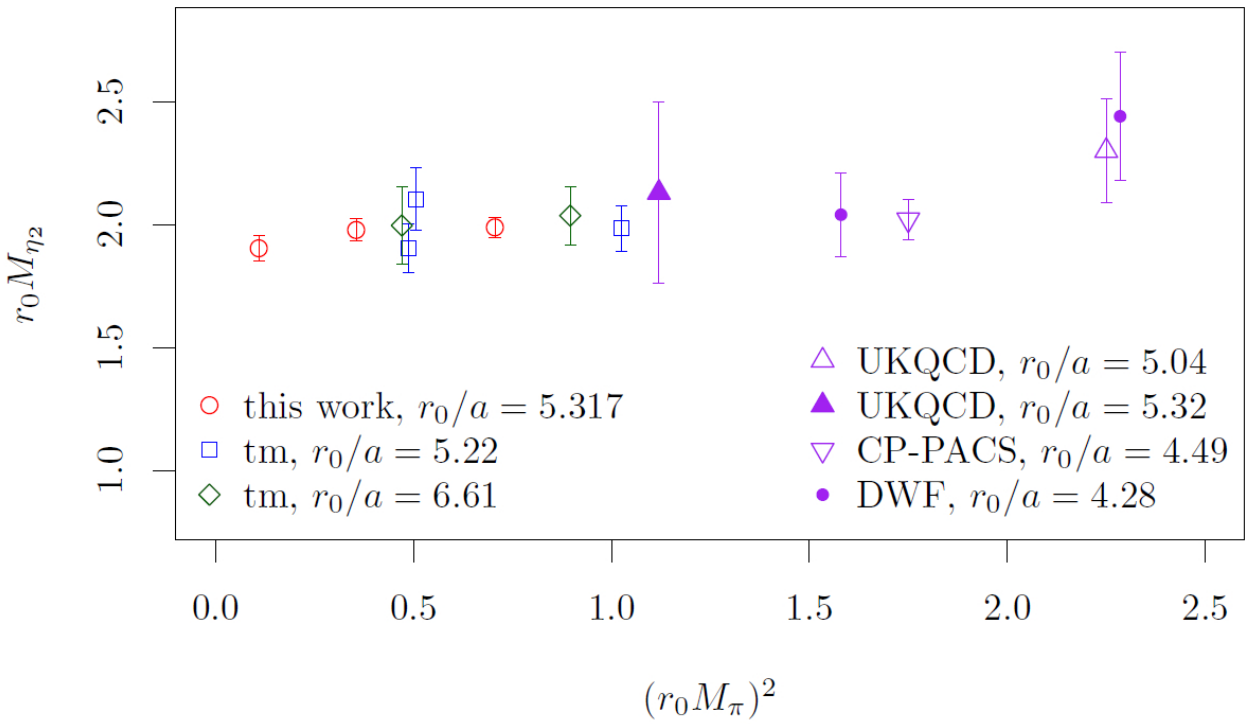


- $r_0 = 0.4907(5)$ fm gives

$$M_{\eta_2} = 768(24) MeV$$

summary of pseudoscalar results

	$P(x)$	$q(x)$	O_G
$N_f = 0$	——	2563(34)MeV A.Chowdhury, PRD91(2015)	2590(40)(130)MeV Y.Chen, PRD73(2006)
$N_f = 2$	768(24)MeV C.Urbach, Lattice2017	890(38)MeV this work $m_\pi = 650$MeV	2605(52)MeV this work $m_\pi = 650$MeV
$N_f = 2+1$	947(142)MeV N.Christ, PRL105(2010)	1019(119)MeV JLQCD, PRD92(2015)	——
$N_f = 2+1+1$	1006(54)(38)MeV C.Michael, PRL111(2013)	——	——

- P(x): $\bar{\psi}\gamma_5\psi$
- q(x): topological charge density
- O_G: glueball operators

continuum form of operators

- The continuum form of our pseudoscalar glueball operator is

$$\phi^{A_1^{-+}}(\mathbf{x},t) \sim \epsilon_{ijk} Tr B_i(\mathbf{x},t) D_j B_k(\mathbf{x},t) + O(a_s^2)$$

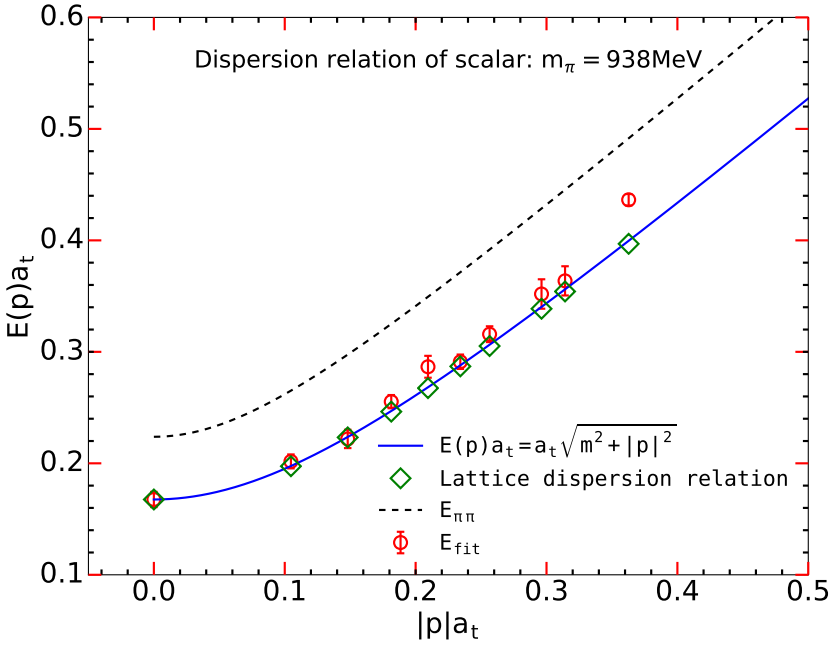
- Topological charge density operator goes like

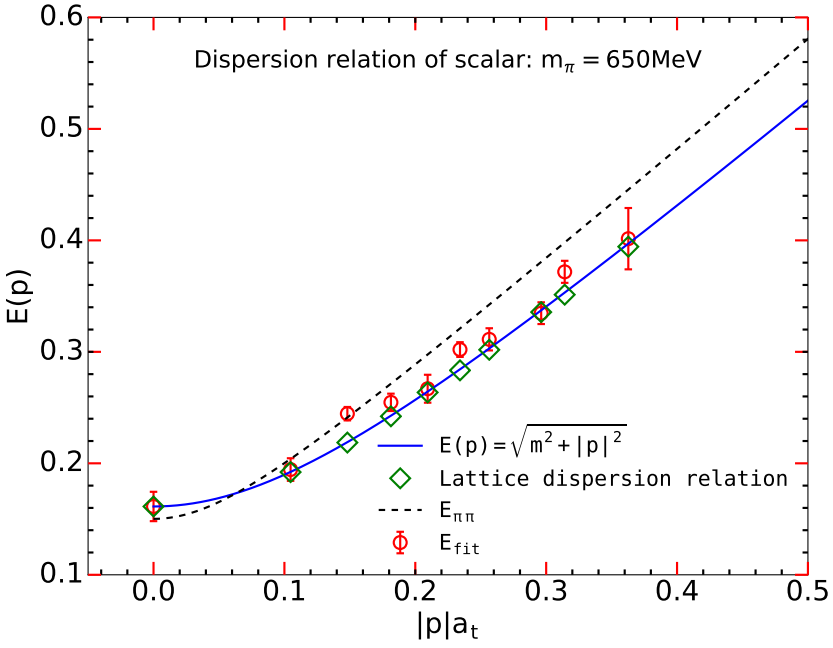
$$q(x) \propto \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \propto \mathbf{E}(x) \cdot \mathbf{B}(x)$$

- The large difference of our glueball and $\eta'(\eta_2)$ mass can be explained.

Preliminary results on dispersion relation

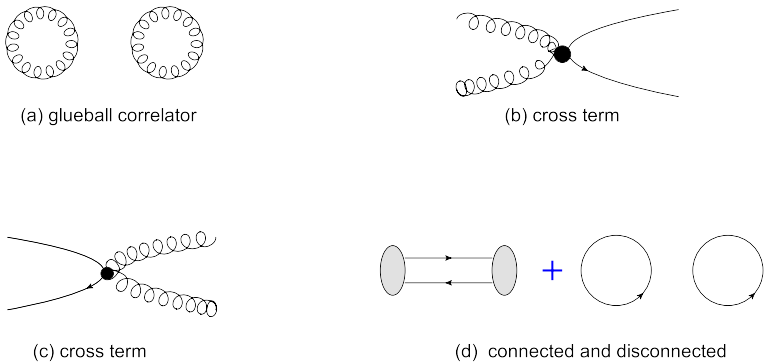
- Single particle or multi-particle state?
- Dispersion relation of one-particle and lowest free two pion state





Mixing of flavor singlet $q\bar{q}$ meson and glueball in progress

- The scalar $q\bar{q}$ meson (actually isovector a_0) mass on our lattice is very close to the obtained scalar glueball mass.
- Mixing of glueball and isosinglet $q\bar{q}$ meson may need be considered.
- Disconnected quark loops can be calculated by wall source technique without gauge fixing.



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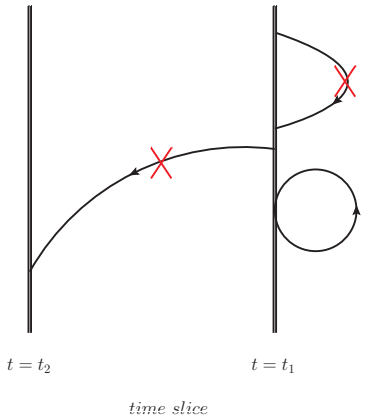

Summary

- The lowest-lying spectrum of 0^{++}, 0^{-+} and 2^{++} glueballs have been calculated in $N_f = 2$ lattice QCD with $m_\pi \sim 650$ MeV and $m_\pi \sim 938$ MeV.
- Results of 2^{++} and 0^{-+} states are consistent with quenched lattice results.
- The 0^{-+} glueball mass are much larger(this work) than ground state flavor singlet 0^{-+} meson.(this work & C.Urbach)
- Mixing of scalar glueball and flavor singlet scalar mesons is in progress.

Thank you!

Glueball operators construction

- Operators with R^{PC} quantum number are linear combinations of Wilson loops
  - $P = \pm,\ C = +$

$$\phi_i^{R^{PC}} = \sum_{g \in O} c_R Re Tr[g \circ W_i(\mathbf{x}, t) \pm \mathcal{P} g \circ W_i(\mathbf{x}, t) \mathcal{P}^{-1}]$$

  - $P = \pm,\ C = -$

$$\phi_i^{R^{PC}} = \sum_{g \in O} c_R Im Tr[g \circ W_i(\mathbf{x}, t) \pm \mathcal{P} g \circ W_i(\mathbf{x}, t) \mathcal{P}^{-1}]$$

where $W_i(\mathbf{x}, t)$ denote prototype of Wilson loop, g is vector representation of group O, $\mathcal{P}$ is space inversion, c_R is combination coefficient based on group theory.

Lattice study on J/ψ radiatively decay into glueballs

For scalar glueball candidates $f_0(1500)$ and $f_0(1710)$, the experimental result of J/ψ radiative decay are as following: [C. Patrignani et al.(Particle Data Group), Chin. Phys. C, 40, 100001 (2016)]

decay channel	branching ratio
$J/\psi \longrightarrow \gamma f_0(1500) \longrightarrow \gamma\pi\pi$	$(1.09 \pm 0.24) \times 10^{-4}$
$J/\psi \longrightarrow \gamma f_0(1500) \longrightarrow \gamma\eta\eta$	$(1.7^{+0.6}_{-1.4}) \times 10^{-5}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma K\bar{K}$	$(1.00^{+0.11}_{-0.09}) \times 10^{-3}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma\pi\pi$	$(3.8 \pm 0.5) \times 10^{-4}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma\omega\omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma\eta\eta$	$(2.4^{+1.2}_{-0.7}) \times 10^{-4}$

Lattice study on J/ψ radiatively decay into glueballs

- Branching ratio of $J/\psi \longrightarrow \gamma X$ (X for f_0 etc.):

$$Br(J/\psi \longrightarrow \gamma X) = \frac{Br(J/\psi \longrightarrow \gamma X \longrightarrow \gamma PP)}{Br(X \longrightarrow PP)}$$

Lattice study on J/ψ radiatively decay into glueballs

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$$Br(J/\psi \longrightarrow \gamma X) = \frac{Br(J/\psi \longrightarrow \gamma X \longrightarrow \gamma PP)}{Br(X \longrightarrow PP)}$$

decay channel	branching ratio	$Br(J/\psi \longrightarrow \gamma f_0(1500))$
$f_0(1500) \longrightarrow \pi\pi$	$(34.9 \pm 2.3)\%$	3.1×10^{-4}
$f_0(1500) \longrightarrow \eta\eta$	$(5.1 \pm 0.9)\%$	3.3×10^{-4}
decay channel	branching ratio	$Br(J/\psi \longrightarrow \gamma f_0(1710))$
$f_0(1710) \longrightarrow K\bar{K}$	0.36 ± 0.12	2.8×10^{-3}
$f_0(1710) \longrightarrow \eta\eta$	0.22 ± 0.12	1.1×10^{-3}
$f_0(1710) \longrightarrow \pi\pi$	0.15	2.5×10^{-3}

Lattice study on J/ψ radiatively decay into glueballs

decay channel	branching ratio	$Br(J/\psi \longrightarrow \gamma f_0(1500))$
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$f_0(1710) \longrightarrow \pi\pi$	0.15	2.5×10^{-3}

- Quenched Lattice QCD predicted the J/ψ radiatively decay into scalar glueball with branching ratio :
  [Long-Cheng Gui et al, CLQCD Collaboration, Phys.Rev.Lett. 110 (2013) no.2, 021601]

$$Br(J/\psi \longrightarrow \gamma G_{0^{++}}) = 3.8(9) \times 10^{-3},$$

which suggested the $f_0(1710)$ as scalar glueball candidate.

tensor glueball & $f_2(2340)$

- Flavour blindness of glueball

$$\Gamma(G \longrightarrow \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3 : 4 : 1 : 0 : 1$$

$$\Gamma(G \longrightarrow \eta\eta)/\Gamma(G \longrightarrow PP) \sim 10\%$$

- Pseudo-Pseudoscalar final states in tensor glueball decay in D-wave, considering centrifugal barrier effect,

$$\Gamma(G \longrightarrow M\bar{M}) = \eta\alpha\frac{k^{2L+1}}{m_G^{2L}} = \frac{\eta\alpha}{m_G}(\frac{k}{m_G})^{2L+1}$$

$$\frac{k}{m_G} = \frac{1}{2}\sqrt{1 - (\frac{2m_M}{G})^2} \sim 0.5 - 0.3$$

tensor glueball & $f_2(2340)$

- Partial width of Glueball decay into pseudoscalar-pseudoscalar final states suppressed to

$$Br(G_{2+} \longrightarrow PP) \sim O(10\%)$$

- BESIII results give

$$Br(J/\psi \longrightarrow \gamma f_2(2340) \longrightarrow \eta\eta) = 5.6(2.3) \times 10^{-5}$$

- Large branch ratio of $f_2(2340)$ in J/ψ decay

$$Br(J/\psi \longrightarrow \gamma f_2(2340)) \sim 10^{-2}$$

- Consistent with quenched LQCD