Factorization, resummation and sum rules for heavy-to-light form factors

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Why heavy-to-light form factors?

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of exclusive B-meson decay amplitudes.
  - Renormalization and asymptotic properties of B-meson DAs.
  - Interplay of different QCD techniques based upon the HQE.

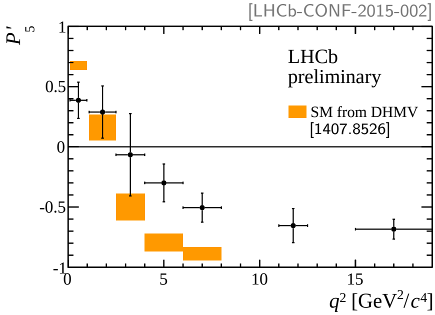
- Precision determination of the CKM matrix element $|V_{ub}|$.
  $B \to \pi \ell \nu$, $B \to \rho \ell \nu$, $\Lambda_b \to p \ell \nu$.

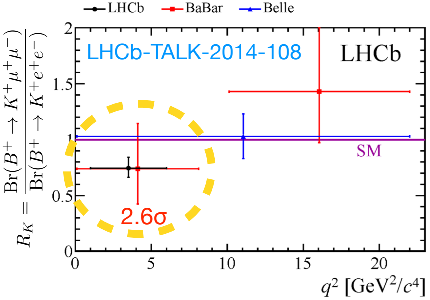
- Fundamental inputs for QCD descriptions of FCNC decays and hadronic decays.
  $B \to K^* \ell\ell$, $\Lambda_b \to \Lambda \ell\ell$, $B \to \pi\pi$, $\Lambda_b \to p \pi$.
  - Sensitive to the BSM physics.
  - CP violating asymmetries and the CKM angles.
  - More complicated than FFs.

- Crucial to understand the flavour puzzles.
  - Important source of theory uncertainties.
  - Systematical treatment of sub-leading power/twist contributions.

Anomalies in FCNC processes

A few “anomalies" exist in $B \to K^{(*)} \ell^+ \ell^-$.

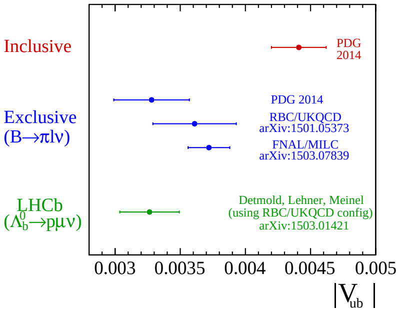


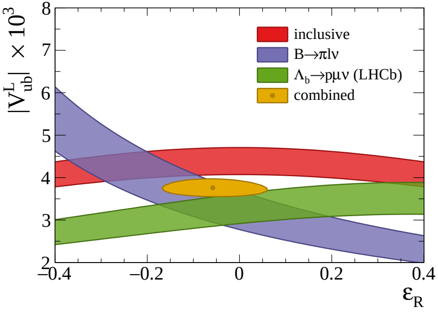


- Indication of BSM physics or ignorance of QCD dynamics?
- P'_5 anomaly below $6\,\mathrm{GeV}^2$ more serious [power corrections].
- Violation of the lepton flavor universality [QED corrections].
- Need more data and theoretical efforts.

$|V_{ub}|$ puzzle

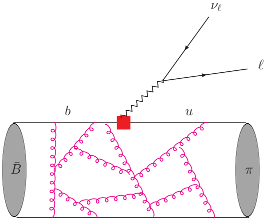
$3\,\sigma$ tension between exclusive and inclusive $|V_{ub}|$ [arXiv:1504.01568].





right handed current, underestimate of QCD uncertainties?

Part I: Semileptonic $B \to \pi \ell \nu$ decays



Hadronic matrix element:

$$\langle \pi(p)|\bar{u}\gamma_\mu b|\bar{B}(p+q)\rangle = f^+_{B\pi}(q^2)\left[p_B + p - \frac{m_B^2 - m_\pi^2}{q^2} q\right]_\mu + f^0_{B\pi}(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2} q_\mu \,.$$

- Lepton spectrum:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3 q^4 m_B^2}(q^2 - m_l^2)^2 |\vec{p}_\pi| \times \left[\left(1 + \frac{m_l^2}{q^2}\right) m_B^2 |\vec{p}_\pi|^2 |f^+_{B\pi}(q^2)|^2 + \frac{3 m_l^2}{8q^2}(m_B^2 - m_\pi^2)^2 |f^0_{B\pi}(q^2)|^2\right].$$

- Still the best way to determine $|V_{ub}|$ exclusively in the continuum approach!

- $\Lambda_b \to p \ell \nu$ decays also become important now [LHCb, arXiv:1504.01568].

$$|V_{ub}| = \left(3.27 \pm 0.23\right) \times 10^{-3}\,.$$

$B \to \pi$ form factors in QCD factorization

- QCD factorization of $B \to \pi$ form factors [Beneke, Feldmann, 2001]:

$$F_i(q^2) = C_i(E)\,\xi_P(E) + \Phi_B(\omega) \otimes T_i(E; \ln\omega, v) \otimes \Phi_\pi(v)\,.$$

- QCD correction to $B \to \pi$ form factors:

$$\begin{aligned}
\xi_P(E) &\equiv f_+(q^2)\ [\text{factorization scheme}]\,,\\
f_0 &= \frac{2E}{M}\,\xi_P\left(1 + \frac{\alpha_s C_F}{4\pi}\,[2 - 2L]\right) + \frac{\alpha_s C_F}{4\pi}\,\Delta f_0\,,\\
\Delta f_0 &= \frac{M-2E}{2E}\,\frac{8\pi^2 f_B f_P}{N_C M}\int dl_+ \frac{\phi_+^B(l_+)}{l_+}\int du\,\frac{\phi(u)}{\bar{u}}\,.
\end{aligned}$$

- SCET factorization of $B \to P$ form factors [Beneke, Feldmann, 2003]:

$$\begin{aligned}
F_i(q^2) &= C_i(E)\,\underbrace{\xi_P(E)}_{\langle P(p)|(\bar{\xi}\,W_c)\,h_v|\bar{B}_v\rangle} + C_i^{(B1)}(E,\tau) \otimes \underbrace{\Xi_P(\tau,E)}_{\langle P(p)|(\bar{\xi}\,W_c)\left(W_c^\dagger\, i\not{D}_{c\perp}\,W_c\right)(rn)\,h_v|\bar{B}_v\rangle}\,,\\
\Xi_P(\tau,E) &= J_P(\tau; v,\omega) \otimes \Phi_B(\omega) \otimes \Phi_P(v)\,.
\end{aligned}$$

  - $\xi_P(E)$ defined in $\mathrm{SCET_I}$.
  - Three-particle DAs contribute to $\xi_P(E)$ at LP.

- Factorization of $\xi_P(E)$ in $\mathrm{SCET_{II}}$: missing field modes?
  $\mathrm{SCET_{II}}$ operators describing the endpoint region and overlapping with the pion DAs.

Alternative approaches to $B \to \pi$ form factors

- Traditional QCD light-cone sum rules [Braun et al; Khodjamirian et al]:
  - Replace the B-meson by a space-like interpolating current.
  - QCD factorization of the correlation function at leading twist.

$$\text{correlation function} \sim \sum_n T^{(n)} \otimes \phi_\pi^{(n)} .$$

  Twist-3 factorization only in the asymptotic limit.
  - No separation of hard and hard-collinear scales.
    $\Rightarrow$ No resummation of large logarithms.

- QCD light-cone sum rules with B-meson DAs:
  - Replace the pion by a space-like hard-collinear current.
  - QCD factorization for the vacuum-to-B-meson correlation function.

$$\Pi_i(n\cdot p, \bar{n}\cdot p) \quad \sim \quad \sum_{k=\pm} \underbrace{C_i(k)(n\cdot p, \mu)} \int \frac{d\omega}{\omega - \bar{n}\cdot p} J_i^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega, \mu) .$$

Hard matching coefficients of the QCD weak currents.

  - Reproduce the structure of QCD factorization for $B \to \pi$ form factors.
  - Can be formulated in SCET [De Fazio, Feldmann and Hurth, 2005, 2008].

$B \to \pi$ form factors from LCSRs with B-meson DAs

- Starting point: correlation function [Y.M.W and Y.L. Shen, 2015]

$$\Pi_\mu(p,q) = \int d^4x\, e^{ip\cdot x} \langle 0 | T\left\{ \bar{d}(x) \not{n} \gamma_5 u(x), \bar{u}(0) \gamma_\mu b(0) \right\} | \bar{B}(p+q) \rangle$$

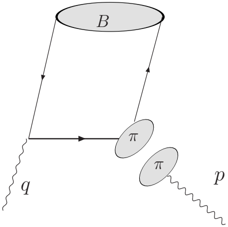
$$= \Pi(n\cdot p, \bar{n}\cdot p) n_\mu + \widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) \bar{n}_\mu ,$$

$$n\cdot p = \frac{m_B^2 + m_\pi^2 - q^2}{m_B}, \qquad \bar{n}\cdot p \sim O(\Lambda), \qquad p+q \equiv m_B v = \frac{m_B}{2}(n+\bar{n}) .$$

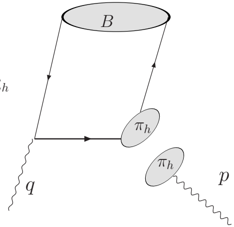
Similar to $B \to \gamma \ell \nu$ decay: replacing the pion current by the e.m. current.

- Inserting complete set of pion states $\Rightarrow$ hadronic sum:

$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) =$

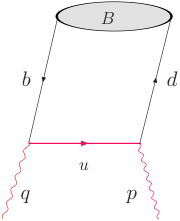


$+ \quad \Sigma_h$



$$\underbrace{\frac{f_\pi\,(n\cdot p)\,m_B}{2(m_\pi^2 - p^2)} \left[\frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p) + f_{B\pi}^{0}(n\cdot p) \right]}_{\text{relative sign changes for } \Pi(n\cdot p,\, \bar{n}\cdot p)} \qquad \int_{\omega_s}^{+\infty} d\omega' \frac{\widetilde{\rho}^h(n\cdot p, \omega')}{\omega' - \bar{n}\cdot p}$$

OPE calculation of the correlation function



Factorization at tree level:

$$\widetilde{\Pi}^{(0)}(n\cdot p, \bar{n}\cdot p) = \tilde{f}_B\, m_B \int_0^{+\infty} d\omega' \, \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p - i0},$$

$$\Pi^{(0)}(n\cdot p, \bar{n}\cdot p) = 0,$$

$$\Rightarrow f_{B\pi}^0(n\cdot p) = \frac{n\cdot p}{m_B} f_{B\pi}^+(n\cdot p) + O(\alpha_s).$$

- Light-cone DAs of the B-meson [Grozin and Neubert, 1996]:

$$i\, m_B \tilde{f}_B\, \phi_B^+(\omega) = \frac{1}{2\pi}\int dt\, e^{i\omega t} \langle 0|(\bar{q}_s\, Y_s)(t\bar{n})\, \not{\bar{n}}\, \gamma_5\, (Y_s^\dagger\, b_v)(0)|\bar{B}(v)\rangle .$$

  - One-loop renormalization of $\phi_B^+(\omega,\mu)$ [Lange and Neubert, 2003].
  - Renormalization of $[\bar{q}_s(t\bar{n})\,\Gamma\, b_v(0)]$ does not commute with the shot-distance expansion [Braun, Ivanov and Korchemsky, 2004].

$$[(\bar{q}_s\, Y_s)(t\bar{n})\not{\bar{n}}\,\Gamma\,(Y_s^\dagger\, b_v)(0)]_R \neq \sum_{p=0} \frac{t^p}{p!}\, \left[\bar{q}_s(0)\,(n\cdot \overleftarrow{D})^p\, \not{\bar{n}}\,\Gamma\, b_v(0)\right]_R .$$

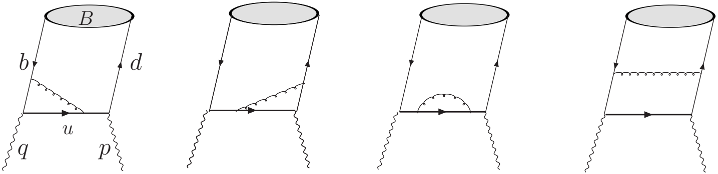
  - Eigenfunctions of the Lange-Neubert kernel [Bell, Feldmann, YMW and Yip, 2013].
  - $\phi_B^-(\omega)$ defined in a similar way, renormalization kernel available [Bell and Feldmann, 2008].

- QCD correction involving $\phi_B^+(\omega')$ at NLO must be IR finite.

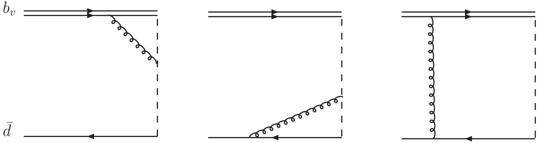
- Symmetry breaking of the form-factor relations at NLO must be IR finite.

Factorization of the correlation function

- Light-cone OPE: $|\bar{n}\cdot p| \sim O(\Lambda_{\rm QCD})$.



- Cancellation of the soft divergences.



- Diagrammatic factorization:

$$\begin{aligned}\Pi_\mu &= \Pi_\mu^{(0)} + \Pi_\mu^{(1)} + \ldots = \Phi_B \otimes T \\ &= \Phi_B^{(0)} \otimes T^{(0)} + \left[\Phi_B^{(0)} \otimes T^{(1)} + \Phi_B^{(1)} \otimes T^{(0)}\right] + \ldots .\end{aligned}$$

$$\Downarrow$$

$$\Phi_B^{(0)} \otimes T^{(1)} = \Pi_\mu^{(1)} - \Phi_B^{(1)} \otimes T^{(0)} .$$

Sample calculation: the weak vertex diagram

- Strategy:
  (i) Identify the leading regions of the QCD amplitudes.
  (ii) Evaluate the leading contributions with the method of regions [Beneke and Smirnov, 1997].
  (iii) Perform the soft subtraction [the same as the QCD amplitude in the soft region].

- QCD amplitude:

$$\Pi^{(1)}_{\mu,weak} = \frac{g_s^2\, C_F}{2(\bar{n}\cdot p-\omega)} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[(p-k+l)^2+i0][(m_b v+l)^2-m_b^2+i0][l^2+i0]}$$
$$\bar{d}(k)\, \not{n}\, \gamma_5\, \not{\bar{n}}\, \underbrace{\gamma_\rho\, (\not{p}-\not{k}+\not{l})\, \gamma_\mu\, (m_b \not{v}+\not{l}+m_b)\, \gamma^\rho}_{\text{soft} \Downarrow \text{region}}\, b(p_b)\,,$$
$$2\, n\cdot p\, m_b\, \gamma_\mu$$

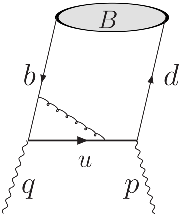
  - Leading contributions from the hard, hard-collinear and soft regions.
  - Important that the collinear region absent at leading power.

- Soft subtraction [Wilson-line Feynman rules]:

$$\Phi^{(1)}_{B,weak} \otimes T^{(0)} = \frac{g_s^2\, C_F}{2(\bar{n}\cdot p-\omega)} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[\bar{n}\cdot(p-k+l)+i0][v\cdot l+i0][l^2+i0]}$$
$$\bar{d}(k)\, \not{n}\, \gamma_5\, \not{\bar{n}}\, \gamma_\mu\, b(p_b)\,.$$

Precise cancellation of the soft contribution.

Sample calculation: the weak vertex diagram



Hard contribution:

$$\Pi^{(1),h}_{\mu,weak} = \frac{\alpha_s C_F}{4\pi} \left\{ \bar{n}_\mu \left[\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \left(2 \ln \frac{\mu}{n\cdot p} + 1 \right) + 2 \ln^2 \frac{\mu}{n\cdot p} + 2 \ln \frac{\mu}{m_b} - \ln^2 r - 2\,\mathrm{Li}_2 \left(-\frac{\bar{r}}{r} \right) + \frac{2-r}{r-1} \ln r + \frac{\pi^2}{12} + 3 \right] + n_\mu \left[\frac{1}{r-1} \left(1 + \frac{r}{\bar{r}} \ln r \right) \right] \right\} \widetilde{\Pi}^{(0)}(n\cdot p, \bar{n}\cdot p) \,.$$

Hard function only from the weak vertex diagram and renormalization of the external b-quark field.

- Hard-collinear contribution:

$$\Pi^{(1),hc}_{\mu,weak} = \frac{g_s^2 C_F}{2(\bar{n}\cdot p - \omega)} \int \frac{d^D l}{(2\pi)^D} \frac{2 m_b\, n\cdot(p+l)}{[n\cdot(p+l)\,\bar{n}\cdot(p-k+l) + l_\perp^2 + i0][m_b\, n\cdot l + i0][l^2 + i0]} \bar{d}(k)\, \not{n}\, \gamma_5\, \not{\bar{n}}\, \gamma_\mu\, b(p_b) \,.$$

  Can be also obtained from the hard-collinear contribution in $B \to \gamma \ell \nu$.

- Compute the hard-collinear contribution with the light-cone projector [Beneke, Feldmann, 2001]:

$$\mathcal{M}^B_{\beta\,\alpha} = -\frac{i\tilde{f}_B\, m_B}{4} \left\{ \frac{1+\not{v}}{2} \left[\phi_B^+(\omega)\not{n} + \phi_B^-(\omega)\not{\bar{n}} - \omega\, \phi_B^-(\omega)\, \gamma_\perp^\mu \frac{\partial}{\partial k_\perp^\mu} \right] \right\}_{\alpha\beta} \,.$$

Factorization of the correlation function

- Factorization of the correlation function.

$$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) = \tilde{f}_B\, m_B \sum_{k=\pm} \widetilde{C}^{(k)}(n\cdot p, \mu) \int \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega, \mu)\,.$$

Similar factorization formula for $\Pi(n\cdot p, \bar{n}\cdot p)$.

- Hard functions:

$$C^{(+)}(n\cdot p, \mu) = \tilde{C}^{(+)}(n\cdot p, \mu) = 1, \qquad C^{(-)}(n\cdot p, \mu) = \frac{\alpha_s C_F}{4\pi}\,\frac{1}{\bar{r}}\left[\frac{r}{\bar{r}}\ln r + 1\right], \qquad r = \frac{n\cdot p}{m_b},$$

$$\tilde{C}^{(-)}(n\cdot p, \mu) = 1 - \frac{\alpha_s C_F}{4\pi}\left[2\ln^2\frac{\mu}{n\cdot p} + 5\ln\frac{\mu}{n\cdot p} - \ln^2 r - 2\mathrm{Li}_2\left(\frac{r-1}{r}\right) + \frac{2-r}{r-1}\ln r + \frac{\pi^2}{12} + 5\right].$$

- Hard matching coefficient of the QCD weak current [Bauer et al, 2001; Beneke et al, 2004]:

$$\bar{q}\,\gamma_\mu\, b \to \left[C_4\,\bar{n}_\mu + C_5\, v_\mu\right] \bar{\xi}_{\bar{n}}\, b_v + \ldots.$$

Perturbative matching coefficients independent of the external states $\Rightarrow$

$$\boxed{C^{(-)} = \frac{1}{2}C_5, \qquad \tilde{C}^{(-)} = C_4 + \frac{1}{2}C_5\,.}$$

Factorization of the correlation function

- Jet functions [Y.M.W and Y.L. Shen, 2015]:

$$
\begin{aligned}
J^{(+)}(\bar{n}\cdot p,\omega,\mu) &= \frac{1}{r}\tilde{J}^{(+)}(\bar{n}\cdot p,\omega,\mu) = \frac{\alpha_s C_F}{4\pi}\left(1-\frac{\bar{n}\cdot p}{\omega}\right)\ln\left(1-\frac{\omega}{\bar{n}\cdot p}\right), \\
J^{(-)}(\bar{n}\cdot p,\omega,\mu) &= 1\,, \\
\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) &= 1+\frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)} - 2\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}\ln\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)}\right. \\
&\left. -\ln^2\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p} - \left(1+\frac{2\bar{n}\cdot p}{\omega}\right)\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p} - \frac{\pi^2}{6} - 1\right].
\end{aligned}
$$

In agreement with the jet functions computed in SCET [De Fazio, Feldmann and Hurth, 2008].

- Cancellation of the factorization-scale dependence:

$$
\begin{aligned}
\frac{d}{d\ln\mu}\tilde{C}^{(-)}(n\cdot p,\mu) &= -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{n\cdot p}+5\right]\tilde{C}^{(-)}(n\cdot p,\mu)\,, \\
\frac{d}{d\ln\mu}\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) &= \frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu^2}{n\cdot p\,\omega}\right]\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) \\
&+\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma^{(1)}(\omega,\omega',\mu)\,\tilde{J}^{(-)}(\bar{n}\cdot p,\omega',\mu)\,, \\
\frac{d}{d\ln\mu}\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] &= -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{\omega}-5\right]\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] \\
&-\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma^{(1)}(\omega,\omega',\mu)\,\left[\tilde{f}_B\,\phi_B^-(\omega',\mu)\right],
\end{aligned}
$$

NLL resummation for $B \to \pi$ form factors

- No common scale μ to avoid the large logarithms in the hard functions, the jet functions, $\tilde{f}_B(\mu)$ and the B-meson DAs.
- Resummation for the hard functions [see also, Beneke and Rohrwild, 2011]:

$$\tilde{C}^{(-)}(n \cdot p, \mu) = U_1(n \cdot p, \mu_{h1}, \mu)\, \tilde{C}^{(-)}(n \cdot p, \mu_{h1}), \qquad \tilde{f}_B(\mu) = U_2(\mu_{h2}, \mu) \tilde{f}_B(\mu_{h2}).$$

RG evolutions at NLL:

$$\frac{d}{d \ln \mu} U_1(n \cdot p, \mu_{h1}, \mu) = \Big[- \underbrace{\Gamma_{\rm cusp}(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^3)} \ln \frac{\mu}{n \cdot p} + \underbrace{\gamma(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)} \Big] U_1(n \cdot p, \mu_{h1}, \mu),$$

[Bonciani et al, 2008; Asatrian et al, 2008; Bell, 2008; Beneke et al, 2008]

$$\frac{d}{d \ln \mu} U_2(\mu_{h2}, \mu) = \underbrace{\tilde{\gamma}(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)} U_2(\mu_{h2}, \mu).$$

[Ji and Musolf, 1991; Broadhurst and Grozin 1991]

- Resummation of parametrically large logarithms in the B-meson DAs ignored.

$$\frac{d \phi_B^-(\omega, \mu)}{d \ln \mu} = - \Big[\Gamma_{\rm cusp}(\alpha_s) \ln \frac{\mu}{\omega} + \underbrace{\gamma_-(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)} \Big] \phi_B^-(\omega, \mu) - \omega \int_0^\infty d\eta\, \underbrace{\Gamma(\omega, \eta, \alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)} \phi_B^-(\eta, \mu).$$

  - Unclear whether the structure of the renormalization kernel holds at $\mathcal{O}(\alpha_s^2)$.
  - Whether Bessel functions are still the eigenfunctions of the evolution kernel at $\mathcal{O}(\alpha_s^2)$?

$B \to \pi$ form factors from the B-meson LCSR

- B-meson LCSR @ NLL:

$$f_\pi \, e^{-m_\pi^2/(n\cdot p\, \omega_M)} \left\{ \frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p)\,,\, f_{B\pi}^{0}(n\cdot p) \right\}$$

$$= \left[U_2(\mu_{h2},\mu)\tilde{f}_B(\mu_{h2})\right] \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \Bigg[r\, \widetilde{C}^{(+)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{+}(\omega',\mu)$$

$$+ \left[U_1(n\cdot p,\mu_{h1},\mu)\, \widetilde{C}^{(-)}(n\cdot p,\mu_{h1}) \right] \phi_{B,\mathrm{eff}}^{-}(\omega',\mu)$$

$$\pm \frac{n\cdot p - m_B}{m_B} \Bigg(C^{(+)}(n\cdot p,\mu)\, \underbrace{\phi_{B,\mathrm{eff}}^{+}(\omega',\mu)}_{\text{“hc” correction}} + \underbrace{C^{(-)}(n\cdot p,\mu)}_{\text{hard correction}}\, \phi_B^{-}(\omega',\mu) \Bigg) \Bigg].$$

- Effective DAs:

$$\phi_{B,\mathrm{eff}}^{+}(\omega',\mu) = 0 + \frac{\alpha_s C_F}{4\pi} \int_{\omega'}^{\infty} \frac{d\omega}{\omega}\, \phi_B^{+}(\omega,\mu)\,,$$

$$\phi_{B,\mathrm{eff}}^{-}(\omega',\mu) = \phi_B^{-}(\omega',\mu) + \frac{\alpha_s C_F}{4\pi} \Bigg\{ \int_0^{\omega'} d\omega \left[\frac{1}{\omega-\omega'} \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} - 4\ln\frac{\omega'-\omega}{\omega'} \right) \right]_+ \phi_B^{-}(\omega,\mu)$$

$$- \int_{\omega'}^{\infty} d\omega \left[\ln^2\frac{\mu^2}{n\cdot p\,\omega} - \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} + 3 \right) \ln\frac{\omega-\omega'}{\omega'} + 2\ln\frac{\omega}{\omega'} + \frac{\pi^2}{6} - 1 \right] \frac{d\phi_B^{-}(\omega,\mu)}{d\omega} \Bigg\}.$$

- Power counting: $\omega \sim \Lambda, \qquad \omega_s \sim \omega_M \sim O(\Lambda^2/m_b) \Rightarrow \omega' \sim O(\Lambda^2/m_b)\,,$

$$\Rightarrow \ln\left((\omega-\omega')/\omega'\right) \sim \ln\left(\omega/\omega'\right) \sim \ln(m_b/\Lambda)\,.$$

The B-meson LCDAs

- Light-cone distribution amplitudes of the B meson:

$$\phi_{B,\mathrm{I}}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,, \qquad \text{[Grozin and Neubert, 1997]}$$

$$\phi_{B,\mathrm{II}}^{+}(\omega,\mu_0) = \frac{1}{4\pi\,\omega_0}\,\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\,\ln k\right], \; k = \frac{\omega}{1\ \mathrm{GeV}}\,, \qquad \text{[Braun et al, 2004]}$$

$$\phi_{B,\mathrm{III}}^{+}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\,\omega_1^2}\, e^{-(\omega/\omega_1)^2}\,, \; \omega_1 = \frac{2\omega_0}{2\sqrt{\pi}}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

$$\phi_{B,\mathrm{IV}}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0\,\omega_2}\,\frac{\omega_2-\omega}{\sqrt{\omega(2\omega_2-\omega)}}\,, \; \omega_2 = \frac{4\omega_0}{4-\pi}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

Perturbative constraints on the B-meson DAs at large ω [Feldmann, Lange and Y.M.W, 2014].

- The shape of $f_{B\pi}^{+}(q^2)$ less model dependent.

  blue curve from pion LCSR, solid, dotted, dashed and dot-dashed curves from Model-I, II, III and IV.

  fitting $f_{B\pi}^{+}(q^2=0) = 0.28 \pm 0.03$ from pion LCSR $\Rightarrow$

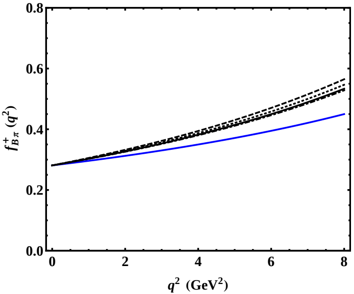
  Model-I: $\omega_0 = 360^{+40}_{-30}\,\mathrm{MeV}$,

  Model-II: $\omega_0 = 375^{+40}_{-35}\,\mathrm{MeV}$,

  Model-III: $\omega_0 = 395^{+35}_{-30}\,\mathrm{MeV}$,

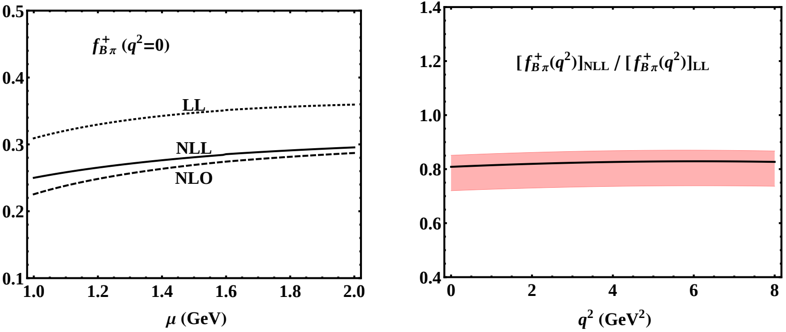
  Model-IV: $\omega_0 = 310^{+40}_{-30}\,\mathrm{MeV}$.

  Determination of ω_0 from $B \to \gamma\ell\nu$.



$B \to \pi$ form factors from the B-meson LCSR

- Factorization scale dependence and radiative correction:



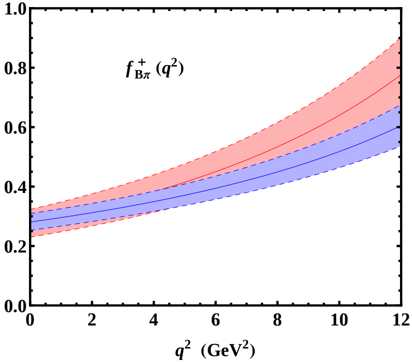
  - Dominant radiative effect from the NLO QCD correction instead of the QCD resummation.
  - Resummation improvement does stabilize the factorization scale dependence.
  - Radiative effect can induce 20 % reduction of the form factor.

$B \to \pi$ form factors from the B-meson LCSR

- The predicted form factor $f^+_{B\pi}(q^2)$:

Pink band: B-meson LCSR @ NLO,
Blue band: pion LCSR @ NLO.

Rapidly increasing $f^+_{B\pi}(q^2)$ from B-meson LCSR:
(i) Different pattern of higher power/twist contributions?
(ii) Different quark-hadron quality ansatz?



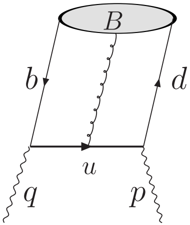
- Exclusive $|V_{ub}|$ from B-meson LCSR @ NLO [Y.M.W and Y.L. Shen, 2015]:

$$|V_{ub}| = \left(3.05^{+0.54}_{-0.38}\Big|_{\rm th.} \pm 0.09\Big|_{\rm exp.}\right) \times 10^{-3}.$$

- Exclusive $|V_{ub}|$ from $B \to \tau\nu$ [Belle, combined two tagging methods, arXiv: 1503.05613]:

$$|V_{ub}| = \left(3.28^{+0.37}_{-0.42}\right) \times 10^{-3}.$$

Three-particle B-meson DA's contributions



- Quark propagator in the background gluon field [Balitsky and Braun, 1988]:

$$\langle 0|T\{q(x)\,,\bar{q}(0)\}|0\rangle|_G \supset -\frac{i}{16\pi^2}\,\frac{1}{x^2}\int_0^1 du\,\left[\not{x}\,\sigma_{\alpha\beta} - 4\,i\,u\,x_\alpha\,\gamma_\beta\right] \times \underbrace{G^{\alpha\beta}(u\,x)}_{\equiv g_s\,T^a\,G^a_{\mu\nu}}\,.$$

- Three-particle B-meson DA's contributions [Khodjamirian, Mannel and Offen, 2007]:

$$\langle 0|\bar{u}_\alpha(x)\,G_{\lambda\rho}(u\,x)\,b_v(0)|B^-(v)\rangle\Big|_{x^2=0} = \frac{F_{\rm stat}(\mu)}{4}\int_0^\infty d\omega\int_0^\infty d\xi\,e^{-i(\omega+u\,\xi)\,v\cdot x}\Bigg[(1+\not{v})\Big\{(v_\lambda\,\gamma_\rho - v_\rho\,\gamma_\lambda)\,[\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)] - i\,\sigma_{\lambda\rho}\,\Psi_V(\omega,\xi) - \frac{x_\lambda\,v_\rho - x_\rho\,v_\lambda}{v\cdot x}\,X_A(\omega,\xi) + \frac{x_\lambda\,\gamma_\rho - x_\rho\,\gamma_\lambda}{v\cdot x}\,Y_A(\omega,\xi)\Big\}\,\gamma_5\Bigg]\,.$$

See also [Kawamura, Kodaira, Qiao and Tanaka, 2001; Geye and Witzel, 2013]. For a complete decomposition at twist-6, see [Braun, Ji and Manashov, 2017].

- Work in the coordinate space, compute the $\int d^4x\,e^{ip\cdot x}$ integral, and do the power counting.

Three-particle B-meson DA [Braun, Manashov and Offen, 2015]

- One-loop renormalization of the three-particle DA $\tilde{\Psi}_3(z_1, z_2)$:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g_s) \frac{\partial}{\partial g_s} + \frac{\alpha_s}{2\pi} \mathscr{H}\right] F_{\rm stat}(\mu) \tilde{\Psi}_3(z_1, z_2, \mu) = 0 ,$$

$$\tilde{\Psi}_3(z_1, z_2) \equiv \tilde{\Psi}_A(z_1, z_2) - \tilde{\Psi}_V(z_1, z_2) , \qquad \mathscr{H} = N_c H_0 + N_c^{-1} \delta H .$$

  An additional "hidden" symmetry for H_0: $[\hat{Q}_1, \hat{Q}_2] = [\hat{Q}_1, H_0] = [\hat{Q}_2, H_0] = 0.$

- Eigenfunctions:

$$H_0 Y_{s,x}(z_1, z_2) = E(s,x) Y_{s,x}(z_1, z_2) , \qquad Y_{s,i/2}(z_1, z_2) = \frac{i s^2}{z_1^2 z_2^3} \int_0^1 du\, u\, \bar{u}\, e^{is(u/z_1 + \bar{u}/z_2)} ,$$

$$\Delta E = \underbrace{E(s,0)}_{\text{continuous spectrum}} - \underbrace{E(s,i/2)}_{\text{ground state}} = 2\psi(3/2) - \psi(2) - \psi(1) .$$

- Expansion, "asymptotics" and RGE of ϕ_B^-:

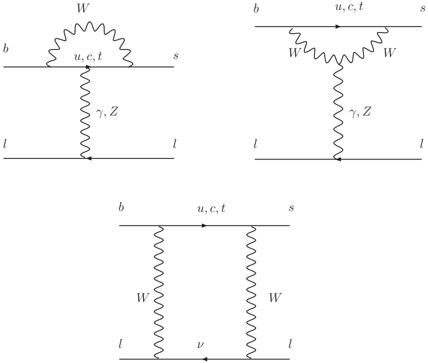
$$\tilde{\Psi}_3(z_1, z_2, \mu) = \int_0^\infty ds \left[\underbrace{\eta_0(s, \mu) Y_{s,i/2}(z_1, z_2)}_{\text{“asymptotical” behaviour}} + \frac{1}{2} \int_{-\infty}^{+\infty} dx\, \eta(s,x,\mu) Y_{s,x}(z_1, z_2)\right] .$$

$$\Psi_3^{\rm asy}(\omega_1, \omega_2, \mu) = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2} [f_1(\omega_1 + \omega_2) - f_0(\omega_1 + \omega_2)] + \omega_1 [f_1(\omega_1 + \omega_2) - f_1(\omega_1)] .$$

$$\phi_B^-(\omega, \mu) = \int_0^\infty ds \left[\hat{\phi}_B^+(s, \mu) + \underbrace{\eta_0(s, \mu)}_{\text{continuous spectrum of } \tilde{\Psi}_3(z_1, z_2, \mu) \text{ irrelevant}}\right] J_0(2\sqrt{\omega s}) .$$

Part II: FCNC $\Lambda_b \to \Lambda \ell \ell$ decays

- Partonic $b \to s\ell^+\ell^-$ diagrams in the SM:



  - Effective Hamiltonian [Buchalla, Buras and Lautenbacher, 1996].
  - Semileptonic and magnetic operators:

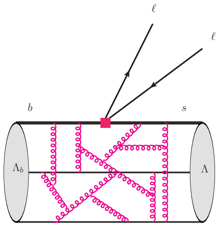
$$
\begin{aligned}
O_{7\gamma} &\propto \bar{s}\sigma_{\mu\nu}(m_s L + m_b R) b F^{\mu\nu} \,, \\
O_{8g} &\propto \bar{s}_i \sigma_{\mu\nu}(1+\gamma_5) T^a_{ij} b_j G^{a\mu\nu} \,, \\
O_{9,10} &\propto (\bar{s}b)_{V-A} \left(\bar{\ell}\ell\right)_{V,A} \,.
\end{aligned}
$$

- Four-quark operators:

$$
\begin{aligned}
O_1 &= (\bar{s}p)_{V-A}(\bar{p}b)_{V-A} \,, \qquad O_2 = \left(\bar{s}^j p^i\right)_{V-A}\left(\bar{p}^i b^j\right)_{V-A} \;(p = u, c) \,, \\
O_{3,5} &= (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V\mp A} \,, \qquad O_{4,6} = \left(\bar{s}^i b^j\right)_{V-A}\sum_q \left(\bar{q}^j q^i\right)_{V\mp A} \,.
\end{aligned}
$$

Naive factorization

- Heavy-to-light form factors:



$$\langle \Lambda(p',s')\ell^+\ell^- | (\bar{s}b)_{V-A} \left(\bar{\ell}\ell\right)_{V,A} |\Lambda_b(p,s)\rangle$$
$$= \underbrace{\langle \Lambda(p',s')| (\bar{s}b)_{V-A} |\Lambda_b(p,s)\rangle}_{\Lambda_b \to \Lambda \text{ form factors!}} \langle \ell^+\ell^- | \left(\bar{\ell}\ell\right)_{V,A} |0\rangle \,.$$

- Ten independent $\Lambda_b \to \Lambda$ form factors in QCD.
  QCD factorization, SCET factorization, LCSR, Lattice QCD.

- Parameterizations of $\Lambda_b \to \Lambda$ matrix elements (helicity-based) [Feldmann and Yip, 2011]:

$$\begin{aligned}\langle \Lambda(p',s')|\bar{s}\,\gamma_\mu\, b|\Lambda_b(p,s)\rangle \quad = \quad & \bar{\Lambda}(p',s')\Big[f^0_{\Lambda_b\to\Lambda}(q^2)\,\frac{m_{\Lambda_b}-m_\Lambda}{q^2}\,q_\mu \\ & + f^+_{\Lambda_b\to\Lambda}(q^2)\,\frac{m_{\Lambda_b}+m_\Lambda}{s_+}\left((p+p')_\mu - \frac{m^2_{\Lambda_b}-m^2_\Lambda}{q^2}\,q_\mu\right) \\ & + f^T_{\Lambda_b\to\Lambda}(q^2)\left(\gamma_\mu - \frac{2\,m_\Lambda}{s_+}p_\mu - \frac{2\,m_{\Lambda_b}}{s_+}p'_\mu\right)\Big]\Lambda_b(p,s)\,.\end{aligned}$$

Similar decompositions for the other matrix elements.

Symmetry relations of $\Lambda_b \to \Lambda$ form factors

- Form factors in HQET limit [Manohar and Wise, 2000]:

$$\langle \Lambda(p',s') | \bar{s}\, \Gamma\, b | \Lambda_b(p,s) \rangle = \bar{\Lambda}(p',s')\, (F_1 + F_2 \not{v})\, \Gamma\, \Lambda_b(v,s)\,.$$

  implying the relations

$$f^0_{\Lambda_b\to\Lambda}(q^2) \simeq g^+_{\Lambda_b\to\Lambda}(q^2) \simeq g^T_{\Lambda_b\to\Lambda}(q^2) \simeq \tilde{h}^+_{\Lambda_b\to\Lambda}(q^2) \simeq \tilde{h}^T_{\Lambda_b\to\Lambda}(q^2) \simeq F_1 + F_2\,,$$
$$f^+_{\Lambda_b\to\Lambda}(q^2) \simeq f^T_{\Lambda_b\to\Lambda}(q^2) \simeq g^0_{\Lambda_b\to\Lambda}(q^2) \simeq h^+_{\Lambda_b\to\Lambda}(q^2) \simeq h^T_{\Lambda_b\to\Lambda}(q^2) \simeq F_1 - F_2\,.$$

- Form factors in SCET limit [Mannel and Y.M.W, 2011; Feldmann and Yip, 2011]:

$$\langle \Lambda(p',s') | \bar{s}\, \Gamma\, b | \Lambda_b(p,s) \rangle = F_1\, \bar{\Lambda}(p',s')\Gamma\Lambda_b(v,s)\,.$$

  Only a single (soft) form factor in the heavy quark and the large recoil limit.

- Factorization formula of the $\Lambda_b \to \Lambda$ form factors [Wei Wang, 2011]:

$$F^i_{\Lambda_b\to\Lambda}(q^2) = \Phi_{\Lambda_b}(\omega_i) \otimes H(\omega_i, x_i) \otimes \Phi_\Lambda(x_i) + \mathcal{O}(\Lambda_{QCD}/E_\Lambda)\,.$$

  - Spectator interaction with two-gluon exchange is of leading power parametrically.
  - Leading power contribution calculable in QCDF and free of end-point divergence.
  - Symmetry relations still hold when including the hard spectator interaction.
  - Soft contribution is power suppressed, but numerically dominant.

$\Lambda_b \to \Lambda$ form factors from LCSRs with Λ_b-meson DAs

- Starting point: correlation function [Y.M.W and Y.L. Shen, 2015]

$$\Pi_{\mu,a}(p,q) = i \int d^4x\, e^{iq\cdot x} \langle 0|T\{j_\Lambda(x), j_{\mu,a}(0)\}|\Lambda_b(p)\rangle\,,$$
$$j_{\mu,a} = \bar{s}\,\Gamma_{\mu,a}\, b\,, \qquad j_\Lambda = \varepsilon_{ijk}\left(u_i^{\mathrm{T}}\, C\, \gamma_5 \not{n}\, d_j\right) s_k\,.$$

Different choices of the Λ-baryon current possible [Braun, Lenz and Wittmann, 2006].

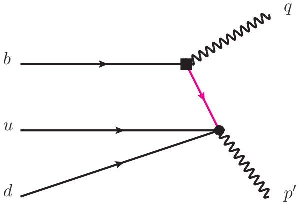
- Hadronic dispersion relations:

$$\Pi_{\mu,V}(p,q) = \frac{f_\Lambda(\mu)\,(n\cdot p')}{m_\Lambda^2/n\cdot p' - \bar{n}\cdot p' - i0}\, \frac{\not{\bar{n}}}{2} \left[f^T_{\Lambda_b\to\Lambda}(q^2)\,\gamma_{\perp\,\mu} + \frac{f^0_{\Lambda_b\to\Lambda}(q^2) - f^+_{\Lambda_b\to\Lambda}(q^2)}{2\,(1 - n\cdot p'/m_{\Lambda_b})}\, n_\mu \right.$$
$$\left. + \frac{f^0_{\Lambda_b\to\Lambda}(q^2) + f^+_{\Lambda_b\to\Lambda}(q^2)}{2}\, \bar{n}_\mu \right] \Lambda_b(p) + \int_{\omega_s}^{+\infty} d\omega'\, \frac{1}{\omega' - \bar{n}\cdot p' - i0}$$
$$\times \frac{\not{\bar{n}}}{2} \left[\rho^h_{V,\perp}(\omega', n\cdot p')\,\gamma_{\perp\,\mu} + \rho^h_{V,n}(\omega', n\cdot p')\, n_\mu + \rho^h_{V,\bar{n}}(\omega', n\cdot p')\,\bar{n}_\mu \right] \Lambda_b(p)\,.$$

Similar expressions for the other correlation functions.

- Potential “contamination" from the negative-parity baryons [Khodjamirian, Klein, Mannel and Y.M.W, 2011].

OPE calculation of the correlation function



Factorization at tree level:

$$\Pi^{(0)}_{\mu,V(A)}(p,q) = f^{(2)}_{\Lambda_b}(\mu) \int_0^{+\infty} d\omega'_1 \int_0^{+\infty} d\omega'_2 \, \frac{\psi_4(\omega'_1,\omega'_2)}{\omega'_1+\omega'_2-\bar{n}\cdot p' - i0} \times (1,\gamma_5)\, \frac{\not{\bar{n}}}{2}\, (\gamma_{\perp\mu}+\bar{n}_\mu)\, \Lambda_b(v) + O(\alpha_s)\,,$$

Kinematics: $n\cdot p' \sim \mathcal{O}(m_b)\,,\ |\bar{n}\cdot p'| \sim \mathcal{O}(\Lambda)$.

- Light-cone DAs of the Λ_b-baryon [Ball, Braun and Gardi, 2008]:

$$f^{(2)}_{\Lambda_b}(\mu)\, \psi_4(\omega'_1,\omega'_2) = \int \frac{dt_1}{2\pi} \int \frac{dt_2}{2\pi}\, e^{i(\omega'_1 t_1+\omega'_2 t_2)} \times \varepsilon_{ijk}\, \langle 0|u_i^T(t_1\bar{n})\,[0,t_1\bar{n}]\, C\gamma_5\not{n}\, d_j(t_2\bar{n})\,[0,t_2\bar{n}]\, b_k(0)|\Lambda_b(v)\rangle\,.$$

[$\psi_2(\omega'_1,\omega'_2,\mu)$ defined in a similar way.]

- Power counting: $F^i_{\Lambda_b\to\Lambda}(q^2) \sim 1/(n\cdot p')^3$.
Leading power contribution from the two-gluon-exchange diagrams.

Factorization of the correlation function

- Aim: Factorization of the correlation function [Y.M.W and Y.L. Shen, 2015]

$$
\begin{aligned}
\Pi_{\mu,V(A)} &= (1,\gamma_5)\,\frac{\bar{\not n}}{2}\left[\Pi_{\perp,V(A)}\,\gamma_{\perp\mu}+\Pi_{\bar{n},V(A)}\,\bar{n}_\mu+\Pi_{n,V(A)}\,n_\mu\right]\Lambda_b(v)\,, \\
\Pi_{\perp,V(A)} &= f^{(2)}_{\Lambda_b}(\mu)\,C_{\perp,V(A)}(n\cdot p',\mu)\int_0^\infty d\omega_1\int_0^\infty d\omega_2\,\frac{1}{\omega_1+\omega_2-\bar{n}\cdot p'-i0} \\
& \quad J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'}\right)\psi_4(\omega_1,\omega_2,\mu)\,.
\end{aligned}
$$

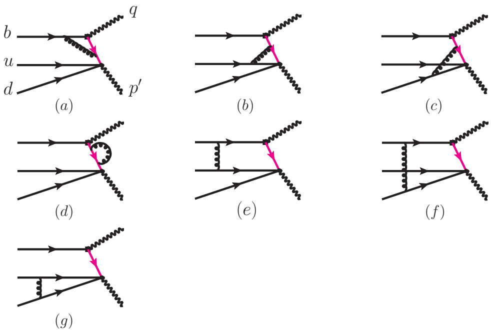
Similar factorization formulae for the other invariant functions.

- Strategy:
  (i) Extract the hard and the hard-collinear contributions using the method of regions.
  (ii) Prove the factorization-scale independence of the correlation functions.

- Theory issues:
  - Involved light-cone projector of the Λ_b-baryon (and more diagrams).
  - Explicit RGE of $\psi_4(\omega_1,\omega_2,\mu)$ unknown.
  - The Λ-baryon current is not renormalization invariant.

Factorization of the correlation function

- QCD diagrams at one loop:



- Light-cone projector [Bell, Feldmann, Y.M.W and Yip, 2013]:

$$
\begin{aligned}
M_2(\omega_1', \omega_2') \quad &= \quad \frac{\not{n}}{2}\, \psi_2(\omega_1', \omega_2') + \frac{\not{\bar{n}}}{2}\, \psi_4(\omega_1', \omega_2') \\
& - \frac{1}{D-2}\, \gamma_\perp^\mu \left[\psi_{\perp,1}^{+-}(\omega_1', \omega_2')\, \frac{\not{n}\not{\bar{n}}}{4}\, \frac{\partial}{\partial k_{1\perp}^\mu} + \psi_{\perp,1}^{-+}(\omega_1', \omega_2')\, \frac{\not{\bar{n}}\not{n}}{4}\, \frac{\partial}{\partial k_{1\perp}^\mu} \right] \\
& - \frac{1}{D-2}\, \gamma_\perp^\mu \left[\psi_{\perp,2}^{+-}(\omega_2', \omega_2')\, \frac{\not{n}\not{\bar{n}}}{4}\, \frac{\partial}{\partial k_{2\perp}^\mu} + \psi_{\perp,2}^{-+}(\omega_1', \omega_2')\, \frac{\not{\bar{n}}\not{n}}{4}\, \frac{\partial}{\partial k_{2\perp}^\mu} \right] .
\end{aligned}
$$

Factorization of the correlation function

- Hard functions:

$$
\begin{aligned}
C_{\perp,V(A)}(n\cdot p',\mu) \quad &= \quad 1-\frac{\alpha_s(\mu)\,C_F}{4\pi}\left[2\,\ln^2\frac{\mu}{n\cdot p'}+5\,\ln\frac{\mu}{m_b}-2\,\mathrm{Li}_2\left(1-\frac{1}{r}\right)\right. \\
&\left. -\ln^2 r+\frac{3r-2}{1-r}\,\ln r+\frac{\pi^2}{12}+6\right].
\end{aligned}
$$

Similar for the other hard functions, but all are already known at two loops.

- Jet function [Y.M.W and Y.L. Shen, 2015]:

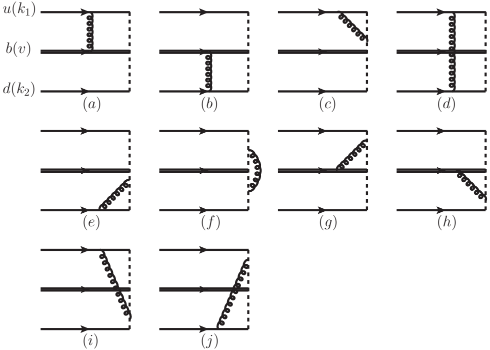
$$
\begin{aligned}
& J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'}\right) \\
&= 1+\frac{\alpha_s(\mu)}{4\pi}\,\frac{4}{3}\left\{\ln^2\frac{\mu^2}{n\cdot p'\,(\omega-\bar{n}\cdot p')}-2\,\ln\frac{\omega-\bar{n}\cdot p'}{\omega_2-\bar{n}\cdot p'}\,\ln\frac{\mu^2}{n\cdot p'\,(\omega-\bar{n}\cdot p')}\right. \\
&\quad -\frac{1}{2}\,\ln\frac{\mu^2}{n\cdot p'\,(\omega-\bar{n}\cdot p')}-\ln^2\frac{\omega-\bar{n}\cdot p'}{\omega_2-\bar{n}\cdot p'}+2\,\ln\frac{\omega-\bar{n}\cdot p'}{\omega_2-\bar{n}\cdot p'}\left[\frac{\omega_2-\bar{n}\cdot p'}{\omega_1}-\frac{3}{4}\right] \\
&\quad \left. -\frac{\pi^2}{6}-\frac{1}{2}\right\}.
\end{aligned}
$$

Symmetry property of $\psi_4(\omega_1,\omega_2,\mu)$ already used to reduce the expression.

Factorization of the correlation function

- Factorization-scale dependence:

$$\frac{d}{d\ln\mu}\Pi_{\perp,V(A)} = \frac{\alpha_s(\mu)}{4\pi}\,\frac{4}{3}\int_0^\infty d\omega_1\int_0^\infty d\omega_2\,\frac{1}{\omega_1+\omega_2-\bar{n}\cdot p'-i0}$$
$$\times\left[4\ln\frac{\mu}{\omega-\bar{n}\cdot p'}-4\ln\frac{\omega-\bar{n}\cdot p'}{\omega_2-\bar{n}\cdot p'}-6\right]\left[f_{\Lambda_b}^{(2)}(\mu)\,\psi_4(\omega_1,\omega_2,\mu)\right]$$
$$+\int_0^\infty d\omega_1\int_0^\infty d\omega_2\,\frac{1}{\omega_1+\omega_2-\bar{n}\cdot p'-i0}\,\frac{d}{d\ln\mu}\left[f_{\Lambda_b}^{(2)}(\mu)\,\psi_4(\omega_1,\omega_2,\mu)\right].$$



Renormalization of $\psi_4(\omega_1,\omega_2,\mu)$ yields

$$-\frac{\alpha_s(\mu)}{4\pi}\,\frac{4}{3}\int_0^\infty d\omega_1\int_0^\infty d\omega_2\,\frac{1}{\omega-\bar{n}\cdot p'-i0}$$
$$\times\left[4\ln\frac{\mu}{\omega-\bar{n}\cdot p'}-4\ln\frac{\omega-\bar{n}\cdot p'}{\omega_2-\bar{n}\cdot p'}-5\right]$$
$$\times\left[f_{\Lambda_b}^{(2)}(\mu)\,\psi_4(\omega_1,\omega_2,\mu)\right],$$

implying

$$\frac{d}{d\ln\mu}\left[\frac{\Pi_{\perp,V(A)}(n\cdot p',\bar{n}\cdot p',\mu)}{f_\Lambda(\mu)}\right]=0.$$

Resummation of large logarithms

- Distinguish the factorization scale μ from the renormalization scales for the Λ-baryon current (ν) and the weak current in QCD (ν').

$$
\begin{aligned}
J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'},\nu\right) &= J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'}\right)+\delta J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'},\nu\right),\\
C^A_{T(\tilde{T})}(n\cdot p',\mu,\nu') &= C^A_{T(\tilde{T})}(n\cdot p',\mu)+\delta C^A_{T(\tilde{T})}(n\cdot p',\mu,\nu').
\end{aligned}
$$

- Determination of ν and ν' dependence:

$$
\begin{aligned}
\frac{d}{d\ln\nu}\ln\delta J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'},\nu\right) &= -\sum_k\left(\frac{\alpha_s(\mu)}{4\pi}\right)^k\gamma_\Lambda^{(k)},\\
\frac{d}{d\ln\nu'}\ln\delta C^A_{T(\tilde{T})}(n\cdot p',\mu,\nu') &= -\sum_k\left(\frac{\alpha_s(\mu)}{4\pi}\right)^k\gamma_{T(\tilde{T})}^{(k)}.
\end{aligned}
$$

  Renormalization conditions:

$$
\delta J\left(\frac{\mu^2}{\bar{n}\cdot p'\,\omega_i},\frac{\omega_i}{\bar{n}\cdot p'},\mu\right)=0,\qquad \delta C^A_{T(\tilde{T})}(n\cdot p',\mu,\mu)=0.
$$

- NLL resummation for the hard functions by solving the evolution equations in μ and ν'.

NLL resummation of $\Lambda_b \to \Lambda$ form factors

- Resummation improved form factors [Y.M.W and Y.L. Shen, 2015]:

$$
\begin{aligned}
& f_\Lambda(\nu)\,(n\cdot p')\, e^{-m_\Lambda^2/(n\cdot p'\,\omega_M)} \left\{ f^T_{\Lambda_b\to\Lambda}(q^2),\, g^T_{\Lambda_b\to\Lambda}(q^2) \right\} \\
&= f^{(2)}_{\Lambda_b}(\mu)\left[U_1(n\cdot p'/2,\mu_h,\mu)\, C_{\perp,V(A)}(n\cdot p',\mu_h)\right] \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \psi_{4,\mathrm{eff}}(\omega',\mu,\nu)\,, \\
& f_\Lambda(\nu)\,(n\cdot p')\, e^{-m_\Lambda^2/(n\cdot p'\,\omega_M)} \left\{ f^0_{\Lambda_b\to\Lambda}(q^2),\, g^0_{\Lambda_b\to\Lambda}(q^2) \right\} \\
&= f^{(2)}_{\Lambda_b}(\mu)\left[U_1(n\cdot p'/2,\mu_h,\mu)\, C_{\bar{n},V(A)}(n\cdot p',\mu_h)\right] \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \psi_{4,\mathrm{eff}}(\omega',\mu,\nu) \\
&\quad + f^{(2)}_{\Lambda_b}(\mu)\left(1-\frac{n\cdot p'}{m_{\Lambda_b}}\right) C_{n,V(A)}(n\cdot p',\mu_h) \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \tilde{\psi}_4(\omega',\mu)\,.
\end{aligned}
$$

Similar for the remaining six form factors.

- Only symmetry breaking effect from the hard gluon contribution taken into account.

- Including the hard-collinear symmetry breaking effect [Feldmann and Yip, 2011]

$$
\begin{aligned}
f^T_{\Lambda_b\to\Lambda}(q^2) &= [...] + \frac{2m_{\Lambda_b}}{n\cdot p'}\,\Delta\xi_\Lambda(n\cdot p')\,, \qquad g^T_{\Lambda_b\to\Lambda}(q^2) = [...] - \frac{2m_{\Lambda_b}}{n\cdot p'}\,\Delta\xi_\Lambda(n\cdot p')\,, \\
f^0_{\Lambda_b\to\Lambda}(q^2) &= [...] - \frac{2m_{\Lambda_b}}{n\cdot p'}\,\Delta\xi_\Lambda(n\cdot p')\,, \qquad g^0_{\Lambda_b\to\Lambda}(q^2) = [...] + \frac{2m_{\Lambda_b}}{n\cdot p'}\,\Delta\xi_\Lambda(n\cdot p')\,.
\end{aligned}
$$

$\Delta\xi_\Lambda$ defined by the B-type SCET current, calculable with power-suppressed correction function.

The Λ_b-baryon LCDAs

- Light-cone distribution amplitudes of the Λ_b-baryon [Ball, Braun and Gardi, 2008]:

$$\phi_4^{\rm I}(\omega,\mu_0) = \frac{1}{\omega_0^2}\, e^{-\omega/\omega_0}\,,$$

$$\phi_4^{\rm II}(\omega,\mu_0) = \frac{1}{\omega_0^2}\, e^{-(\omega/\omega_1)^2}\,, \qquad \omega_1 = \sqrt{2}\,\omega_0\,,$$

$$\phi_4^{\rm III}(\omega,\mu_0) = \frac{1}{\omega_0^2}\left[1-\sqrt{\left(2-\frac{\omega}{\omega_2}\right)\frac{\omega}{\omega_2}}\right]\theta(\omega_2-\omega)\,, \qquad \omega_2 = \sqrt{\frac{12}{10-3\,\pi}}\,\omega_0\,.$$

- The shape of $F^i_{\Lambda_b\to\Lambda}(q^2)$ less model dependent.

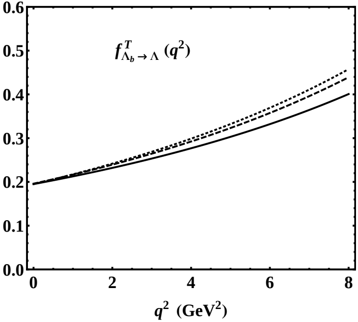
Solid, dotted, and dashed curves from Model-I, II and III.

Fitting $f^+_{\Lambda_b\to\Lambda}(0) = 0.18 \pm 0.04$
$\Rightarrow$

Model-I: $\omega_0 = 280^{+47}_{-38}\,\text{MeV}$,

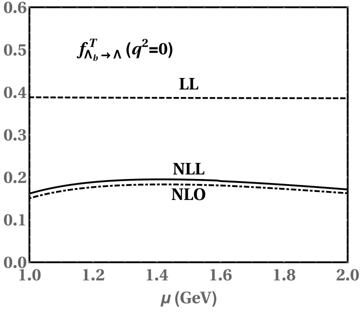
Model-II: $\omega_0 = 386^{+45}_{-37}\,\text{MeV}$,

Model-III: $\omega_0 = 273^{+38}_{-29}\,\text{MeV}$.



$\Lambda_b \to \Lambda$ form factors from the Λ_b-baryon LCSR

- Factorization scale dependence and radiative correction:



  - Dominant radiative effect from the NLO QCD correction instead of the QCD resummation.
  - NLO correction dominated by the hard-collinear contribution instead of the hard fluctuation. $\Rightarrow$ A complete calculation of the NLO jet function important!
  - Radiative effect can induce 50 % reduction of the form factor.

$\Lambda_b \to \Lambda$ form factors from the Λ_b-baryon LCSR

- The z-series expansion:

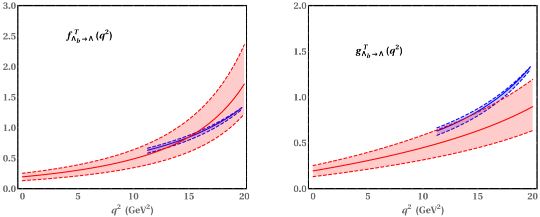
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_+ = (m_{B_s} + m_\pi)^2 < (m_{\Lambda_b} + m_\Lambda)^2 .$$

Parameterizations of $\Lambda_b \to \Lambda$ form factors:

$$f^T_{\Lambda_b \to \Lambda}(q^2) = \left[f^T_{\Lambda_b \to \Lambda}(0) / \left(1 - q^2/m^2_{B_s^*}\right)\right] \left\{ 1 + b_1^{f^T_{\Lambda_b\to\Lambda}} \left[z(q^2, t_0) - z(0, t_0) \right] \right\},$$

$$g^T_{\Lambda_b \to \Lambda}(q^2) = g^T_{\Lambda_b \to \Lambda}(0) \left\{ 1 + b_1^{g^T_{\Lambda_b\to\Lambda}} \left[z(q^2, t_0) - z(0, t_0) \right] \right\}.$$

- The predicted form factors [Y.M.W and Y.L. Shen, 2015]:

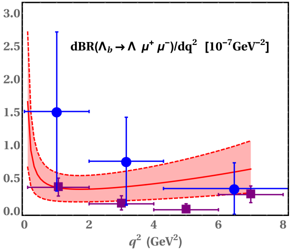


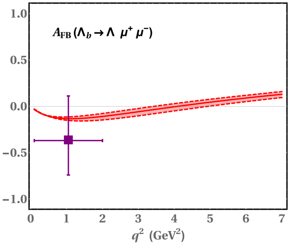
Pink and blue bands predicted from LCSR and Lattice QCD [Detmold et al, 2012].
Reasonable agreement but different shapes, only HQET form factors from Lattice QCD.
Similar observation for the remaining form factors.

Physical observables in $\Lambda_b \to \Lambda \ell \ell$ decays

- Predictions in the factorization limit:

$$\frac{d^2\Gamma(\Lambda_b \to \Lambda \ell^+ \ell^-)}{dq^2 d\cos\theta} = \frac{3}{8}\left[(1+\cos^2\theta) H_T(q^2) + 2\cos\theta H_A(q^2) + 2(1-\cos^2\theta) H_L(q^2)\right].$$

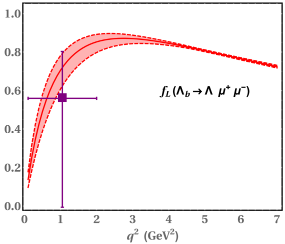




Pink bands predicted by LCSR.

Purple squares from LHCb (2015).

Blue circles from CDF (2012).



Concluding Remarks

- Heavy-to-light form factors as fundamental inputs of describing heavy hadron decays.
  - Factorization properties not fully understood in QCD.
  - Can reproduce the factorization structure in QCD light-cone sum rules.
  - Diagrammatic factorization of the correlation functions with the method of regions.
  - Different $B \to \pi$ form factor shapes from different sum rules at leading twist.
  - Hard-collinear correction important for $\Lambda_b \to \Lambda$ form factors.

- Further developments:
  - Higher Fock-state contributions to $B \to \pi$ form factors.
    $\Rightarrow$ Renormalization properties of three-particle B-meson DAs.
  - Power suppressed contributions from the B-meson LCSR.
  - Factorization of $\xi_P(E)$ in $\mathrm{SCET_{II}}$.
  - Non-form-factor contributions to $\Lambda_b \to \Lambda \ell \ell$ decays at one loop.
    $\Rightarrow$ Renormalization properties of higher twist Λ_b-baryon DAs.
  - QCD factorization for $\Lambda_b \to \Lambda \ell \ell$ at leading power.