Phase diagram, fluctuations, thermodynamics and hadron chemistry

Claudia Ratti
University of Houston (USA)

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Quantum Chromodynamics

QCD describes interactions among quarks q and gluons g

Quarks are described in terms of Dirac fields $\psi_\alpha^{ir}(x)$, where:

$$\begin{array}{ll} \alpha: & \text{Dirac spinor index} \\ i: & SU(3)\ \text{color index}\ (1,2,3) \\ r: & \text{flavor index}\ (u,d,s,c,b,t) \end{array}$$

Gluons are described in terms of vector fields $A_\mu^a(x)$, where:

$$\begin{array}{ll} \mu: & \text{Lorentz vector index} \\ a: & \text{color index}\ (1,2,\ldots 8) \end{array}$$

Gell-Mann matrices λ^a: 3×3 matrices in color space: $(\lambda^a)_{ij},\ i,j = 1,2,3$

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$$

The QCD Lagrangian

Local $SU(3)$ color transformation: $U(x) = \exp\left[i\frac{\lambda^a}{2}\theta^a(x)\right] = \exp\left[\frac{i}{2}\vec{\lambda}\cdot\vec{\theta}(x)\right]$

$U(x) = 3 \times 3$ matrix in the color space

$$\psi(x) \to U(x)\psi(x) \qquad \psi^\dagger(x) \to \psi^\dagger(x)U^\dagger(x)$$

$$\psi_i(x) = U_{ij}(x)\psi_j(x) \qquad \psi_i^*(x) = \psi_j^*(x)U_{ji}^*(x)$$

QCD Lagrangian: $\mathcal{L}_{QCD} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \bar{\psi}^r\left(i\not{D} - m_r\right)\psi^r$, where:

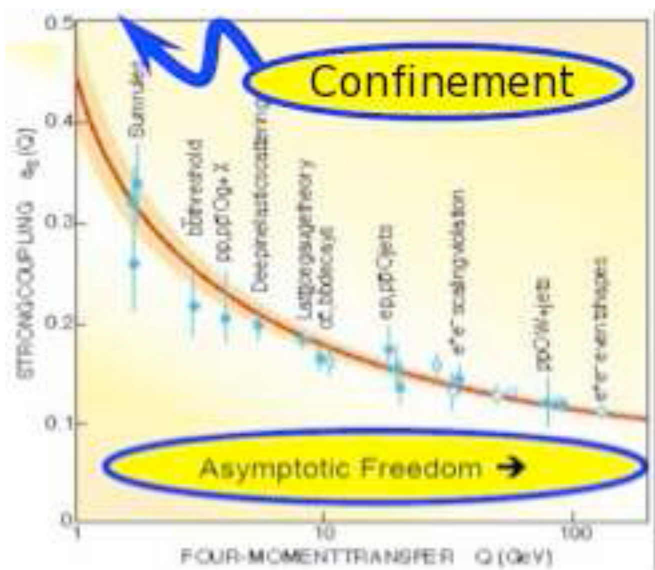
$$\begin{aligned} F_{\mu\nu}^a &\equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \\ F_{\mu\nu} &\to U F_{\mu\nu} U^\dagger \qquad \left(F_{\mu\nu} \equiv F_{\mu\nu}^a \frac{\lambda^a}{2}\right) \\ D_\mu &= \partial_\mu - igA_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A_\mu^a \\ D_\mu^{ab} &= \delta_{ab}\partial_\mu - ig\left(\frac{\lambda^c}{2}\right)_{ab} A_\mu^c \end{aligned}$$

- m_r: quark mass for flavor r; $m = diag(m_u, m_d, m_s, m_c, m_b, m_t)$

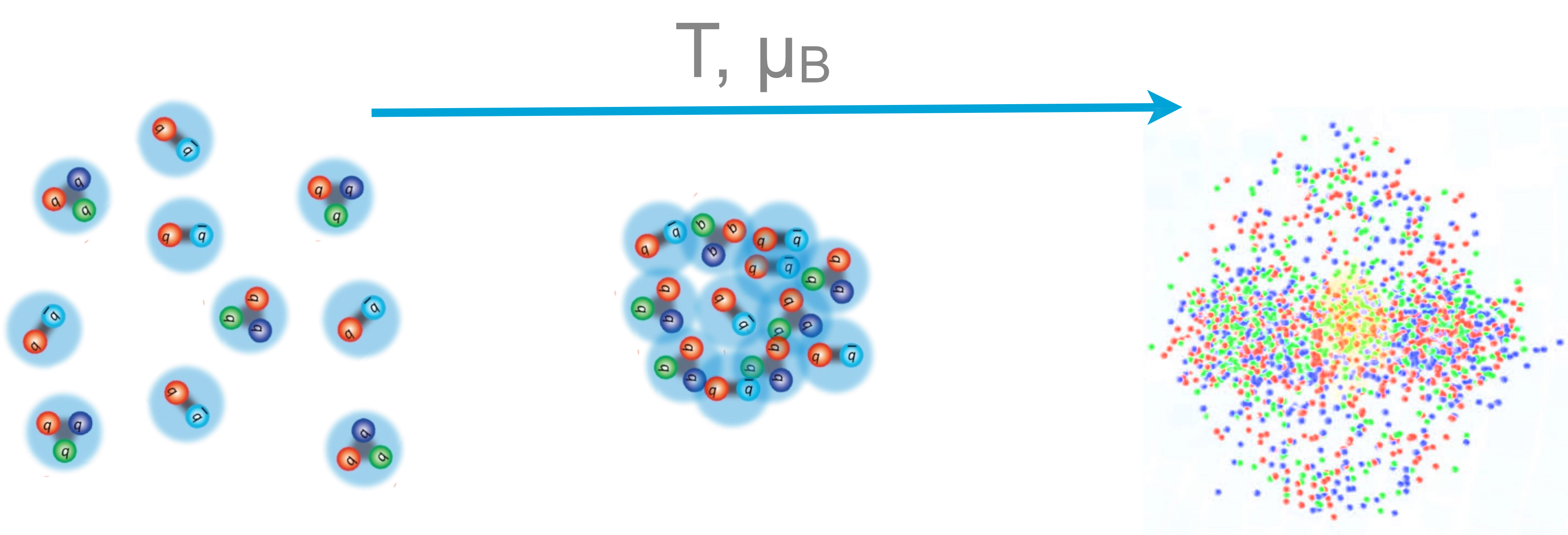
- g: coupling constant, with $g^2/(4\pi) = \alpha_s$

$\mathcal{L}_{QCD}$ invariant for local $SU(3)$ color transformations

QCD Thermodynamics



* Confinement
  * At large distances the effective coupling is large
  * Free quarks are not observed in nature
* Asymptotic freedom
  * At short distances the effective coupling decreases
  * Quarks and gluons appear to be quasi-free



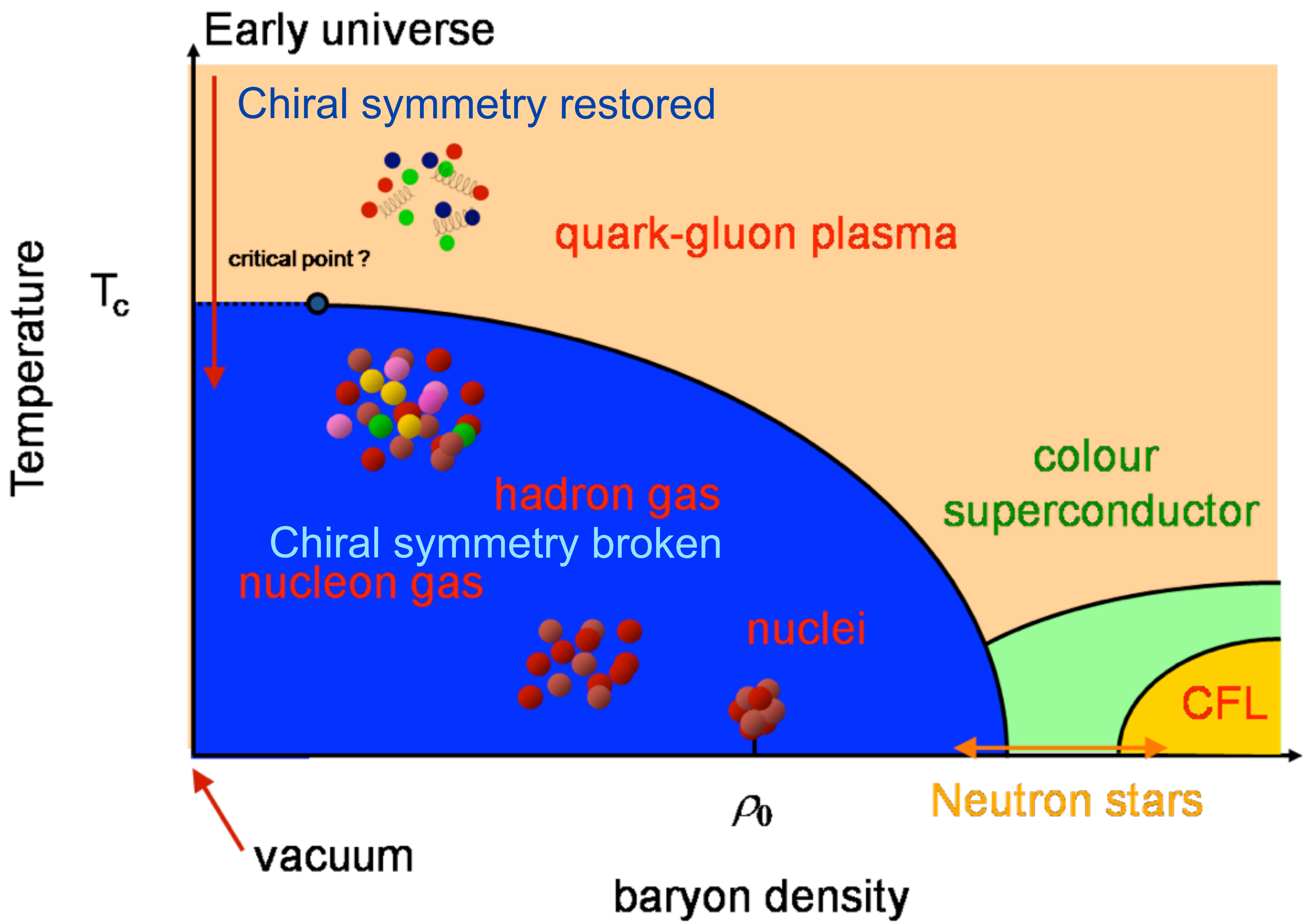
Hadron Gas

Quark-Gluon

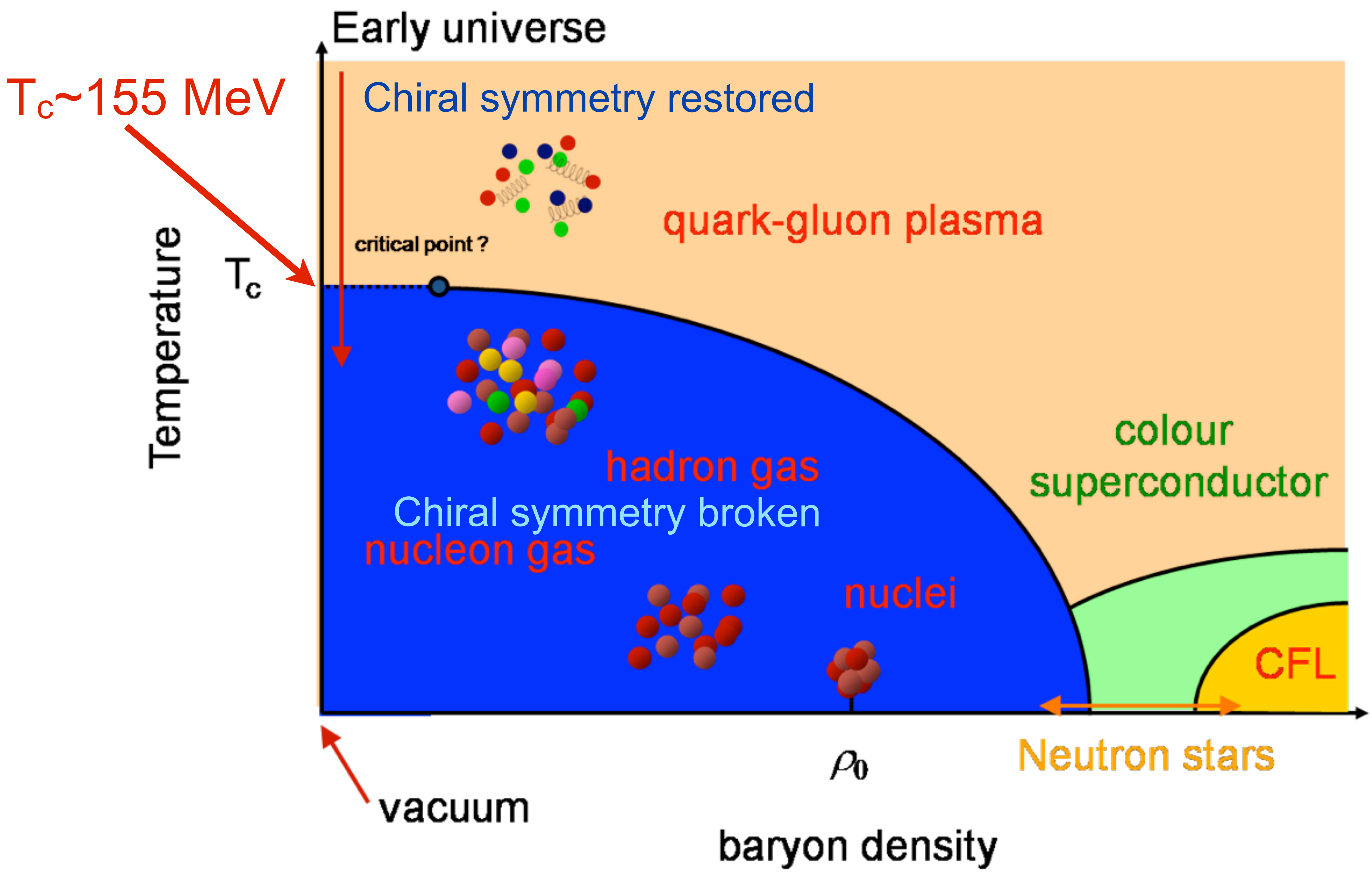
Chiral Symmetry: broken

Chiral Symmetry: restored

QCD Phase Diagram



QCD Phase Diagram



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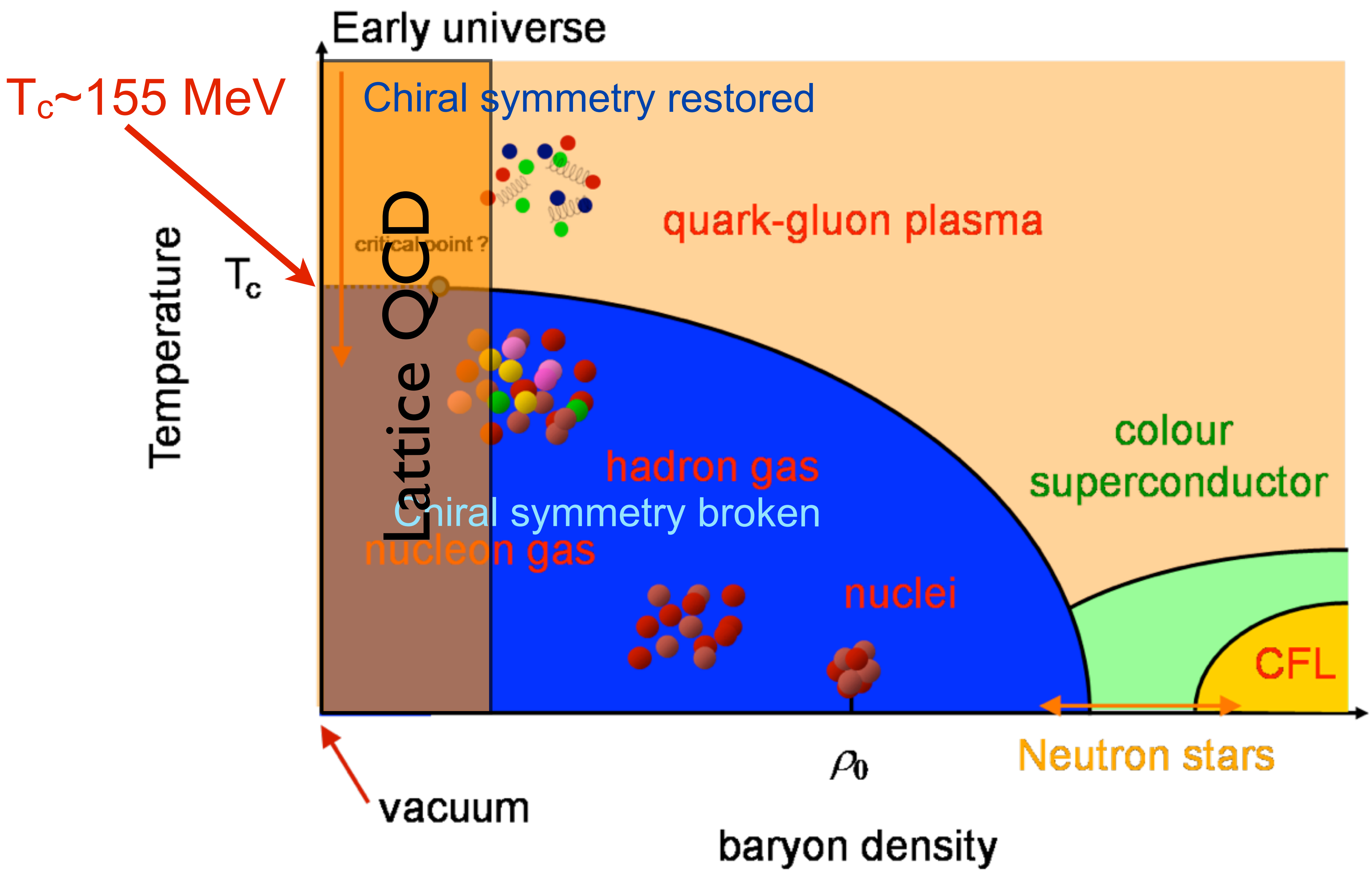

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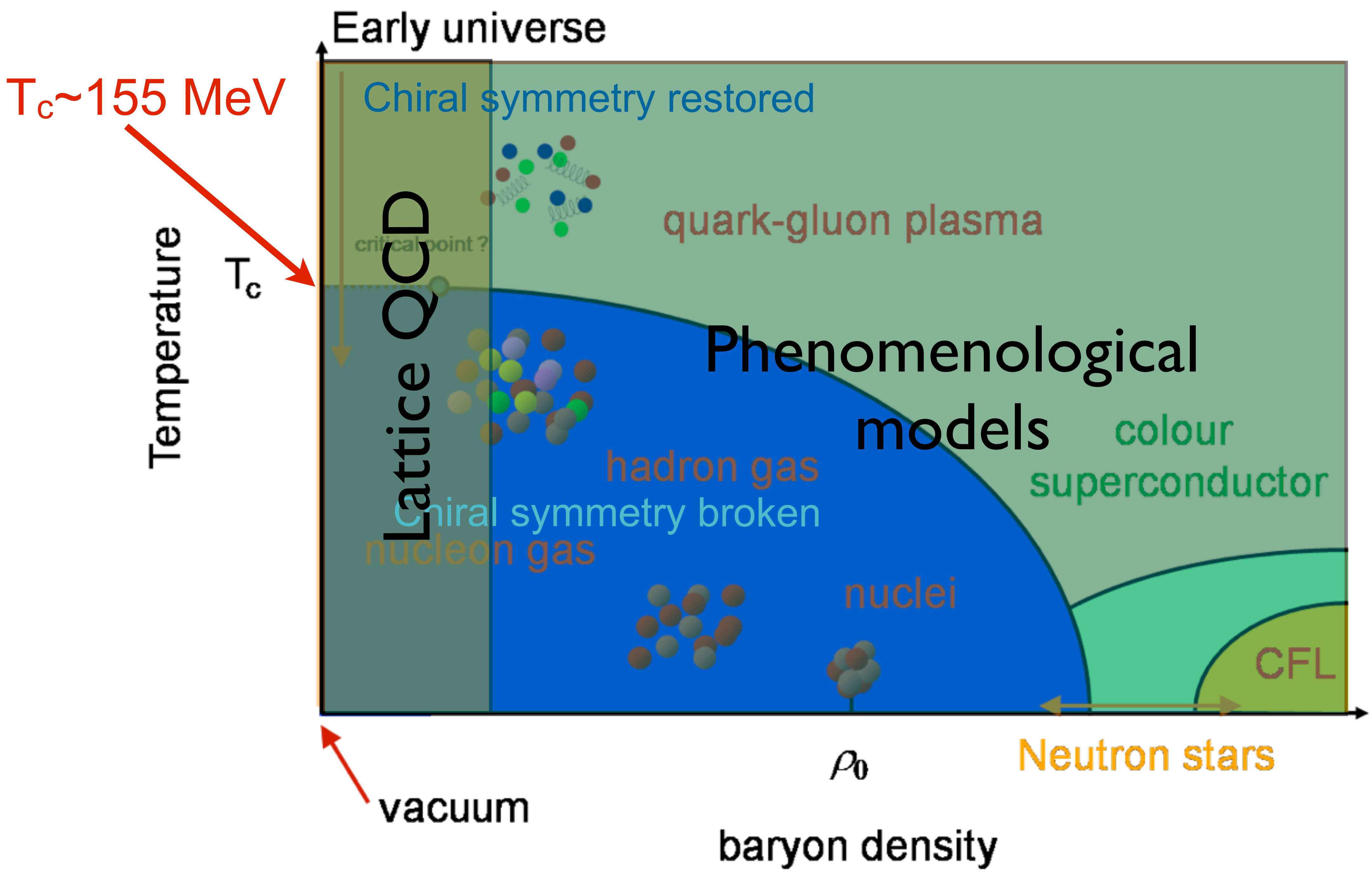
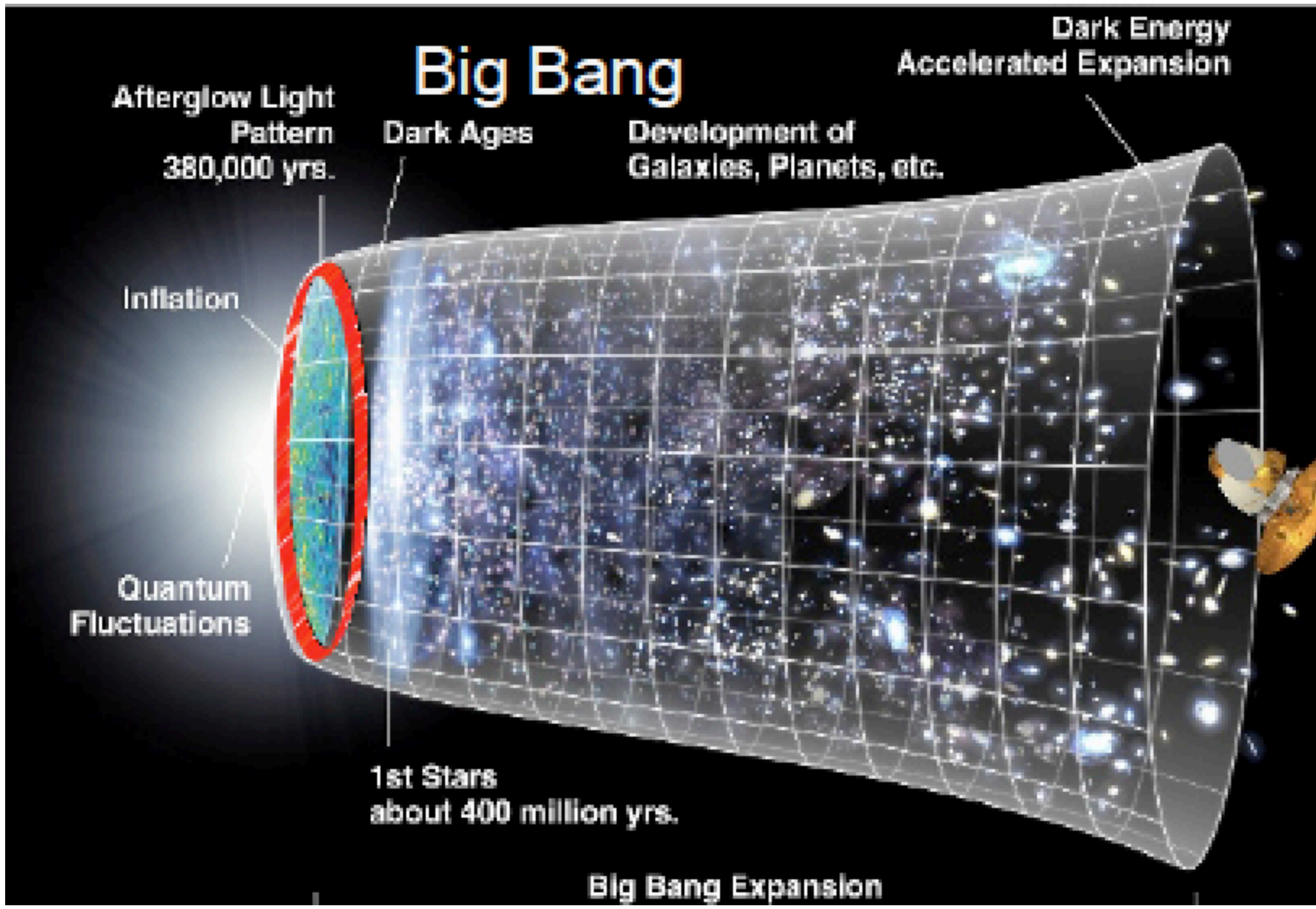
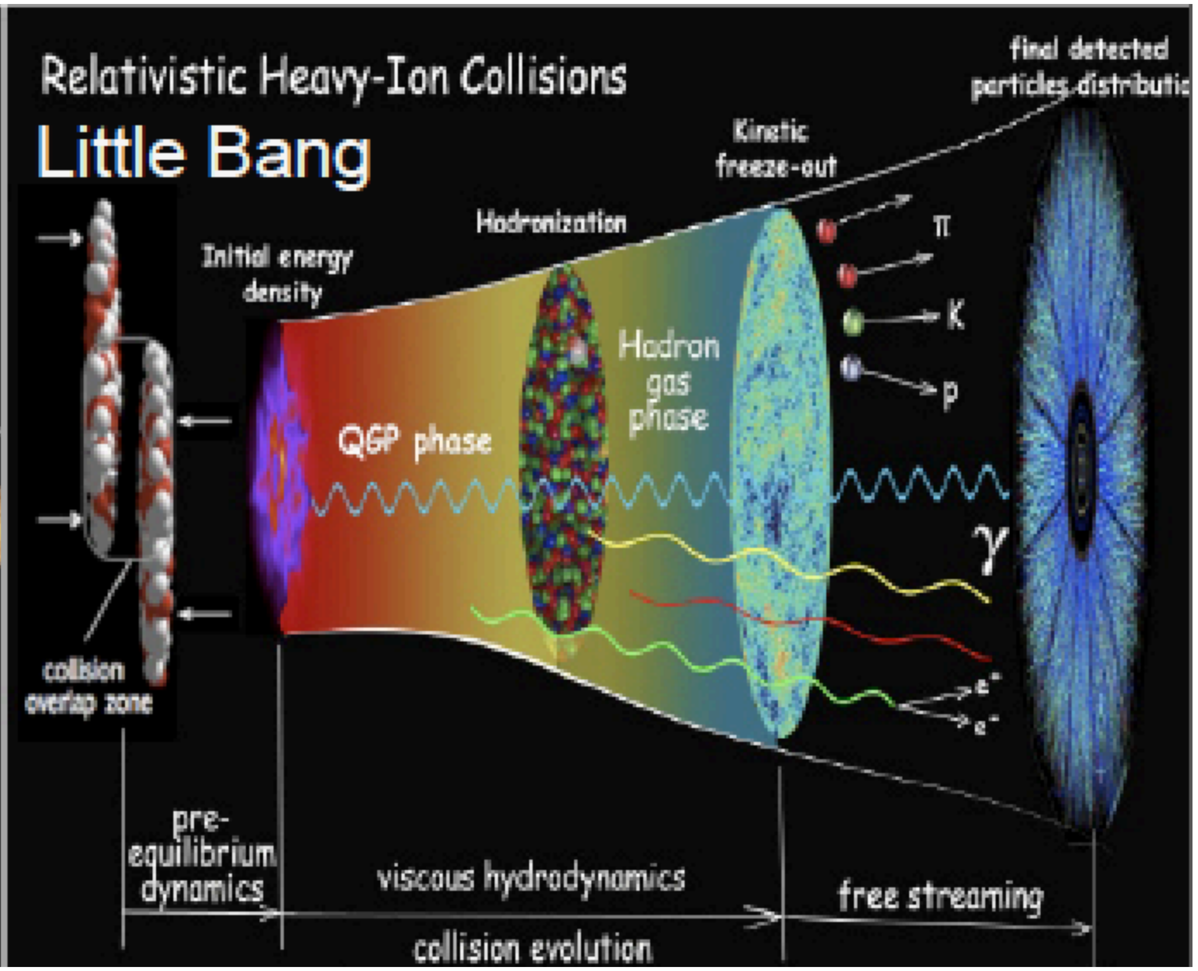


Motivation





- The universe was in a QGP phase at the beginning of its evolution
- We can re-create this phase in the laboratory!

Statistical Mechanics reminder

In equilibrium statistical mechanics one normally encounters three types of ensemble

- microcanonical ensemble: used to describe an isolated system with fixed $E,\ N,\ V$
- canonical ensemble: used to describe a system in contact with a heat bath with fixed $T,\ N,\ V$
- grand canonical ensemble: used to describe a system in contact with a heat bath, with which it can exchange particles: fixed $T,\ V,\ \mu$

In a relativistic quantum system, where particles can be created and destroyed, one computes observables in the grand canonical ensemble

Partition function

The grand canonical PARTITION FUNCTION reads:

$$Z = Tr\left[e^{-\beta(\hat{H}-\mu_i \hat{N}_i)}\right]$$

with $\hat{H}$ Hamiltonian of the system, $\hat{N}_i$ set of conserved number operators and $\beta = 1/k_B T$

$Z = Z(V, T, \mu_1, \mu_2, ...)$ is the most important function in thermodynamics

All other standard thermodynamic properties may be determined from it:

$$P = T\frac{\partial \log Z}{\partial V}; \qquad N_i = T\frac{\partial \log Z}{\partial \mu_i};$$

$$S = \frac{\partial T \log Z}{\partial T} \qquad E = -PV + TS + \mu_i N_i$$

Phase transitions and order parameters

- We want to study the transition from hadrons to the QGP: deconfinement and chiral symmetry restoration

- A phase transition is the transformation of a thermodynamic system from one phase or state of matter to another

- During a phase transition of a given medium certain properties of the medium change, often discontinuously, as a result of some external conditions

- The measurement of the external conditions at which the transformation occurs is called the phase transition point

- Order parameter: some observable physical quantity that is able to distinguish between two distinct phases

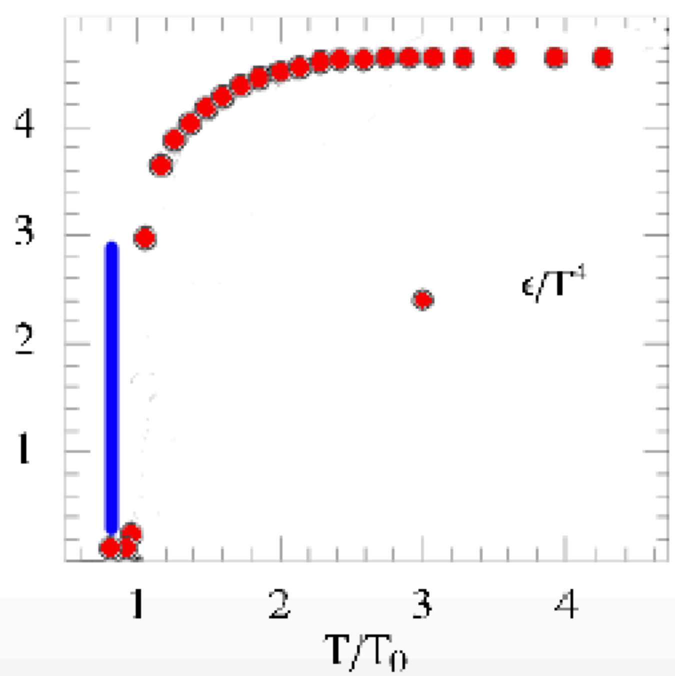
- We need to find observables which allow us to distinguish between confined/deconfined system and between chirally broken/restored phase

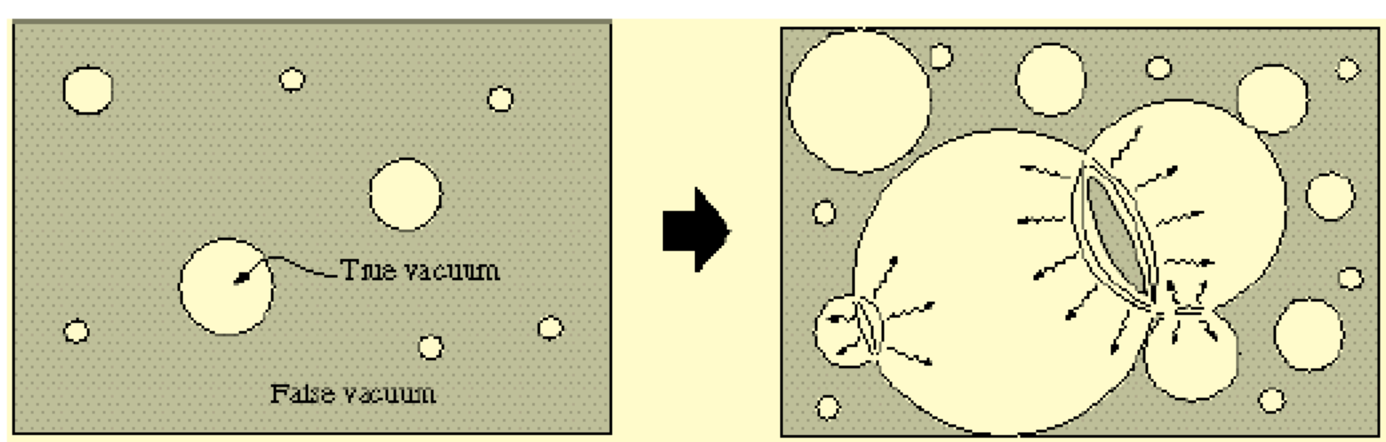
Phase transition classification (I)

Paul Ehrenfest classified phase transitions based on the behavior of the thermodynamic free energy as a function of other thermodynamic variables

First-order phase transitions exhibit a discontinuity in the first derivative of the free energy with respect to some thermodynamic variable

- involve a latent heat
- the system is in a "mixed-phase regime" in which some parts of the system have completed the transition and others have not





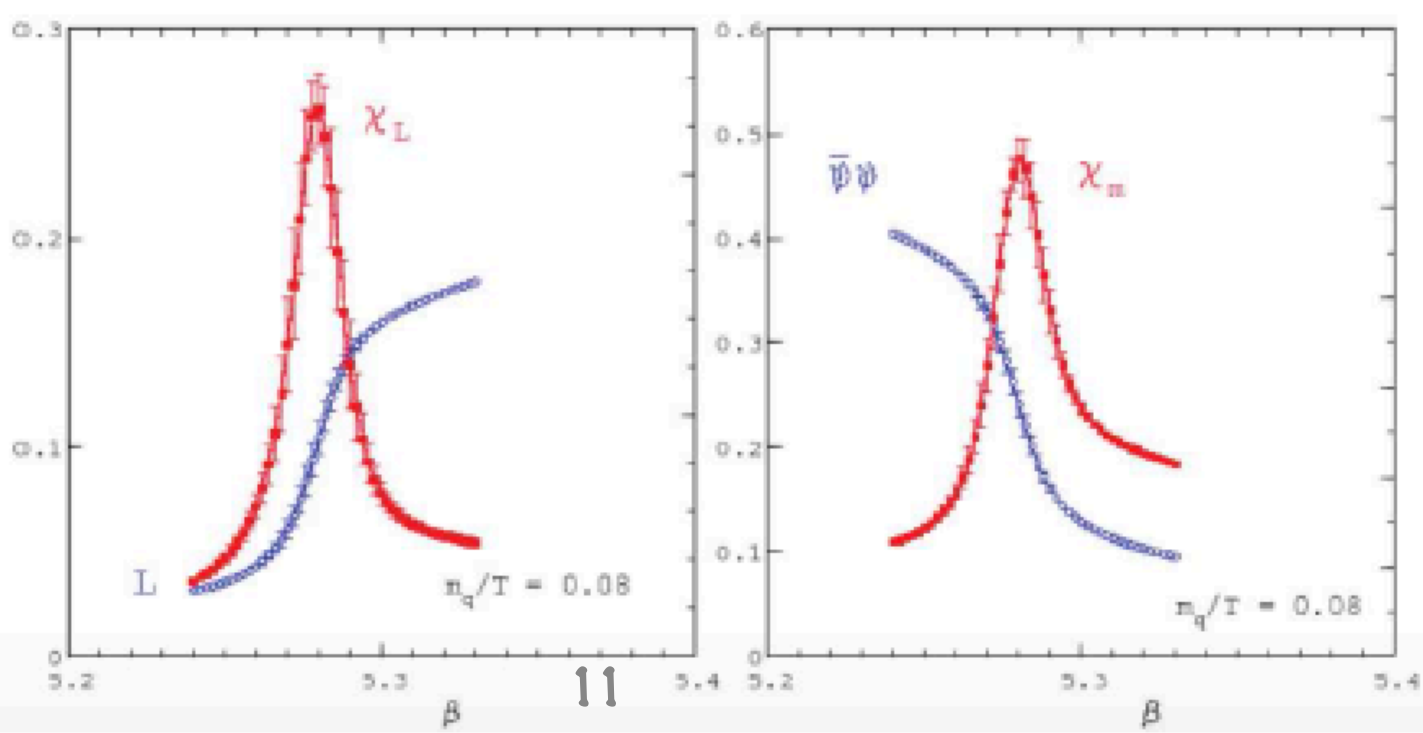
Phase transition classification (II)

Second-order phase transitions: free energy and its first derivative continuous at T_c

- A new state grows continuously out of the previous one
- for $T \to T_c$ the two states become quantitatively the same
- second derivative can be discontinuous or diverge at T_c
- power law behavior in $|1 - T/T_c|$ at T_c

Analytic crossover: free energy and all its derivative continuous at T_c

- System changes smoothly from one phase to the other
- Phase transition point identified by peak of susceptibility



Different limits

$m_q \to \infty$: pure gauge QCD

- only gluons are relevant degrees of freedom
- Z_3 symmetry of QCD: confinement/deconfinement phase transition: first order
- Polyakov loop: order parameter

$m_q \to 0$: chiral limit of QCD

- chiral symmetry spontaneously broken
- chiral condensate: order parameter; second order

real world: $m_q \neq 0$: but small for $u,\ d,\ s$ quarks

- chiral symmetry explicitly broken by the finite quark mass
- Z_3 symmetry: explicitly broken by the presence of quarks
- QCD transition is analytic crossover

Polyakov loop: deconfinement

* We consider a system of gluons in which we put a heavy quark-antiquark pair as a probe

$$<\Phi>\sim e^{-F/T}$$

➡ How much energy F is needed to extract the heavy quark from the system?

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Confined system
Infinite energy is needed

$$\langle \Phi \rangle = 0$$

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$$<\Phi> \longrightarrow 1$$

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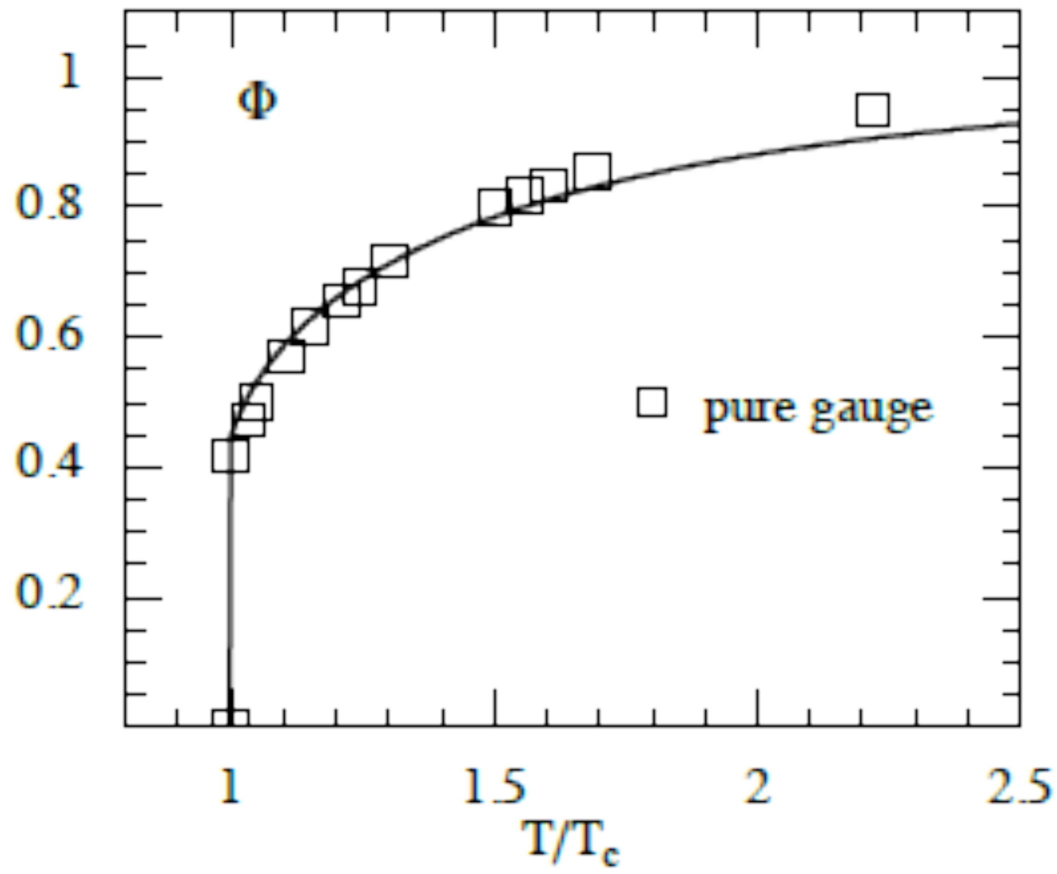

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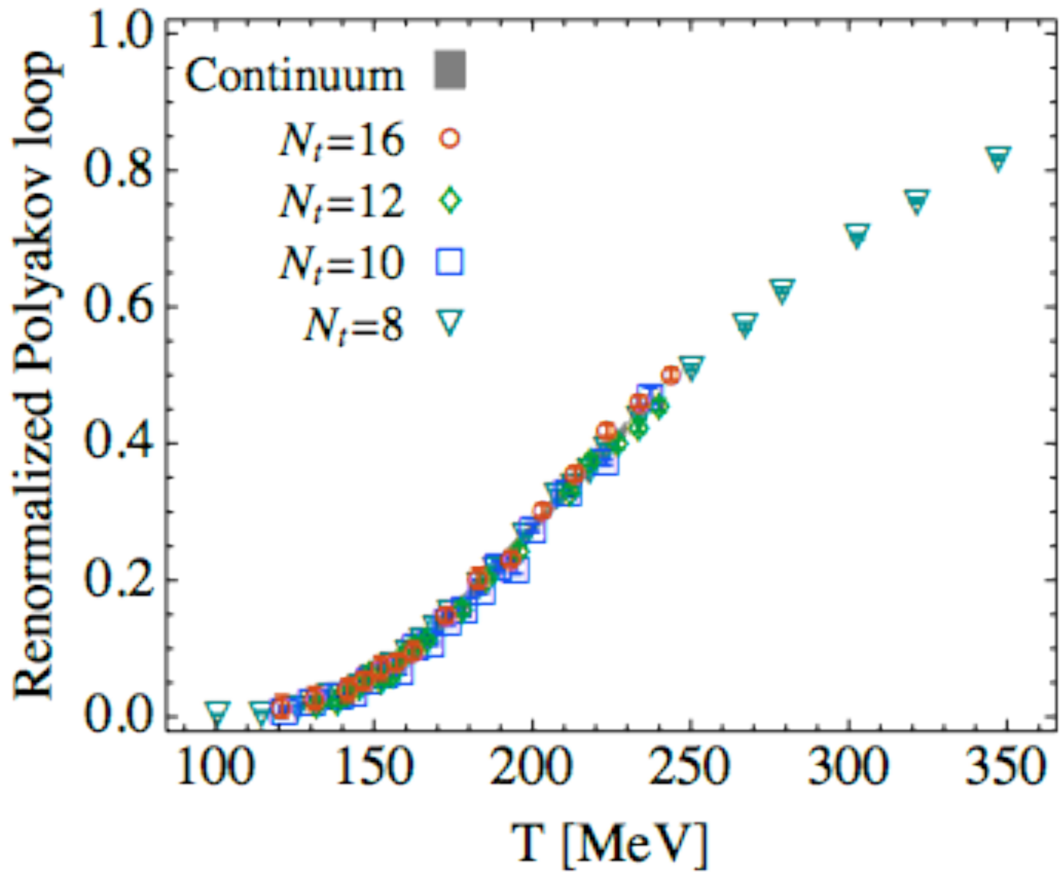

For QCD with physical quark masses the transition is a smooth crossover

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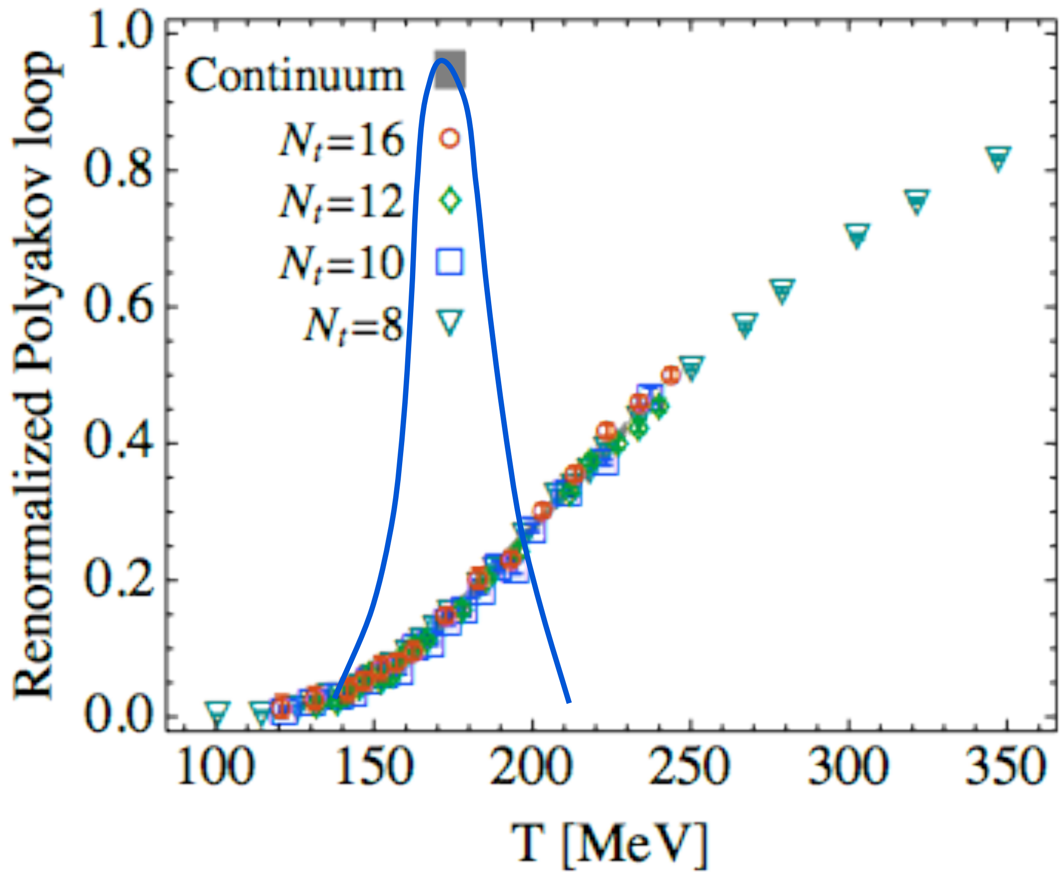

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Polyakov loop: order parameter for deconfinement

Chiral condensate: chiral transition

- The chiral condensate $\langle\bar{\psi}\psi\rangle$ is the vacuum expectation value of the operator $\psi\psi$

- The magnitude of the constituent quark mass is proportional to it

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Chirally broken system
Large effective quark mass

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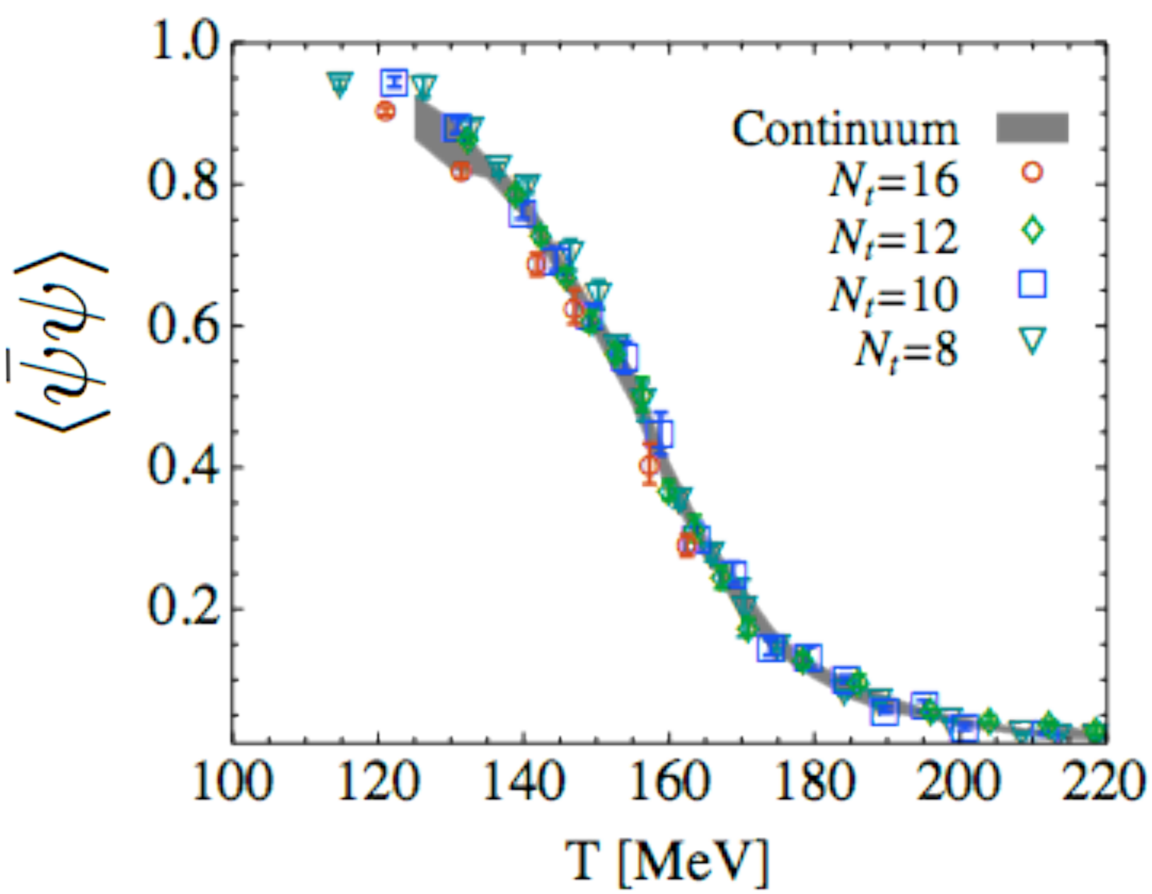

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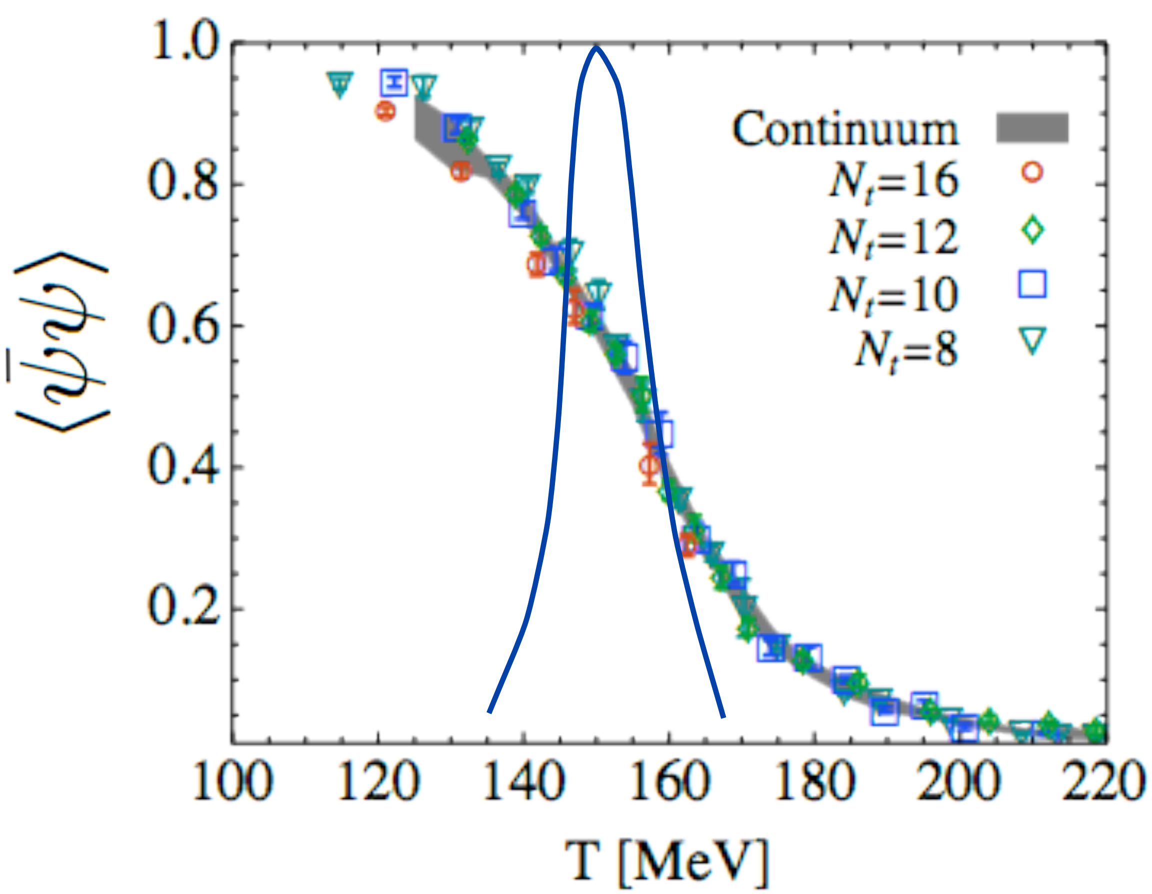


For QCD with physical quark masses the transition is a smooth crossover

Chiral condensate: order parameter for chiral phase transition

Stefan-Boltzmann limit

- Simplest possible system: non-interacting gas of massless quarks and gluons

- we expect QCD thermodynamics to reach this limit at $T \to \infty$

$$n = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{p/T} \pm 1} = \nu \frac{\zeta(3)}{\pi^2} T^3 \qquad \nu = \begin{cases} 1 & bosons \\ \frac{3}{4} & fermions \end{cases}$$

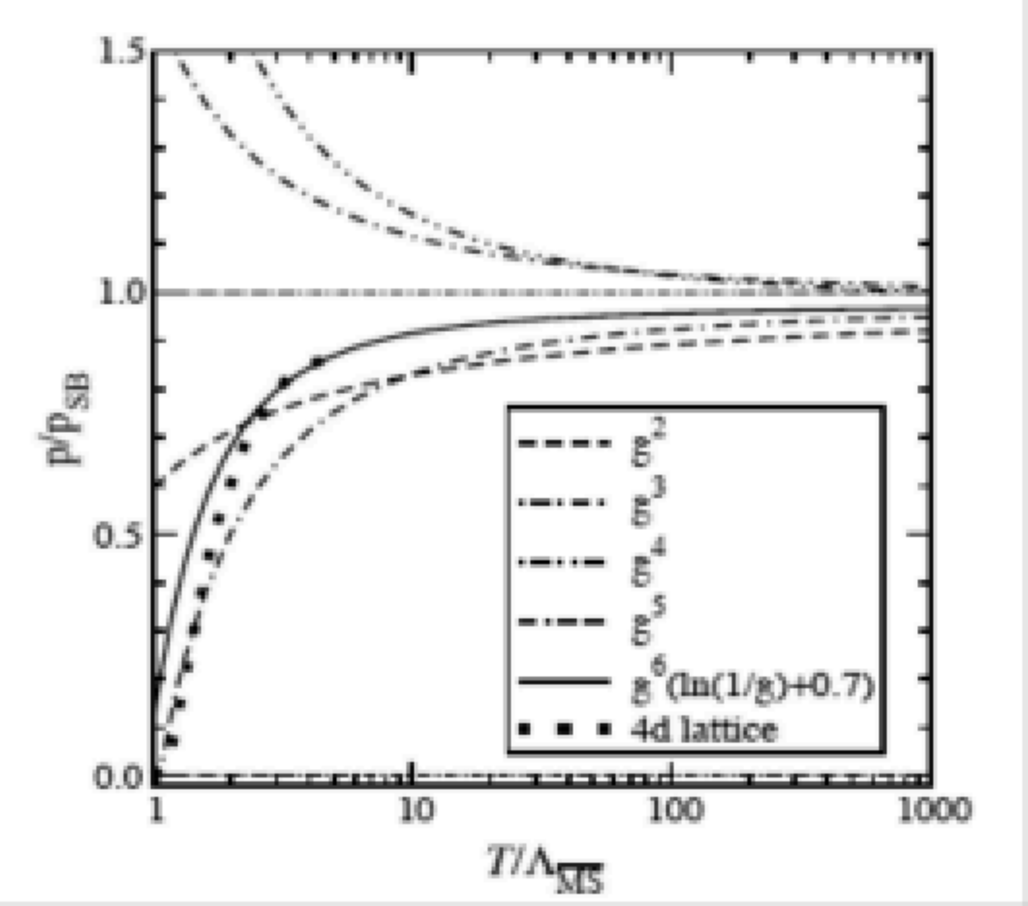
where $\zeta(3) = 1.202$ (Riemann ζ function)

$$\epsilon = \int \frac{d^3p}{(2\pi)^3} \frac{p}{e^{p/T} \pm 1} = \nu' \frac{\pi^2}{30} T^4 \qquad \nu' = \begin{cases} 1 & bosons \\ \frac{7}{8} & fermions \end{cases}$$

pressure: $p = \frac{\epsilon}{3}$ entropy density: $Ts = \epsilon + P = \frac{4}{3}\epsilon \implies s = \frac{4}{3}\frac{\epsilon}{T} = 2\nu' \frac{\pi^2}{45} T^3$

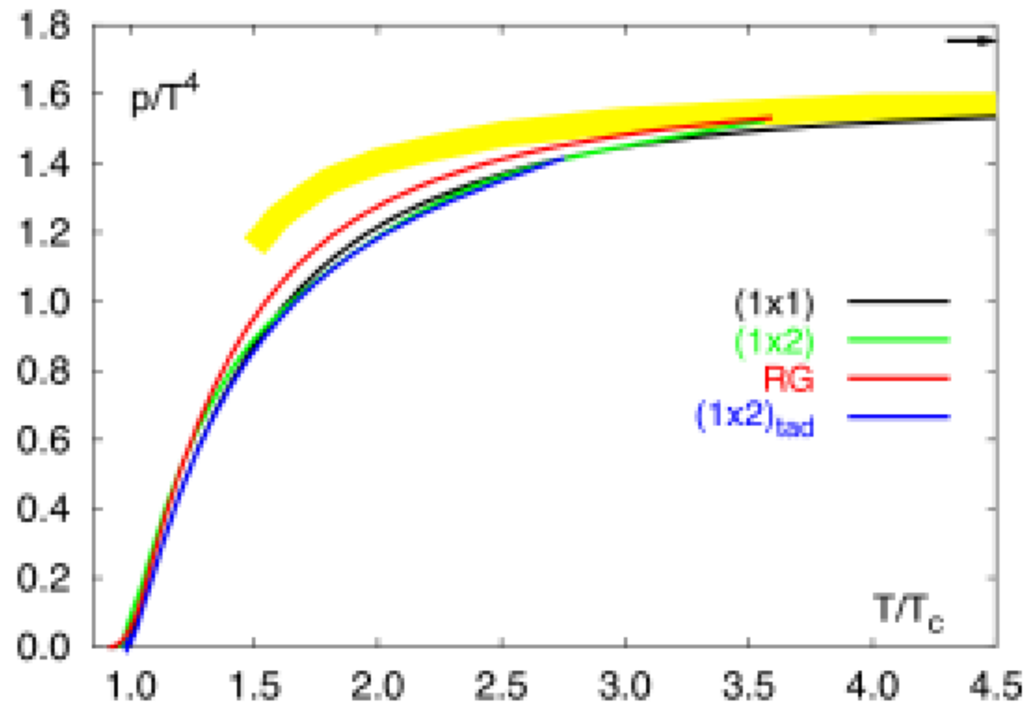
$\frac{n}{T^3}, \frac{p}{T^4}, \frac{s}{T^3}, \frac{\epsilon}{T^4}$ constant in SB limit!

Switch on interaction: perturbative QCD



- Much effort put into calculating the successive orders of the perturbative expansion for the pressure
- the series is known now up to order $g^6 \log g$
- perturbation theory makes sense only for very small values of the coupling constant
- For not too small values of the coupling, the successive terms in the expansion oscillate

Improve series convergence

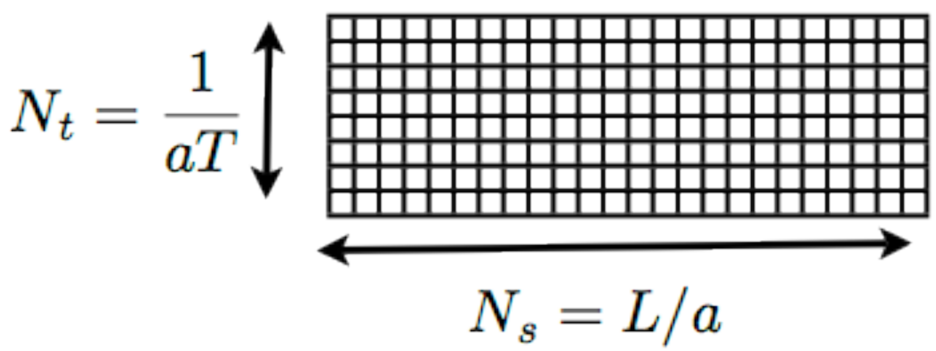


J.P. Blaizot, E. Iancu, A. Rebhan, PLB470

- One can improve the convergence of the series by some clever resummation
- Hard Thermal Loop: Quark and Gluon propagators are dressed by some effective mass
- this improves the series convergence and the agreement to lattice data down to $T \sim 3T_c$

Lattice QCD

- Analytic or perturbative solutions in low-energy QCD are hard or impossible due to the highly nonlinear nature of the strong force
- Lattice QCD: well-established non-perturbative approach to solving QCD
- Solving QCD on a grid of points in space and time
- The lattice action is the parameterization used to discretize the Lagrangian of QCD on a space-time grid



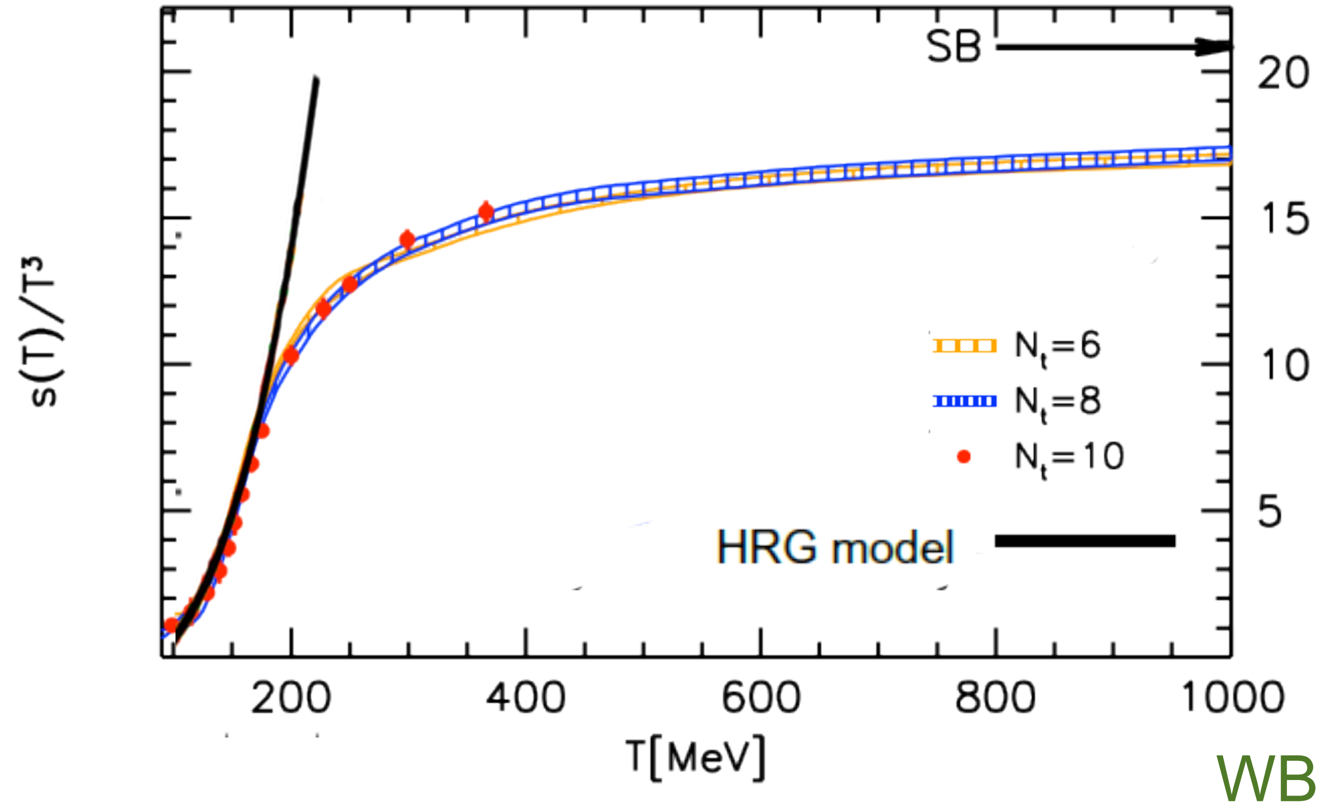
- From the partition function Z, knowledge of all the thermodynamics

$$Z_W = \mathrm{Tr}\exp(-H/T) = \int [dU][d\bar{\psi}d\psi]\exp(-S) \qquad S = S_G + S_F$$

$$S_G = \frac{6}{g^2}\sum_{x,\mu<\nu}[1 - \mathrm{Re}\,\mathrm{Tr}\,U_P(x;\mu,\nu)/3]$$

$$S_F = \sum_x \bar{\psi}(x)\psi(x) - \kappa\sum_{x,\mu}[\bar{\psi}(x)(1+\gamma_\mu)U_\mu(x)\psi(x+\hat{\mu}) + \bar{\psi}(x+\hat{\mu})(1-\gamma_\mu)U^\dagger_\mu(x)\psi(x)]$$

Transition from QCD Thermodynamics



WB Collab., JHEP (2010)

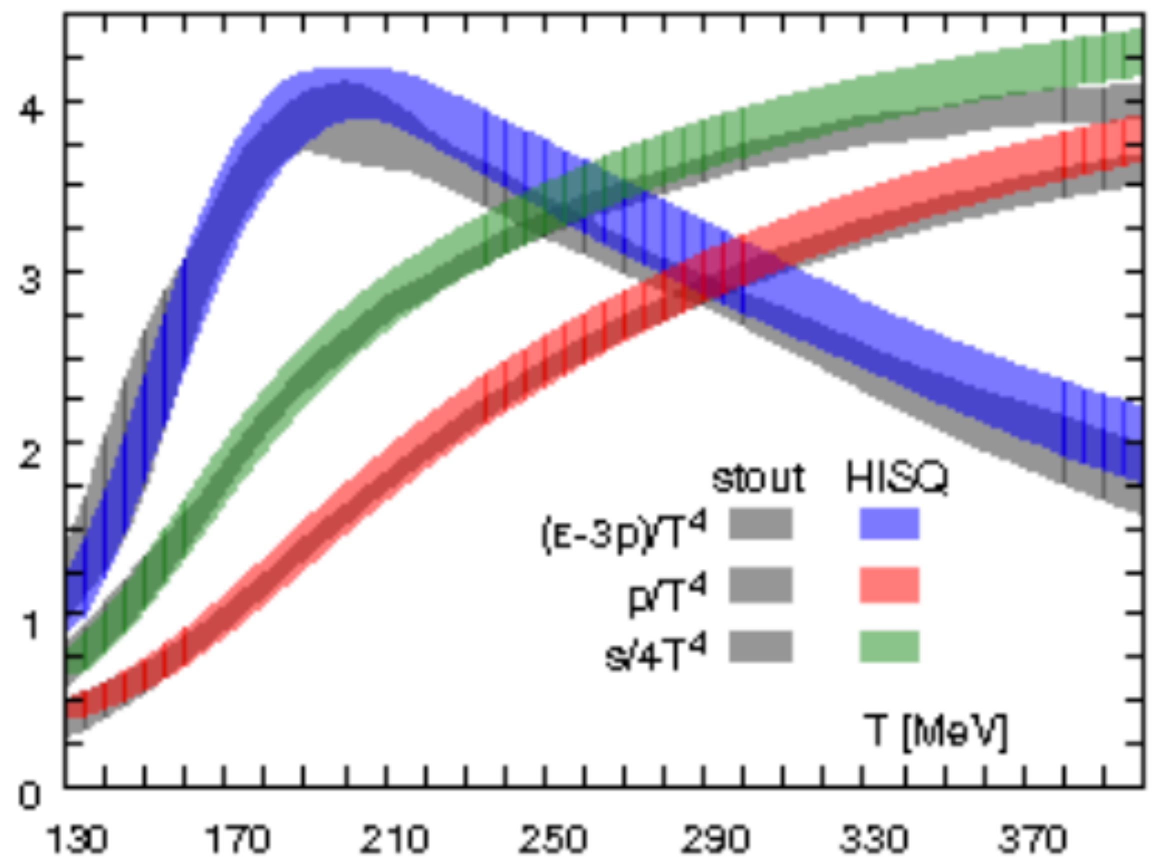
- s/T^3 indicates the number of particle species
- Rapid rise = liberation of degrees of freedom
- Compare to an ideal gas of quarks and gluons

$$s = \frac{4g}{\pi^2} T^3$$

- This gives us an idea of how strong the interaction is

Equation of state for Nf=2+1

Two independent and compatible results for the $\mu = 0$ and $N_f = 2 + 1$



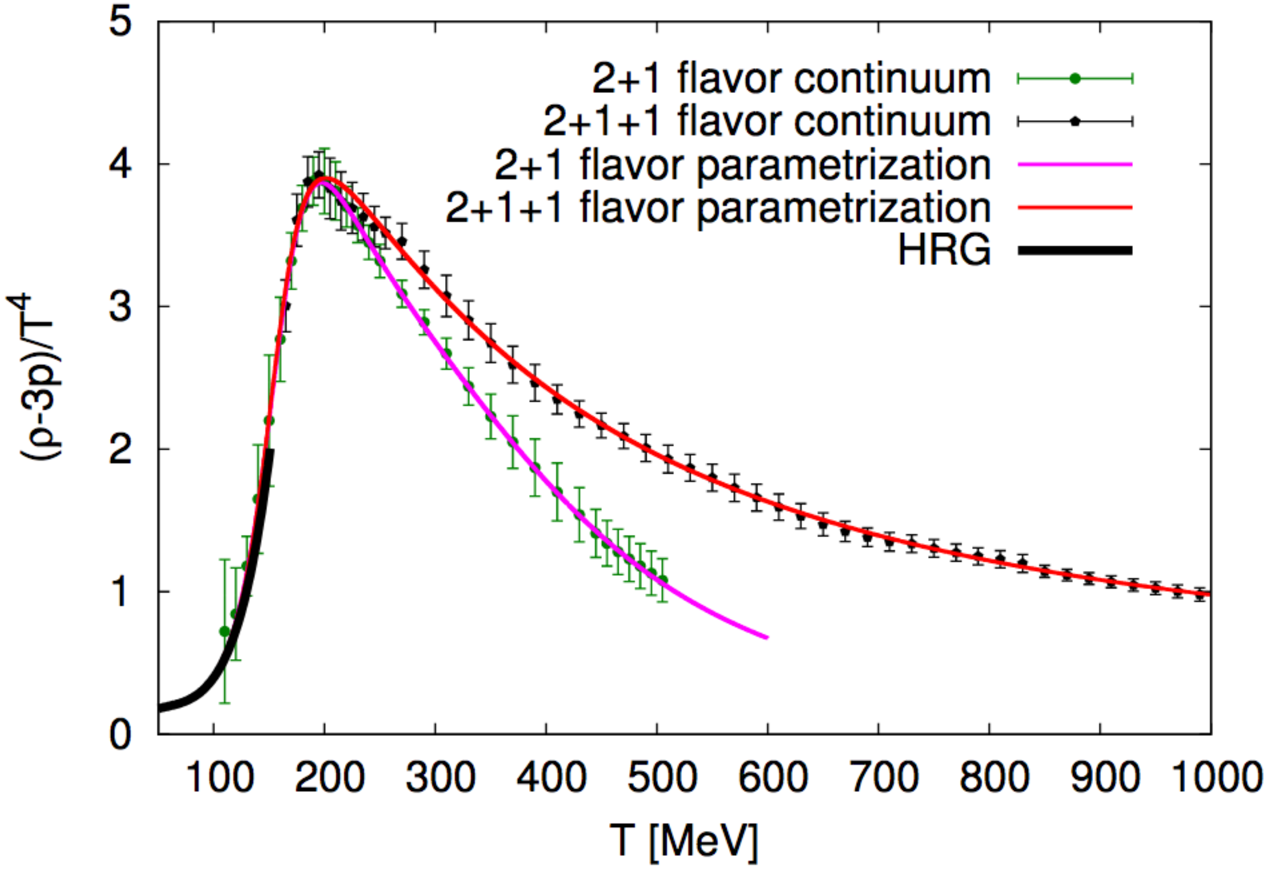
WB: S. Borsanyi et al., 1309.5258, PLB (2014)
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

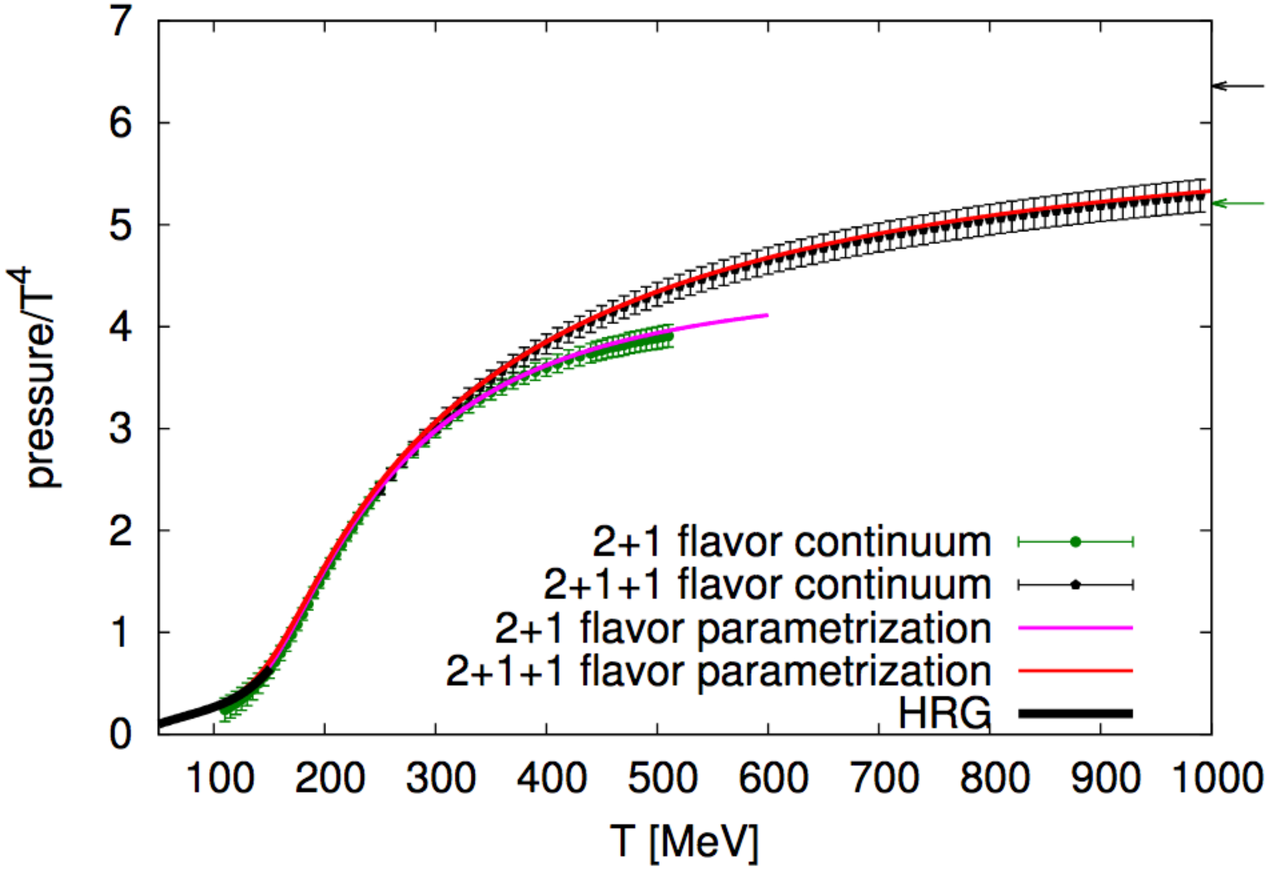
This is nice, but...

(1) Heavy ion physics: needs $\mu > 0$

(2) Cosmology: needs higher temperature, therefore more quark flavours

Equation of state with dynamical charm





WB (S. Borsanyi et al.), Nature 2016

QCD in the grand canonical ensemble

Grand canonical partition function:

$$e^{-F/T} = \mathcal{Z}(T; \mu_u, \mu_d, \mu_s) = \mathrm{Tr}\left(e^{-\beta(H-\mu_u N_u - \mu_d N_d - \mu_s N_s)}\right)$$

$\implies$ 4D phase diagram.

$$\text{Quark number density} \qquad \langle n_q \rangle = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_q}$$

$$\text{Baryon number density} \qquad \langle n_B \rangle = \frac{1}{3}\left(\langle n_u \rangle + \langle n_d \rangle + \langle n_s \rangle\right)$$

$$\text{Isospin density} \qquad \langle n_I \rangle = \frac{1}{2}\left(\langle n_u \rangle - \langle n_d \rangle\right)$$

$$\text{Electric charge} \qquad \langle n_Q \rangle = \frac{2}{3}\langle n_u \rangle - \frac{1}{3}\langle n_d \rangle - \frac{1}{3}\langle n_s \rangle$$

Both for heavy ion physics and neutron star physics, we need:

- $\langle n_I \rangle < 0$ not a problem
- $\langle n_B \rangle > 0$ complex action problem

} I will shortly review why

Euclidean path integral

Start with a grand canonical partition function: $Z = \mathrm{Tr}\left(e^{-(H-\mu N)/T}\right)$

Notation: $H - \mu N \to H$ for simplicity

The partition function $Z = \mathrm{Tr}\left(e^{-H/T}\right)$ written as a path integral:

$$Z = \sum_{c_1, c_2, \ldots} \langle c_1 | e^{-aH} | c_2 \rangle \langle c_2 | e^{-aH} | c_3 \rangle \ldots \langle c_{n-1} | e^{-aH} | c_1 \rangle =: \sum_{[c]} w[c]$$

- Maps the quantum system to a classical system, with configurations c
- $w(c) =$ weight of a configuration c
- $w(c) \geq 0 \implies$ can use Monte Carlo
- $w(c)$ can be negative or complex $\implies$ sign or complex action problem
- Sign problem property of the system **AND** the basis we used

Euclidean path integral for QCD

$$\mathcal{Z} = \mathrm{Tr}\left(e^{-\beta(\hat{H}-\mu_u\hat{N}_u-\mu_d\hat{N}_d-\mu_s\hat{N}_s)}\right)$$
$$= \int \mathcal{D}U\mathcal{D}\bar{\Psi}\mathcal{D}\Psi e^{-S_{YM}-(\bar{\Psi}M\Psi)}$$
$$= \int \mathcal{D}U e^{-S_{YM}[U]} \det M[U;m_u,\mu_u] \det M[U;m_d,\mu_d]$$

Fermion determinant

If positive, we can treat $w := e^{-S_{YM}} \det M \det M$ to be a probability distribution, we can generate configurations of the gauge field with these probabilities and calculate observable on these configurations

The complex action problem at finite μ

$$[\det M(\mu)]^* = \det M(-\mu^*) \in \mathbb{C}$$

$w := e^{-S_{YM}} \det M \det M$ complex $\implies$ cannot use importance sampling

Cases without a complex action/sign problem:

- $\mu = 0$ Calculate derivatives of $\log \mathcal{Z}$ at $\mu = 0$ } $\implies$ small $\mu_B > 0$
- Purely imaginary μ
- Isospin chemical potential $\mu_u = -\mu_d \implies$ Study pion condensation.
- The Dirac operator can have an additional symmetry, e.g. SU(2) gauge theory with N_f =even

Taylor expansion of the pressure

Suppose we either:

- fix $\mu_S = 0 = \mu_Q$ for simplicity
- fix $\langle S \rangle = 0$ and $\langle Q \rangle = 0.4 \langle B \rangle$ for HIC

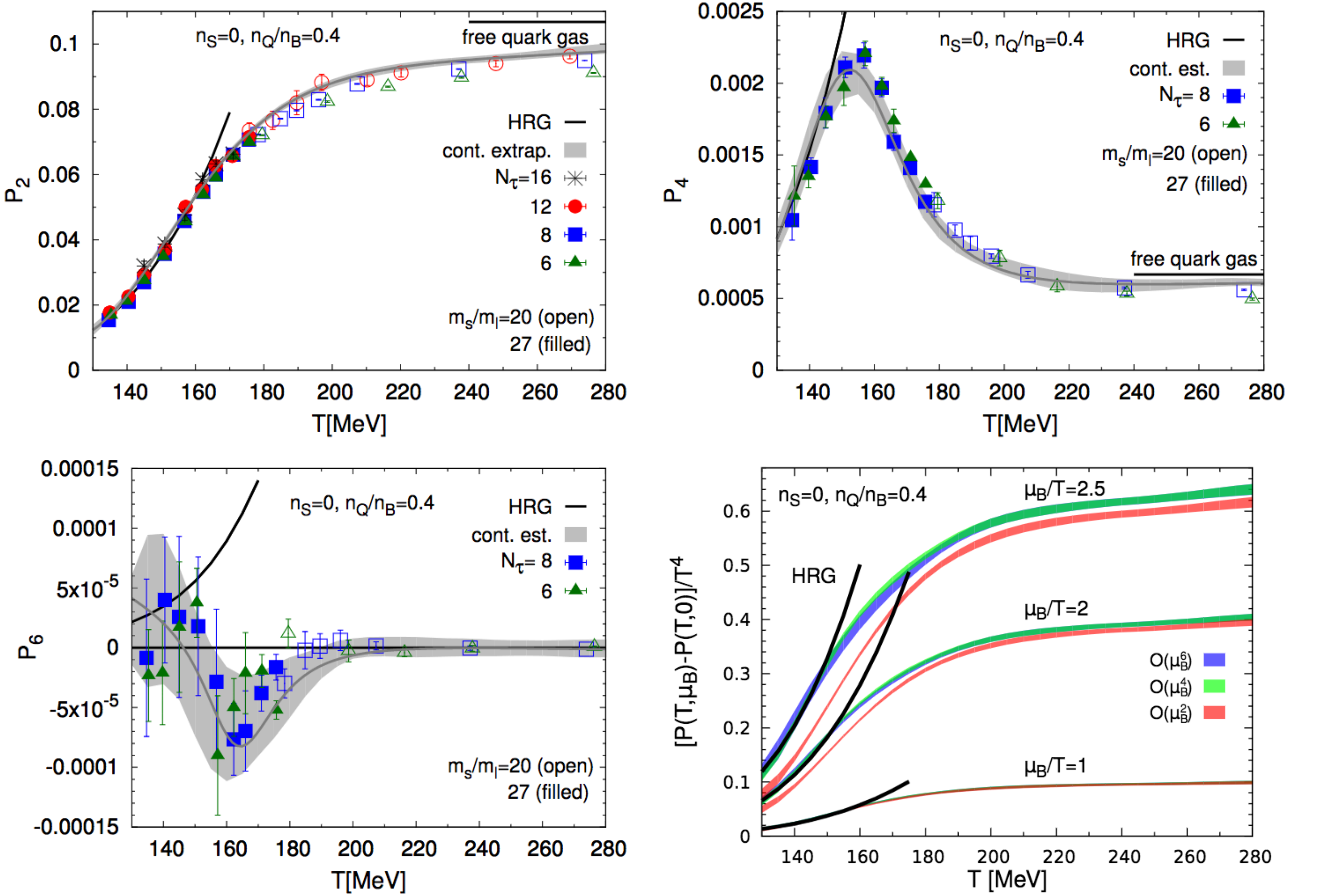
The pressure is now:

$$\frac{P}{T^4} = P(T, \mu = 0) + \sum_{k=1}^{\infty} c_{2k} \left(\frac{\mu_B}{T} \right)^{2k}$$

Alternatively, I can fix nothing and calculate the 3 variable Taylor expansion. The coefficients contain lots of info:

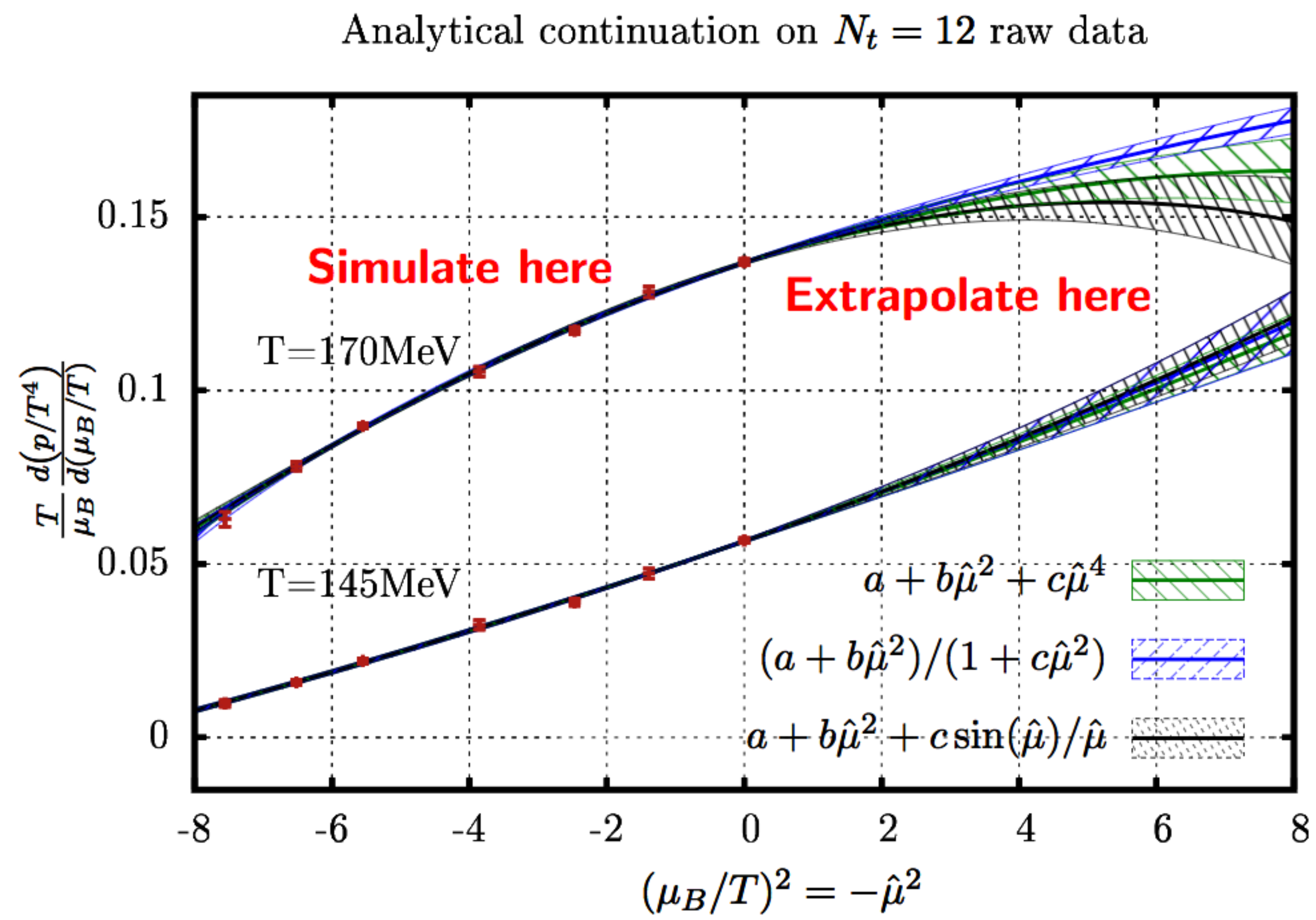
- $T_c(\mu)$
- EoS
- Lower limit on location of critical point
- ...

Taylor expansion of the pressure

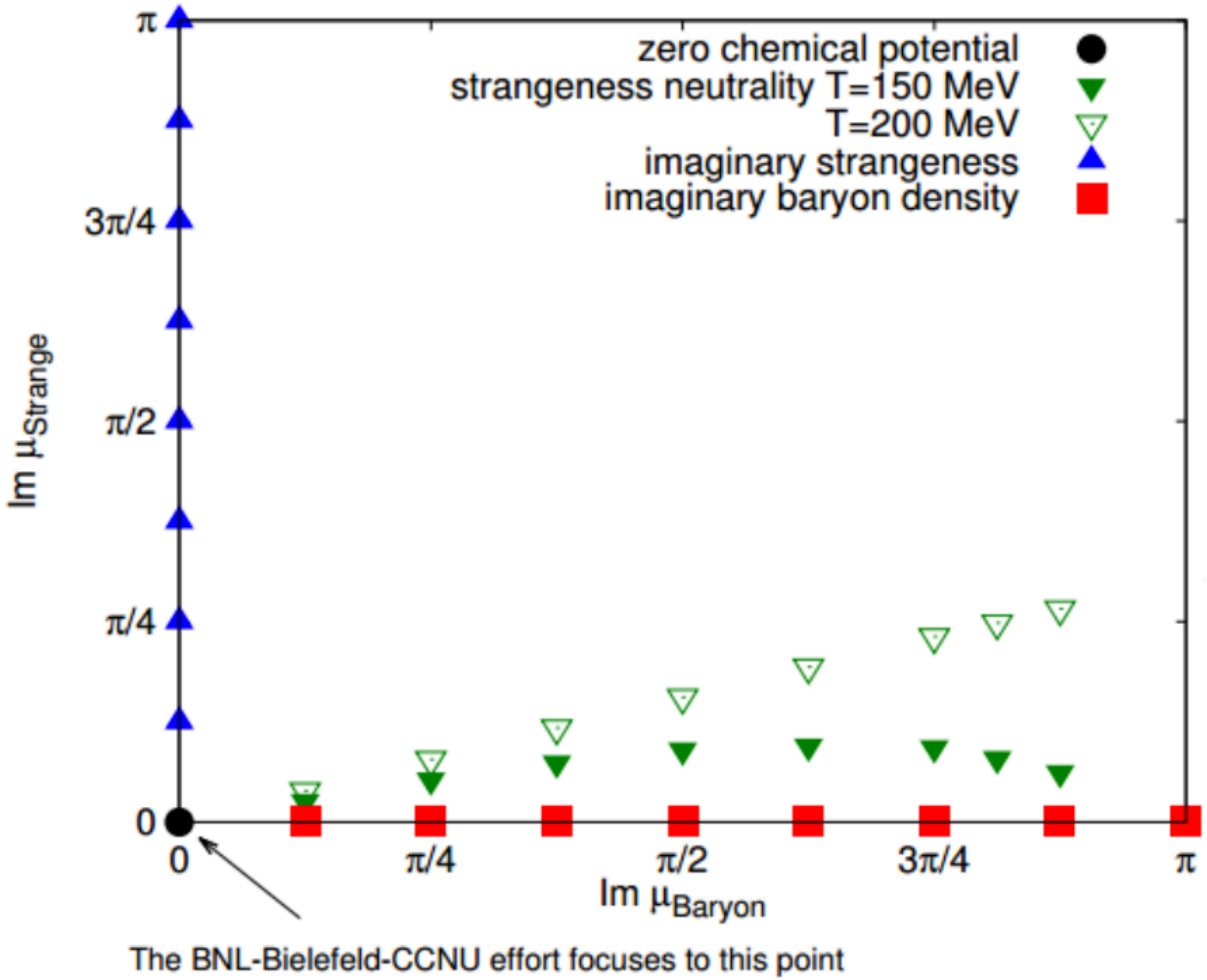


A. Bazavov et al., PRD (2017)

Analytical continuation



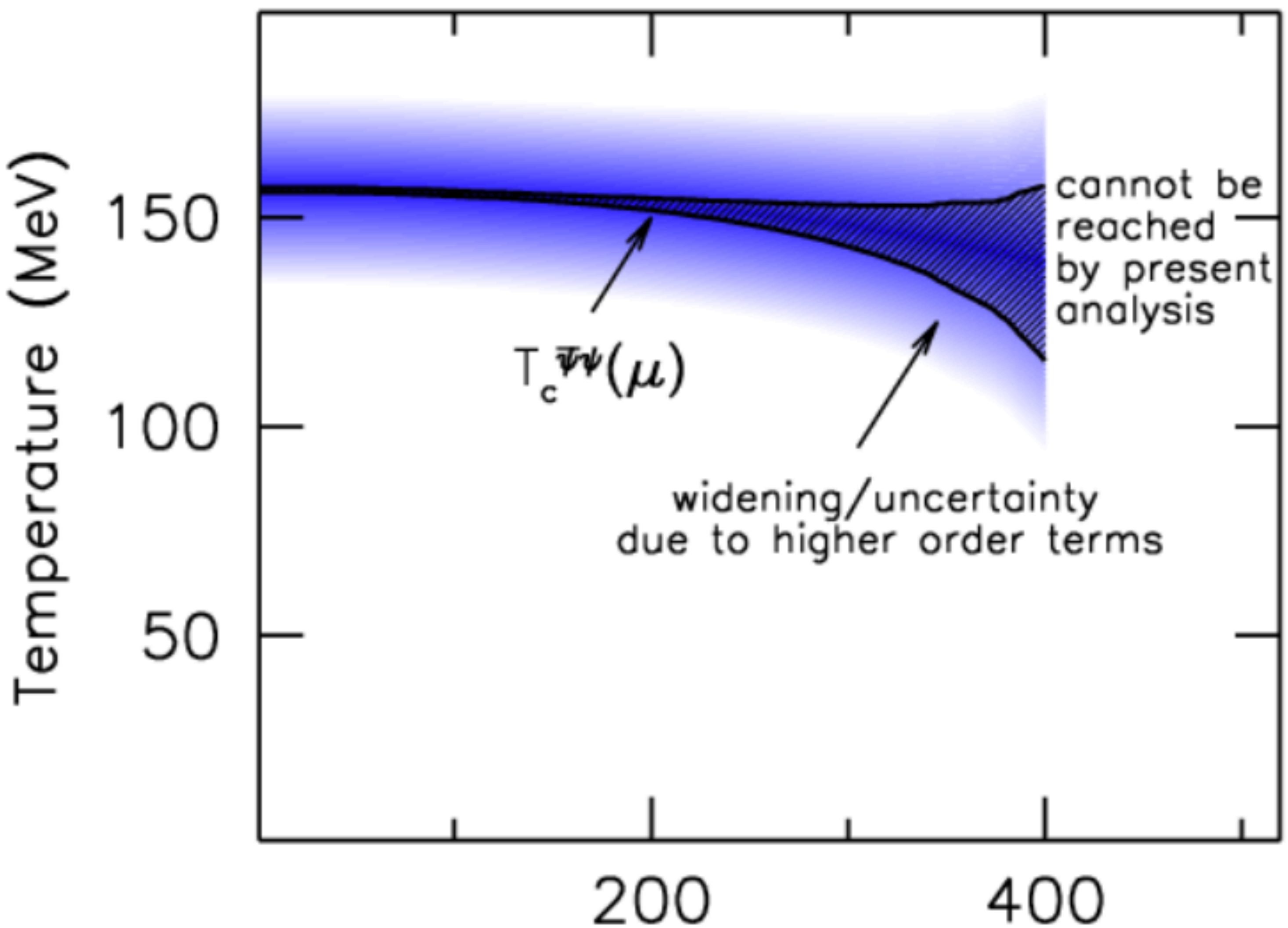
Simulation landscape



The crossover line from analytical continuation

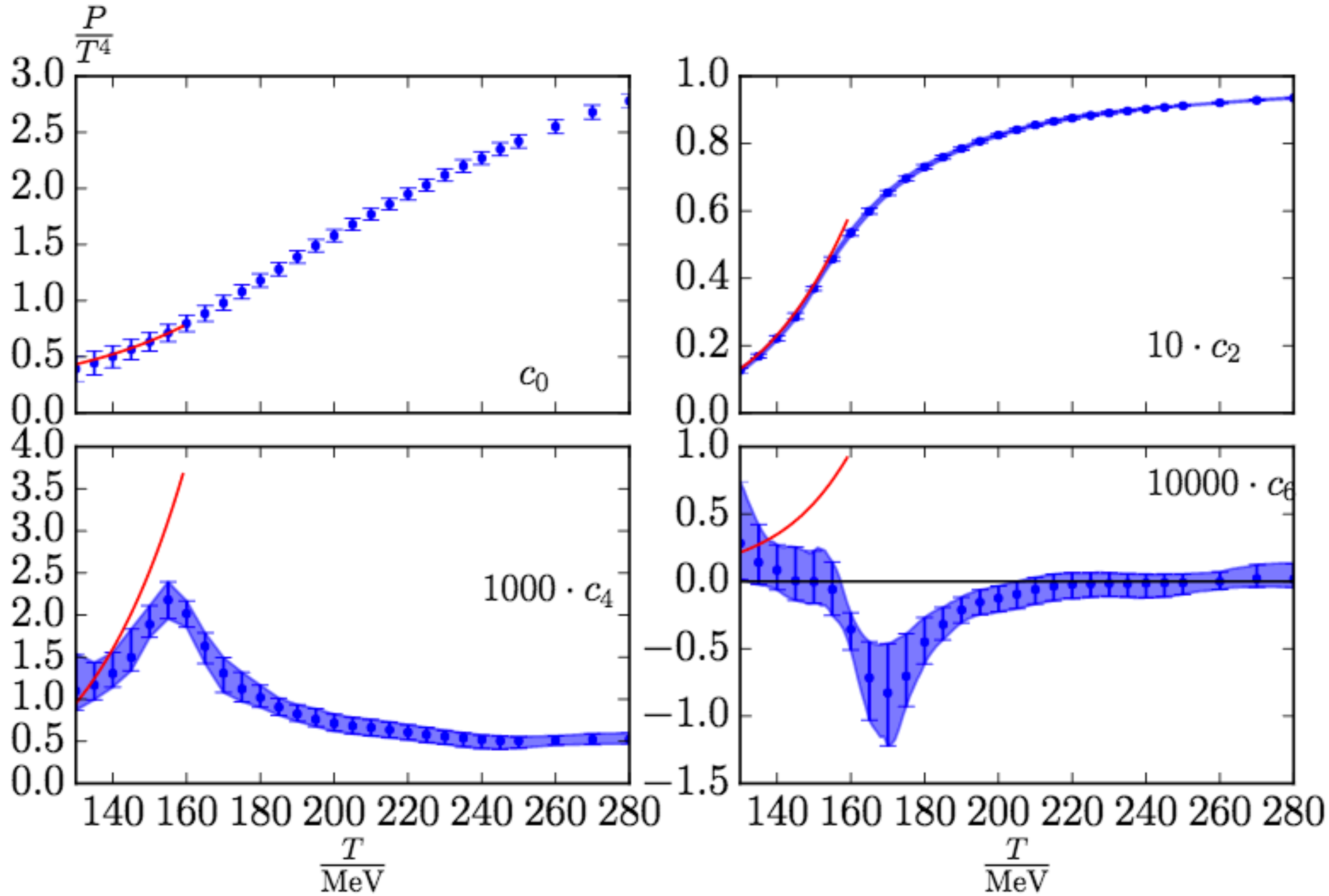
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa\,(\mu_B/T_c(\mu_B))^2 \qquad \kappa = 0.0149(21)$$

WB: hep-lat/1507.07510 μ_S and μ_Q from $\langle S\rangle = 0$ and $\langle Q\rangle = 0.5\,\langle B\rangle$



Equation of state from analytical continuation

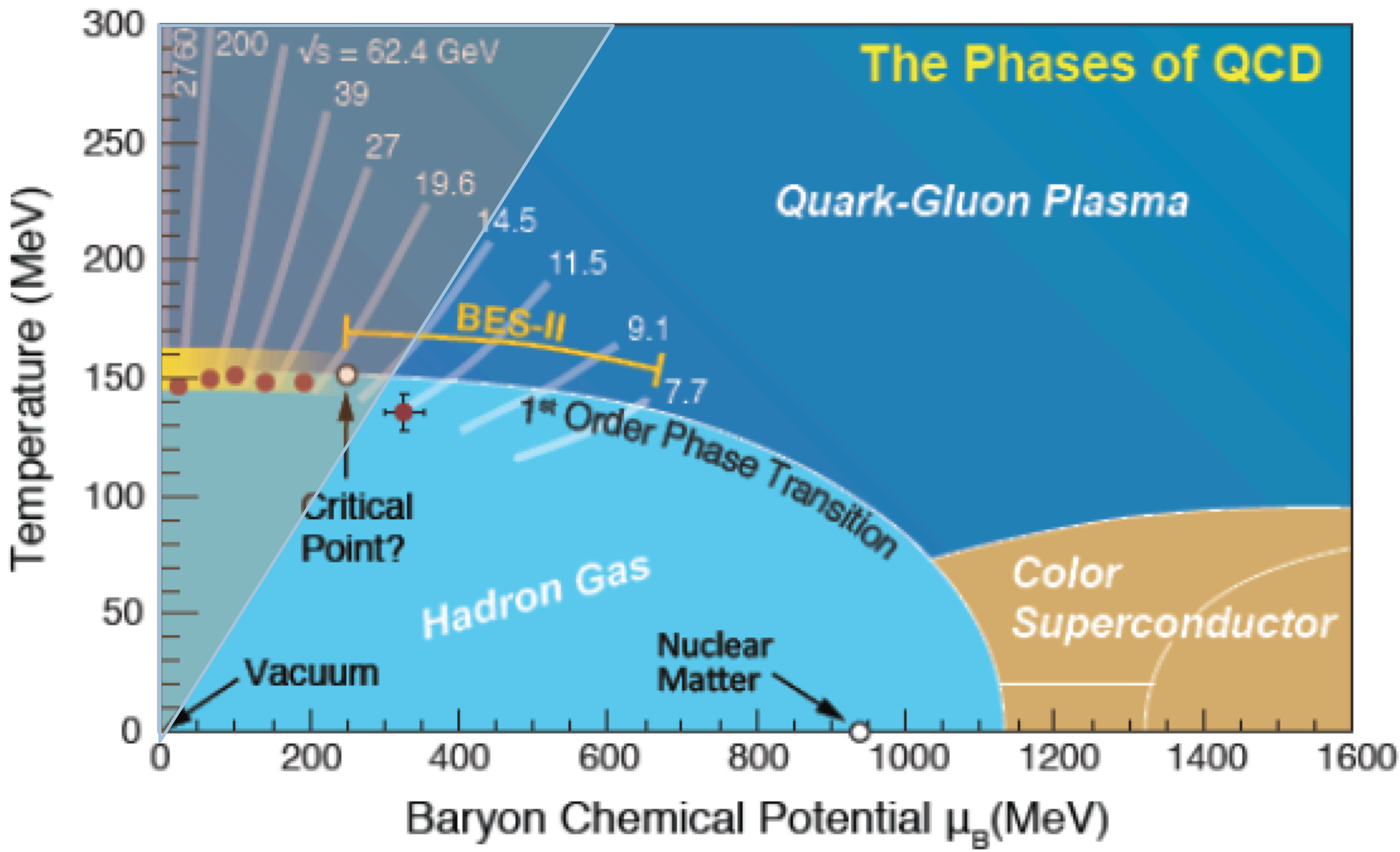
WB: hep-lat/1507.07510 μ_S and μ_Q from $\langle S \rangle = 0$ and $\langle Q \rangle = 0.4 \langle B \rangle$



It appears Taylor expansion is under control for $\mu_B/T \leq 2$. This is not so bad. It means it can be used for RHIC energies:

$$\sqrt{s} = 200,\ 62.4,\ 39,\ 27,\ 19.6,\ 14.5\mathrm{GeV}$$

Equation of state from analytical continuation



What happens below T_c?

- At low T and μ_B=0, QCD thermodynamics is dominated by pions

- as T increases, heavier hadrons start to contribute

- Their mutual interaction is suppressed:

$$n_i n_k \sim \exp[-(M_i + M_k)/T]$$

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting gas of hadronic resonances

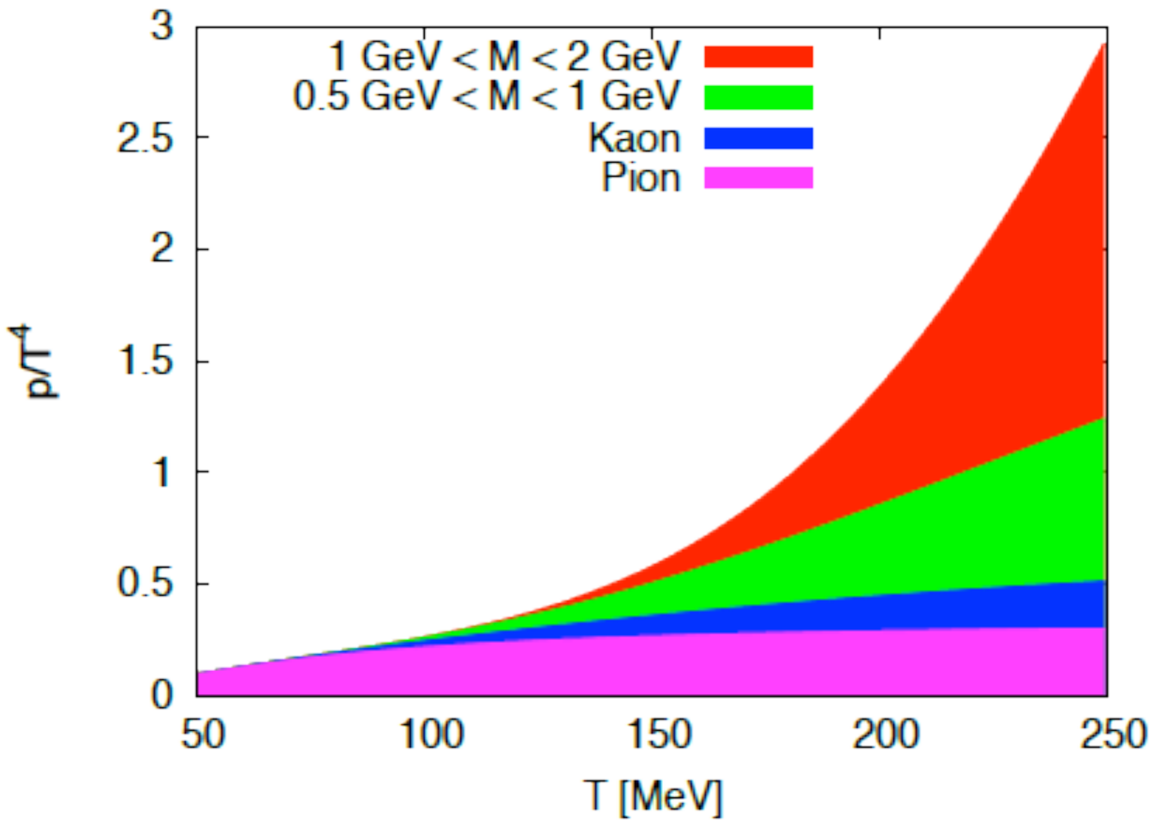
$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in\ mesons} \ln \mathcal{Z}^M_{m_i}(T, V, \mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in\ baryons} \ln \mathcal{Z}^B_{m_i}(T, V, \mu_{X^a}) ,$$

with $\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \qquad \varepsilon_i = \sqrt{k^2 + m_i^2}$

$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right)$ and X^a are all conserved charges.

R. Hagedorn, N. Cabibbo and G. Parisi

How many resonances do we include?

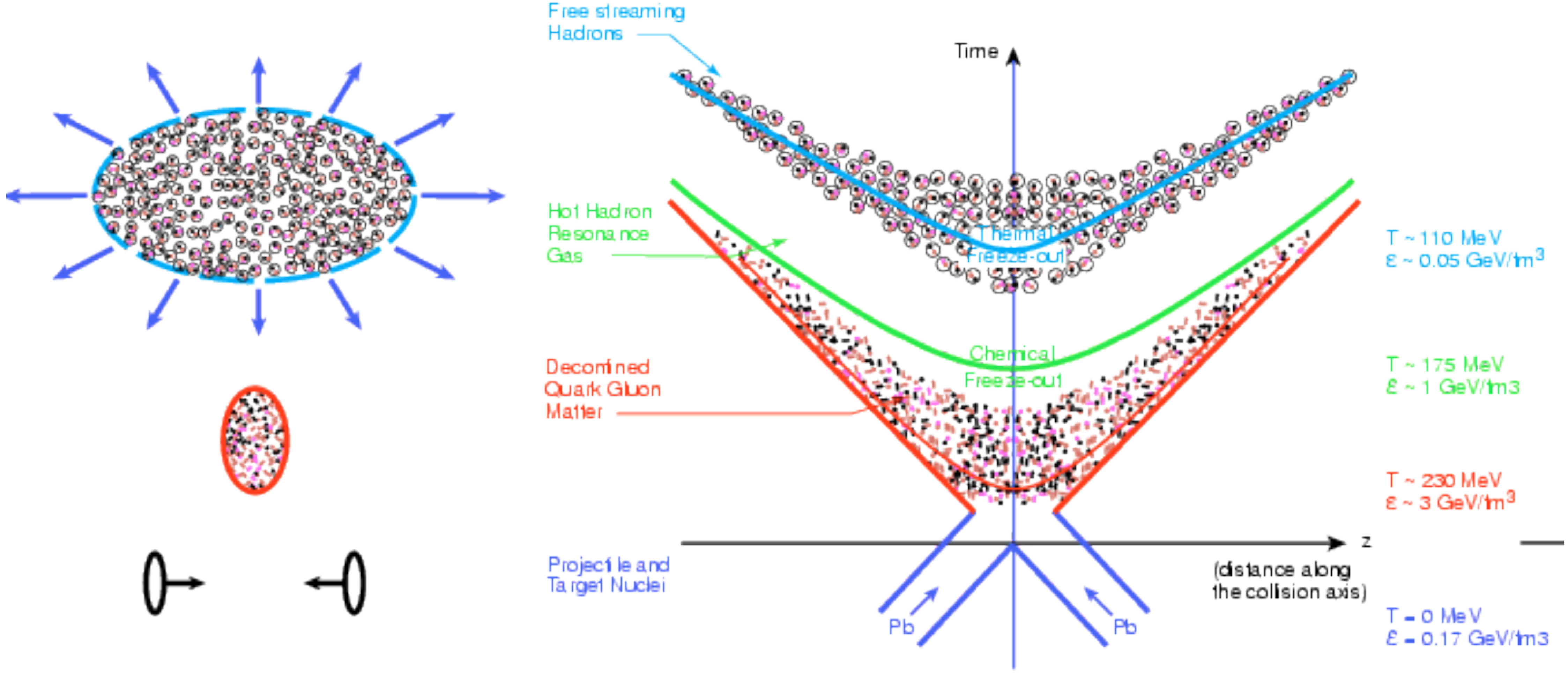


- With different mass cut-offs we can separate the contribution of different particles

- Known resonances up to M=2.5 GeV

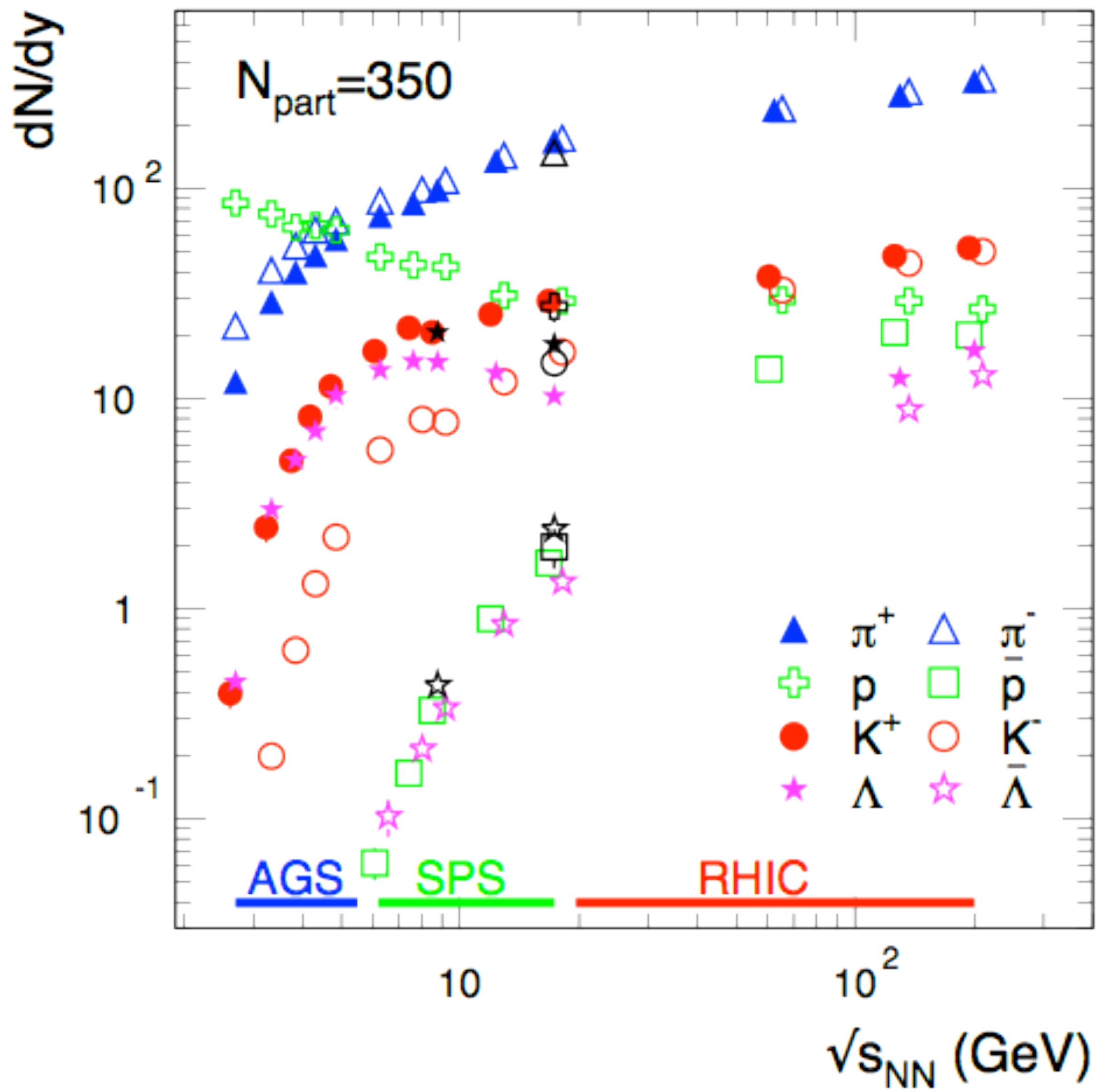
- ~170 different masses ⟷ 1500 resonances

Evolution of a heavy-ion collision



- **Chemical freeze-out**: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- **Kinetic freeze-out**: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Hadron yields

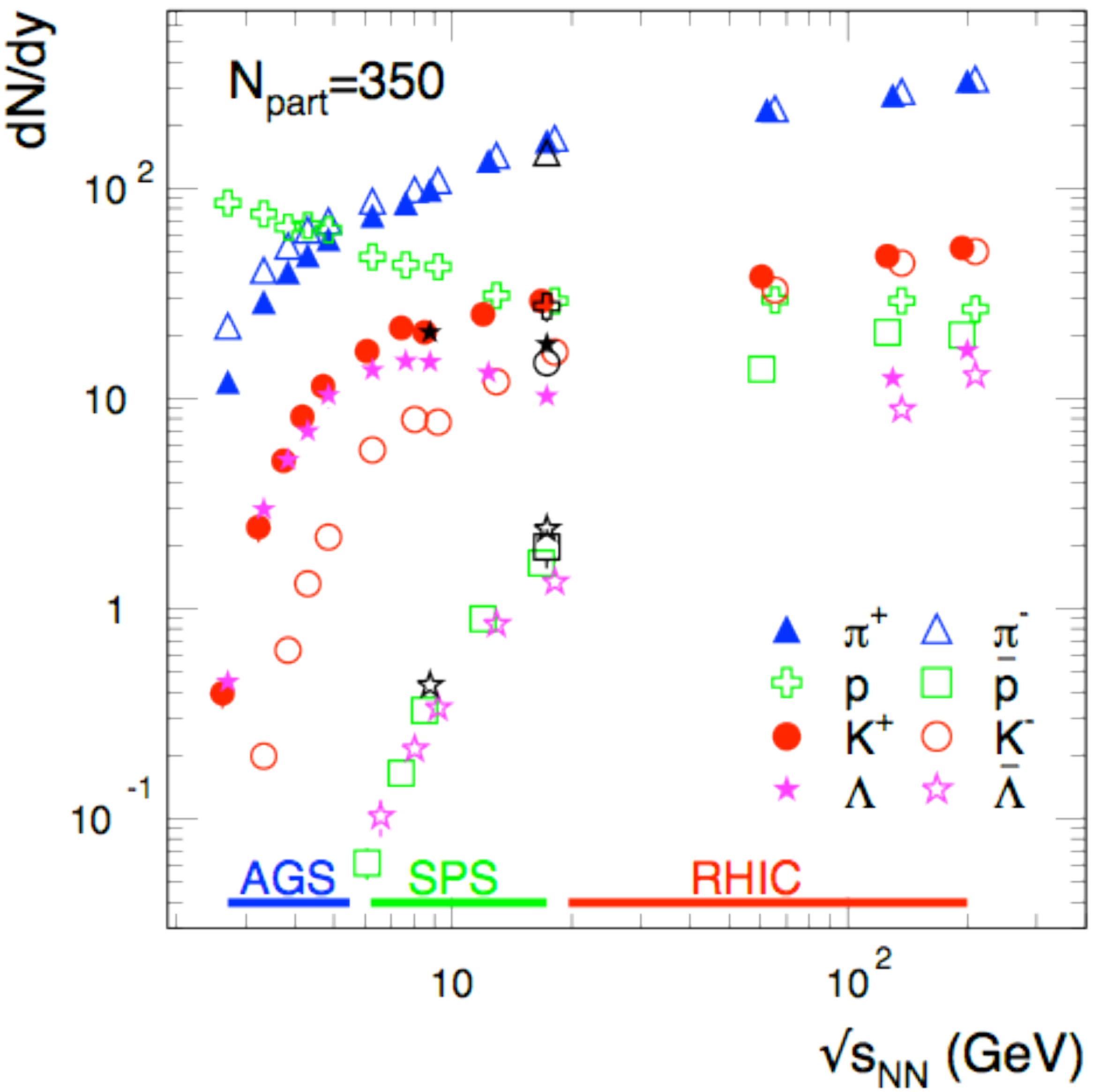


- E=mc²: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays

- HRG model: test hypothesis of hadron abundancies in equilibrium

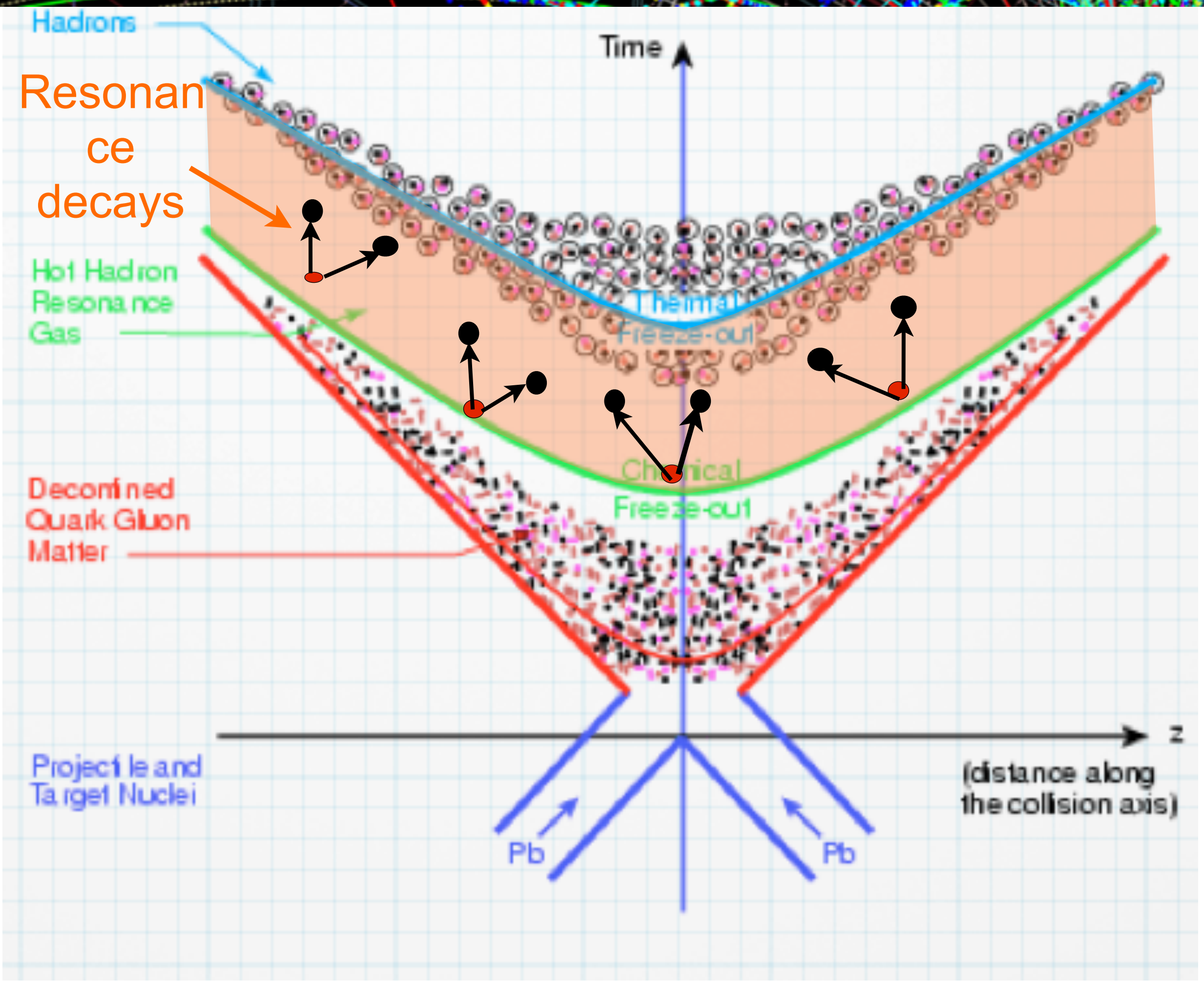
$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

Hadron yields



- E=mc^2: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays
- HRG model: test hypothesis of hadron abundancies in equilibrium
- We need:
  - a complete hadron spectrum
  - control the hadron fraction from decays

Decays



- Most hadrons are subject to strong and electromagnetic decays

$$\Delta \to p(n) + \pi,\ \rho \to \pi + \pi$$

- e.g. pions: 1/4 primordial, 3/4 from strong decays
- Weak decays can be treated too:

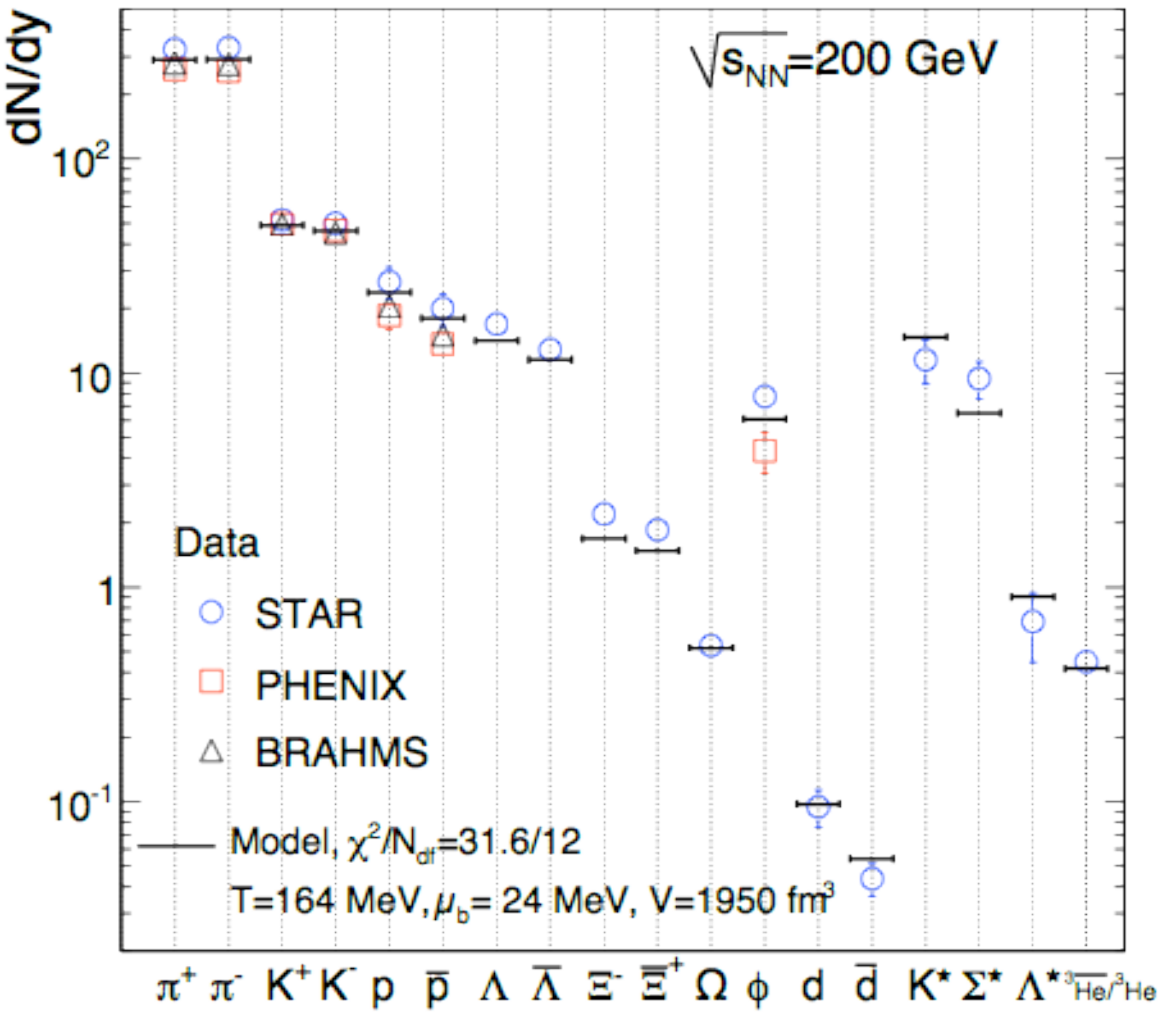
$$\Sigma \to \Lambda + \gamma$$

- after chemical freeze-out: only elastic and quasi-elastic scatterings take place:

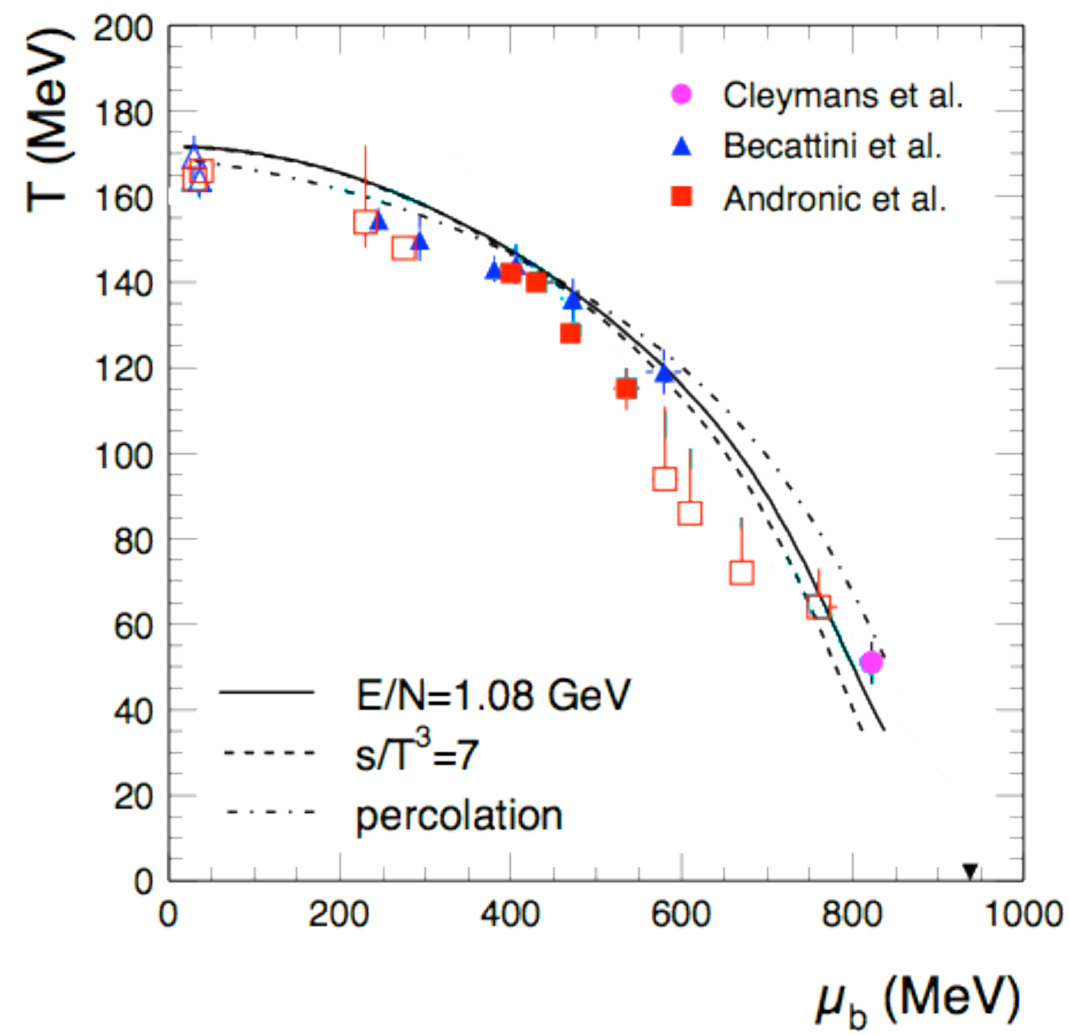
$$\pi\pi \to \rho \to \pi\pi \qquad p\pi \to \Delta \to p\pi \qquad K\pi \to K^* \to K\pi$$

$$\bar{N}_i = N_i + \sum_r d_{r\to i} N_r$$

The thermal fits



- Fit is performed minimizing the χ^2
- Fit to yields: parameters T, μ_B, V
- Fit to ratios: the volume V cancels out



- Changing the collision energy, it is possible to draw the freeze-out line in the T, μ_B plane

Cleymans et al, Becattini et al, Andronic et al.

Caveats

- These results are model-dependent
  - they depend on the particle spectrum which is included in the model
  - possibility of having heavier states with exponential mass spectrum
  - not known experimentally but can be postulated
  - their decay modes are not known

Caveats

- These results are model-dependent
  - they depend on the particle spectrum which is included in the model
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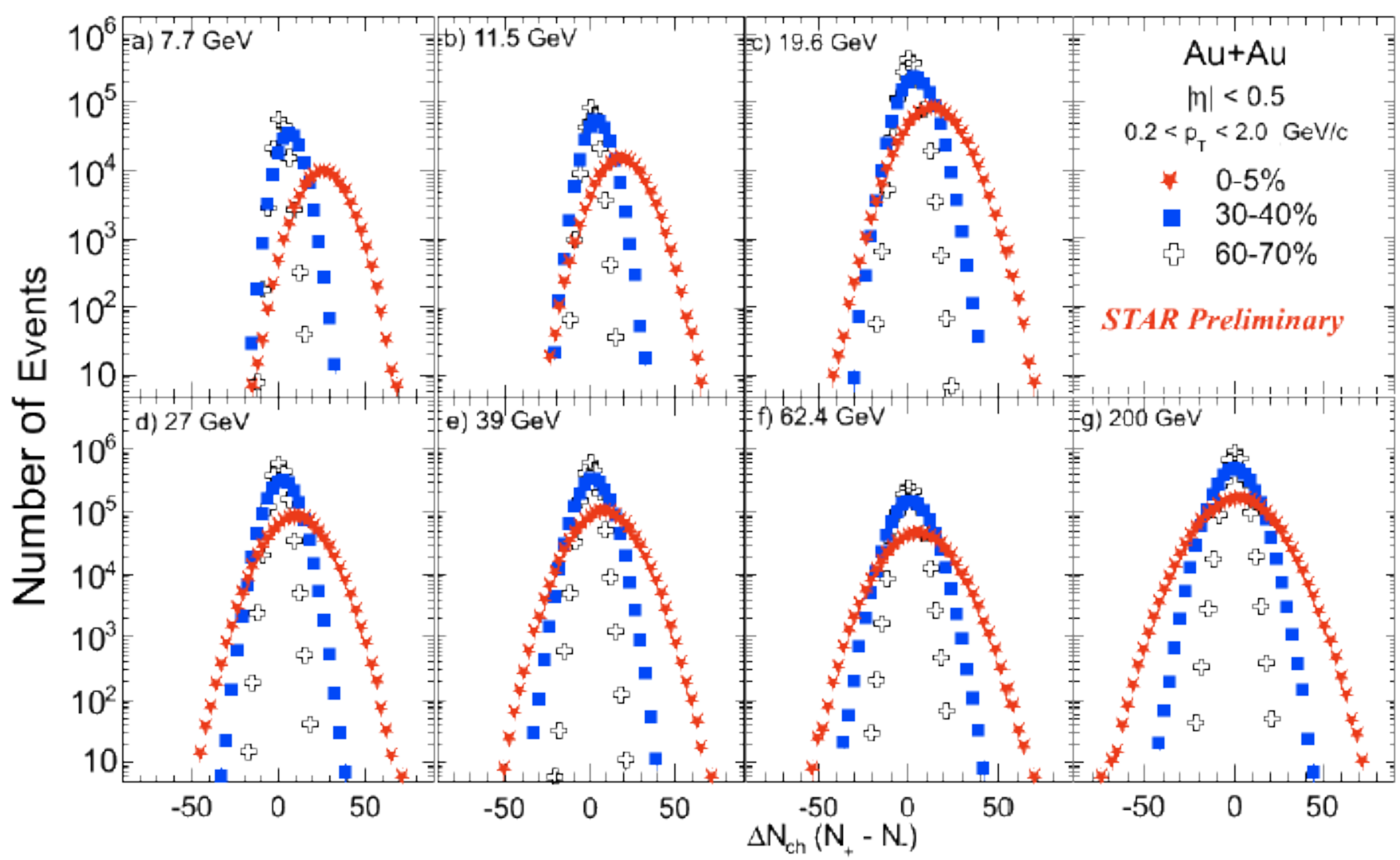
  - not known experimentally but can be postulated
  - their decay modes are not known
- Purpose: extract freeze-out parameters from first principles
  - direct comparison between experimental measurement and lattice QCD results
  - observable: fluctuations of conserved charges (electric charge, baryon number and strangeness)
  - directly related to moments of multiplicity distribution (measured)
  - lattice QCD looks at conserved charges rather than identified particles

Fluctuations of conserved charges

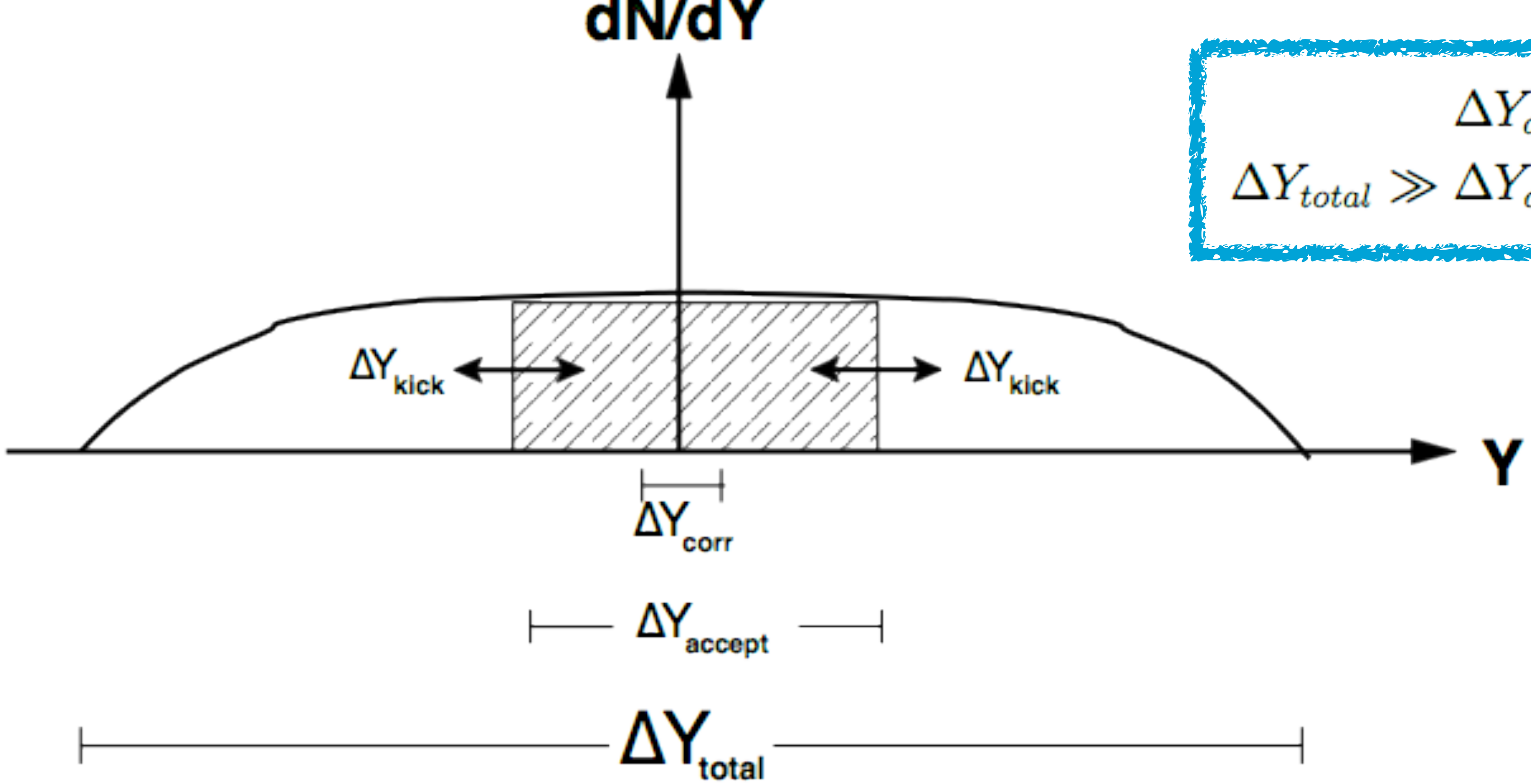
- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to **measure its event-by-event distribution**



STAR Collab.: 1402.1558

Fluctuations of conserved charges???

- If we look at the entire system, none of the conserved charges will fluctuate
- By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{accept} \gg \Delta Y_{corr}$$
$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

- **ΔY_{total}: range for total charge multiplicity distribution**
- **ΔY_{accept}: interval for the accepted charged particles**
- **ΔY_{corr}: charge correlation length characteristic to the physics of interest**
- **ΔY_{kick}: rapidity shift that charges receive during and after hadronization**

V. Koch: 0810.2520

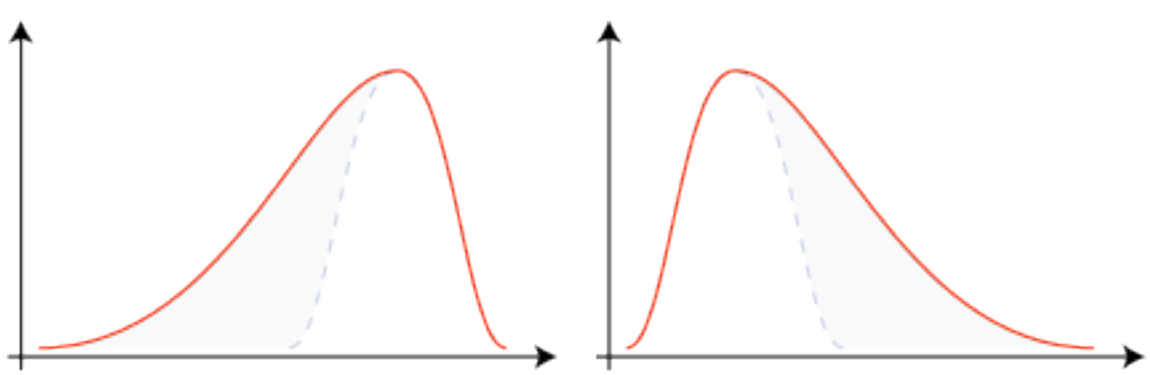
Cumulants of multiplicity distribution

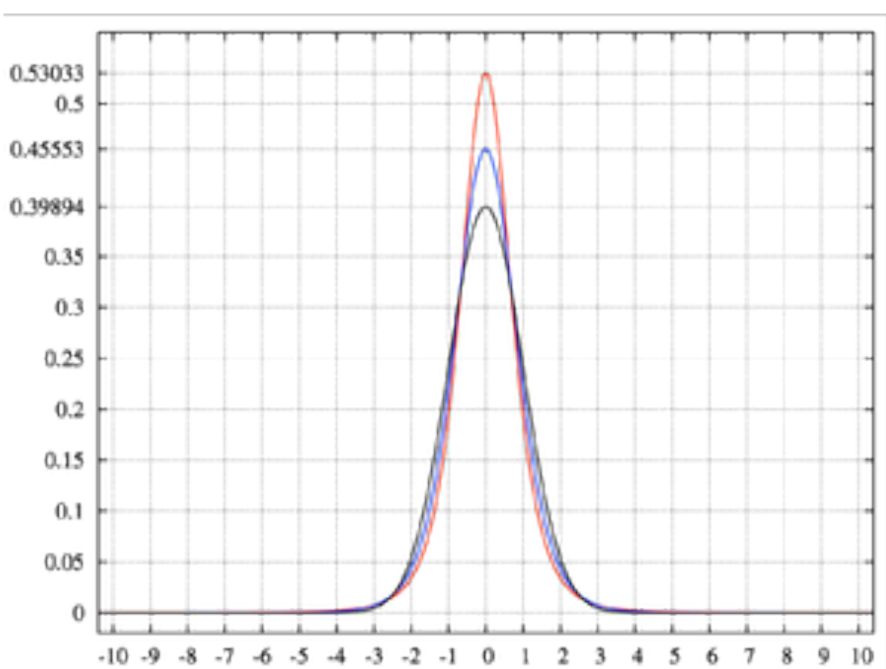
- Deviation of N_Q from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$
- The cumulants of the event-by-event distribution of N_Q are:

$$\kappa_2=\langle(\delta N_Q)^2\rangle \qquad \kappa_3=\langle(\delta N_Q)^3\rangle \qquad \kappa_4=\langle(\delta N_Q)^4\rangle-3\langle(\delta N_Q)^2\rangle^2$$

- The cumulants are related to the central moments of the distribution by:

variance: $\sigma^2=\kappa_2$ *Skewness:* $S=\kappa_3/(\kappa_2)^{3/2}$ *Kurtosis:* $\kappa=\kappa_4/(\kappa_2)^2$

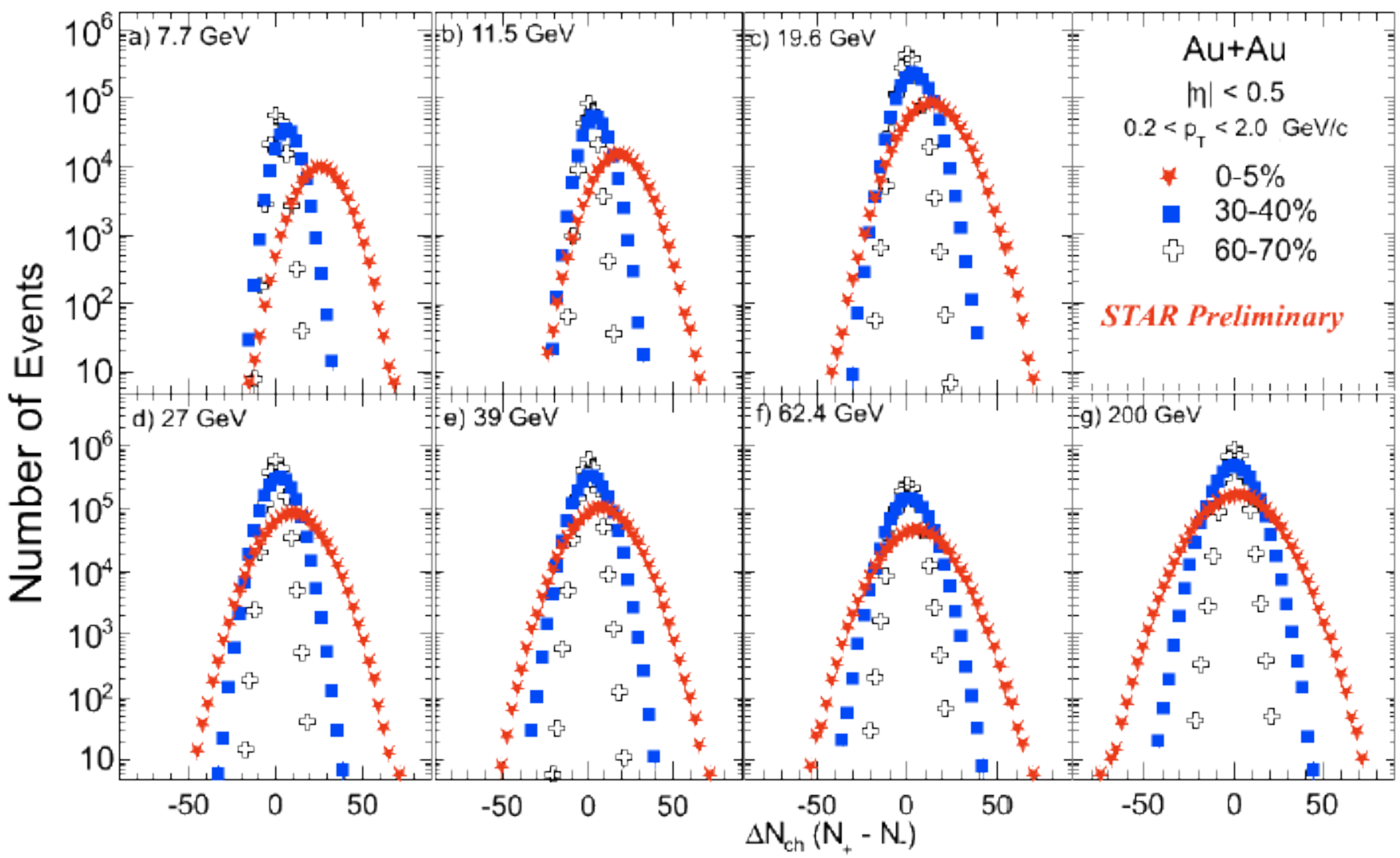




Experimental measurement

* Volume-independent ratios:

$M/\sigma^2 = K_1/K_2 \quad S\sigma = K_3/K_2 \quad \kappa\sigma^2 = K_4/K_2 \quad S\sigma^3/M = K_3/K_1$



STAR Collab.: 1402.1558

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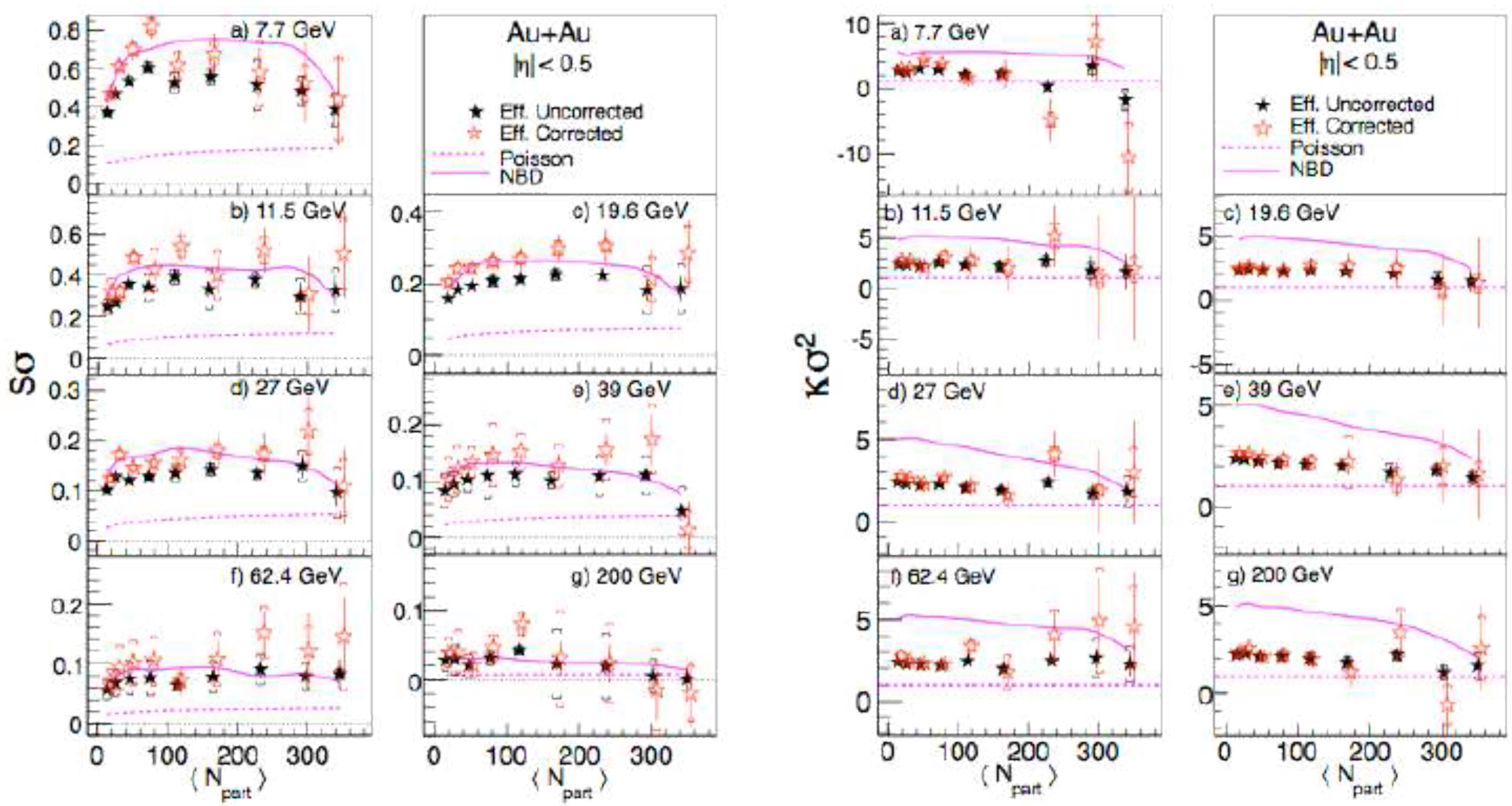


STAR Collab.: 1402.1558

Susceptibilities of conserved charges

- Susceptibilities of conserved charges

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Susceptibilities of conserved charges are the cumulants of their event-by event distribution

$$\text{mean}: \; M = \chi_1 \qquad \text{variance}: \; \sigma^2 = \chi_2$$

$$\text{skewness}: \; S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \; \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

- Lattice QCD results are functions of temperature and chemical potential
  - ➡ By comparing lattice results and experimental measurement we can extract the freeze-out parameters from first principles

F. Karsch: Centr. Eur. J. Phys. (2012)

Baryometer and thermometer

* Let us look at the Taylor expansion of R^B_{31}

$$R^B_{31}(T,\mu_B) = \frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)} = \frac{\chi^B_4(T,0) + \chi^{BQ}_{31}(T,0)q_1(T) + \chi^{BS}_{31}(T,0)s_1(T)}{\chi^B_2(T,0) + \chi^{BQ}_{11}(T,0)q_1(T) + \chi^{BS}_{11}(T,0)s_1(T)} + \mathcal{O}(\mu^2_B)$$

* To order μ^2_B it is independent of μ_B: it can be used as a thermometer

* Let us look at the Taylor expansion of R^B_{12}

$$R^B_{12}(T,\mu_B) = \frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)} = \frac{\chi^B_2(T,0) + \chi^{BQ}_{11}(T,0)q_1(T) + \chi^{BS}_{11}(T,0)s_1(T)}{\chi^B_2(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu^3_B)$$

* Once we extract T from R^B_{31}, we can use R^B_{12} to extract μ_B

Caveats

* Effects due to volume variation because of finite centrality bin width
* Finite reconstruction efficiency
* Spallation protons
* Canonical vs Gran Canonical ensemble
* Proton multiplicity distributions vs baryon number fluctuations
* Final-state interactions in the hadronic phase

Caveats

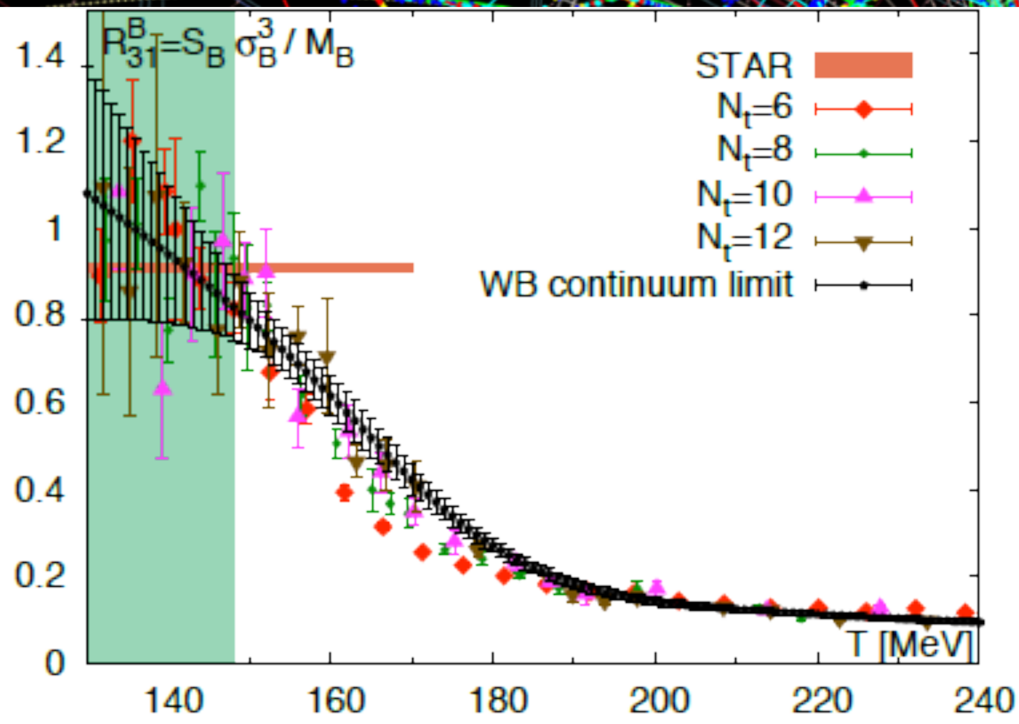
- Effects due to volume variation because of finite centrality bin width
  - ➡ Experimentally corrected by centrality-bin-width correction method
- Finite reconstruction efficiency
  - ➡ Experimentally corrected based on binomial distribution

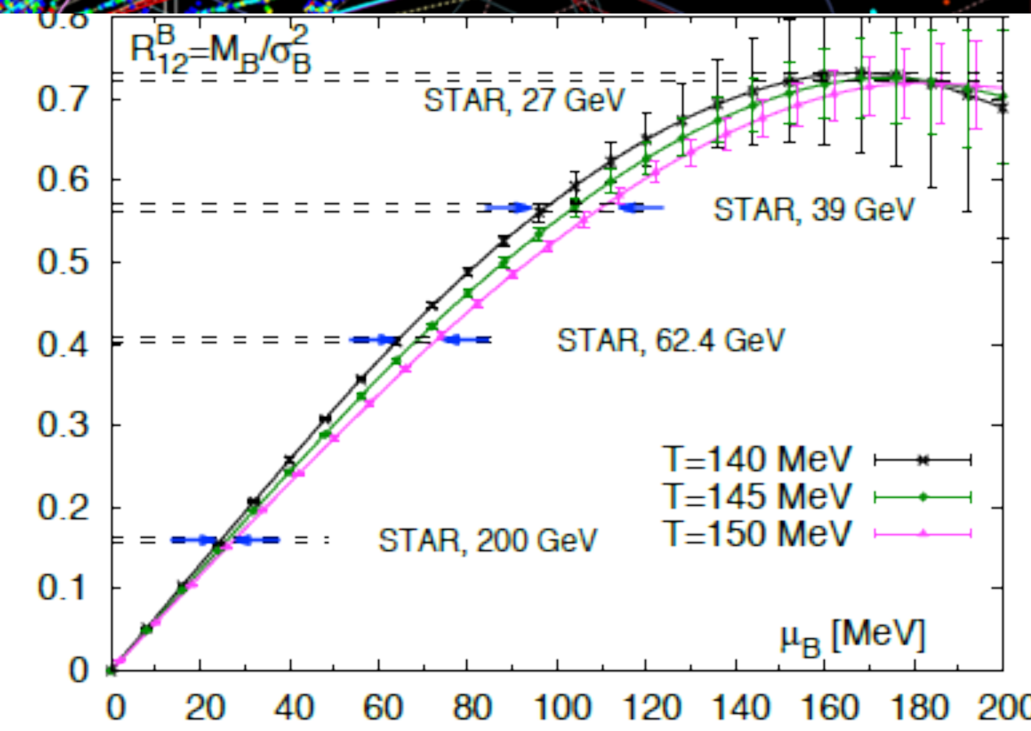
    A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - ➡ Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - ➡ Experimental cuts in the kinematics and acceptance   V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - ➡ Numerically very similar once protons are properly treated

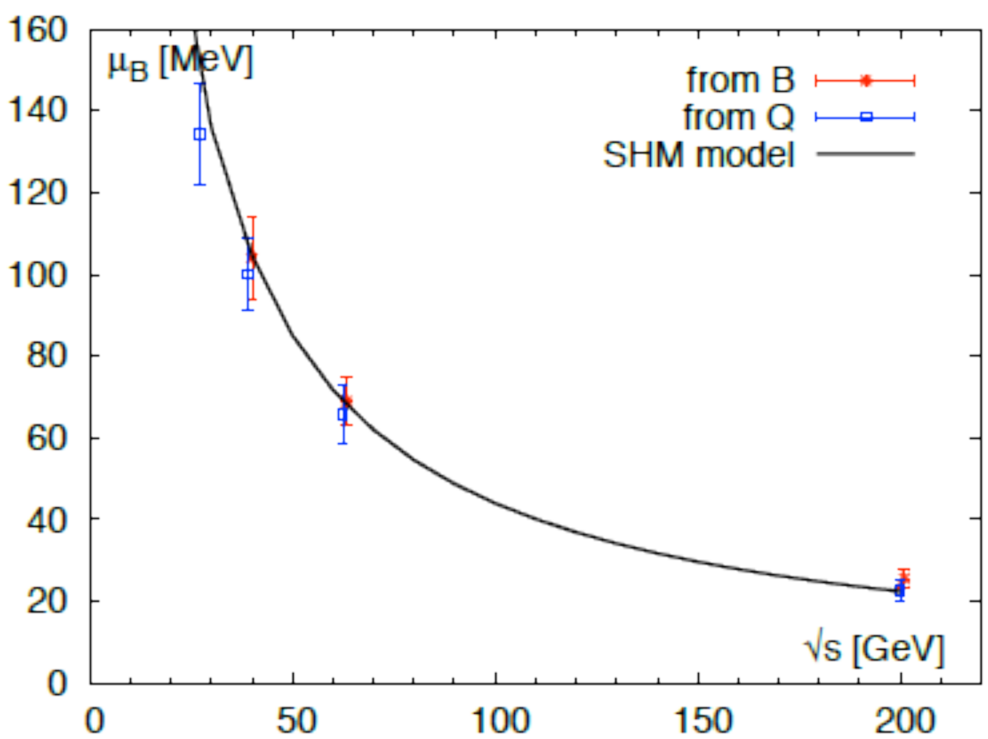
    M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238
- Final-state interactions in the hadronic phase

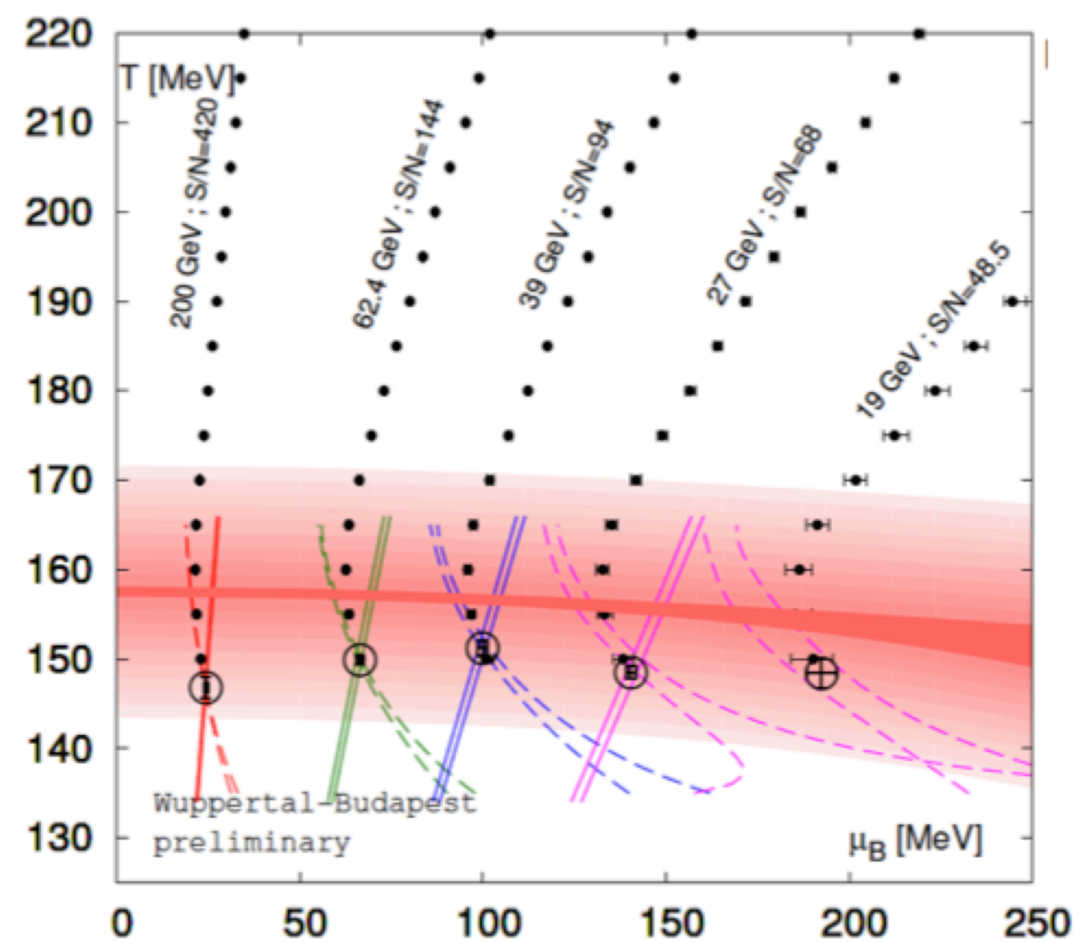
  J.Steinheimer et al., PRL (2013)
  - ➡ Consistency between different charges = fundamental test

Results





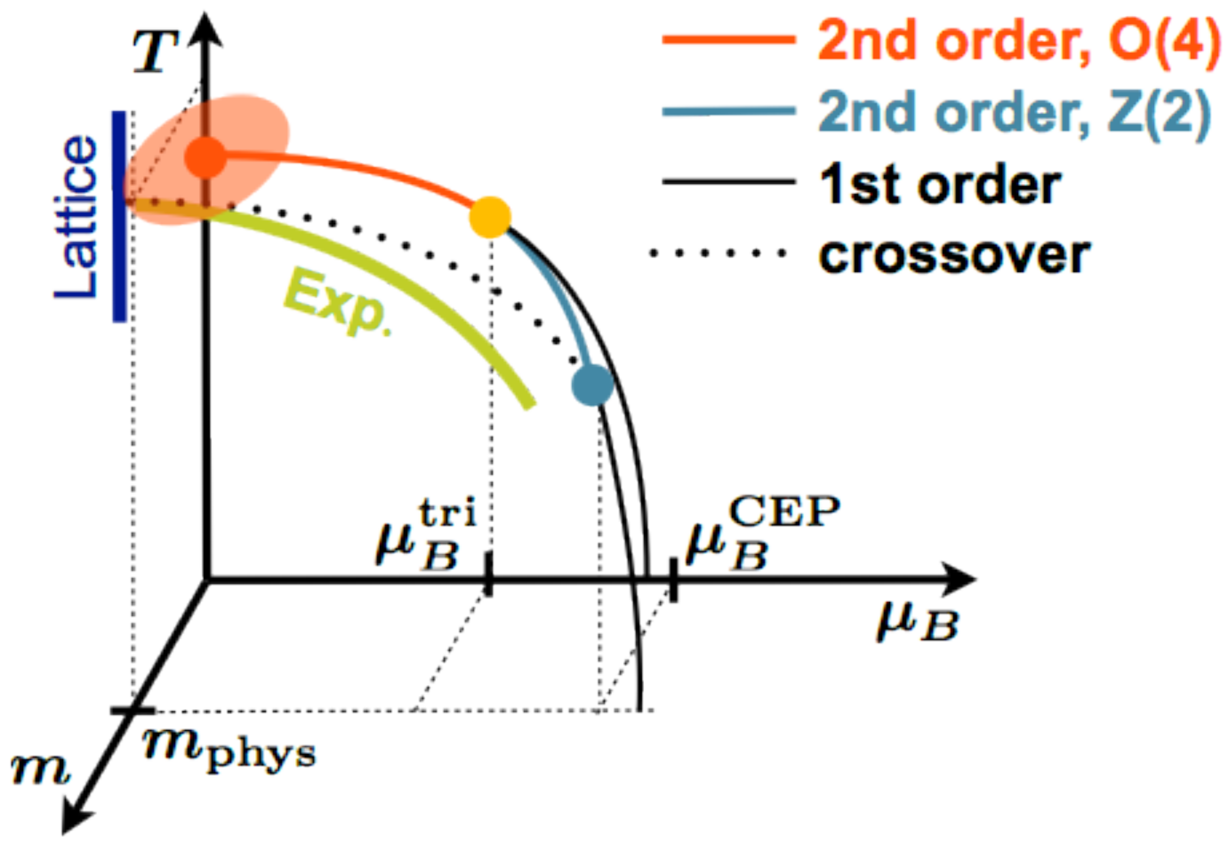




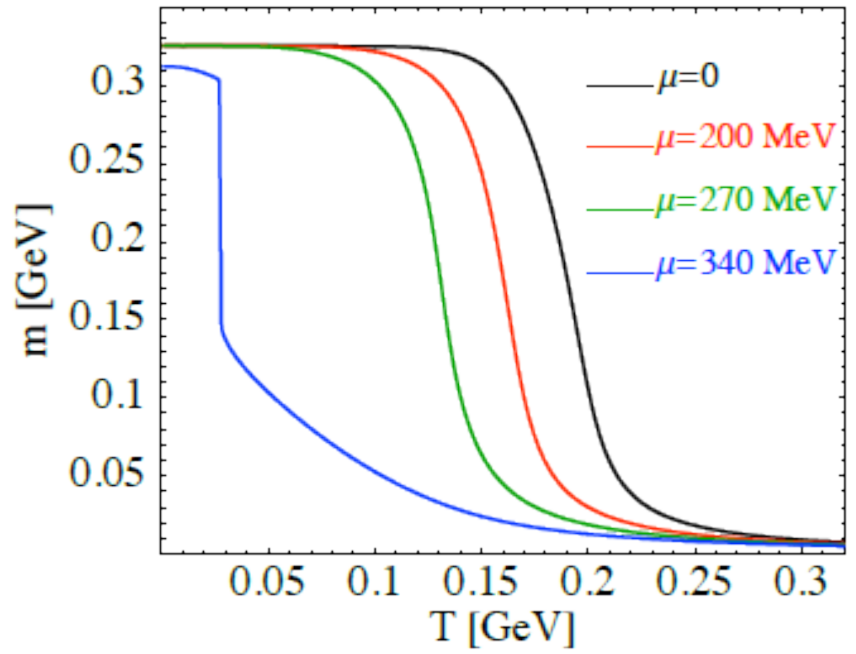
WB Collab.: PRL (2014)

Our world is not ideal:

neither chiral symmetry (m_q=0) nor confinement (m_q=∞) is well defined.



Existence of QCD critical point predicted by models



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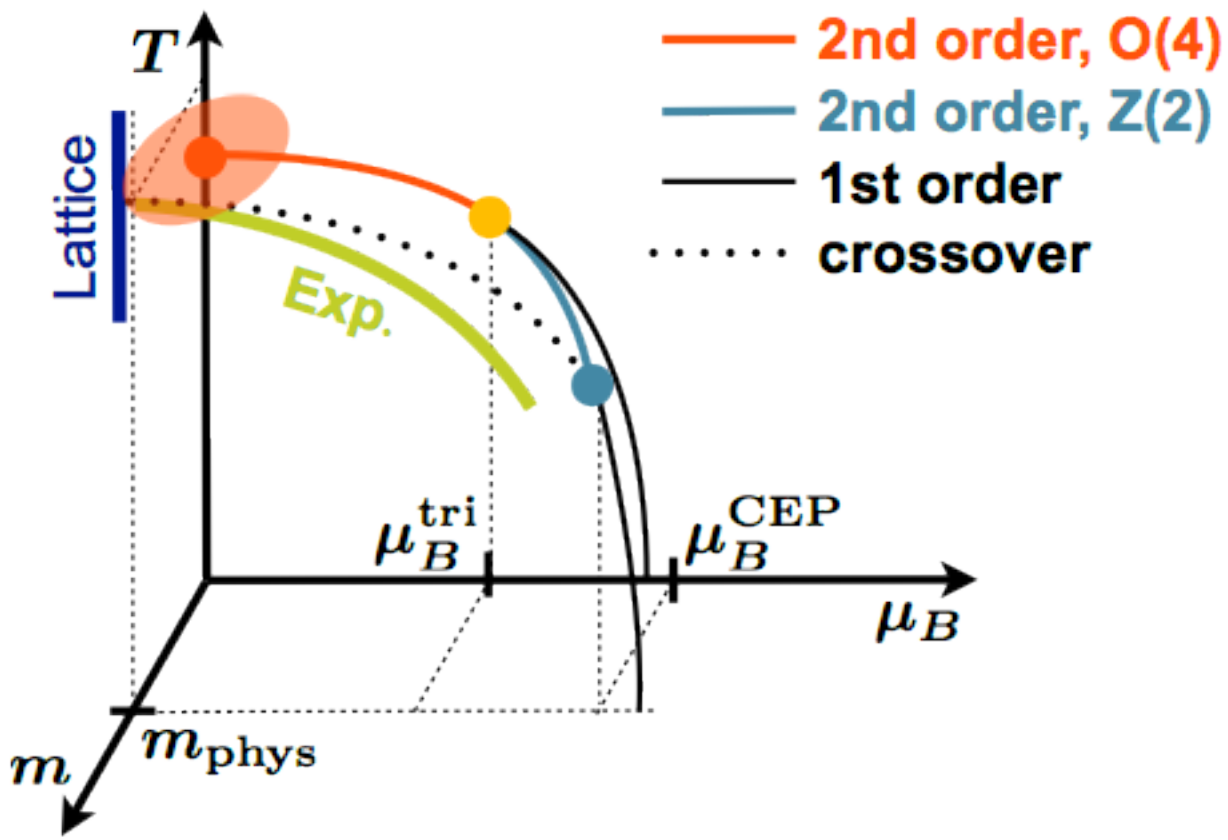

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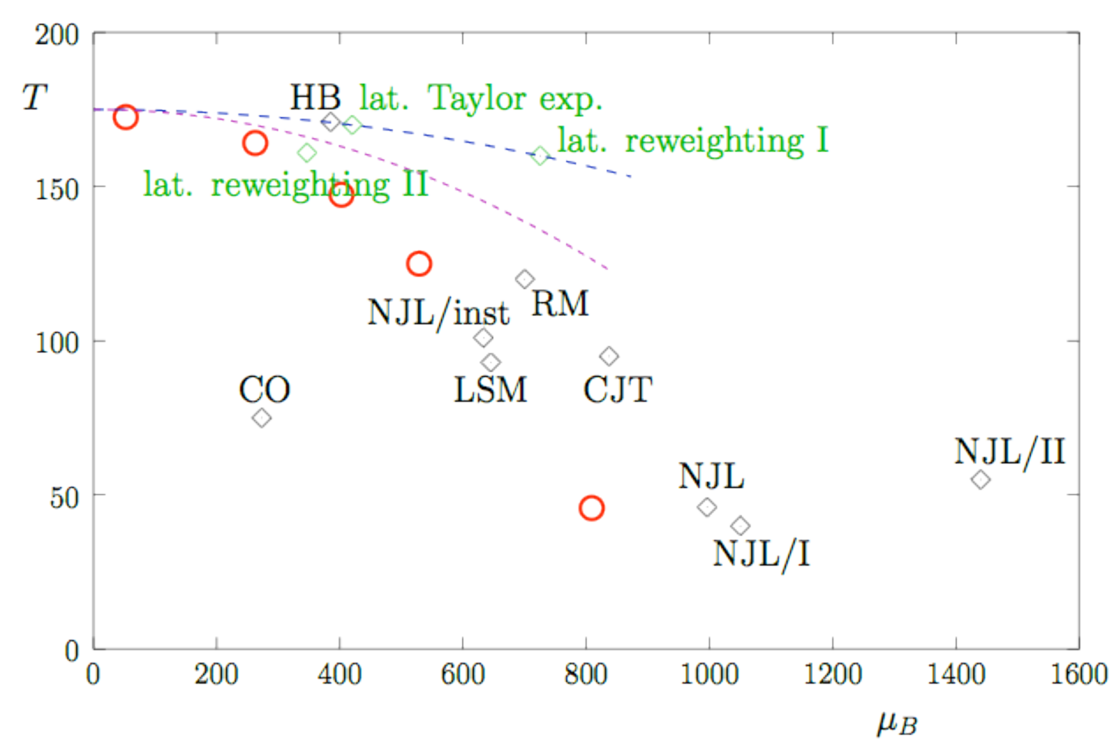
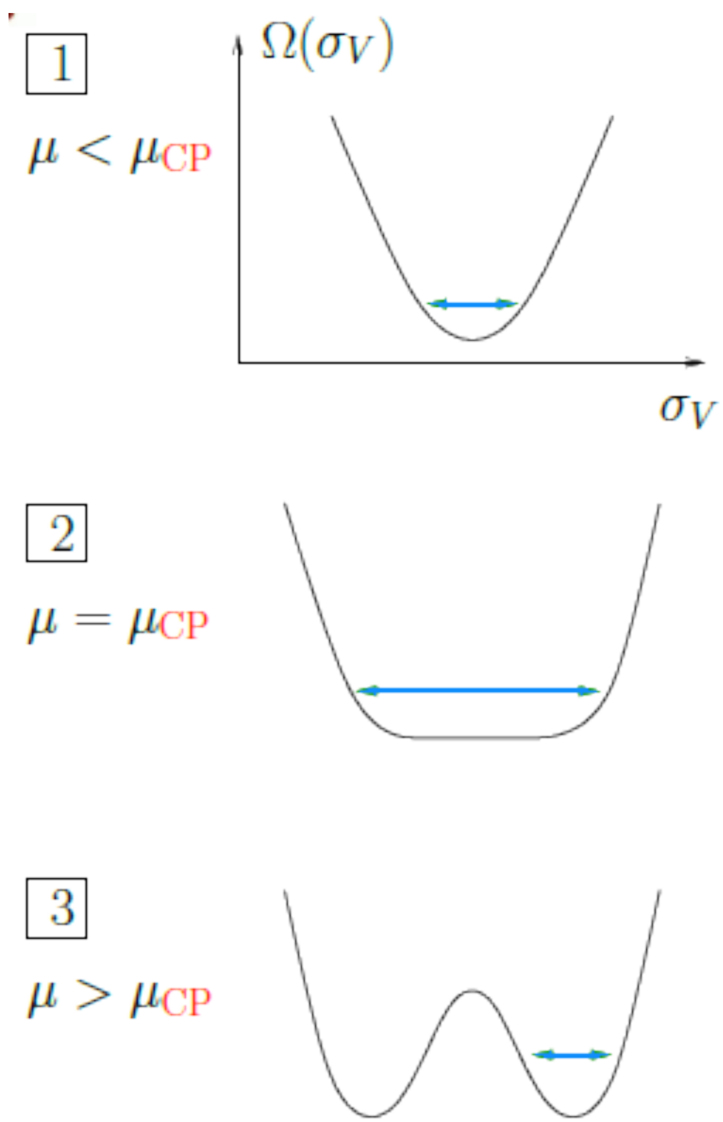


Fluctuations at the critical point



Consider the order parameter for the chiral phase transition σ ~< $\bar{\psi}\psi$ >

It has a probability distribution of the form:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right].$$

where: $m_\sigma \equiv \xi^{-1}$

and, near the critical point, ξ->∞:

$$\lambda_3 = \tilde{\lambda}_3 \, T \, (T\,\xi)^{-3/2}, \quad \text{and} \quad \lambda_4 = \tilde{\lambda}_4 \, (T\,\xi)^{-1}$$

$\kappa_3 = \langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}$ $\kappa_4 = \langle \sigma_V^4 \rangle = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7$

M. Stephanov (2009)

correlation length ξ is limited due to critical slowing down, together with the finite time the system has to develop the correlations: ξ<2-3 fm

Berdnikov-Rajagopal

Experimental fluctuations

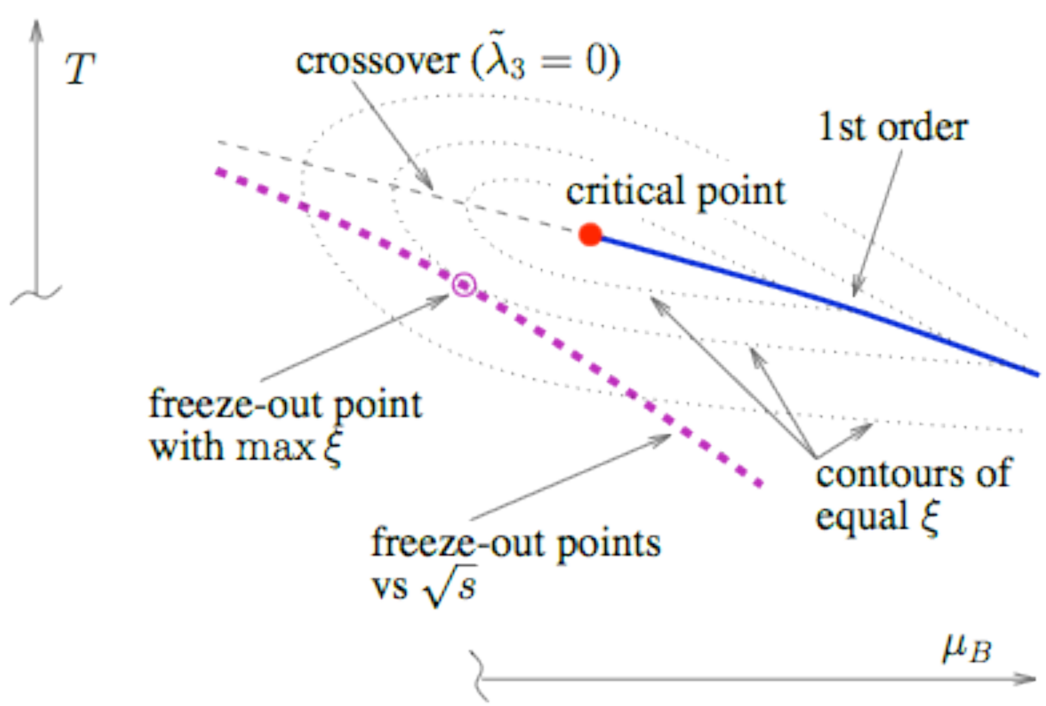
We consider the fluctuation of an observable (e.g. proton multiplicity)

$$\delta N = \sum_{\boldsymbol{p}} \delta n_{\boldsymbol{p}}$$

At the critical point, it receives both a regular and a singular contribution. The latter comes from the coupling to the σ field:

$$\delta n_{\boldsymbol{p}} = \underbrace{\delta n^0_{\boldsymbol{p}}}_{\text{statistical}} + \underbrace{\frac{\partial \bar{n}_{\boldsymbol{p}}}{\partial m} g\, \delta\sigma}_{\text{critical}}$$

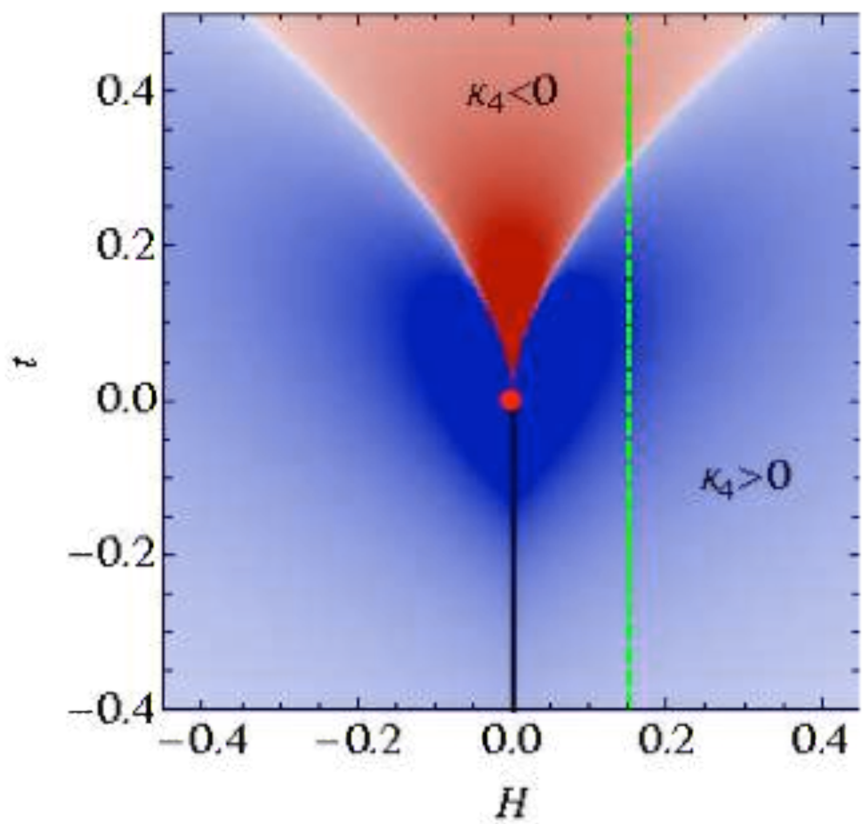
(Poisson)



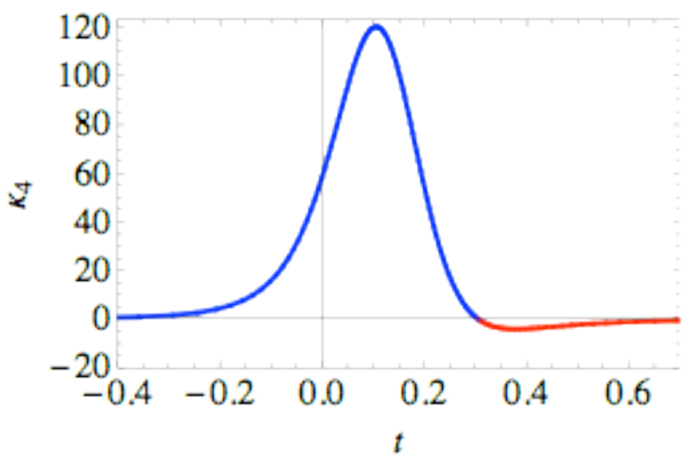
Higher order moments have stronger dependence on ξ: they are more sensitive signatures for the critical point

M. Stephanov

Sign of kurtosis



(a)



M. Stephanov (2011)

The 4th order cumulant becomes negative when the critical point is approached from the crossover side: from Ising model:

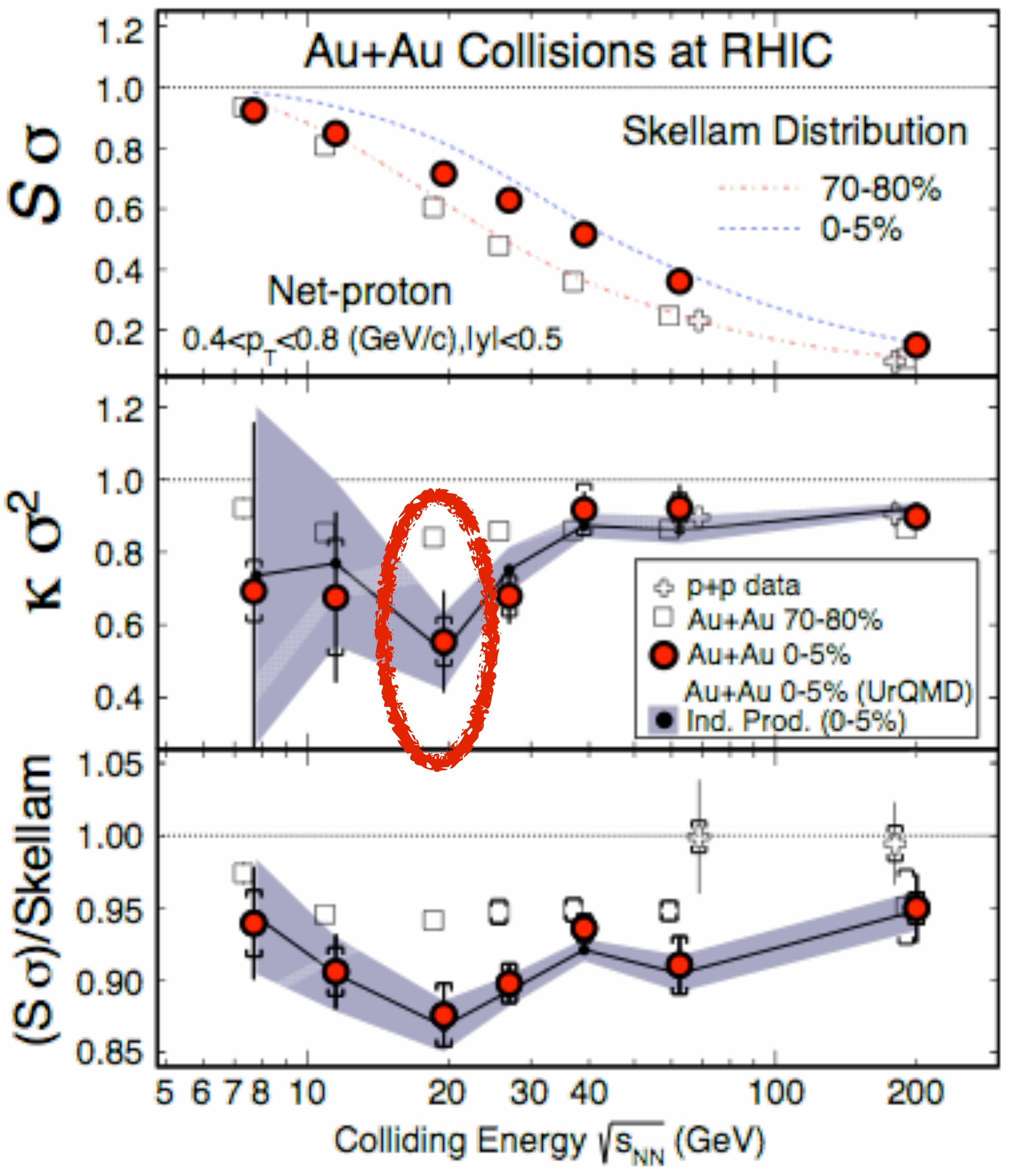
$M=R^{\beta}\theta$, $t=R(1-\theta^2)$, $H=R^{\beta\delta}h(\theta)$

$\kappa_4=\langle M^4\rangle$ $(t,H)\text{->}(\mu-\mu_{CP}, T-T_{CP})$

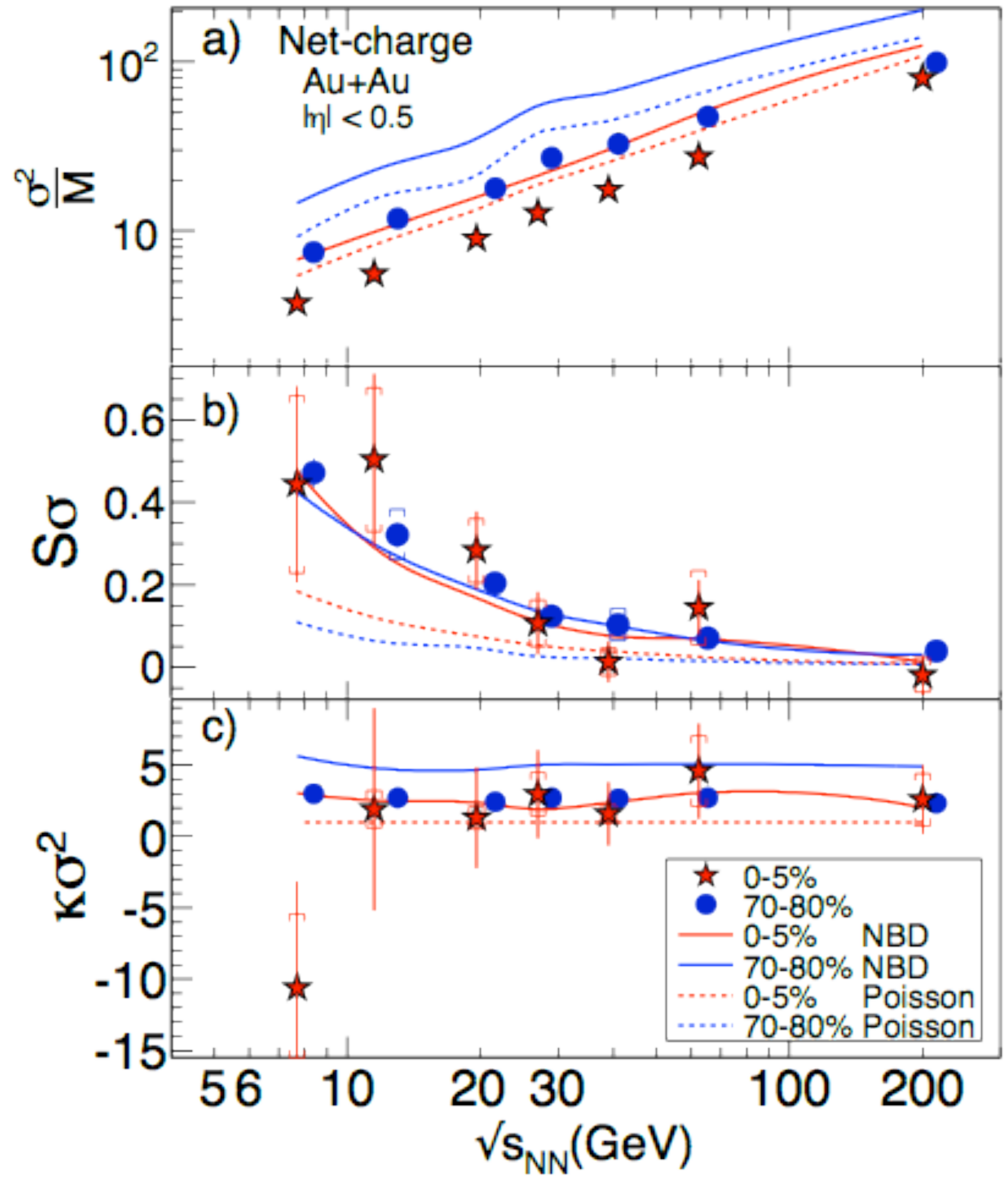
Consequently, the experimental 4th order fluctuation will be smaller than its Poisson value (precise value depends on ξ, on how close the freeze-out occurs to the critical point...)

$$\langle(\delta N)^4\rangle=\langle N\rangle+\langle\sigma^4{}_V\rangle\ldots$$

Experimental results on kurtosis



STAR Collab.: 1309.5681



STAR Collab.: 1402.1558

Kurtosis of Net-protons shows anomalous dip at $\sqrt{s_{NN}}$ = 19 GeV. Not confirmed by kurtosis of Net-charge.

Conclusions

- QCD transition: a smooth crossover at μ_B=0; expected to become first order at large μ_B (critical point)

- Lattice QCD simulations: equilibrium, thermodynamic quantities at small μ_B.

- HRG model: good description below the transition. Fit of hadron yields and ratios -> freeze-out parameters

- Alternative: fluctuations of conserved charges. Determination of freeze-out parameters from first principles

- Fluctuations at the critical point: expected to scale with some power of correlation length

- The kurtosis changes sign in the vicinity of the critical point